MATHEMATISCHES FORSCHUNSINSTITUT OBERWOLFACH

Tagungsbericht 04/1999

Ganzzahlige quadratische Formen und Gitter

24.01.-30.01.1999

The meeting was organised by Wilhelm Plesken (Aachen), Heinz-Georg Quebbemann (Oldenburg) and Boris Venkov (St. Petersburg).

The meeting was centred around the classification and structure theory of positive definite integral quadratic forms with special emphasis on constructive methods.

Topics included

- construction of dense lattices
- applications of the geometry of numbers to lattices
- applications of representation theory of finite groups to the theory of lattices
- spherical designs and lattices
- connections to coding theory
- modular forms and modular lattices
- connections with number theory (Arakelov theory) and constructive number theory
- connections with algebraic geometry (Mordell-Weil-lattices, K_3-surfaces)
- hyperbolic lattices and Kac-Moody groups.

The presence of experts in various neighboured fields created a stimulating atmosphere.

Maximal integral triangulations and curvature

Roland Bacher

A maximal integral triangulation of some subset in $\mathbb{R}^2$ is a triangulation with triangles having integral vertices and area $1/2$. The orbit of such a triangulation (which we suppose always locally finite) under $\mathrm{Aff}^+(\mathbb{Z}^2) = \mathrm{SL}_2(\mathbb{Z}) \ltimes \mathbb{Z}^2$ can be encoded by a curvature flow. This is an application ϕ from the set of oriented edges into $\mathbb{Z}$ which satisfies $\phi(\vec{e}) = -\phi(\overleftarrow{e})$. Denote by $w(\vec{e})$ the endpoint of an oriented edge.

The aim of the talk was to prove

$$\sum_{w(\vec{e})=v} \phi(\vec{e}) = 2(6 - \deg v)$$

where v is an interior vertex (in the 1-skeleton of the triangulation).

Designs, lattices and codes

Christine Bachoc

In this talk we discuss the notion of combinatorial block designs and of spherical designs, in connection respectively with binary linear codes and lattices.

There is a unified characterisation of such designs in terms of harmonic discrete functions, respectively harmonic polynomials:

$$X \text{ is a } t\text{-design} \iff \sum_{x \in X} P(x) = 0 \ \forall x \in \mathrm{Harm}_k \forall k = 1, 2, \ldots, t$$

In the setting of lattices, we are concerned with the X of the form $X = L_{2m} = \{x \in L, (x,x) = 2m\}$ and a natural tool is the so called spherical theta series: $\Theta_{L,P} := \sum_{x \in L} P(x) q^{(x,x)/2}$. These are known from Hecke to be modular forms for a certain congruence group and character, and was used in previous work by Boris Venkov in his study of unimodular lattices. In a common work we have extended this work to the case of l-modular lattices.

In the case of binary linear codes, I define, by analogy with the $\Theta_{L,P}$, some "harmonic weight enumerators" $W_{c,f}$ which generalize the classical weight enumerator W_C and satisfies a Mac-William type identity. In the case of type II codes for example they provide a whole family of relative invariant polynomials.

In both cases, one can make use of zonal spherical functions to provide a method to compute the so called Jacobi forms, respectively Jacobi polynomials, and we can derive in some particular cases classification results.

Positive quadratic forms over number fields

R. Baeza

Let $K/\mathbb{Q}$ be a totally real number field, $m = [K : \mathbb{Q}]$, $\mathcal{O}_K$ = ring of integers, d_K = discriminant.

A tuple $S = (S_1, \dots, S_m)$ of $n \times n$ real positive definite symmetric matrices is called a Humbert form $/K$. $\mathrm{GL}(n, \mathcal{O}_K)$ acts on such forms via: $U \in \mathrm{GL}(n, \mathcal{O}_K)$ $S[U] := (S_1[U^{(1)}], \dots, S_m[U^{(m)}])$, where $U^{(i)}$ is the i-th conjugate of U. Let $[S]$ be the class of S.

Define the determinant of S, $|S| = \prod_1^m |S_i|$, the minimum of S by $m(S) = \mathrm{Inf}\,\{S[u] | 0 \neq u \in \mathcal{O}_K^n\}$, where $S[u] = \prod_1^m S_i[u^{(i)}]$ = value of S at u.

The function $\gamma_K(S) = \frac{m(S)}{|S|^{1/n}}$ is bounded and we set $\gamma_{K,n} = \mathrm{Sup}_S \gamma_K(S)$. For example is $\gamma_{\mathbb{Q},n} = \gamma_n$ = Hermite's constant.

A vector $u \in \mathcal{O}_K^n$ is minimal for S if $m(S) = S[u]$. Let $M(S)$ be the set of minimal vectors of S. Set $\langle u \rangle$ = the ideal generated by the components of u. Then, if $h(K) = 1$, every $u \in M(S)$ is unimodular, i.e. $\langle u \rangle = \mathcal{O}_K$, but this is not true in general. The following number $M_{K,n}$ measures a kind of obstruction for the unimodularity of minimal vectors: For any S define

$$N(S) = \mathrm{Inf}_{u \in M(S)} |N_{K/\mathbb{Q}}(\prod_{u_i \neq 0} u_i)| \geq 1$$

$$N[S] = \mathrm{Inf}_{T \in [S]} N(T)$$

Then $N[S]$ is bounded for all S.

Define $M_{K,n} = \mathrm{Sup}_S N[S]$. Thus $h(K) = 1 \Rightarrow M_{K,n} = 1$ for all n. We prove $h(K) = 1 \iff M_{K,2} = 1$, and hence all $M_{K,n} = 1$.

