Reelle Analysis

July 11-17, 1999

This conference was organised by

D. Müller	(*Kiel*),
E.M. Stein	(*Princeton*)

and

H. Triebel	(*Jena*).

There were 28 plenary sessions, including six (invited) double lectures (given by *M. Christ, G. Grubb, S. Müller, A.J. Nagel, A. Seeger and L. Skrzypczak*) and 16 further talks about the following topics :

- Fourier analysis (convergence, maximal operators, oscillatory integrals, singular integrals)

- PDE (Strichartz estimates, hypoellipticity, wave equation, Sturm-Liouville operators)

- Applications of harmonic analysis (crystal microstructures, porous medium equation, Ornstein-Uhlenbeck semigroup)

- Analysis on Lie groups and symmetric spaces

- Pseudodifferential operators

- Function spaces (embeddings, decomposition techniques)

- Problems of complex analysis, tackled with real methods (Bergman projection, universal covering maps)

- closely related questions (Paley type inequalities for orthogonal series, random trigonometric polynomials, sparse spectra)

In addition, some informal lectures took place in the afternoons, where 8 participants presented their latest results.

The organisers and all the participants of this conference are greatly indebted to the Oberwolfach institute for providing a stimulating atmosphere for discussions and the exchange of ideas concerning common research interests.

The following abstracts are ordered alphabetically by the author.

OLIVIER BRACCO
Spectral measure for a certain class of singular Sturm-Liouville operators

We consider differential operators of the following type :

$$L = \frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{A'}{A}(x)\frac{\mathrm{d}}{\mathrm{d}x} \ .$$

Such operators arise for example as radial parts of Laplacians on Euclidean or on some Riemannian symmetric spaces. The basic properties we suppose on A are :

- $A \in \mathcal{C}^1\Big(]0, +\infty[\Big)$ and $A > 0$,

- at 0, $\dfrac{A'}{A}(x) = \dfrac{2\alpha + 1}{x} + B(x)$, with $\alpha \geq -\frac{1}{2}$ and B neglectable,

- at $+\infty$, $\dfrac{A'}{A}(x) \to 2\varrho \geq 0$.

We study two questions :
1)   the estimations of the eigenfunctions of L,
2)   the estimation of the spectral measure of L.

In the first part of the talk, we present the technique of perturbation of differential equations, which we used to attack the first question. We explain how it leads to an expression of the spectral measure.

In the second part we present the different methods we found to know precisely the behaviour of the spectral measure in the whole set where it is defined.

ANTHONY P. CARBERY
A conditional form of Stein's conjecture   *(joint work with F. Soria and A. Vargas)*

A (variant of a) conditional form of Stein's conjecture concerning Bochner-Riesz means was given. Let ϕ_δ be a smooth bump function associated to a 1-neighbourhood of $\delta^{-1}S^1 \subseteq \mathbb{R}^2$. Let $(S^\delta f)^\wedge(u) = \phi_\delta(u)\hat{f}(u)$. Let

$$M_\delta f(x) = \sup_{R \ni x} \frac{1}{|R|} \int_R f \ ,$$

the $\sup$ being over all rectangles of eccentricity $\leq \delta^{-1}$.
Let Φ_δ be a smooth bump function, $\int \Phi_\delta = 1$, associated to $B(0,\delta)$. Then

$$\int \left| S^\delta f \right|^2 w \leq c \left(\log\frac{1}{\delta} \right)^M \int |f|^2 M_{\delta^{1/2}} \circ \left[\Phi_{\delta^{1/2}} \times \left(M_{\delta^{1/2}} w \right)^2 \right]^{1/2}$$

for some $M \in \mathbb{N}$, all $f \in \mathcal{S}$, some $w \geq 0$, PROVIDED that whenever $A, B \in L^2(S^1)$ are supported in 1-separated 1-intervals with bisector the x_i-axis, we have

$$\int \left| \widehat{A\,\mathrm{d}v}(Ry)\ \widehat{B\,\mathrm{d}v}(Ry) \right| w(y)\,\mathrm{d}y \leq \frac{C}{R}\,\|A\|_2\,\|B\|_2\,\frac{\sup w(T)}{\varepsilon^{-3/4}R^{-1}}\,, \qquad\qquad (*)$$

where the $\sup$ is taken over all tubes T of dimensions $\varepsilon^{-1/2}R^{-1} \times \varepsilon^{-1}R^{-1}$, $R^{-1} \leq \varepsilon \leq 1$, with long direction making angle $\leq \frac{\pi}{2} - \frac{1}{100}$ with the x_i-axis.

A discussion of the likelihood of $(*)$ holding, and some potential consequences, was given.

