MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Reelle Methoden in der Komplexen Analysis

28. Febr. - 6. März '99

Complex Analysis is a lively field worldwide. There have been important developments during the last 10 years, in particular, due to the applications of deep methods originally coming from real analysis and geometry. Modern Complex Analysis is a field of interaction of many parts of mathematics. This also was reflected by the conference at the Mathematische Forschungsinstitut Oberwolfach dedicated to Complex Analysis with emphasis on methods from the theory of partial differential equations. It was organized by K. Diederich, Wuppertal, T. Ohsawa, Nagoya, and by E.L. Stout, Seattle. It has found a large interest and was attended by 49 mathematicians from 8 countries. In fifty minute lectures 21 researchers reported on their recent work, and there were many additional informal activities with lectures and discussion groups. The topics covered belonged to the following areas:

Hulls of holomorphy, foliations, the $\overline{\partial}$*-Neumann problem, complex dynamics, CR-geometry, the Cauchy-Riemann equations, the tangential Cauchy-Riemann equation, the Bergman kernel, pluripotential theory, Serre duality, the Levi problem on complex manifolds, the Oka principle, the Michael problem,* $K3$*-surfaces, singularities, Paley-Wiener theory, and uniformization theory.*

Abstracts

David E. Barrett

Diffusion and Analytic Continuation

In the talk I began with a discussion of Brian Birger's result that the space M of smooth Jordan curves in the Riemann sphere has a unique conformally invariant symmetric affine connection ∇.

The Levi form of a surface

$$\Sigma_f = \{(z,w): \ z \in \Delta, \, w \in f(z)\}$$

obtained from a map $f : \Delta \longrightarrow M$ may be written as

$$\Delta f - 2\mathcal{J}[\mathcal{J}\frac{\partial f}{\partial x}, \mathcal{J}\frac{\partial f}{\partial y}],$$

where Δf is the harmonic mapping Laplacian and $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are understood as vector fields normal to the curve $f(z)$.

The heat flow attached to this operator performs a type of analytic continuation.

Examples of the heat flow were discussed, and it was conjectured that a modification of this heat flow could be used to provide a new proof of a recent analytic continuation result of Chirka.

Eric Bedford

Quasi-Expansion in Polynomial Diffeomorphism of $\mathbb{C}^2$

We let

$$f(x,y) = (y, p(y) - ax),$$

where $a \in \mathbb{C} \setminus \{0\}$, and $p(y) = y^d + ...$ is a polynomial of degree $d \geq 2$. Let $\mathcal{S}$ denote the set of saddle points of f. If p is a saddle point, then

$$Df_p^n = \begin{pmatrix} \chi^+ & 0 \\ 0 & \chi^- \end{pmatrix}, \ |\chi^-| < 1 < |\chi^+|, \ f^n p = p.$$

We uniformize the unstable manifold $W^n(p)$ as $\psi : \mathbb{C} \longrightarrow W^n(p) \subset \mathbb{C}^2$. It follows that $\psi(\chi^+\zeta) = f^n\psi(\zeta)$. Let

$$G^+(x,y) = \lim_{n\to\infty} \frac{1}{d^n} \log^+ |f^n(x,y)|.$$

We may use G^+ to specify ψ (almost) uniquely: $\psi(0) = p$, and

$$\max_{|\zeta|\leq 1} G^+ \circ \psi(\zeta) = 1.$$

The mapping f will be called "quasi-expanding" if the normalized uniformizations $\{\psi_p : \mathbb{C} \longrightarrow \mathbb{C}^2 : \ p \in \mathcal{S}\}$ are a normal family. In this case, (i.e. when f is quasi-expanding) the sets $\psi(\mathbb{C})$, (where ψ is any subsequential limit: $\psi_{p_j} \longrightarrow \psi$) play the role of unstable manifolds.

In joint work with John Smillie we are trying to develop a theory of quasi-hyperbolic mappings which will allow tools that are analogous to those in the (uniformly) hyperbolic theory to be applied to more general contexts. This, among other things, should allow us to understand what happens, in certain cases, when hyperbolicity breaks down.

BO BERNDTSSON

L^2-estimates for $\overline{\partial}$

In the talk we discussed some variations on the theme of Hörmander's L^2-estimates with an eye on possibilities to extend part of these L^2-estimates to uniform norms.

