Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 22/1999

Schnelle Löser für partielle Differentialgleichungen

30.05. – 05.06.1999

The meeting was organized by Randolph E. Bank (La Jolla), Wolfgang Hackbusch (Leipzig) and Gabriel Wittum (Heidelberg). Each presentation was about 40 min followed by 5–10 min for discussion and questions. Informal evening and afternoon discussions gave the meeting a workshop character. The conference was divided into sessions on

- **Parallelization**
  Espedal, Holst, Jung, Mitchell
- **Domain Decomposition**
  Hoppe, Kuznetsov, Lube, Mandel, Widlund
- **Convection-Diffusion**
  Johannsen, Reusken
- **Wavelets**
  Dahmen, Pflaum, Stevenson
- **Algebraic Multigrid**
  Chan, Stüben, Wagner
- **Continuum Mechanics**
  Brenner, Kornhuber, Wieners
- **Preconditioners**
  Grote, Hackbusch, Khoromskij, Neuss, Xu
- **Miscellaneous topics**
  Björstad, Braess, Oosterlee, Schulz, Wesseling

Due to a public holiday the traditional Wednesday afternoon walk was postponed to Thursday afternoon.

The conference provided an interesting mixture of mathematical theory and applications for fast solvers for partial differential equations. Brand-new developments like $\mathcal{H}$-matrices and new ideas in modern fields like wavelets and algebraic multigrid have been presented as well as new results in the classic domain decomposition and multigrid area (e.g. for convection-diffusion equations). Parallelization especially for clusters of workstations or PCs turned out to be an important issue of the conference. The meeting showed that fast solvers for partial differential equations are still an important and interesting research field with a large variety of different aspects.

Abstracts

Fast Spectral Solvers

Petter Björstad and Jan Moldekleiv

We report on the efficient design of second and forth order PDE computational kernels based on the Galerkin-spectral formulation of Jie Shen. We discuss cache performance and parallel performance — portability. Spectral discretizations with several thousand grid points are reported and compared with the $O(N^2 \log N)$ finite difference package Fishpack.

A Cascadic Multigrid Algorithm for Mortar Elements

Dietrich Braess

We consider a cascadic multigrid method for the saddle point formulation of mortar elements. A modified Richardson iteration admits to stay in the kernel of the constraint operator, and a cg-iteration can be based on it. A central point is the transfer between the grids. A duality argument applies to Fortin interpolation and makes that the extra terms due to non-nestedness is damped on the next level.

The general theory can be applied to the mortar elements since ellipticity, the inf-sup condition, and L_2 error estimates are established with mesh-dependent norms.

Multigrid for Stress Intensity Factors and Singular Solutions

Susanne C. Brenner

In this talk we discuss new algorithms that can provide good approximate solutions to elliptic boundary value problems in the presence of corner singularities or interface singularities using only the P1 finite element on uniform grids. The key idea is to combine the full multigrid methodology and the extraction formulas for stress intensity factors.

An Energy Minimizing Approach to Robust Multigrid Interpolation

Tony Chan, Justin Wan and Barry Smith

We propose an unifying framework for deriving robust interpolation operators in multigrid methods. Our goal is to provide an unifying view of characterizing interpolation operators which can be applied in an uniform way to problems with jumps in coefficients, oscillatory coefficients, unstructured grids, anisotropy and SPD problems which are not necessarily elliptic (e.g. convolution operators). The principle is based on requiring the coarse space to possess stability and approximation properties. This leads to a variational characterization of minimizing the sum of the A-energies of the coarse basis subject to a global constraint of preserving the zero energy mode of the operator. We show that this variational problem can be solved efficiently by a preconditioned iterative method. In practice, high accuracy is not needed and only a few iterations are needed. Numerical results show that our goal can be achieved.