MICHAEL CHRIST
Remarks on hypoellipticity

Consider a linear partial differential operator $L = \sum_j X_j^2$ where X_j are real vector fields with coefficients in C^∞ or C^ω. Under what conditions is L hypoelliptic in one of the function spaces C^ω, C^∞ or G^s ?

We list a number of the examples already known, and seek to provide a unifying framework. This is formulated by defining a metric ϱ, closely adapted to $\{X_j\}$, in the cotangent bundle; the definition involves the ratios $\mathcal{H}_j/\widetilde{\sigma}$, where $\mathcal{H}_j$ is the Hamiltonian vector field associated to the principal symbol of iX_j, and $(\widetilde{\sigma})^2$ is the 'effective symbol' of $-L$. A certain inequality comparing ϱ to a power (or logarithm) of a standard metric characterizes hypoellipticity in C^ω, G^s, C^∞ for all of the examples that we have listed. We ask whether this condition or some closely related one characterizes hypoellipticity in full generality, or at least provides a sort of first-order approximation to a characterization. We explain how the definition of ϱ is related to the commutator method which, in various guises, has been used by many authors to prove the known positive results in the subject.

In the second half of the lecture we announce partial results concerning C^∞ hypoellipticity for $\overline{\partial}_b \circ \overline{\partial}_b^*$, for 3-dimensional pseudoconvex CR structure possessing cylindrical symmetry, for which the set of weakly pseudoconvex points consists of a single curve transverse to the complex direction. Hypoellipticity holds under various rather weak supplementary hypotheses, but we outline a counterexample showing that it fails in general. We explain how this is predicted by the general framework discussed in the first half of the lecture.

ANTHONY H. DOOLEY
De Leeuw's Theorems for the contraction of K to $\overline{N}M$

De Leeuw's Theorems study the relationship between L^p-multipliers on $\mathbb{T}$ or $\mathbb{R}$ induced by the homomorphisms $\pi_\lambda : x \mapsto e^{-\lambda x}$, $\lambda \in \mathbb{R}_+$; if Φ is a function on $\mathbb{R}$ $(= \widehat{\mathbb{R}})$, define $\Phi^{(\lambda)}$ on $\mathbb{Z}$ $(= \widehat{\mathbb{T}})$ by $\Phi^{(\lambda)}(k) = \Phi\left(\frac{k}{\lambda}\right)$. Then De Leeuw showed that

(1) if Φ is a multiplier of $L^p(\mathbb{R})$, then $|||\Phi^{(\lambda)}|||_{p,\mathbb{T}} \leq |||\Phi|||_{p,\mathbb{R}}$ for all λ ; and

(2) if for each λ, $\Phi^{(\lambda)}$ is a multiplier of $L^p(\mathbb{T})$ with $\limsup\limits_{\lambda \to \infty} |||\Phi^{(\lambda)}|||_{p,\mathbb{T}} \leq K < \infty$, then

$\qquad |||\Phi|||_{p,\mathbb{R}} < K$.

We prove versions of De Leeuw's Theorems where π_λ is replaced by a contraction mapping

between two Lie groups G_1, G_2, that is, a family $\{\pi_\lambda\}_{\lambda>0}$ such that

$$x \cdot_{G_1} y = \lim_{\lambda \to \infty} \pi_\lambda^{-1} \left(\pi_\lambda x \cdot_{G_2} \pi_\lambda y \right) \ .$$

In this talk, we surveyed these results for the contraction of a semisimple group to its Cartan motion group, and described some new results obtained jointly with S.K. Gupta, extending these to the contraction of K to $\overline{N}M$ within the Iwasawa decomposition of a rank one semisimple group $G = KAN$. (The case $G = SU(2,1)$ gives the contraction of $SU(2)$ to the Heisenberg group.)

The essential technical problem to be overcome is the definition of $\Phi^{(\lambda)}$ as an 'appropriate intertwining' between the representations of the groups G_1 and G_2. We also gave a new definition of 'restriction' so that both directions of De Leeuw's Theorem could be proved — this leads to a resolution of Herz' asymmetry problem.

JOSE GARCIA-CUERVA

Paley type inequalities for orthogonal series with vector-valued coefficients *(joint work with K.S. Kazarian and V.I. Kolyada)*

We investigate the extension to Banach-space-valued functions of the classical inequalities due to Paley for the Fourier coefficients with respect to a general uniformly bounded orthonormal system Φ. This leads us to introduce the notions of Paley Φ-type and Φ-cotype for a Banach space and some related concepts.

We study the relations between these notions of type and cotype and those previously defined. We also analyse how the interpolation spaces inherit these characteristics from the original spaces and use them to obtain sharp coefficient estimates for functions taking values in Lorentz spaces.