The following theorems were discussed:

Theorem 1. *Let $\mathcal{D} = \{\rho < 0\} \subset\subset \mathbb{C}$, where $\Delta\rho \geq 1$. Let φ be subharmonic in $\mathcal{D}$, and let u be the $L^2(e^{-\varphi})$-minimal solution to the equation $\overline{\partial} u = f$. Then*

$$\sup_{\partial\mathcal{D}} \frac{|u|e^{-\varphi/2}}{|\partial\rho|} \leq 2 \sup_{\mathcal{D}} \frac{|f|e^{-\varphi/2}}{(-\rho)\Delta\varphi + 2}.$$

Theorem 2. *Let $\mathcal{D}$ be pseudoconvex in $\mathbb{C}^n$, and $(-w), \varphi$, and ψ plurisubharmonic functions on $\mathcal{D}$. Suppose that ψ satisfies the Donnelly-Fefferman condition*

$$\partial\psi \wedge \overline{\partial\psi} \leq \partial\overline{\partial}\psi.$$

Let u be the $L^2(e^{-\varphi})$-minimal solution to the equation $\overline{\partial} u = f$, where f is a $\overline{\partial}$-closed $(0,1)$-form. Then, for any $0 \leq r < 1$ we have

$$(1-r) \int_{\mathcal{D}} |u|^2 w e^{r\psi - \varphi} \leq \int_{\mathcal{D}} |f|^2_{\partial\overline{\partial}\varphi} w e^{r\psi-\varphi}.$$

We also stated the

Conjecture: If f is a $\overline{\partial}$-closed $(0,1)$-form in the unit ball $I\!B$ in $\mathbb{C}^n$, such that

$$\sup_{I\!B} \left(|f|_\omega^2 + |\partial f|_\omega \right) \leq 1,$$

for some Kaehler metric ω with bounded potential then there exists a solution u to the equation $\overline{\partial} u = f$ with $\sup_{I\!B} |u| \leq C$.

Ewgeni Chirka

Holomorphic motion and simultaneous uniformization

A holomorphic family of Riemann surfaces is a triple (M, p, B), where M, B are complex manifolds and $p : M \longrightarrow B$ is a holomorphic surjective mapping with the one-dimensional fibres $M_z = p^{-1}(z)$. We assume further that rank $(p) = \dim_{\mathbb{C}}(B)$ and the M_z are connected.

The general *problem* is, assuming that M_z are conformally equivalent to a domain in the Riemann sphere, to find a meromorphic function in M which gives holomorphic coordinates (with values on $\widehat{\mathbb{C}}$) on each fibre M_z.

Simple examples show that some pseudoconvexity conditions must be assumed, so we assume that M is Stein. The problem is still open even in the case when the base B and the fibre M_z are conformally the unit disc $I\!D$ in $\mathbb{C}$.

For $M_z \approx \mathbb{C}$ (conformally equivalent) it was solved by T. Nishino in 1969 with the (essentially necessary) assumption that there exists a holomorphic section of p over $I\!D$.

We discussed the methods in the problem related with the notion of holomorphic motion and the tools from quasiconformal theory developed for its study. The crucial thing is the following result (essentially proved by Z. Slodkowski (Math. Ann. (1997)):

If $p : M \longrightarrow B$ (the base B is arbitrary) is traced by a holomorphic motion, then there is a holomorphic embedding $f : M \longrightarrow \Omega$ into a domain $\Omega \subset B \times \mathbb{C}$, such that $p_{st} \circ f = p$ (here $p_{st} : B \times \mathbb{C} \longrightarrow B$ is the standard projection).

With a natural normalization the map is uniquely defined by $p : M \longrightarrow B$, and it gives a possibility of the analytic continuation of ϕ (canonical) along paths in B. For the normalization, what one needs is two disjoint global sections of p over B, so we have the following:

Let $p : M \longrightarrow B$ be a (regular) family of Riemann surfaces which is locally (over the elements of some covering $B = \cup_\alpha B_\alpha$) traced by a holomorphic motion, and which admits two disjoint holomorphic sections over B. Then there exists a holomorphic embedding $f : M \longrightarrow \Omega \subset B \times \mathbb{C}$ commuting with the projections, assuming that some M_a is conformally equivalent to a domain in $\mathbb{C}$.