Adaptive Wavelet Methods - Convergence Rates

Wolfgang Dahmen, A. Cohen and R. DeVore

First linear operator equations $Au = f$ are considered where A is selfadjoint and boundedly invertible as an operator from some Sobolev space into its dual. This covers boundary value problems for elliptic partial differential equations as well as many classical singular integral operators such as the single layer potential, double layer potential and hypersingular operator. The efficient numerical treatment of such problems is obstructed by several factors such as the size of the resulting discrete problems, a possibly growing ill conditioning when the operator has an order different from zero or by the fact that densely populated matrices arise in connection with integral operators. An adaptive wavelet scheme is outlined that aims at determining in the course of the solution process a possibly small set of wavelets needed to recover the solution within some desired error tolerance. It is based on a-posteriori error estimates for the current approximate solution in terms of residuals. The main result is its asymptotic optimality in the sense that (within a certain range of Besov regularity) the convergence rate of best N-term approximation is achieved at a computational expense which stays proportional to the number N of significant degrees of freedom provided that full information on the given data is available. As a consequence one observes an asymptotic gain in accuracy and efficiency over a-priorily given uniform refinements if the solution lacks the order of Sobolev regularity that corresponds to the order of the discretization error for uniform refinements. A crucial step in the analysis is to prove the uniform boundedness of the residuals evolving during the adaptive process with respect to discrete norms for certain Lorentz sequence spaces. The main ingredients of the analysis are norm equivalences for Sobolev and Besov spaces induced by wavelet expansions, related preconditioning effects, the near sparseness of the wavelet representations of the operators under consideration and the elements of Besov spaces, new fast approximate matrix-vector multiplication schemes suggested by the analysis and a judicious use of intermediate thresholding of current approximate solutions. The theoretical results are illustrated by first numerical experiments. Finally it is indicated how to extend the scheme with the aid of certain stable least squares formulations to a wider scope of problems covering systems of operator equations and indefinite problems.

Parallelization of a Compositional Reservoir Simulation

Magne Espedal

Flow of fluids in a porous media is subject to a wide interest in several fields: reservoir engineering, subsurface hydrology, etc. This paper is based on a finite volume formulation of thermal , multiphase, compositional model in three space dimension.

We have chosen to use a sequential solution procedure solving for pressure velocity in a first step and updating temperature and molar mass in a second step. Saturations are calculated from a thermodynamical model in a third step. Conservation of fluxes and continuity in the potential are essential for accuracy and this is obtained by an O-method. An important part of the work is on adaptive local refinement based on domain decomposition methods. This type of technique is implemented for the pressure solver as well as for the flow part and makes the code well suited for parallel computing.

Numerical results are presented for a model with 2.5 million unknown, including fractures and faults. The code scales very well on a parallel computer.

Preconditioning with Sparse Approximate Inverses

MARCUS GROTE

The SPAI preconditioner for the solution of sparse linear systems of equations is presented. Unlike incomplete factorization methods, like ILU, the SPAI algorithm directly computes an approximate inverse for use as a preconditioner with iterative methods. Next, the block-SPAI algorithm, which greatly reduces the cost of computing the preconditioner, is presented. Finally, the usefulness of approximate inverses as robust smoothers for multigrid methods is discussed.

$\mathcal{H}$-Matrices

WOLFGANG HACKBUSCH

A class of matrices is introduced with the following properties: (i) They are data sparse, i.e. the storage is almost $O(n)$, (ii) the matrix-vector multiplication needs almost $O(n)$ operations, (iii) sums, products and the inverse can be performed approximately in again almost $O(n)$ operations, (iv) finally, this set of matrices is ϵ-dense in the set of discrete elliptic matrices, i.e. for all ϵ there is an $k \geq 0$ and an $\mathcal{H}$-matrix A' from $\mathcal{M}_{\mathcal{H},k}$ with $||A - A'|| \leq \epsilon$.