The preprint can be obtained in dvi form from the web page of the Universidad Autónoma de Madrid at the address : http://www.uam.es.

DIRK GORGES

Convergence a.e. of Bochner-Riesz means on the Heisenberg group

Let L denote the sub-Laplacian on the Heisenberg group $\mathbb{H}_n$ and $T_r^\lambda := (1 - rL)_+^\lambda$ the corresponding Bochner-Riesz operator. Furthermore let Q denote the homogeneous dimension of $\mathbb{H}_n$.

We prove convergence a.e. of the Bochner-Riesz means $T_r^\lambda f$ as $r \to 0$ for all $f \in L^p(\mathbb{H}_n)$ such that

$$\text{either} \qquad \lambda \le \frac{3}{2} \quad \text{and} \qquad \frac{Q - \frac{8}{3}\lambda}{2Q} < \frac{1}{p} \le \frac{1}{2}$$

$$\text{or} \qquad \lambda > \frac{3}{2} \quad \text{and} \qquad \frac{Q - 2\lambda - 1}{2Q} < \frac{1}{p} \le \frac{1}{2} \ .$$

GERD GRUBB
Parametric pseudodifferential calculi allowing full asymptotic trace expansions *(joint work with E. Schrohe)*

After having recalled the classical parameter-dependent operators $(P-\lambda)^{-1}$, $(P-\lambda)^{-k}$, e^{-tP}, P^{-s}, where P is, say, a strongly elliptic differential operator on a closed manifold – it can also be replaced by P_T, a realization of P on a compact manifold with boundary defined by an elliptic boundary condition $Tu = 0$ – we accounted for cases where pseudodifferential elements come in. Then the traces have asymptotic expansions in the parameter with not just powers of λ or t, but also logarithmic terms $\lambda^{-\sigma}\log\lambda$ resp. $t^{\sigma}\log t$, and the traces of functions of s have double poles.

A survey was given of the weakly polyhomogeneous calculus by the author and Seeley (*Inventiones 1995*) for ψdo's on closed manifolds, and its application to the Atiyah-Patodi-Singer problem.

The second talk was concerned with two recent developments :

1. A weakly polyhomogeneous calculus for pseudodifferential boundary operators of Boutet de Monvel type, allowing complete trace expansions with logarithms.

2. An analysis of $\operatorname{Tr}\left[(P_+ + G)(P_{1,T} - \lambda)^{-k}\right]$ and $\operatorname{Tr}\left[(P_+ + G)P_{1,T}^{-s}\right]$, where $P_+ + G$ is in the Boutet de Monvel calculus and $P_{1,T}$ is a realization of a second order strongly elliptic differential operator, showing how the noncommutative residue of $P_+ + G$ (defined by Fedosov, Golse, Leichtnam and Schrohe, *JFA 1996*) appears as the coefficient of the first logarithmic term $\lambda^{-k}\log\lambda$ in $\operatorname{Tr}\left[(P_+ + G)(P_{1,T} - \lambda)^{-k}\right]$ or the first double pole (at $s = 0$) of $\Gamma(s)\operatorname{Tr}\left[(P_+ + G)P_{1,T}^{-s}\right]$, thus connecting the noncommutative residue with the original residue formula of Wodzicki (1984) in the boundaryless case.

DOROTHEE D. HAROSKE
Embeddings in spaces of Lipschitz type, entropy and approximation numbers *(joint work with David E. Edmunds)*

We consider (sharp) embeddings of certain Besov and Triebel-Lizorkin spaces in spaces of Lipschitz type. The prototype of such embeddings arises from the BRÉZIS-WAINGER result (*Comm. PDE, 1980*) about the 'almost' Lipschitz continuity of elements of the Sobolev spaces $H_p^{1+n/p}(\mathbb{R}^n)$ when $1 < p < \infty$. Thus we were led to the introduction of logarithmically adapted spaces of Lipschitz type, that is spaces $\operatorname{Lip}^{(1,-\alpha)}$, $\alpha \geq 0$, containing continuous functions with

$$\left\| f | \operatorname{Lip}^{(1,-\alpha)}(\mathbb{R}^n) \right\| = \| f | L_\infty(\mathbb{R}^n) \| + \sup \frac{|f(x) - f(y)|}{|x - y| \, |\log|x - y||^\alpha} < \infty,$$

where the supremum is taken over all $x, y \in \mathbb{R}^n$, with $0 < |x - y| < \frac{1}{2}$. Then the BRÉZIS-WAINGER result reads as $H_p^{1+n/p}(\mathbb{R}^n) \hookrightarrow \operatorname{Lip}^{(1,-1/p')}(\mathbb{R}^n)$, where $1 < p < \infty$, and $\frac{1}{p} + \frac{1}{p'} = 1$, as usual. We can show that the log-exponent $\frac{1}{p'}$ is sharp (i.e. the smallest possible one)