Bert Fischer

Hoelder Estimates on convex domains of finite type

There is already a long history in Hoelder estimates for the $\overline{\partial}$-equation on weakly pseudoconvex domains in $\mathbb{C}^n$. Besides many other results there are two interesting special ones. In 1976 Range proved Hoelder-$\frac{1}{m}$-estimates for complex ellipsoids of type m, and in 1986 Diederich-Fornaess-Wiegerinck proved Hoelder-$\frac{1}{m}$-estimates for real ellipsoids of type m. The main difference between these two results is, that in the case of a real ellipsoid there might be in any complex line real lines with higher order of contact with the boundary. This is particularly bad for the estimates. So the main task is to find a support function which corrects this order of contact.

In 1998 Diederich - Fornaess constructed a "good" support function on arbitrary convex domains of finite type m in $\mathbb{C}^n$ and proved certain estimates for the real part of this function. It turned out that this support function can be used to construct a solution operator for the Cauchy-Riemann equation that satisfies the best possible, namely Hoelder-$\frac{1}{m}$-estimates.

The same support function can also be used to construct solution operators with good estimates in L^p-spaces. The result is a bounded linear operator

$$T : L^p_{0,r+1} \longrightarrow L^q_{0,r}$$

for $1 < p < mn + 2$, where q satisfies

$$\frac{1}{q} = \frac{1}{p} - \frac{1}{mn + 2}.$$

Franc Forstneric

Oka's Principle for Holomorphic Submersions with Sprays

In the talk I presented the outline of proof of the following result which was announced by M. Gromov in 1989 [J. Amer. Math. Soc. **2**, 851 - 897 (1989)]:

Theorem: *Let X be a Stein manifold and $h : Z \longrightarrow X$ a holomorphic submersion such that for each point $x \in X$ there is an open neighborhood $x \in U \subset X$ with the property that $Z|_U = h^{-1}(U)$ admits a fiber-dominating spray. Then the sections $f : X \longrightarrow Z$ of h satisfy the homotopy principle in the sense that each continuous section can be homotopically deformed to a holomorphic section, and any two holomorphic sections which are homotopic through continuous sections are also homotopic through holomorphic sections.*

I discussed some special cases and applications.

Josip Globevnik

Discs in Stein manifolds

In the talk I presented an outline of the proof of the following

Theorem: *Let M be a Stein manifold, dim $M \geq 2$. Given a point $p \in M$ and a vector X tangent to M at p, there is a proper holomorphic map f from the open unit disc in $\mathbb{C}$ to M such that $f(0) = p$ and $f'(0) = \lambda X$ for some $\lambda > 0$.*

Gregor Herbort

On the pluricomplex Green function on smooth bounded pseudoconvex domains

The subject of my talk was the boundary behavior of the pluricomplex Green function

$$\begin{aligned} G_D(z,w) \quad := \quad & \sup\{u(z) \ : \ u < 0, \text{ plurisubharmonic on } D, \\ & \zeta \longmapsto u(\zeta) - \log|\zeta - w| \text{ is bounded from above near } w\} \end{aligned}$$

under approach of w towards the boundary of the pseudoconvex bounded domain $D \subset \mathbb{C}^n$. This has useful applications in the theory of the Bergman kernel function, as results of Ohsawa (Nagoya Math. J. **129**, 43-52 (1993)) and more recently Błocki-Pflug (Nagoya Math. J. **151**, 221 - 225 (1998)) (resp. Herbort) show. I gave sketches of the proofs of the following two theorems which resulted from joint work with K. Diederich :

Theorem 1. *Assume that $\partial D \in C^2$ and $\rho : D \longrightarrow [-1, 0)$ is a plurisubharmonic exhaustion function satisfying $|\rho| \approx \delta_D^\alpha$, were $0 < \alpha < 1$ and δ_D denotes the boundary distance function on D. Then, given an arbitrarily small number $t > 0$, there exists a constant $C_t \gg 1$ such that*

$$\sup_{z \in K} |G_D(z,w)| \leq C_t \left((\frac{\delta_D(w)^{1-t}}{\delta_D(K)})^{\alpha/n} + \delta_D(w)^{t\alpha} \right)$$

holds for any compact subset $K \subset D$ and all $w \in D \setminus K$ such that $\delta_D(w) < C_t^{-1} \delta_D(K)$.

Theorem 2. *(Application to the Bergman metric) Let D be as in theorem 1. If additionally any $q \in \partial D$ is a plurisubharmonic peak point, then for every $w^0 \in \partial D$, and any $X \in \mathbb{C}^n$ with $|X| = 1$, we have for the Bergman differential metric B_D of D:*

$$B_D(w, X) \longrightarrow +\infty, \text{ when } w \longrightarrow w^0.$$

Finally an outlook to a possible generalization of the pluricomplex Green function was given.