Remark: $M_{K,n} = 1$ means that any S has at least one unimodular vector.

The constant $M_{K,n}$ appears in the following version of Mordell's theorem:

$$\gamma_{K,n}^{n-2} \leq \gamma_{K,n-1}^{n-1} M_{K,n}^{\frac{2(n-1)}{n}}$$

Thus it seems interesting to estimate $M_{K,n}$. The proof of the above result shows

$$M_{K,n} \leq \mathrm{Sup}_{\mathcal{A}\text{ ideal of minimal norm in its class}} \left[\mathrm{Inf}_{\mathcal{A}=(\alpha,\beta)} |N(\alpha)N(\beta)|\right]$$

Now use the following corollary of a result of Davenport: there exist $\alpha, \beta \in \mathcal{A}$ =ideal in $\mathcal{O}_K$ with $\mathcal{A} = (\alpha, \beta)$ and $|N(\alpha)| \leq c_m^2 |d_K| N(\mathcal{A})|$, $|N(\beta)| \leq c_m |d_K|^{1/2} N(\mathcal{A})$, where c_m is a constant depending only on m. One concludes $M_{K,n} \leq C_m |d_K|^{5/2}$, where C_m is a constant.

Question: Do exist Humbert forms S such that the associated ideals to the minimal vectors of S give different ideal classes (or the whole class group)?

Some results on modular forms

Eiichi Bannai

1) We give the classification of finite index subgroups Γ of $\mathrm{SL}(2,\mathbb{Z})$ such that the space $\mathcal{M}(\Gamma)$ of modular forms of integral weights is isomorphic to a polynomial ring. In particular, if $\mathcal{M} \cong \mathbb{C}[f_1, f_2]$ (weight$f_1 = 1$ and weight$f_2 = 1$), then there are exactly 6 such Γ up to conjugacy in $\mathrm{SL}(2,\mathbb{Z})$.

2) If the space $\mathcal{M}^{\frac{1}{2}}(\Gamma)$ of modular forms of half integral weights is isomorphic to $\mathbb{C}[f_1, f_2]$ (weight$f_1 = \frac{1}{2}$ and weight$f_2 = \frac{1}{2}$), then there are 191 possible such Γ up to conjugacy in $\mathrm{SL}(2,\mathbb{Z})$. (It is unknown which of them actually satisfy the condition.)

3) Similar studies can be made for the space of modular forms of $\frac{1}{l}$-integral weight. An interesting example is $l = 5$ and $\Gamma = \Gamma(5)$. In this case, we explicitly give the candidate of f_1 and f_2 of $\mathcal{M}^{\frac{1}{5}}(\Gamma(5)) \cong \mathbb{C}[f_1, f_2]$ (weight$f_1 = \frac{1}{2}$ and weight$f_2 = \frac{1}{2}$) and we discuss the connections with the work of F. Klein and with the 2-dimensional unitary reflection group of order 600.

(The above 1 and 2 are joint work with Akihiro Munemasa. 3 is joint work with Masao Koike and Jiro Sekiguchi.)

Extreme symplectic lattices

Christoph Bavard

In this talk we discuss the density theory for symplectic lattices. We give some general geometric setting to extend the usual Voronoi theory. We define 2-parameter families of symplectic lattices which have a good Voronoï theory (in particular there is a Voronoï type algorithm for them). This construction leads to several infinite sequences of extremal symplectic lattices, and gives in low dimension many new examples of such lattices which are not extreme in the usual sense. We can also recover some classical lattices, such as D_4, E_8, Λ_{10} or L_{24}.

Lattices and number fields

E. Bayer-Fluckiger

Let K be a number field with an involution $\overline{} : K \to K$, and let F be the fixed field of $\overline{}$. Let $\mathcal{D}$ be the be the inverse different of $K/\mathbb{Q}$. If $\alpha \in F$ and I an ideal such that $\alpha I \bar{I} \subset \mathcal{D}$, then

$$b : \begin{array}{ccc} I \times I & \to & \mathbb{Z} \\ (x, y) & \mapsto & \mathrm{Tr}_{K/\mathbb{Q}}(\alpha x \bar{y}) \end{array}$$

is an integral symmetric bilinear form.

The talk discussed the questions

1) Given K, $\overline{}$, which lattices arise in this way?

2) Given an integral symmetric bilinear form, is it possible to realize it as above?

Such lattices naturally arise in many different questions: Euclidean fields, information theory, topology,... The talk also mentioned some of these applications.

Inequalities related to duality

Anne-Marie Bergé

Ten years ago, Martinet and Bergé introduced for Euclidean lattices an invariant taking into account together the densities of a lattice and its dual: let (Λ, Λ^*) be a pair of n-dimensional dual lattices, their dual Hermite invariant is $\sqrt{\gamma(\Lambda)\gamma(\Lambda^*)}$, where $\gamma(\lambda) = \frac{\min\Lambda}{\det\Lambda^{1/n}}$ is the usual Hermite invariant; the dual Hermite constant for dimension n is $\gamma'_n = \sup_{\mathrm{rk}\Lambda=n} \sqrt{\gamma(\Lambda)\gamma(\Lambda^*)}$. One can derive an "a la Voronoi" theory of extremality for this invariant, and the enumeration of dual extreme lattices was done up to dimension 4 by combinatorial methods. A dual extreme positive matrix is entirely determined up to equivalence by its minimal vectors and those of its inverse. We classify positive definite matrices by the set of their minimal vectors and the set of minimal vectors of their inverses. This was done for 5-dimensional matrices invariant under the regular representation of the cyclic group of order 5; there appeared an irrational dual extreme lattice, and the conjecture $\gamma'_5 = \sqrt{5}$ was proved for these invariant matrices; it was also proved that the so-called $m\mathbb{D}_5$ lattice constructed by Conway and Sloane is the densest isodual G-invariant lattice.