Decoupling Adaptive Finite Element Methods using Local Estimates

MICHAEL HOLST AND RANDOLPH BANK

We describe a new algorithm for solving elliptic equations adaptively, based on decoupling the problem using local a priori and posteriori error estimates. The algorithm resolves the load-balancing issue in an a priori way using error estimates as work predictors. It involves the solution of a sequence of subproblems which can be done in parallel, and requires almost no communication. We present several examples with an implementation of the algorithm using a new 2D/3D finite element code called MC. These examples include 2D and 3D nonlinear elasticity, and the four-component nonlinear elliptic constraints in the Einstein equations. While the algorithm appears to ignore the globally coupled nature of elliptic problems, we outline some recent results of Xu and Zhou on local a priori and a posteriori error estimation, and the earlier interior estimates of Schatz et al., which provide justification for the new algorithm.

Electromagnetic Field Computation by Domain Decomposition Methods on Non-matching Grids

RONALD H. W. HOPPE

We consider the computation of eddy currents as the solution of the quasistationary limit of Maxwell's equations. The spatial discretization is done by edge elements with respect to a nonoverlapping, geometrically conforming decomposition of the computational domain featuring individual triangulations of the subdomains regardless the situation on the interfaces between them. In particular, we concentrate on the proper specification of the multiplier spaces for the Lagrangian multipliers that guarantee weak continuity of the tangential components on the skeleton of the decomposition. Further, for the iterative solution of the resulting saddle point problem we suggest a multigrid algorithm whose main feature is a special hybrid smoothing process. Appropriate Helmholtz decompositions of the tangential trace spaces and the edge element spaces play a central role in the analysis.

Robust Multigrid Methods for Convection Diffusion Equations with Closed Characteristics

Klaus Johannsen

We consider the two-dimensional convection diffusion equation with a convection containing one or more vortices. Special emphasis is laid on problems with dominant convection. We present a detailed analysis for a representative number of model problems. The investigation is based on Fourier analysis in the case of a very simple model problem and on numerical experiments for more realistic situations. The latter cases are discretized by means of the control volume finite element method using continuous, piecewise linear ansatz functions on triangular meshes. These experiments have been carried out on the software platform UG using grids with up to 4.10^5 unknowns to investigate the asymptotic behavior. To construct a robust multigrid scheme we have to modify the components of the multigrid.

ROBUST SMOOTHER The method is based on a Gauss-Seidel type smoother using an ordering strategy. In the simple case of one vortex a one-dimensional set of feedback vertices are removed from the matrix graph yielding a cyclefree subgraph. A block Gaussian elimination on the resulting 2×2 block system is used to treat the cyclic dependencies. The Schur complement is inverted by a (robust) frequency filtering iteration.

COARSE GRID CORRECTION We will show that the robust smoothing property cannot be complemented by a robust approximation property. Instead we will use an improved coarse grid correction which provides a sufficient approximation to make robust multigrid convergence feasible. The improvement is obtained by a scaling of the convection on the coarser grids.

MULTIGRID SCHEME A robust multigrid scheme is realized using a WW-cycle ($\gamma = 4$) or a VW-cycle ($\gamma = 3$). A modification of the multigrid procedure can be used leading to a significant improvement of the convergence rate (with a neglectable amount of additional work).

The presented multigrid procedure is of (nearly) optimal complexity ($O(n \log^2 n)$ resp. $O(n \log n)$) and is robust w.r.t. to the Peclet number as well as to the amount of crosswind diffusion introduced by the discretization scheme.

For practical use (problems with non-vanishing crosswind diffusion on grids up to 10^6 unknowns) we show that a modification based on Krylov space methods using a W-cycle multigrid ($\gamma = 2$) leads to a robust method of optimal complexity. The resulting algorithm is well suited for more complex problems in which dominant convection plays an important role. An application to the density driven groundwater flow is discussed.

Comparison of Parallel Multilevel Solvers for Elliptic Boundary Value Problems

Michael Jung

In recent years many efficient solvers for partial differential equations were developed. Examples for such solvers are the multigrid method or the preconditioned conjugate gradient (pcg) method with multilevel preconditioners (preconditioners derived from classical multigrid methods, additive multilevel preconditioners, algebraic multilevel iteration preconditioners, domain decomposition preconditioners). In the talk, we discuss the parallelization of these methods based on a non-overlapping domain decomposition data distribution. We show some advantages and disadvantages of these methods and give a comparison by means of several examples.