Kengo Hirachi

Local Sobolev - Bergman kernels of strictly pseudoconvex domains

One of the most important properties of the Bergman kernel is its transformation law under biholomorphic maps. In this talk I defined Sobolev - Bergman kernels as an analogy to the Bergman kernel so that they satisfy a biholomorphic transformation law.

The main tool is Kashiwara's microlocal analysis of the Bergman kernel. Using his theory I constructed, for strictly pseudoconvex domains $\Omega = \{r > 0\} \subset \mathbb{C}^n$, a kernel function K_Ω^m, satisfying the following properties (for $m = 0, 1, ..., n$):

(SB 1). *If* $\Phi : \Omega_1 \longrightarrow \Omega_2$ *is biholomorphic, then*

$$K_{\Omega_1}^m = K_{\Omega_2}^m \circ \Phi \cdot |\det \Phi'|^{2(n+1-m)/n+1}$$

(SB 2) *The following asymptotic expansion holds:*

$$K_\Omega^m = \begin{cases} \phi_m r^{m-n-1} + \psi_m \log r & \text{for} \quad m < n+1 \\ \phi_m r^m \log r & \text{for} \quad m \geq n+1 \end{cases}$$

with functions ϕ_m, ψ_m, *that are smooth up to the boundary,*

(SB 3) *If* $\partial\Omega$ *is in normal form*

$$2\mathrm{Re}\, z_n = |z'|^2 + \sum A^l_{\alpha\bar\beta} z'^\alpha \overline{z'^\beta} (\mathrm{Im}\, z_n)^l,$$

then

$$K_\Omega^m(0,t) \approx_{t\searrow 0} \sum_{j=-\infty}^{-1} P_j(A) + \sum_{j=0}^{\infty} P_j(A) t^j \log t$$

where $P_j(A)$ *are polynomials in* $A = (A^l_{\alpha\bar\beta})$.

I also showed that if $m \in \mathbb{Z}\backslash\{0, 1, ..., n+1\}$, there is no non-trivial domain functional $K^m = (K_\Omega^m)_{\Omega \text{ strictly pseudoconvex}}$ satisfying (SB 1), (SB 2), and (SB 3).

Burglind Jöricke

Local polynomial hulls of discs near isolated parabolic points

Let Δ be a C^2-disc imbedded into $\mathbb{C}^2$ with an isolated parabolic point. The problem was considered whether sufficiently small closed neighbourhoods of this point on the disc are polynomially convex. This problem remained open after a classical paper of E. Bishop. Generically the index of the parabolic point is zero and the answer is "yes".

However, there is an explicit example for the index zero case, where the answer is "no", in contrast to what one would like to expect. In such a case for any small enough closed disc K on Δ containing the parabolic point, the set

$$K_{tr} := K \cap \overline{(\widehat{K} \setminus K)}$$

has the structure of an "onion". The "coates" of the onion bound analytic discs. Here $\widehat{K}$ denotes the polynomial hull of K. Methods of dynamical systems are applied. The dynamical system uses the characteristic vector field obtained from imbedding the disc into a strictly pseudoconvex boundary.

JOACHIM MICHEL

C^∞-regularity for $\overline{\partial}_b$ on pseudoconvex domains of Levi flat submanifolds of $\mathbb{C}^n$

(Joint work with Mei Chi Shaw, University of Notre Dame)

Let $\Omega \subset\subset \mathbb{C}^n$ be a pseudoconvex domain with a piecewise smooth boundary. Let L be a real hypersurface defined in a neighborhood of $\overline{\Omega}$ which divides Ω into two parts Ω_+ and Ω_-. We shall call L *admissible with respect to* Ω if there exist two smoothly bounded pseudoconvex domains D_+ and D_-, each on one side of L such that $\Omega_+ \subset D_+$ and $\Omega_- \subset D_-$. In this case the part of L which is in Ω is Levi flat.

We set $M_1 = L \cap \Omega$. From previous results of Michel and Michel-Shaw it follows that we can solve the $\overline{\partial}$-equation $\overline{\partial} u = f$ with $u \in C^\infty_{0,q-1}(\overline{X})$, if $f \in C^\infty_{0,q}(\overline{X})$, $\quad q \geq 1$, and $\overline{\partial} f = 0$, for $X = \Omega, \Omega_+, \Omega_-$.