Linear relations among theta series

Siegfried Böcherer

Let D be a quaternion algebra over $\mathbb{Q}$ ramified in q and ∞, $R_1, \ldots, R_T$ a complete set of non-isomorphic maximal orders (T = type number). Let $R_i^0 := \{x \in \mathbb{Z} + 2R_i | \text{tr}(x)\}$, $\hat{R}_i^0 := \{x \in R_i | \text{tr}(x) = 0\}$.

Then there is an intimate relation between linear relations among theta series attached to these quaternary and ternary lattices.

<u>Theorem</u> (conjectured by Hashimoto)

$$\sum x_i \theta_{R_i} = 0 \iff \sum x_i \theta_{R_i^0} = 0 \iff \sum x_i \theta_{\hat{R}_i^0} = 0$$

The main difficulty is the first "$\Rightarrow$". The obstruction against the validity of "$\Rightarrow$" is described by the space of cusp form

$$S_4(\Gamma_0(q))^1 := \{f \in S_4(\Gamma_0(q)) | \text{ord}_0(f) > \dim S_4(\Gamma_0(q))\}.$$

(This is due to J. Kramer.)

We prove the theorem by showing $S_4(\Gamma_0(q))^1 = \{0\}$; for this we use some considerations about modular forms mod q.

A characterization of extreme Humbert forms

Renaud Coulangeon

The classical notion of Hermite number of a quadratic form has been extended to *Humbert-forms* over number fields by M.I. Icaza and R. Baeza (if K is a totally real number field, a n-ary Humbert form is simply a tuple $(A_1, \ldots, A_r)$, $r = [K : \mathbb{Q}]$, of positive definite quadratic forms in n variables). Among others, the first author gave an explicit bound for the Hermite-Humbert number of such a Humbert form, depending on the field K and the dimension n of the form, and proved that the supremum $\gamma_{n,K}$ is attained. In analogy with the case of quadratic forms, Humbert forms at which the Hermite-Humbert's function achieves a local (respectively global) maximum are called *extreme* (respectively *critical*).

A well-known theorem of Voronoï characterizes extreme quadratic forms as those which are both *perfect* and *eutactic* .

The aim of the talk is to give an analoguous result for extreme Humbert forms, with suitable notions of *perfection* and *eutaxy*.

Universal and Regular Positive Quadratic Lattices

Andrew G. Earnest

The systematic search for positive definite integral quadratic forms for which the represented value set is as large as possible can be traced to the first half of this century, when a landmark result of Ramanujan on diagonal quaternary quadratic forms appeared. Recent renewed interest in this venerable subject has centered on two families of forms - universal forms (those forms f which represent all positive integers) and regular forms (those forms f which represent all positive integers n for which $f \equiv n \pmod{m}$ is solvable for every $m \in \mathbb{N}$).

These properties can be formulated in the contemporary general context of positive definite quadratic $\mathcal{O}_F$-lattices over the ring of integers $\mathcal{O}_F$ of an arbitrary totally real algebraic number field F. This talk focuses on the fundamental questions of existence, finiteness and enumeration of the universal or regular $\mathcal{O}_F$-lattices of given rank over a given field. For example, it is shown that, for any given F and n, there exist at most finitely many inequivalent universal $\mathcal{O}_F$-lattices of rank n which contain no universal sublattice of smaller rank.

Automorphims of even unimodular lattices of dimension 24

Wolfgang Ebeling

We consider Coxeter elements which are defined by certain starshaped generalized Coxeter-Dynkin diagrams. K. Saito has classified the graphs for which the corresponding Coxeter element is quasiunipotent, i.e., all eigenvalues are roots of unity, and 1 is not an eigenvalue. There are 38 cases. Each such graph determines an even indefinite integral lattice of signature $(2, \mu - 2)$ which primitively embeds in a unique way in the even unimodular lattice Γ of signature $(4, 20)$. One can show that the orthogonal complement is again defined by a Coxeter-Dynkin diagram of this type or of a similar type. This defines a mirror symmetry between these graphs which is related to the mirror symmetry of $K3$-surfaces. The product of the Coxeter elements of a mirror pair defines an automorphism of the 24-dimensional indefinite lattice Γ. Surprisingly, it turns out that the characteristic polynomials of these automorphisms are the same as the characteristic polynomials of certain automorphisms of the Leech lattice.

Characteristic Vectors of Unimodular $\mathbb{Z}$-lattices

Mark Gaulter

All the lattices under discussion here are understood to be integral unimodular $\mathbb{Z}$-lattices in $\mathbb{R}^n$. A characteristic vector of a lattice L is a vector $w \in L$ such that $v \cdot w \equiv |v|^2$ $(mod\ \mathbb{Z})$ for every $v \in L$. Elkies has considered the minimal (squared) norm of the characteristic vectors in a modular lattice. He asked: "For any $k > 0$, is there $\mathcal{N}_k$ such that every integral unimodular lattice all of whose characteristic vectors have norm $\geq n - 8k$ is of the form $L_0 \perp \mathbb{Z}^r$ for some lattice L_0 of rank $\leq \mathcal{N}_k$?"

Elkies solved his question in the cases $k = 0$ and $k = 1$. Here we solve the question for the cases $k = 2$ and $k = 3$.