Furthermore, we present cg-like iterative solvers which are especially designed for the application of additive preconditioners of the type $B^{-1} = c_1 B_1 + c_2 B_2 + \cdots + c_n B_n$ with symmetric, positive semidefinite operators B_i and real numbers c_i, $i = 1, 2, \ldots, n$. The scaling factors c_i are computed automatically within each iteration step of the cg-like methods.

Data-Sparse Hierarchical Approximation of Nonlocal Operators

Boris N. Khoromskij and Wolfgang Hackbusch

A class of hierarchical matrices ($\mathcal{H}$-matrices) was introduced in [1] which are data-sparse and allow a matrix arithmetic of almost linear complexity. Several types of $\mathcal{H}$-matrices were considered in [1], [2], [3], which are able to approximate the integral and pseudodifferential operators in the case of quasi-uniform unstructured meshes.

In this talk, we discuss the consistency error and general complexity estimates for data-sparse hierarchical approximations to integral operators defined on domains/manifolds in R^d, $d = 2, 3$. The local kernel expansions by using both the Taylor and Legendre polynomials are analyzed. The resulting $\mathcal{H}$-matrices retain the approximation power of the exact Galerkin scheme, on the one hand, and provide the asymptotically linear complexity (w.r.t. the problem size) for matrix-vector/matrix-matrix multiplications and matrix-inversion, on the other. Emphasis will be placed upon the sharp complexity bounds regarding the basic parameters of our techniques.

[1] W. Hackbusch: *A Sparse Matrix Arithmetic based on $\mathcal{H}$-Matrices. Part I: Introduction to $\mathcal{H}$-Matrices.* 1998, Computing, to appear.

[2] W. Hackbusch and B. N. Khoromskij: *A Sparse $\mathcal{H}$-Matrix Arithmetic. Part II. Application to Multi-Dimensional Problems.* Preprint No. 22, Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig, 1999, submitted.

[3] W. Hackbusch and B. N. Khoromskij: *A Sparse $\mathcal{H}$-Matrix Arithmetic: General Complexity Estimates.* Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig, 1999. To be submitted.

Monotone Multigrid Methods for the Signorini Problem in Linear Elasticity

Ralf Kornhuber and Rolf Krause

We present a new class of multigrid methods for the Signorini problem in linear elasticity. The methods are based on inexact subspace correction with respect to d-dimensional subspaces associated with the nodes of the underlying triangulation. $d = 2, 3$ is the dimension of the reference configuration. The subspaces are intended to represent the high and low frequency contributions of the error. Monotonically decreasing energy provides global convergence of the algorithms. We consider inexact solution of the local subproblems in order to preserve optimal complexity of each iteration step. The difficulty behind is that Signorini conditions on the fine grid cannot be represented on the coarse grid.

In case of constant normal directions, we obtain straightforward block versions of well-known monotone multigrid methods for scalar obstacle problems. Special care has to be taken in case of varying normal directions, because corresponding slip boundary conditions then are no longer resolved by the coarse grid. Efficiency and reliability of monotone multigrid methods for the Signorini problem is illustrated by numerical experiments in $d = 2, 3$ space dimensions.

Fictitious Domain Methods with Distributed Lagrange Multipliers

Yuri Kuznetsov

A historical overview of various approaches to fictitious domain methods is presented. We discuss the connection between the original algorithm by V. Saul'ev, and the recent technique based on distributed Lagrange multipliers. A particular attention is given to elliptic problems with

Dirichlet boundary conditions. It is shown that with the latter approach we can easily design preconditioners spectrally equivalent to the original saddle-point matrix, which has the optimal order of arithmetical complexity.