and extend the above result in the framework of Triebel-Lizorkin spaces $F_{p,q}^s(\mathbb{R}^n)$, $s \in \mathbb{R}$, $0 < p < \infty$, $0 < q \leq \infty$. We study similar limiting embeddings of type $B_{p,q}^{1+n/p}(\mathbb{R}^n) \hookrightarrow \mathrm{Lip}^{(1,-\alpha)}(\mathbb{R}^n)$ and obtain that this embedding holds, if, and only if, $\alpha \geq (1 - \frac{1}{q})_+$, where $0 < p, q \leq \infty$. This outcome (in case of the Besov spaces) is somehow surprising in our opinion, involving the usually less important q-parameter essentially.

Furthermore we investigate a variety of related compact embeddings (the spaces are now defined on bounded domains) and study their entropy and approximation numbers. Two-sided estimates are obtained, providing thus an opportunity to apply our results, for instance, when estimating the eigenvalue distribution of certain (degenerate) pseudodifferential or elliptic operators.

ANDRZEJ HULANICKI
Martin boundary for homogeneous Riemannian manifolds of negative curvature at the bottom of the spectrum *(joint work with Ewa Damek, and Roman Urban)*

Let $\mathcal{L}$ be a second order subelliptic differential operator on a Riemannian manifold of negative sectional curvature K with $-a^2 \leq K \leq -b^2$. In the case when $\mathcal{L}$ is weakly coercive, i.e. if for $\epsilon > 0$ the operator $\mathcal{L} + \epsilon I$ has the Green function, the Martin boundary has been described by A. Ancona as a sphere at infinity. For general negatively curved manifolds for $\mathcal{L}$ not weakly coercive, e.g. when $\mathcal{L} = \Delta + \alpha I$, where Δ is the Laplace-Beltrami operator and $-\alpha$ is the bottom of the spectrum of Δ no description of the Martin boundary is known.

We show that in the case of *homogeneous* Riemannian manifolds of negative curvature the Martin boundary is the sphere at infinity also for noncoercive invariant second order operators such like $\Delta + \alpha I$.

Since the Ancona methods do not apply in this case, we develop another approach.

NETS KATZ
Recent progress on the Kakeya maximal operator *(joint work with Terry Tao)*

We improve the following Lemma of Bourgain.

Lemma [Bourgain]. *Let Z be a torsion free Abelian group. Let A, B, $C \subset Z$ be finite sets with $\#(A) = \#(B) = \#(C) = N$. Let $G \subset A \times B$. Define maps $+(a, b) = a + b$ and $-(a, b) = a - b$. Suppose $-$ is one to one on G and $+ : G \to C$. Then $\#(G) \leq N^{2 - \frac{1}{13}}$*
.

Lemma [Katz - Tao]. *Indeed under these hypotheses $\#(G) \leq N^{2 - \frac{1}{6}}$* .

Lemma [Katz - Tao]. *Define $+_2(a, b) = a + 2b$. Suppose in addition $D \subset Z$, $\#(D) = N$, $+_2 : G \to D$. Then $\#(G) \leq N^{2 - \frac{1}{4}}$* .

These lemmata imply lower bounds of the dimension d of a Besicovitch set in $\mathbb{R}^n$. Indeed, regardless of the number of finite sets, an estimate of the form $\#(G) \leq N^{2 - \alpha}$ implies

$(2 - \alpha)(d - 1) \geq n - 1$. Therefore the final lemma gives us

$$d \geq \frac{4n + 3}{7}$$

which is best known for $n > 8$. If we could reach $\alpha = 1$, the Kakeya conjecture could be solved and the grand destiny of Spanish harmonic analysis would be redeemed.

HERBERT KOCH
Singular integrals and the porous medium equation

The theory of singular integrals, which was developed by Calderón and Zygmund around 1950 in $\mathbb{R}^n$, had a profound impact on various areas of analysis. That theory relies on few properties of the Euclidean geometry and can be adapted to different geometric structures. Examples are operators which occur in homogenization, elliptic equations with strong drift, as well as operators which come from linearizing the porous medium equation

$$\rho_t - \Delta \rho^m = 0 \quad \text{in} \quad \mathbb{R}^n \times \mathbb{R}, \qquad m > 1.$$

The main result, regularity of the free boundary (the boundary of the support) for large times under weak assumptions on initial data, follows from modified Gaussian estimates of the fundamental solution of degenerate parabolic equations, which imply Harnack inequalities and fit into the theory of singular integrals.