Quadratic Forms over Global Function Fields

Larry J. Gerstein

I showed in Amer. J. Math (1969) that for every Hasse domain $\mathcal{O}$ in a global field of char $\neq 2$ there is a positive integer $n_{\mathcal{O}}$ such that every quadratic $\mathcal{O}$-lattice L of rank $n \geq n_{\mathcal{O}}$ has a nontrivial splitting $L = L_1 \perp L_2$. (A Hasse domain of a global field F is obtained by taking a set S consisting of almost all the nontrivial spots on F and taking the intersection $\mathcal{O} = \mathcal{O}(S) = \{\alpha \in F | \mathrm{ord}_{\mathfrak{p}}\alpha \geq 0\ \forall \mathfrak{p} \in S\}$.) I also showed that while no $n_{\mathcal{O}}$ works for the Hasse domains of all number fields simultaneously, $n_{\mathcal{O}} = 7$ always works in the function field case.

Then and subsequently (J. Number Theory (1988)) I showed that $n_{\mathcal{O}} = 5$ suffices to guarantee splitting in the function field case in special circumstances; e.g., when $\mathcal{O}$ has odd class number or when -1 is a square in F.

In my present talk I showed the number 7 to be best possible in the function field case. Explicitly, I proved the following result:

Let k be a finite field in which -1 is nonsquare, let $\mathfrak{q}$ be the $(x^2 + 1)$-adic spot on the rational function field $F = k(x)$, and let S be the set of all nontrivial spots on F except $\mathfrak{q}$. Then there is an indecomposable $\mathcal{O}(S)$-lattice of rank 6 on an appropriate quadratic F-space.

Prime divisors of the determinant of a Steinberg lattice of a group of Lie type

Roderick Gow

The Steinberg module of an (almost simple) finite group of Lie type is realizable over the rational field $\mathbb{Q}$ and thus integral Steinberg lattices exist. Steinberg (1957) has described an explicit such lattice. We describe the Gram matrix for Steinberg's original lattice and calculate a factor of its determinant, which is known to contain all primes that divide the order of the group.

We also discuss primes which must divide the determinant of any Steinberg lattice (not just that defined by Steinberg) and consider the case of a rank 1 Chevalley group in some detail.

Class numbers of positive definite ternary quaternian hermitian forms

Ki-ichiro Hashimoto

We discuss first a general method to obtain a closed formula for the class numbers of positive definite quadratic forms, hermitian forms, and quaternian hermitian forms of arbitrary rank. The formula we obtain is, for such form F and genus $\mathrm{gen}(L)$:

$$H = \sum_{C(g)} H(g), \quad H(g) = \sum_{\{W_A\}} M_{G(g)}(W_A) \times \prod_{p<\infty} C_p(g, W_p, U_p). \tag{1}$$

Here we denote by $G = GU(F)$ the group of positive unitary similitudes; $U_p =$ stabilizer of $L_p \cong L \otimes Z_p$, and $C(g)$ is running over the $G_{\mathbb{Q}}$-conjugacy classes of all possible torsion elements; $G(g)$ is the centralizer of g in G, and $\{W_A\}$ is running over the $G(g)_{\mathbb{Q}}$-conjugacy classes of idelic open subgroups of $G(g)_A$. $M_{G(g)}(W_A)$ denotes the mass of W_A which is defined as the sum $\sum_{j=1}^{h} \frac{1}{\sharp\Delta_j}$, where $G(g)_A = \coprod_{j=1}^{h} G(g)_{\mathbb{Q}}\eta_j W_A$, $\Delta_j = G(g)_{\mathbb{Q}} \cap \eta_j W_A \eta_j^{-1}$ (finite group). The factor $C_p(g, W_p, U_p)$ is the number of U_p conjugacy classes $x^{-1}gx$ of g such that $x^{-1}gx \in U_p$, $G(g)_p \cap xU_p x^{-1} \sim_{G(g)_p} W_p$.

The above formula (1) is derived by a combinatorial argument of counting the average of $\sharp C(g) \cap \Gamma_i / \sharp\Gamma_i$, where Γ_i is the stabilizer of L_i, $\{L_1, \ldots, L_H\}$ being the set of representatives of the classes in $\mathrm{gen}(L)$:

$$H(g) = \sum_{i=1}^{H} \frac{\sharp(C(g) \cap \Gamma_i)}{\sharp\Gamma_i}. \tag{2}$$

We next discuss the classification problem of conjugacy classes in $G_{\mathbb{Q}}$, and the structure of $G(g)$. Using these results, we can obtain explicit formulas in the case of ternary quaternian hermitian forms. The table below exhibits the known cases for explicit class number formulas:

quadratic forms	$\mathrm{rank}_{\mathbb{Z}} L = n$	explicit formula for $n \leq 5$
hermitian forms	$2n$	$n \leq 3$
quaternian hermitian forms	$4n$	$n \leq 3$

Designs and lattices

Jacques Martinet

The talk was devoted to results, largely due to Boris Venkov, relative to restricted notions of eutaxy and extremality, which correspond to the fact that the set S of minimal vectors of a lattice is a 3- or a 5-spherical design. A symmetric finite set $X \subset \mathbb{R}^n$ is a spherical $(2\ell+1)$-design if there exists a identity

$$\sum_{x \in X/\pm 1} (x, \alpha)^{2\ell} = c_{n,\ell}(x,x)^{\ell}(\alpha,\alpha)^{\ell} \quad \forall \alpha \in \mathbb{R}^n,$$

where $c_{n,\ell}$ is some constant, indeed

$$c_{n,\ell} = \Delta \frac{1 \cdot 3 \cdots (2\ell - 1)}{n(n+2)\cdots(n+2(\ell-1))} \text{ with } \Delta = \frac{1}{2}|X|.$$