A Non-Overlapping DDM for Finite Element Approximations of the Oseen Equations using Inf-Sup Stable Elements

Gert Lube

We analyze a non-overlapping domain decomposition algorithm of Robin-Robin type which can be applied to the discretized Oseen equations appearing as a linearized variant of the incompressible Navier-Stokes equations. Here we consider finite element approximations using velocity/pressure pairs which satisfy the Babuska-Brezzi stability condition.

First we prove coercivity and strong convergence of the method. Then we derive an a-posteriori error estimate which controls convergence of the discrete subdomain solutions to the global discrete solution by measuring the jumps of the velocities at the interface. Additionally we obtain information how to design some parameter function within the Robin type interface condition which essentially influences the convergence speed. Some numerical results confirm the theoretical results.

Note that we may consider the case of stabilized finite element approximations including equal order interpolation of velocity/pressure in a similar way.

Convergence Bounds for Iterative Substructuring by Lagrange Multipliers

Jan Mandel, Radek Tezaur and Charbal Farhat

We present an overview of convergence theory for the so-called FETI method both for second and forth order problems. The bounds are based on algebraic arguments involving discrete dual norms and functional analysis tools developed previously for the Balancing Domain Decomposition method and substructuring in general by Dryja, Widlund, Bramble, Pasciak, Schatz, Letallec, Mandel and others.

Coarse Grain Parallel Adaptive Multilevel Methods

William F. Mitchell

This talk will present recent results in the development of parallel techniques for adaptive multilevel methods for partial differential equations. These techniques are suitable for low-bandwidth/high-latency environments like clusters of workstations or PCs. The techniques decrease the granularity of the parallel communication by using the "full domain partition" with subdomain overlap on each refinement level.

A new Sparse Matrix Storage System for Adaptive Parallel Solving of Systems of Large Diffusion-Convection-Reaction Equations

Nicolas Neuss

We introduce a new matrix storage scheme and examine its performance by careful tests. Applications inside the PDE tool UG lead to a powerful scheme for adaptive parallel multigrid for systems of diffusion-convection-reaction equations.

Fourier Analysis of GMRES(m) Preconditioned by Multigrid

Kees Oosterlee, R. Wienands and T. Washio

This paper deals with convergence estimates of GMRES(m) preconditioned by multigrid. Fourier analysis is a well-known and useful tool in the multigrid community for the prediction of two-grid convergence rates. This analysis is generalized here to the situation in which multigrid is a preconditioner, since it is possible to obtain the whole spectrum of the two-grid iteration matrix. A preconditioned Krylov subspace method like GMRES(m) implicitly builds up a minimal residual polynomial. The determination of the polynomial coefficients is easily possible and can be done explicitly since, from Fourier analysis, a simple block diagonal two-grid iteration matrix results. Based on the GMRES(m) polynomial, sharp theoretical convergence estimates can be obtained which are compared with estimates based on the spectrum of the iteration matrix. Several numerical scalar test problems are computed in order to validate the theoretical predictions.

Generalized Prewavelets

Christoph Pflaum

The convergence rate of a multiplicative multilevel algorithm with a subspace correction on a complementary space W_i depends on the constant in the strengthened Cauchy-Schwarz inequality γ between the coarse-grid space and the complementary space. Here, the fine-grid space is a direct sum of the coarse-grid space and the complementary space. One has to construct the complementary space W_i such that the constant γ is as small as possible and such that W_i is spanned by functions with a small support. Generalized prewavelets are an optimal basis for such a construction. First we study a multilevel algorithm with semi-coarsening, line-relaxation and 1D-optimized generalized prewavelets. It is proved that the convergence rate of this multilevel algorithm is less than 0.201 for a W(1,2)-cycle and 3 line relaxations. This holds for elliptic differential equations with anisotropies in x- and y-direction, highly variable coefficients in y-direction, and for some non H^1-elliptic differential equations. Furthermore, we construct problem dependent generalized prewavelets by a local orthogonalization of nodal basis functions. The multilevel algorithm based on these functions is robust for a large class of differential equations with discontinuous coefficients.