STEFANO MEDA
Functional calculus for the Ornstein-Uhlenbeck semigroup (*joint work with J. García Cuerva, G. Mauceri, P. Sjögren, and J.L. Torrea*)

The Ornstein–Uhlenbeck operator $-\frac{1}{2}\Delta + x \cdot \nabla$ is essentially self-adjoint in $L^2(\gamma)$, where $\mathrm{d}\gamma(x) = \pi^{-d/2}\, e^{-x^2}\, \mathrm{d}x$. We denote by $\mathcal{L}$ its self-adjoint extension. Then for every f in the domain of $\mathcal{L}$ we have $\mathcal{L}f = \sum_{n=0}^{\infty} n\, \mathcal{P}_n f$, with eigenvectors given by the (d-dimensional) Hermite polynomials.

Suppose that $M : \mathbb{N} \to \mathbb{C}$ is a bounded sequence. By the spectral theorem we may form the operator $M(\mathcal{L})f = \sum_{n=0}^{\infty} M(n)\, \mathcal{P}_n f$ for every f in $L^2(\gamma)$.

Suppose that α is a nonnegative integer and that ψ is in $(0, \pi/2)$. We denote by $H^\infty(\mathbf{S}_\psi; \alpha)$ the Banach space of all M in $H^\infty(\mathbf{S}_\psi)$ such that $M(\cdot\, e^{\pm i\psi})$ satisfy a Hörmander condition of order α. We prove the following

Theorem. *Suppose that $1 < p < \infty$, $p \neq 2$, and define ψ_p to be $\frac{\pi}{2} - \arctan \frac{2\sqrt{p-1}}{|p-2|}$. Let $M : \mathbb{N} \to \mathbb{C}$ be bounded and suppose that there exists a bounded holomorphic function $\widetilde{M}$ such that*
$$\widetilde{M}(k) = M(k) \qquad k = 1, 2, 3, \dots$$

The following hold:

(i) *if $\alpha > 3$ and $\widetilde{M}$ is in $H^\infty(\mathbf{S}_{\psi_p}; \alpha)$, then $M(\mathcal{L})$ extends to a bounded operator on $L^q(\gamma)$, $|1/q - 1/2| \leq |1/p - 1/2|$;*

(ii) *if $\widetilde{M}$ is in $H^\infty(\mathbf{S}_{\psi_p})$, then $M(\mathcal{L})$ extends to a bounded operator in $L^q(\gamma)$, $|1/q - 1/2| < |1/p - 1/2|$.*

PAUL F.X. MÜLLER
Universal covering Maps and radial Variation *(joint work with Peter W. Jones)*

We let $E \subseteq \mathbb{C}$ be a closed set with two or more points. By the uniformization theorem there exists a Fuchsian group of Moebius transformations such that $\mathbb{C} \setminus E$ is conformally equivalent to the quotient manifold $\mathbb{D}/G$. The universal covering map $P : \mathbb{D} \to \mathbb{C} \setminus E$ is then given by $P = \tau \circ \pi$, where π is the natural quotient map onto $\mathbb{D}/G$ and τ is the conformal bijection between $\mathbb{C} \setminus E$ and $\mathbb{D}/G$. We will show that there exists $e^{i\beta} \in \mathbb{T}$ such that

$$\int_0^1 |P''(re^{i\beta})| \, \mathrm{d}r < \infty.$$

Considering $u = \log |P'|$, one obtains this from variational estimates.

Theorem. *There exists $e^{i\beta} \in \mathbb{T}$ and $M > 0$ such that for $r < 1$,*

$$u(re^{i\beta}) < -\frac{1}{M} \int_0^r |\nabla u(\rho e^{i\beta})| \, \mathrm{d}\rho + M.$$

Clearly, the class of universal covering maps contains two extremal cases: The case where $\mathbb{C} \setminus E$ is simply connected and the case where E consists of two points. (We considered the simply connected case separately when we solved Anderson's conjecture. The second case follows from estimates for the Poincaré metric on the triply punctured sphere.) In the course of the proof of the Theorem we measure the thickness of E at *all* scales, and we are guided by the following philosophy. If, at some scale, the boundary E appears to be thick then, locally, the universal covering map behaves like a Riemann map. On the other hand, if E appears to be thin, then, locally, the Poincaré metric of $\mathbb{C} \setminus E$ behaves like the corresponding Poincaré metric of $\mathbb{C} \setminus \{0, 1\}$. With the right estimates for the *transition* from the thick case to the thin case, this philosophy leads to a rigorous proof. Our proof also shows the existence of a very large set of angles β for which the Theorem holds.