For $\ell = 1$ (respectively 2) and $X = S(\Lambda)$, we obtain the notion of a strongly eutactic lattice (respectively of a strongly perfect lattice); strongly perfect lattices are extreme in the sense of Korkine and Zolotareff. I have explained some classification results for strongly perfect lattices (in dimension $n \leq 9$ or $n = 11$; Λ integral of minimum $m \leq 3$; $\Delta(\Lambda) = \frac{n(n+1)}{2}$, the smallest possible value). I have also described the the known strongly perfect lattices, a list which includes some celebrated lattices, such as the Coxeter-Todd, the Barnes-Wall or the Leech lattice, and given the classification of integral lattices of minimum $m \leq 5$ such that $S(\Lambda)$ is a 7-design.

These results can be found in three papers on my web page at www.math.u-bordeaux.fr/ ˜ martinet.

Perfect isometries, F-lattices and hermitian forms

Maurice Mischler

A perfect isometry of a unimodular $\mathbb{Z}$-lattice is an isometry t such that $\mathrm{char}(t)(1) = 1$. Only even lattices can have perfect isometry. In dimension $8, 16, 24$, the list of lattices having perfect isometry is known.

In dimension 32 and higher, the list is not known, but it is possible to give some results about the mass in the sense of Siegel of such lattices. For this, we need the notion of F-lattice. If $F \in \mathbb{Z}[X]$, an F-lattice is a unimodular $\mathbb{Z}$-lattice having an isometry with characteristic polynomial F. We denote by $\bar{\mathcal{E}}(F)$ the set up to isometry of F-lattices. For every F-lattice, we associate hermitian forms, and we prove that the mass of $\bar{\mathcal{E}}(F)$ is smaller than a sum including the mass of genera of these hermitian forms. In dimension 32, we can say that a lattice has a perfect isometry if and only if it is a F-lattice, where F belongs to a small list of polynomial. We give explicit examples of mass formula giving good estimates for the mass of $\bar{\mathcal{E}}(F)$. In the last part, we explain the link between genus and invariant factors. We shall give an example of calculation of the number of genera of hermitian forms having given invariant factor: Let $E = \mathbb{Q}(\zeta_r)$ and $B = \mathbb{Z}[\zeta_r]$ and $U(r) =$ the number of genera of B-hermitian unimodular lattices. We have $U(r) = \begin{cases} 1 & \text{if } r \not\equiv \bmod 4 \\ 2 & \text{otherwise} \end{cases}$. In dimension 4, we have identity and $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & i \\ 0 & 1 & 2 & 1 \\ 0 & -i & 1 & 2 \end{pmatrix}$. If $E = \mathbb{Q}(\zeta_{2^r})$ and $B = \mathbb{Z}[\zeta_{2^r}]$, $r > 2$, then $U(r) = \begin{cases} 1 & \text{if } r \not\equiv \bmod 2 \\ 0 & \text{otherwise} \end{cases}$, and in dimension 2, we have identity and $\begin{pmatrix} 2-\sqrt{2} & 1 \\ 1 & 2+\sqrt{2} \end{pmatrix}$, where $\sqrt{2} = \zeta_8 + \bar{\zeta}_8$. We have also general formulas.

The last part is joint work with P. Calame.

Unimodular G-lattices

Gabriele Nebe

Let G be a finite group, K a finite extension of $\mathbb{Q}_p$ and R its ring of integers. I consider the problem to determine all KG-modules that contain a G-invariant R-lattice that is unimodular with respect to a G-invariant regular quadratic form. This question can easily be answered, if one constructs the simple KG-modules.

But here a description of the group ring RG is used to obtain (for certain groups) enough information on the invariant quadratic forms on the simple KG-modules. This information is encoded in a Witt-decomposition matrix that allows to read off the KG-modules containing a unimodular G-lattice.

Lorentzian Kac-Moody algebras and integral quadratic forms

V. V. Nikulin

We consider problems on integral quadratic forms which are interesting from the point of view of the theory of Lorentzian Kac-Moody algebras. This theory was recently developed in papers by R. Borcherds and V. Gritsenko - V. Nikulin.

Lorentzian Kac-Moody algebras are defined by the data

(1) Hyperbolic lattice S;

(2) Reflection group $W \subset \mathrm{O}(S)$;

(3) Set $P = P(\mathcal{M})$ of roots, orthogonal to the fundamental chamber $\mathcal{M}$ of W with Weyl vector $\rho \in S \otimes \mathbb{Q} : (\rho, \alpha) = -\frac{\alpha^2}{2}$ for all $\alpha \in P(\mathcal{M})$;

(4) Automorphic form $\Phi(z)$, $z \in \Omega(V^+(S)) = \Omega(T)$, $T = U(k) \oplus S$, $U(k) = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$, with Fourier expansion

$$F(z) = \sum_{w \in W} \det(w) \left[\exp(2\pi i(w(\rho), z)) - \sum_{a \in S \cap \mathbb{R}^+ \mathcal{M}} m(a) \exp(2\pi i(w(\rho + a), z)) \right],$$

$m(a) \in \mathbb{Z}$.

(5) Φ is reflective: $(\Phi)_0 = \bigcup_{\mathrm{root}\, \alpha \in T} D_\alpha = \{\mathbb{C}w \in \Omega(T) | (w, \alpha) = 0\}$, D_α is quadratic divisor $\perp$ root $\alpha \in T$.