A Note on Basic Iterative Methods for Convection-Diffusion Equations

Arnold Reusken and Jürgen Bey

We consider matrices which result from known finite element or finite volume discretization methods applied to convection-diffusion equations. In general, these matrices do not have the M-matrix property. Based on inspection these stiffness matrices we introduce the following matrix classes: For $A \in \mathbb{R}^{n\times n}$

$$Z := \{\, A \,|\, a_{i,j} \leq 0 \ \forall i \neq j \,\},$$

$$SPD := \{\, A \,|\, A = A^T > 0 \,\},$$

$$PD := \{\, A \,|\, A + A^T > 0 \,\},$$

$$M := \{\, A \,|\, A \in Z \text{ and } Re(\lambda) > 0 \,\},$$

$$M_0 := PD \cap Z,$$

$$SPD.M := \{\, A \,|\, A = A_d + A_c \text{ with } A_d \in SPD,\ A_c \in M \,\},$$

$$SPD.M_0 := \{\, A \,|\, A = A_d + A_c \text{ with } A_d \in SPD,\ A_c \in M_0 \,\}.$$

With this notation, the following strict inclusion hold

$$\begin{array}{ccccc} M_0 & \subset & M & \subset & Z \\ \cap & & \cap & & \\ SPD.M_0 & \subset & SPD.M & & \\ \cap & & & & \\ PD & & & & \end{array}$$

We consider the question whether for the damped Jacobi or damped Gauß-Seidel method convergence can be proved for matrices from the classes in the second and third row of this scheme. For $A \in SPD.M_0$ a new contraction result for the damped Jacobi method is proved and compared with known results for the class PD. For a matrix $A \in SPD.M_0$ it is shown that for every real damping parameter the damped Gauß-Seidel method has an iteration matrix with spectral radius larger than one. For the class PD a new hybrid Jacobi-Gauß-Seidel type of method is proposed.

Multigrid Methods for Optimization Problems in PDE

Volker Schulz

Optimization problems are frequently encountered in inverse modeling, shape optimization or process control problems. Typically these problems evolve from simulation tasks defining some output states to be influenced by some input controls. Recent efficient numerical methods typically rely on the *direct discretization approach*, which treat the states and controls together as unknowns of a discretized finite dimensional constrained optimization problem and apply iterative methods of sequential quadratic programming (SQP) type. Despite the resulting large number of variables this approach has proven very successful since it enables a simultaneous solution of the optimization and the simulation problem.

At the core of all SQP type methods lie Karush-Kuhn-Tucker (KKT) systems. They can be considered special saddlepoint problems which, however, differ from the ones from Stokes or Navier-Stokes discretizations. In this talk a novel numerical multigrid approach is presented to the numerical solution of such KKT systems. This approach is based on iterative null-space methods employing so-called transforming smoothers.

A multigrid convergence proof for a model problem as well as numerical results for practical optimization problems are given.

Element-by-Element Construction of Wavelets Satisfying Stability and Moment Conditions

Rob Stevenson

A construction is presented of locally supported wavelets in standard finite element spaces on unstructured meshes on domains or manifolds. As main application we have in mind the solution of discretized differential- and singular integral operators. Then in order to get uniformly well-conditioned stiffness matrices, the wavelet system should be a Riesz basis in a relevant Sobolev space. Moreover, in case of integral operators, to compress the dense matrix to a sparse one without reducing the order of convergence, the wavelets should have sufficiently many vanishing moments, or more generally cancellation properties. In particular, the latter condition specialized to the case of integral operators of negative order rules out constructions based on orthogonal space decompositions, and therefore we consider the more general concept of *biorthogonal space decompositions* instead. With wavelets satisfying both aforementioned conditions, the discretized operators can be solved in linear time.