STEFAN MÜLLER
Possible connections between harmonic analysis and crystal microstructures

In the first lecture I outlined some mathematical problems that arise in the analysis of microstructures in crystals that undergo solid-solid phase transformations and the connection of these problems with harmonic analysis. In the second lecture I considered a specific problem whose resolution involves seemingly new estimates for the Haar coefficients in terms of the

Riesz transform.

A key problem that arises in the analysis of crystal microstructure is the following. Given

$$K \subset M^{m \times n}, \quad \text{a subset of } m \times n \text{ matrices}, \quad m, n \geq 2$$
$$\Omega \subset \mathbb{R}^n \qquad \text{bounded, open,}$$

characterize sequences $u^j : \Omega \to \mathbb{R}^m$, $|\nabla u^j| \leq C$, $u^j \xrightarrow{\;*\;} u$ in $W^{1,\infty}$ such that

$$\operatorname{dist}\left(\nabla u^j, K\right) \to 0 \quad \text{in} \quad L^p.$$

Specific questions are

a) $(compactness)$ $\quad \nabla u^j \to \nabla u$ (strongly) in L^p

b) $(stability)$ $\qquad \nabla u \in K$?

c) $(relaxation)$ $\qquad$ Find smallest set K^{macro} for which $\nabla u \in K^{\mathsf{macro}}$ a.e.

A closely related question is for which integrands the functional $\quad I(u) = \int_\Omega f\left(\nabla u\right) \mathrm{d}x \quad$ is lower semicontinuous with respect to $W^{1,\infty}$ weak-$*$-convergence. Morrey showed that this is the case if, and only if, f is quasiconvex, i.e. $\forall F \in M^{m \times n}$,

$$\int_{T^n} f\left(F + \nabla \varphi(x)\right) \mathrm{d}x \geq f(F) \qquad \forall\, \varphi \in C^1 \quad \text{periodic on} \quad T^n,$$

but more than 40 years after their introduction the class of quasiconvex functions remains largely mysterious. A linearized condition, implied by quasiconvex is convexity along rank-1 lines (rank-1 convexity) which for C^2 functions becomes $\quad D^2 f(F)\left(a \otimes b, a \otimes b\right) \geq 0$.

<u>Major question</u> : Does rank-1 convexity imply quasiconvexity ?

The answer is 'No' for $m \geq 3$ (Šverák '92), $m = 2$, $n \geq 2$ is open and a positive answer would have striking consequences, including an optimal bound for the Beurling transform.

The questions raised above are a special case of the compensated compactness theory, initiated by L. Tartar and F. Murat. Instead of gradients they consider sequences which (almost) belong to the kernel of a general first order operator $\quad A(v) = \sum_k A_k \partial_k v$, $A_k \in \operatorname{Lin}\left(\mathbb{R}^d; \mathbb{R}^p\right)$,

$$v^j \rightharpoonup v \quad \text{in} \quad L^2, \quad A(v^j) \in \text{cpt set } H_{\mathsf{loc}}^{-1}. \qquad\qquad (*)$$

There is a theory largely parallel to that for gradients a as long as a satisfies the constant rank condition,

$$\operatorname{rank} \sum_k A_k\, \xi_k = \text{const} \qquad \text{for} \quad \xi \neq 0.$$

In the second lecture I discuss the simplest example where this conditions fails : $v = (v_1, v_2)$: $\mathbb{R}^2 \to \mathbb{R}^2$, $A(v) = (\partial_2 v_1, \partial_1 v_2)$.

Theorem. *Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ separately convex, $0 \leq f(p) \leq C\left(1 + |p|^2\right)$, v^j satisfies* $(*)$. *Then*

$$\int_\Omega f(v)\, \mathrm{d}x \leq \liminf_{j \to \infty} \int_\Omega f(v^j)\, \mathrm{d}x.$$

9

Thus in this case the counterpart of rank-1 convexity, namely separate convexity, implies weak lower semicontinuity which corresponds to quasiconvexity. A key ingredient is the following estimate for coefficients in the Haar basis $\{h_\lambda\}$: $h_{j,k}^{(\varepsilon)} = h^{(\varepsilon)}(2^j \cdot -k)$, $h^{(1,0)} = h \otimes \mathbb{1}_{[0,1]}$, $h = \mathbb{1}_{[0,\frac{1}{2}]} - \mathbb{1}_{[\frac{1}{2},1]}$, $h^{(0,1)} = \mathbb{1}_{[0,1]} \otimes h$, etc.

Theorem. *For* $u = \sum_\lambda a_\lambda\, h_\lambda$ *and fixed* $\varepsilon \in \{0,1\}^2 \setminus \{(0,0)\}$, *consider the projection* $P^{(\varepsilon)}u := \sum_{j,k} a_{j,k}^{(\varepsilon)} h_{j,k}^{(\varepsilon)}$. *Then*

$$\left\| P^{(\varepsilon)}u \right\|_{L^2} \le C \left\| R_2 u \right\|_{L^2}^{\frac{1}{2}} \left\| u \right\|_{L^2}^{\frac{1}{2}} , \qquad \text{if} \quad \varepsilon \ne (1,0) \ .$$

Similar estimates hold in L^p.