Data (1) - (5) define a Lorentzian Kac-Moody Algebra which is a generalised Kac-Moody superalgebra. Using automorphic properties of Φ, in many cases we can find infinite product expansion

$$\Phi(z) = \exp(2\pi i(\rho, z)) \prod_{\alpha \in \Delta_+} (1 - \exp(2\pi i(\alpha, z)))^{\mathrm{mult}(\alpha)},$$

where $\mathrm{mult}(\alpha) = \dim \mathcal{G}_{\alpha, \bar{0}} - \dim \mathcal{G}_{\alpha, 1}$, $\mathcal{G} = \left(\oplus_{\alpha \in \Delta_+ \subset S} \mathcal{G}_\alpha\right) \oplus \mathcal{G}_o \oplus \left(\oplus_{\alpha \in \Delta_+ \subset S} \mathcal{G}_{-\alpha}\right)$.

Main problem: Find all possible (1)-(5) and $\mathcal{G}$.

(1) - (5) $\Rightarrow$ hyperbolic S is reflective, T is reflective. We consider problems of classification of reflective S and T.

Distinguishing even unimodular lattices in dimension 32 and 40

Michio Ozeki

In this talk we discuss the following classical problem: Given two even unimodular lattices of the same rank n. How can one determine whether these two are isometric or not? We compare two analytic methods. One is Siegel theta series of various degrees. The other is Jacobi theta series of various indices. First I explained a method to construct Jacobi forms from Jacobi polynomials for codes. In the 32 rank case Jacobi theta series of indices 2 and 3 for extremal even unimodular lattices are unique. In index 4 there are 63 possible Jacobi theta series. For 5 extremal even unimodular lattices of rank 32 coming from binary codes, these Jacobi theta series are shown to be enough to solve the distinguishing problem.

Related to this problem we raise two problems. There are at least 239 extremal ternary $[32, 16, 9]$-codes. Will these codes produce non isometric even unimodular lattices of rank 32 by the ternary code construction? Next, are these lattices isometric to the lattices coming from binary codes?

In the rank 40 case, Jacobi theta series of index 2 will be enough invariants for the distinguishing problem. There are more than 1000 non isometric ones. Compare these lattices with the lattices coming from ternary codes. Someone suggested me to use Bacher polynomials for the above problem, but I have some intentions not covered by Bacher polynomials.

Lattices of covariant quadratic forms

Wilhelm Plesken

For a semisimple $\mathbb{Q}$-Algebra $\mathcal{A}$ with positive involution the covariant quadratic forms on a $\mathbb{Z}$-lattice L in V are studied. They can be viewed as symmetric elements of $\mathrm{Hom}_\Lambda(L, L^*)$, where Λ is some $\mathbb{Z}$-order in $\mathcal{A}$ admitting L and L^* as lattices. Equivalence is defined for these $\mathbb{Z}$-lattices $\mathrm{Hom}_\Lambda(L, L^*)$ via the endomorphism ring $\mathrm{End}_\Lambda(L \oplus L^*)$. The group of autoequivalences is studied and the concept of depth is established via the idealizor sequence of $\mathrm{End}_\Lambda(L)$ respectively $\mathrm{End}_\Lambda(L \oplus L^*)$. For the case $\mathrm{End}_{\mathcal{A}_\mathbb{R}}(V_\mathbb{R}) \in \{\mathbb{R}^{2\times 2}, \mathbb{C}^{2\times 2}, \mathbb{H}^{2\times 2}\}$ further details are worked out and examples are discussed which construct interesting modular lattices.

Simultaneous Representations by Ternary Quadratic Forms in Number Fields

M. Pohst

Let $E/\mathbb{Q}$ be an algebraic number field of degree n with maximal order R. Let $Q_i(X,Y,Z) \in R[X,Y,Z]$ and $U_i \in R$ $(i = 1,2)$. We want to determine all $(x,y,z) \in R^3$ satisfying

$$Q_i(x,y,z) = U_i \quad i = 1,2. \tag{1}$$

For this we parametrise all solutions $(x,y,z) \in R^3$ of (1) and $Q(x,y,z) := U_2Q_1(x,y,z) - U_1Q_2(x,y,z) = 0$ in the form $s(x,y,z)^{tr} = C(p^2,pq,q^2)^{tr}$, where $C \in R^{3\times 3}$ is known and s belongs to a finite set of elements of E which can be calculated. Then the initial system of equations becomes

$$F_i(p,q) = Q_i(f_1(p,q), f_2(p,q), f_3(p,q)) = s^2U_i \quad (i = 1,2) \tag{2}$$

with binary quadratic forms f_j $(1 \le j \le 3)$. This requires the solution of a relative quartic Thue equation (in case F_i is is irreducible, otherwise the equation is even simpler to solve).

The solution of relative Thue equations can be obtained by known methods if the total degree $4n$ is not too large, say $n \le 4$. We also present an application to index form equations in relative extensions F of E of degree $[F:E] = 4$.

The shadow theory of modular and unimodular lattices

Eric M. Rains

The Mallows-Odlyzko-Sloane bound for unimodular lattices, while quite strong for even lattices, is rather weak in the odd case. Using the notion of the shadow of a lattice, I show that the bound $n \le 2\lfloor \frac{n}{24} \rfloor + 2$ which they only proved for even lattices, actually holds for odd lattices as well (expect for $n = 23$).

I also discuss the generalisation to lattices similar to their dual (modular lattices), and in particular the lowest-dimensional lattices of minimum 3 or 4. This is joint work with Neil Sloane, and has recently appeared in the Journal of Number Theory.