We proved that we can then solve $\overline{\partial}_b u = f$, if $f \in C^\infty_{0,q}(\overline{M}_1)$, $\ q \geq 1$, $\ \overline{\partial}_b f = 0$, with $u \in C^\infty_{0,q-1}(\overline{M}_1)$.

Now suppose that we are given k hypersurfaces $L_1, ..., L_k$ which are admissible with respect to Ω. We set

$$M_k := \Omega \cap L_1 \cap ... \cap L_k$$

and assume that M_k is a Cauchy-Riemann manifold. Furthermore we assume that the L_i intersect transversally with the other L_j's and with the boundary of Ω. Under the following *working hypothesis* which is not completely proved by the authors but which is true if the L_i intersect complex transversally we showed the main theorem by an induction argument.

Working hypothesis: Let $f \in C^\infty_{0,q}(\overline{M}_k)$ be a $\overline{\partial}_b$-closed form with $q \geq 1$. Then there exists an extension of f, denoted by $\widetilde{f}$, with

α) $\widetilde{f} \in C^\infty_{0,q}(\mathbb{C}^n)$,

and

$\beta\ \overline{\partial}_b \widetilde{f}$ vanishes on $\overline{M}_k$ to infinitely high order.

Main Theorem. *Let M_k be as defined above and $f \in C^\infty_{0,q}(\overline{M}_k)$ a $\overline{\partial}_b$-closed form, with $q \geq 1$. Then there exists $u \in C^\infty_{0,q-1}(\overline{M}_k)$ with*

$$\overline{\partial}_b u = f.$$

Remark. In contrast to many other situations in this context there is no top degree q for solving the $\overline{\partial}_b$-equation.

Jürgen Leiterer

On Serre duality with support conditions

(Joint work with Chr. Laurent-Thiebaut)

It is known (Serre (1955) et al.) that, for any complex manifold X and for all p, q, such that $0 \leq p, q \leq n = \dim_{\mathbb{C}}(X)$, the following two conditions are equivalent:

(i) *$H^{p,q}_c(X)$ is separated;*

(ii) *$H^{n-p,n-q+1}(X)$ is separated;*

In the paper of the authors "On Serre duality" (To appear in Bull. Sci. Math.) it is proved that these two conditions can be completed by the following equivalent condition

(iii) *For any compact set $K \subset X$ the space $\mathcal{D}^{p,q}_K(X) \cap \overline{\partial}\,\mathcal{D}^{p,q-1}_K(X)$ is closed.*

Using this new equivalence (i)$\Longleftrightarrow$ (iii) one can prove some new separation theorems for the Dolbeault cohomology.

Furtheron generalizations to more general families of supports were discussed. To get more insight, answers to the following two problems would be extremely interesting:

Problem 1: Let $A : \widetilde{E} \longrightarrow E$ be a continuous linear operator between two strict LF-spaces such that Im (A) is topologically closed. Is it true that then the operator $A : \widetilde{E} \longrightarrow$ Im(A) is "weakly open" in the following sense: For all weakly open sets $\widetilde{U}$ in $\widetilde{E}$ with Ker $A \subset \widetilde{U}$, the set $A(\widetilde{U})$ is open in Im (A) (with the topology induced from E) ?

Problem 2: Let E be a strict LF-space with the defining sequence $E_1 \subset E_2 \subset ... \subset E$ of Fréchet spaces $(E_k)_{k\in\mathbb{N}}$, and let $L \subset E$ be a linear subspace such that $L \cap E_k$ is topologically closed for all k. Is it true that then L is topologically closed ?

Any answers (positive or negative) are welcome.

Joel Merker

Algebraicity of holomorphic mappings and analyticity of formal CR mappings

In the talk I presented two main theorems about regularity of formal or holomorphic mappings between CR manifolds:

Theorem 1. *Let $M \subset \mathbb{C}^n$, $M' \subset \mathbb{C}^{n'}$ be connected generic real algebraic manifolds; let $p \in M$, let $U \ni p$ be a small polydisc. Assume that M is minimal in the sense of Tumanov. Let $f \in \mathcal{O}(U, \mathbb{C}^{n'})$ be of constant rank, with $f(U \cap M) \subset M'$. Let k be the transcendence degree of f. Finally, let Σ'' be the minimal (for inclusion) real algebraic set with $f(M \cap U) \subset \Sigma'' \subset M'$. Then Σ'' is at least k-algebraically degenerate.*

In other words, Σ'' is flat in the CR - geometric sense, i.e.

$$\Sigma'' \approx \underline{\Sigma''} \times \Delta^k$$

is foliated by k-dimensional polydiscs around a generic point.