ALEXANDER J. NAGEL

The $\overline{\partial}_b$ complex on quadratic CR manifolds *(joint work with Fulvio Ricci and Elias M. Stein)*

We consider an Hermitian map $A : \mathbb{C}^n \to \mathbb{C}^m$, and we want to study the operators associated to the $\overline{\partial}_b$ complex on the manifold

$$\Sigma_A = \{(z, w) \in \mathbb{C}^n \times \mathbb{C}^m \mid \Im m[w] = A(z)\} .$$

This is a generic CR submanifold of CR dimension n. We let $Z_1, \ldots, Z_n$, and $\overline{Z}_1, \ldots, \overline{Z}_n$ be the standard bases for tangential vector fields of type $(1, 0)$ and $(0, 1)$ on Σ_A. The Laplacian is the operator $\Box_b = \overline{\partial}_b \, \overline{\partial}_b^* + \overline{\partial}_b^* \, \overline{\partial}_b$. In general, this is a system of second order PDE's. For simplicity of exposition, we consider $\Box_b = \Box_0$ acting on functions, when it becomes the scalar operator

$$\Box_0(f) = \sum_{j=1}^n Z_j \overline{Z}_j[f] .$$

Σ_A carries the structure of a nilpotent Lie group of step 2.

Theorem 1. *There are convolution operators P_j, Q_j, G_j, H_j, P, and K on Σ_A such that*
(i) *P_j and Q_j are the orthogonal projection of $L^2(\Sigma_A)$ onto the null spaces of $\overline{Z}_j$ and Z_j.*
(ii) *$G_j \overline{Z}_j = I - P_j$, $\overline{Z}_j G_j = I - Q_j$, $H_j Z_j = I - Q_j$, $Z_j H_j = I - P_j$*
(iii) *P is the orthogonal projection of $L^2(\Sigma_A)$ onto the null space of $\Box_0$.*
(iv) *$\Box_0 K = K \Box_0 = I - P$*

Our objective is to study the regularity properties of these operators, and also to study the nature of the singularities of their distribution kernels.

Theorem 2. *For general A, the operators P_j, Q_j, P and $Z_j \overline{Z}_j K$, $\overline{Z}_j \, \overline{Z}_j K$, $\overline{Z}_j Z_j (I - P_j) K$, $Z_j Z_j (I - P_j) K$ are bounded on $L^2(\Sigma_A)$.*

However, the operators $\overline{Z}_j Z_j K$ and $Z_j Z_j K$ are in general <u>not</u> bounded on $L^2(\Sigma_A)$.
To go beyond L^2 theory, we consider the special case when the quadratic forms in A can be simultaneously diagonalized.

Theorem 3. *If A can be diagonalized, the operators in Theorem 2 are bounded in $L^p(\Sigma_A)$ for $1 < p < \infty$.*

We also give a description of the singularities of the distribution kernels of these operators. In general, the kernels have singularities away from the origin. We introduce the notion of 'flag' singularities which is analogous to but more general than a product singularity.

Theorem 4. *If A can be diagonalized, the distribution kernels of the operators in Theorem 2 can be written as sums of flag singularities.*

11

ALEXANDER OLEVSKII
Sparse spectra : approximation and expansions

We consider the following two (connected) problems.

1. How sparse the spectrum $\Lambda \subset \mathbb{R}$ might be, such that for an appropriate $\psi \in L^2(\mathbb{R})$ the set of translates $\{\psi(t - \lambda)\}_{\lambda \in \Lambda}$ spans the space ?

2. Is it possible to decompose any measurable function f on $\mathbb{R}$ into a series

$$f(x) = \sum_{n \in \mathbb{Z}} c(n) \, e^{i\lambda(n)x} \; ,$$

convergent a.e., which involves harmonics with 'almost integer' frequencies,

$$\lambda(n) = n + \mathrm{o}(1) \qquad ?$$

MARCO M. PELOSO
Boundedness of Bergman projections on tube domains over light cones *(joint work with David Bekollé, Aline Bonami, and Fulvio Ricci)*

Let
$$\Gamma = \left\{ y \in \mathbb{R}^n : y_n > (y_1^2 + \cdots + y_{n-1}^2)^{\frac{1}{2}} \right\}$$

be the forward light cone in $\mathbb{R}^n$, $n \geq 3$, and let $\Omega = \mathbb{R}^n + i\Gamma$ be the associated tube domain in $\mathbb{C}^n$. If Q denotes the quadratic form

$$Q(y) = -y_1^2 - \cdots - y_{n-1}^2 + y_n^2 \; ,$$

we denote by L_ν^p, $1 \leq p \leq \infty$, the Lebesgue space $L^p\left(\Omega, Q(y)^{\nu-n} \, \mathrm{d}x \, \mathrm{d}y\right)$.