The construction that we will present satisfied the conditions in the following sense: The wavelet systems are Riesz bases in the Sobolev spaces H^s for $|s| < 1.5$ ($|s| < 1$ on Lipschitz' manifolds), and the wavelets can be arranged to have, in principal, any desired number of vanishing moments. Based on the affine equivalence of the finite elements, the construction of the wavelets consists of two parts: An implicit part involving some computations on a reference element which, for each type of finite element space, have to be performed only once. In addition there is an explicit part which takes care of the necessary adaptations of the wavelets to the actual mesh. The only condition we need for this construction to work is that the refinements of initial elements are uniform.

We will show that the wavelet bases can be implemented efficiently.

[1] Dahmen, W. and Stevenson, R. P., *Element-by-element construction of wavelets satisfying stability and moment conditions* Technical Report 9725 University of Nijmegen, November 1997, To appear in SIAM J. Num. Anal..

Interpolation in Algebraic Multigrid (AMG)

Klaus Stüben

A proper selection of interpolation (and the related coarsening process) is crucial for obtaining fast and robust AMG convergence. This presentation reviews and compares some - purely algebraically defined - approaches, including a couple of more recent ones. The focus is on the AMG solution of (symmetric, positive definite) linear systems which typically arise in the discretization of scalar elliptic PDEs.

The interpolation approaches currently used range from the simplest one, piecewise constant interpolation, to more complex, operator-dependent ones. Piecewise constant interpolation leads to very simple AMG-algorithms. However, without certain improvements — by, for example re-scaling the coarse-level Galerkin operator (Braess), smoothing the interpolation (Vanek/Mandel), or smoothing of corrections — it is usually not practical since it causes very slow (and strongly h-dependent) convergence. On the other hand, relatively simple operator-dependent interpolation yields very robust and fast convergence which, in practice turns out to be essentially h-independent. This has been demonstrated by various large industrial applications (several millions of unknowns), for example, from CFD and oil-reservoir simulation. The AMG solution of corresponding systems is typically faster than that by standard ILU-preconditioned cg-methods by factors between 6 and 20, depending on the concrete type of application.

With increasing effort put into the construction of interpolation, one can (in principle) enforce arbitrarily fast and uniform (i.e. matrix independent) AMG convergence. For corresponding two-level methods this can be proved for various classes of matrices.

On the Algebraic Construction of Multilevel Transfer Operators

Christian Wagner

The standard way to construct coarse grids for algebraic multigrid methods is a heuristic labeling of the nodes as C- and F-nodes. While the F-nodes are eliminated, the C-nodes built the coarse grid. After that, in a separate step, prolongation and restriction operators are constructed.

The basic idea of our new approach is to determine for each node those pairs of nodes which allow an optimal interpolation of the considered node. These pairs of neighbor nodes (in some cases only one node) are called parent nodes. A theoretical analysis shows that the problem of finding these parent nodes for the node i can be reduced to a minimization problem of the form minimize $||Y\,z||$, where Y is a sort of a smoothing operator and z is allowed to have aside from $z_i = -1$ only two non-zero entries. These non-zero entries will be the coefficients in the prolongation/restriction operators and the corresponding nodes are the parent nodes. Additionally, a filter condition $(z, t) = 0$ with a given test vector t can be imposed. The minimization problem can be solved locally and is therefore relatively cheap.

After the possible pairs of parent node have been determined, the nodes are labeled as C- and F-nodes such that each F-node can be interpolated using these pairs of parent nodes and the already computed coefficients. Additionally, a simple heuristic algorithm tries to minimize the number of C-nodes and the number of edges in the coarse grid graph.

The algorithm has been parallelized and shows mesh size independent convergence for standard model problems. Realistic numerical experiments confirm the efficiency of the presented algorithm.