An LLL algorithm for totally positive lattices over number fields

Alexander Schiemann

Let F be a totally real number field, E a quadratic extension and V an n-dimensional E-space with totally positive hermitian form h, i.e. $T := tr_{E/\mathbb{Q}} \circ h$ is positive definite. An $\mathcal{O}_E$-lattice L in V has a representation $L = \sum_{l=1}^{n} \mathfrak{a}_l x_l$ with fractional ideals $\mathfrak{a}_i$ which is called a pseudo-base. Starting with such a base we want to find another one with small $T(x_i, x_i)$ and ideals that are reduced in some sense. It turns out that the conditions given below can be matched by an algorithm very similar to the LLL for $\mathbb{Z}$-lattices and that they imply similar bounds for the quality of the result. This is an advantage compared to LLL-versions suggested by Fieker in a more general context.
For $L = \sum_{l=1}^{n} \mathfrak{a}_l x_l$ let p_i denote the orthogonal projection on $(\sum_{l=1}^{i-1} E x_l)^{\perp}$, $x_i^* := p_i(x_i)$. Let $L_{ij} := \sum_{l=i}^{j} \mathfrak{a}_l x_l$ and for a discrete set L' let $\mu_T(L') := \min\{T(x,x) \mid x \in L' \setminus \{0\}\}$.
The pseudo-base is called (k, q, q_1)-reduced (for constants q, q_1 subject to $0 < q < 1$, $0 < q_1 \leq 1$ and "blocksize" $k \in \{2, \ldots, n\}$) iff

1. $\forall\, i = 1, \ldots, n : \quad \mathfrak{a}_i \supseteq \mathcal{O}_E$
2. $\forall\, i = 1, \ldots, n$ with $b(i) := \min(n, i + k - 1)$:
$$q\, T(x_i^*, x_i^*) \leq \mu_T(p_i(L_{i,b(i)}))$$
3. $\forall\, 1 \leq j < i \leq n$:
$$q_1\, T(p_j(x_i)\,,\, p_j(x_i)) \leq \mu_T\Big(p_j(\{x_i + \alpha x_j \mid \alpha \in \mathfrak{a}_i^{-1}\mathfrak{a}_j\})\Big).$$

By translating everything back to $\mathbb{Z}$-lattices we can prove:

- $\mathrm{N}_{E/\mathbb{Q}}(\mathfrak{a}_i^{-1}) \leq \left(\frac{\gamma_m}{qm}\right)^{m/2} |\, \mathrm{d}_E|^{1/2}$,
where $m = [E : \mathbb{Q}]$ and γ_m is Hermite's constant.
- $T(x_i^*, x_i^*) \leq C_1\, T(x_{i+1}^*, x_{i+1}^*)$ with $C_1 = \left(\frac{\gamma_{2m}}{qm}\right)^2 |\, \mathrm{d}_E|^{2/m}$.
- $T(x_1, x_1) \leq q^{-1}\, C_1^{n-1}\, \mu_T(L)$.
- There is a positive constant $C_2(i)$ not depending on L or T such that $T(x_i, x_i) \leq C_2(i)\, T(x_i^*, x_i^*)$.

An implementation of this algorithm for complex quadratic fields showed much better results than any other method we tried.

Arakelov divisors and lattices

René Schoof

The Picard group $\mathrm{Pic}(X)$ of an algebraic curve X classifies isomorphism classes of line bundles $\mathcal{L}$ on X. The number theoretic analogue of $\mathrm{Pic}(X)$ is the Arakelov divisor class group $\mathrm{Pic}(F)$ of a number field F. It classifies isometry classes of $\mathcal{O}_F$-lattices of rank 1. Here $\mathcal{O}_F$ denotes the ring of integers of F and an $\mathcal{O}_F$-lattice is a projective $\mathcal{O}_F$-module L of rank 1 equipped with a scalar product on the $F\otimes\mathbb{R}$-module $L\otimes\mathbb{R}$ satisfying $\langle\lambda x, y\rangle = \langle x, \overline{\lambda} y\rangle$ for all $x, y \in L\otimes\mathbb{R}$ and all $\lambda \in F\otimes\mathbb{R}$. Here "$\lambda \mapsto \overline{\lambda}$" denotes the canonical involution of the étale $\mathbb{R}$-algebra $F\otimes\mathbb{R}$.

The analogue of the function $h^0(\mathcal{L}) = \dim H^0(X, \mathcal{L})$ on the Picard group of a curve X is given by the function

$$h^0(L) = \log(\sum_{x\in L} e^{-\pi\langle x,x\rangle})$$

on $\mathrm{Pic}(F)$. The function $h^0(L)$ is related to the Hermite constants $\gamma(L)$ of the lattices L. In many respects it behaves like its geometric analogue $h^0(\mathcal{L})$. The function $h^0(L)$ satisfies a Riemann-Roch Theorem. One can show that $h^0(L)$ tends doubly exponentially fast to zero when $\deg L = -\log(\mathrm{coval} L)$ becomes negative. We conjecture that $h^0(L)$ takes its maximum on the compact group $\mathrm{Pic}^0(F)$ of Arakelov divisor classes of degree 0 in the neutral element. This would imply that

$$h^0(L) \le \deg L + h^0(\mathcal{O})_F$$

whenever $\deg L \ge 0$. Here $\mathcal{O}_F$ denotes the ring of integers equipped with the usual scalar product on $\mathcal{O}_F\otimes\mathbb{R}$. Its class is the neutral element in $\mathrm{Pic}(F)$. It would also be interesting to obtain an analogue of Clifford's theorem.