Theorem 2. *Let $n = n'$ and $M, M' \subset \mathbb{C}^n$ be real analytic CR manifolds. Let $p \in M$, $p' \in M'$; let $h : (M, p) \longrightarrow (M', p')$ be a formal invertible CR holomorphic map. If M is minimal and holomorphically non-degenerate, then h converges.*

Two main tools are used: Artin's approximation theorem and propagation along the Segre foliations.

Sergey Pinchuk

Analytic continuation of holomorphic and CR mappings

The talk was focused on principle problems in this area. Here are some of them:

1. *Holomorphic continuation of proper holomorphic maps*

Let $\mathcal{D}, \mathcal{D}' \subset \mathbb{C}^n$ be domains with real-analytic boundaries and $f : \mathcal{D} \longrightarrow \mathcal{D}'$ a proper holomorphic map. Does this imply that f extends holomorphically to a neighborhood of $\overline{\mathcal{D}}$?

2. *Continuation of CR mappingsß*

Let Γ, Γ' be real-analytic hypersurfaces in $\mathbb{C}^n$ of finite type and $f : \Gamma \longrightarrow \Gamma'$ a continuous CR map. Is f analytic ?

3. *Propagations of CR (holomorphic) maps*

Let Γ, Γ' be real-analytic hypersurfaces in $\mathbb{C}^n$, Γ' be compact and ${}_pf : \Gamma \longrightarrow \Gamma'$ be a germ of a holomorphic (CR) map in a point $p \in \Gamma$. When does ${}_pf$ extend analytically along any path in Γ ?

Some partial results about the above the problems were discussed.

Jean Pierre Rosay

Non-linear Paley-Wiener Theory

We discussed the following

Theorem: *Let $K \subset \mathbb{C}^n$ and let Ψ be an analytic functional in $\mathbb{C}^n$. The following are equivalent:*

(1) Ψ is carried by K,

(2) For every $d \in I\!N$ and every neighborhood V of K in $\mathbb{C}^n$ there exists $C_{V,d}$ such that for every polynomial P of degree $\leq d$

$$|\langle \Psi, e^P \rangle| \leq C_{V,d} \sup_V |e^P|.$$

(In the convex case it is enough to take polynomials P of degree 1 (Martineau))

This theorem is an immediate consequence of a dual statement on the span of the exponentials e^P, which is obtained by easy computation in the polydisc case, and by using Oka extension in the general case.

It has applications to study the carrier of analytic families of analytic functionals.

Nessim Sibony

Dynamics of polynomial automorphisms of $\mathbb{C}^k$

Let f be a polynomial automorphism of $\mathbb{C}^k$ and $\overline{f}$ the extension to $I\!P^k$ as a birational map. Let I_+ denote the indeterminacy set of $\overline{f}$ and I_- the indeterminacy set of $\overline{f}^{-1}$. Then we make the

Definition: $f \in \mathrm{Aut}_d(\mathbb{C}^k)$ is *regular* if $I_+ \cap I_- = \emptyset$.

One has the following theorems:

Theorem 1. *Let $f \in \mathrm{Aut}_d(\mathbb{C}^k)$ be a regular automorphism of $\mathbb{C}^k$. Let ω be the Kaehler form on $I\!P^k$. The following limit exists and defines a positive closed current of bidegree $(1,1)$:*

$$T_+ = \lim_{n \to \infty} \frac{1}{d^n} (f^n)^{\circ *} \omega$$

T_+ does not give mass to algebraic varieties. T_+ is an extremal current in the cone of positive closed currents.

Theorem 2. *Let f be a regular automorphism of $\mathbb{C}^k$. Assume that dim $I_- = l-1$. Then*

$$\mu := T_+^l \wedge T_-^{k-l}$$

is an invariant probability measure supported on

$$K = \{z \;:\; \{f^n(z) : \; n \in \mathbb{Z}\} \text{ is bounded}\}.$$

Theorem 3. *Let f be a regular automorphism of $\mathbb{C}^3$. Then the measure μ is mixing.*

Berit Stensønes

The Michael problem

In this talk some ideas of the proof of the following theorem were presented:

Theorem. There exists a sequence $(\Phi_j)_j$ of entire maps $\Phi_j : \mathbb{C}^3 \longrightarrow \mathbb{C}^3$, such that

$$\bigcap_{j=1}^{\infty} \Phi_1 \circ \ldots \circ \Phi_j(\mathbb{C}^3) = \emptyset.$$

Further more it was proved that this gives a positive answer to the

Michael Problem: Let A be a Fréchet algebra and let $\phi : A \longrightarrow \mathbb{C}$ be a multiplicative linear functional. Does it follow that ϕ is continuous ?