Unified Methods for Computing Compressible and Incompressible Flows

Pieter Wesseling

The difficulties arising when applying methods for compressible flows to flows with vanishing Mach number M are elucidated. Improvements may be obtained by preconditioning, and multistage solution methods for the resulting system $P(u)\,u_t + \sum_\alpha f^\alpha(u)_{x^\alpha}$ are reviewed. Extension of staggered schemes for incompressible flows to $M > 0$ is considered. Based on an asymptotic expansion in powers of M^2, the governing equations are made non-dimensional such that the limit $M \to 0$ is regular. The pressure correction method is extended to the fully compressible case. Numerical experiments show that the method is accurate and efficient uniform in M. The method is applied to a nonconvex hyperbolic system $v_t - u_y = 0$, $u_t + p(v)_y = 0$, used to model a hydrodynamic flow with cavitation. The Mach number varies from 0.01 to ≈ 20, making the use of a Mach-uniform method mandatory.

Domain Decomposition Algorithms using Lagrange Multipliers

Olof Widlund and Axel Klawonn

Domain decomposition methods have been developed quite systematically and successfully, in particular, for conforming finite element approximations of elliptic problems. These algorithms are preconditioned conjugate gradient- type methods based on solvers for subregions and certain low-dimensional global models. In many methods the preconditioners are simply built from a large number of much smaller instances of the original finite element problem. The best results show that the number of iterations required to decrease the residual norm by a fixed factor is independent of the number of local subproblems, which form an important part of the preconditioner, and is also independent or only grows very slowly with the dimension of the local problems. These algorithms are designed with parallel and distributed computing in mind and there is now a considerable and positive experience with hard problems. Large software systems have been built based on these ideas.

In this talk, a description will first be given of two successful domain decomposition methods of Schur complement type, namely the balancing and the FETI methods. The FETI algorithm, invented and much developed by Farhat and Roux, has the interesting feature that the continuous finite element solution is approached through a sequence of discontinuous iterates. The jumps across the interface, defined by the boundaries of the subregions are controlled by Lagrange multipliers and when accurate enough values of these dual variables have been found, then the solution on individual subregions can be computed locally. The convergence of the iteration can be monitored by observing the jumps across the interface and when they have decreased sufficiently we are close to convergence.

We have made two contributions to the further understanding of these methods. We have first shown how the FETI method can be redesigned so inexact solvers can be used for the subregions. In such an algorithm both the solution and the Lagrange multipliers are updated in each iteration step after that the problem has been recast as a saddle point problem. This work is based on earlier work by Klawonn and others on preconditioners for saddle point problems. It is interesting to note that the leading principal block of the saddle point matrix is only positive semidefinite. We have also developed new versions of the FETI algorithm for which the convergence is faster and for which existing theory, primarily due to Mandel and Tezaur, also can be improved. We have found a surprisingly simple connection between this theory and that of the balancing method. This has allowed us to simplify and unify the theory and to develop valuable tools that can assist us directly in the design of appropriate variants of the FETI method.

An Abstract Framework for Multigrid Analysis for Finite Elements in Linear Elasticity

Christian Wieners

We establish sufficient criteria for W-cycle convergence of finite elements. This can be applied to conforming, nonconforming, mixed, hybrid, curved, stabilized and mortar elements.

In particular, we formulate an interpolation property which can be verified for averaged locally linear interpolations. In addition, the criteria are fulfilled for an explicit geometry based algebraic multigrid method.

We discuss different methods for constructing robust multigrid methods for linear elasticity in the nearly incompressible case: the formulation as a saddle-point problem by the introduction of a mean-stress variable and the construction of special smoothers with respect to the approximated Schur complement applied to the positive definite system.

Finally, we present numerical experiments for quasi-static elasto-plasticity with exponential isotropic hardening. Here, the algebraic multigrid method for large coarse grids is combined with geometric multigrid for advanced applications on complicated geometries with millions of unknowns.

Partition-of-Unity Methods for Overlapping Grids and for Homogenization

Jinchao Xu and Yunqing Huang

Two new finite element discretization methods are presented in this talk that are based on partition-of-unity technique. The first method is for overlapping grids and the method is proven to admit optimal error estimates for any number of grid patches with minimal overlappings. The second method is for elliptic problems with highly oscillatory coefficients and, compared with related existing methods, it gives much improved accuracy when the finite element grid size gets close to cell size.

Reporter: Christian Wagner

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