This is joint work with Gerand van der Geer (Amsterdam).

Mordell-Weil Lattices of Jacobian Varieties over higher-dimensional Function Fields

Tetsuji Shioda

A. Néron (ICM54) claimed the existence of (a) ∞ family of elliptic curves over $\mathbb{Q}$ of rank ≥ 11, and (b) for all $g \ge 2$ ∞ family of curves $\Gamma/\mathbb{Q}$ of genus g of rank $\ge 3g+7$. (The rank is $r = \mathrm{rank}\ J(\mathbb{Q}),\ J =$ Jacobian of Γ here).

We gave an explicit (effective) version of (a) in 1990 by using Mordell-Weil-Lattices of elliptic curves over one-dimensional function fields. Then Mestre gave a more elementary construction.

In the talk we prove the following result, improving the bound in (b).

Theorem There exists an infinite family of curves Γ with rank $r \geq 4g + 7$.

This is an consequence of

Theorem For all $g \geq 2$, there exists $\tilde{\Gamma}$, a curve of genus g over $\mathbb{Q}(t_1, \ldots, t_n)$ of rank $\geq 4g + 7$, where $t_1, \ldots, t_n$ are independent variables.

The curve $\tilde{\Gamma}$ is constructed à la Mestre which has many rational points. The main idea to prove the indepence of these points is to use the notion of Mordell-Weil-Lattices of abelian varieties over a function field of higher dimension N, plus Galois symmetry. (This new proof is much simpler than the one we gave for $g = 2$ before, using Mordell-Weil-Lattices of elliptic curves over a 1-dimensional base.)

Reference: T. Shioda "Constructing curves with high rank via symmetry", Amer. J. Math.120(98).

Theta series mod l

Nils-Peter Skoruppa

A well-known basic principle of the theory of l-adic modular forms says: "Modulo l modular forms f of level l^n is the same as modular forms of level 1." We gave a lattice theoretic proof of a slightly more precise result of this general fact for the case of f being a theta series associated to a lattice of level l^n. More precisely we proved:

Theorem. *Let $Q(x)$ be an integral, positive definite quadratic form of level l^n (l a prime, $n \geq 1$). Write $Q(x) = \frac{1}{2}x^t F x$ (F a symmetric matrix) and let $e = e(Q)$ be $\frac{1}{2}$ times the sum of the elementary divisors of F. Then there is a modular form g of* level 1, weight e *and with integral Fourier coefficients such that* $\theta_Q := \sum_{x \in \mathbb{Z}^r} q^{Q(x)} \equiv g \bmod l$.

This statement is somewhat sharper then the corresponding one for general modular forms, and its proof develops techniques which may be useful for the study of lattices via their theta series.

We also indicated and discussed briefly possible applications, in particular to the problem of deciding whether a given modular form of level 1 with nonnegative and integral Fourier coefficients is a theta series.

Deus ex Machina: Packing in Grassmannian Spaces

N. J. A. Sloane

Let $G(m,n)$ denote the Grassmannian manifold of n-dimensional subspaces (through 0, not oriented) of $\mathbb{R}^n$. We argue that the best metric on $G(m,n)$ is given by $d(P,Q)^2 = \sin\theta_1 + \cdots + \sin\theta_n$, where $\theta_1,\ldots,\theta_n$ are the principal angles between $P,Q \in G(m,n)$. The talk discusses the problem of finding optimal (or close to optimal) packings of N points in $G(m,n)$ with respect to this metric. We have constructed extensive tables of such packings. For example, we give an optimal packing of 70 4-spaces in $\mathbb{R}^8$. Upper bounds follow from the following

Theorem: For $P \in G(m,n)$ let $\mathcal{P}$ be the orthogonal projection matrix which projects onto P. The map $P \mapsto \mathcal{P}$ is an isometric embedding of $G(m,n)$ into a sphere of radius $\sqrt{n(m-n)/m}$ in $\mathbb{R}^D$, where $D = \binom{m+1}{2} - 1$, such that $d(P,Q) = \frac{1}{2} d_F(\mathcal{P},\mathcal{Q})$, where d_F is the Frobenius distance between matrices.

Corollary (a) The minimal distance d achieved by a packing of N points in $G(m,n)$ satisfies $d^2 \le \frac{n(m-n)}{m}\frac{N}{N-1}$; if $=$ then $N \le D+1$ and the points form a regular simplex on the sphere. (b) For $N > D+1$, $d^2 \le \frac{n(m-n)}{m}$; if $=$ then $N \le 2D$; if $=$ and $N = 2D$ then the points form a regular orthoplex (or generalized octahedron). We are especially interested in packings with $n = [m/2]$ which achieve these bounds, the above example being an instance of this. The automorphism groups of this example turned out by an extraordinary coincidence to be the same as a group which Peter Shor had encountered in quantum computers. This "group out of the machine" has since led us to construct several infinite families of optimal packings, via the orthogonal geometry associated with the group. The group in question is a certain Clifford group of order $|\mathrm{O}^+(2i,2)|2^{2i+1}$, for $i = 1,2,\ldots$. One example of an infinite family of optimal packings gives $2(2^i-1)(2^{i-1}+1)$ points in $G(2^i, 2^{i-1})$, for $i \ge 1$, meeting the bound (b) of the Corollary. This is joint work with R. H. Hardin, J. H. Conway, Peter Shor, A. R. Calderbank and E. M. Rains. The tables (and papers) can be found on the speaker's homepage: www.research.att.com/ ˜ njas/.

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