The proof of this connection is due to Dixon and Esterle.

Emil J. Straube

Compactness of the $\overline{\partial}$-Neumann problem on convex domains

In this talk, I discussed joint work with Siqi Fu concerning compactness of the $\overline{\partial}$-Neumann problem on convex domains. This compactness is an analytic condition. There are two other conditions, one geometric and one potential theoretic that bear on this question. The geometric condition is the absence or presence of analytic varieties in the boundary, the potential theoretic one is the existence of a family of functions with suitable Hessians. On convex domains, these conditions match perfectly:

Theorem: *Let Ω be a bounded convex domain in $\mathbb{C}^n$, and $1 \leq q \leq n$. The following are equivalent:*

(i) $\partial\Omega$ satisfies condition (P_q),

(ii) $\partial\Omega$ contains no analytic variety of dimension greater than or equal to q,

(iii) The Neumann operator N_q (at the level of $(0,q)$-forms) is compact

Here, the condition (P_q) means the following:

(P_q): For all $M > 0$ there exists a C^2-function λ in a neighborhood U of $\partial\Omega$, such that $0 \leq \lambda \leq 1$, and the sum of any q eigenvalues of

$$H_\lambda(z) := \left(\frac{\partial^2\lambda}{\partial z_i\,\partial\overline{z}_j}(z)\right)_{j,k=1}^{n}$$

is greater than or equal to M, for any $z \in U$.

It is known that no such characterization holds on general pseudoconvex domains.

Shigeharu Takayama

The Levi problem and the structure theorem for non-negatively curved complete Kaehler manifolds

We discussed the Levi problem on complex manifolds and a related problem. It is well-known that, if a complex manifold X is holomorphically convex, then there exists a C^∞-smooth plurisubharmonic exhaustion function $\Phi : X \longrightarrow I\!R$. Such manifolds are said to be "weakly 1-complete" after Nakano. We also consider manifolds with a continuous plurisubharmonic exhaustion function. Such manifolds are said to be "pseudoconvex". Then the Levi problem in our case asks whether a weakly 1-complete or pseudoconvex manifold is holomorphically convex, or not.

One of our main results is as follows:

Theorem: *Let X be a pseudoconvex manifold with negative canonical bundle. Then X is holomorphically convex.*

Structure Theorem: *Every complete Kaehler manifold with non-negative sectional curvature and positive Ricci curvature is holomorphically convex. Moreover the Remmert reduction gives a structure of a holomorphic fiber bundle over a Stein manifold with a compact Hermitian symmetric manifold as the typical fibre.*

This gives the complete affirmative answer to a conjecture of Greene-Wu.

Jean-Marie Trépreau

Conic reflection and the classification of germs of resonant diffeomorphisms of $(\mathbb{C}, 0)$

1) We showed how the following questions are related with the classification of resonant diffeomorphisms by Ecalle and Voronin (1981) and the classification of generic pairs of involutions by Voronin (1981).

Problem 1. An anlytic cusp in $\mathbb{C} \approx I\!R^2$ is a germ of a real-analytic singular curve, which is real analytically equivalent to

$$S = \{z = x + iy \ : \ x^2 = y^3\}$$

near 0.

It happens that any two cusps S_1, S_2 are formally complex equivalent, i.e. there exists a formal power series $f(z) = az + \ldots$, with $a \neq 0$, such that $f(S_1) = S_2$. Actually, f is unique.

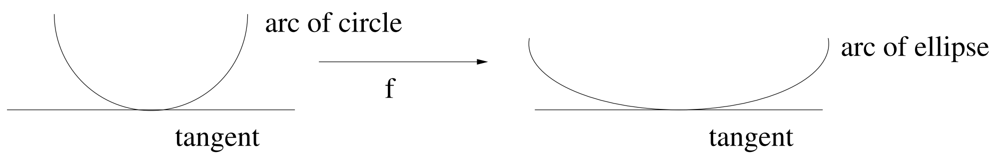
Question. Give " geometric " examples of cusps which are not complex equivalent, i.e. such that f is not convergent.

Problem 2. Classify the pairs consisting of an arc of a smooth analytic curve and a tangent.

Question. In a formal class, give examples of non-conformal pairs, using geometric arguments.

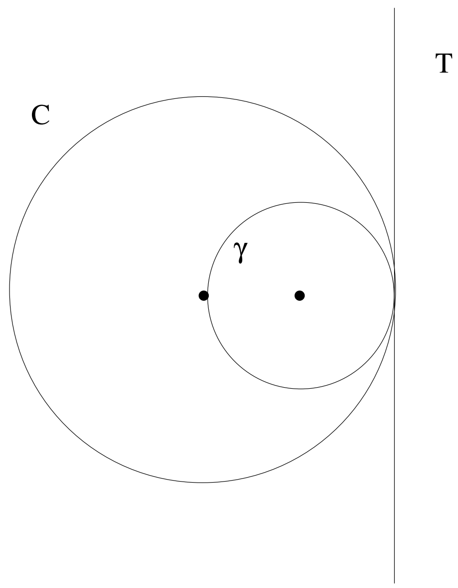
2) As an example it was proved that there exists no local biholomorphism near $0 \in \mathbb{C}$ which transforms

Fig. 1



This is proved by introducing (Schwarz)- reflection through an ellipse.

Fig.2



γ is the inverse of T for the circle C.

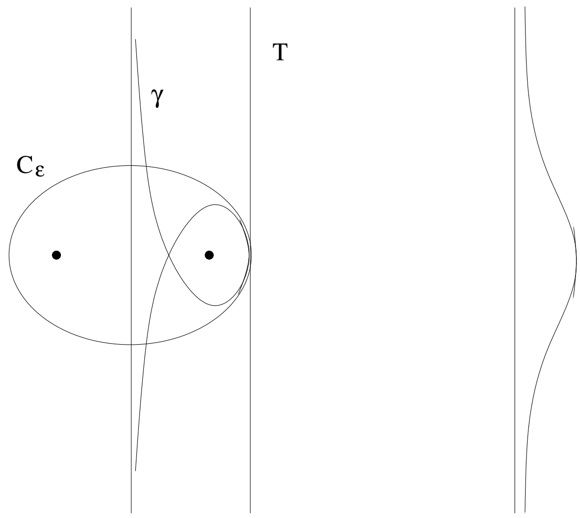


Fig. 3

γ is the inverse of T for an ellipse C_ε with small excentricity, and foci at $\varepsilon, -\varepsilon$.

The two pictures are not quite the same.

Ken-ichi Yoshikawa

Analytic Torsion and Automorphic Forms on the Moduli Spaces

Let (X, ι) be a pair of a $K3$ surface and an anti-symplectic involution (an involution which reverses the holomorphic symplectic form). We call such a pair a 2-elementary $K3$ surface. A family of 2-elementary $K3$ surfaces is parametrized by the invariant lattice

$$H^2_+(X, \mathbb{Z}) = \{l \in H^2(X, \mathbb{Z}) \ : \ \iota^* l = l\}.$$

We call (X, ι) to be of type S, if $H^2(X, \mathbb{Z})$ is isometric to $S \subset L_{K3}$. It is known that S is a primitive hyperbolic 2-elementary lattice in L_{K3}, the $K3$-lattice.

We introduce an invariant of 2-elementary $K3$ surfaces via analytic torsion. For a 2-elementary $K3$ surface (X, ι) of type S, we define

$$\tau_S(X, \iota, \kappa) := \tau(X/\iota, \kappa)^{\frac{14-r}{8}} \left(\tau(X^\iota, \kappa|_{X^\iota}) \mathrm{vol}(X^\iota, \kappa|_{X^\iota})\right)^{1/2},$$

where κ is an ι-invariant Ricci-flat Kaehler metric and X^ι is the fixed curve of ι. $(r = \mathrm{rk}_{\mathbb{Z}} S)$.

Theorem 1. *τ_S is independent of κ and becomes a smooth function on the moduli space which is an arithmetic quotient of a bounded symmetric domain of type IV. τ_S can be identified with an automorphic form on the moduli space.*

Theorem 2. *If* $S = U(2) \oplus E_8(-2)$, $U \oplus E_8(-2)$, *and* $S^{\perp} = U(2) \oplus I_k(2)$, $(0 \leq k \leq 8)$, *then* τ_S *is represented by an automorphic form with infinite product.*

Reported by Gregor Herbort

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