The *weighted Bergman space* A_ν^p is the closed subspace of L_ν^p consisting of holomorphic functions. In order to have a non-trivial subspace, we impose that $\nu > n - 1$.

The *weighted Bergman kernel* $B_\nu(z, w) = c_\nu Q(z - \bar{w})^{-\nu}$ is the reproducing kernel on A_ν^2, and the *weighted Bergman projection*

$$P_\nu f(z) = \int_\Omega B_\nu(z, u + iv) \, f(u + iv) \, Q(v)^{\nu-n} \, \mathrm{d}u \, \mathrm{d}v$$

is the orthogonal projection of L_ν^2 onto A_ν^2. Let P_ν^+ denote the operator

$$P_\nu^+ f(z) = \int_\Omega |B_\nu(z, u + iv)| \, f(u + iv) \, Q(v)^{\nu-n} \, \mathrm{d}u \, \mathrm{d}v \; .$$

The question is whether there are values of p for which P_ν is bounded, but P_ν^+ is unbounded. We obtain the following partial answer to this question.

Theorem. *P_ν is bounded on L_ν^p for* $1 + \frac{n-2}{2(\nu-1)} < p < 1 + \frac{2(\nu-1)}{n-2}$.

We must take advantage of the oscillations of the Bergman kernel. We are so induced to use the Fourier transform in the x variables and consequently to focus on L^2 norms in these variables. For this reason, for $1 \leq p, q \leq \infty$, we consider the spaces $L_\nu^{p,q} = L^p\left(\Gamma, Q(y)^{\nu-n}\, \mathrm{d}y, L^q(\mathbb{R}^n, \mathrm{d}x)\right)$. As before, we call $A_\nu^{p,q}$ the closed subspace of $L_\nu^{p,q}$ consisting of holomorphic functions.

For $q = 2$, we obtain the exact range of p (modulo two endpoints) for which P_ν is bounded. Then the Theorem follows by interpolation with the results of [1].

[1] D. BEKOLLÉ, A. BONAMI, Estimates for the Bergman and Szegö projections in two symmetric domains of $\mathbb{C}^n$, Coll. Math., **68** (1995), 81-100.

[2] D. BEKOLLÉ, A. BONAMI, M. M. PELOSO, F. RICCI, Boundedness of Bergman projections on tube domains over light cones, preprint, 1999.

DUONG H. PHONG
Uniform estimates and stability of oscillatory integrals and oscillatory integral operators   (*joint work with Elias M. Stein and Jacob Sturm*)

We report uniform estimates both on

1. <u>Scalar Oscillatory Integrals</u> : In dimension 3, we show the stability of decay rates for oscillatory integrals

$$\left| \int e^{i\lambda S(x)}\chi(x)\,\mathrm{d}x \right| \leq C\, |\lambda|^{-\delta}\ ,$$

   as long as $\delta < \frac{2}{N}$, where N is the order of vanishing of $S(x)$ at the origin and χ is a cut-off function supported near 0.

2. <u>Oscillatory Integral Operators</u> : we establish uniform estimates for one-dimensional operators of the form

$$T_\lambda f(x) = \int\limits_{-\infty}^{\infty} e^{i\lambda S(x,y)}\chi(x,y)f(y)\,\mathrm{d}y\ ,$$

   where $S(x, y)$ is a polynomial of degree n.

   - When $S(x, y) = S_0(x, y) - 2\,E(x, y)$, then the sharp decay rate $\|T_\lambda\| \leq C|\lambda|^{-\frac{1}{2}\delta}$ (with δ defined by the reduced Newton diagram), is uniform for S_0 and E homogeneous polynomials and $|\alpha|$ sufficiently small.

   - When $\partial_x^k \partial_y^\ell S_{xy}'' > 1$ and $\partial_x^r \partial_y^s S_{xy}'' > 1$, then the estimate $\|T_\lambda\| \leq C|\lambda|^{-\frac{1}{2}\delta}$ holds uniformly (with δ defined by a Newton diagram construction based on the vectors $(k+1, \ell+1)$ and $(r+1, s+1)$), with a constant C depending only on n, k, ℓ, r, s, and χ.

HANS M. REIMANN
Mappings on H-type groups

H-type groups are special step 2 nilpotent Lie groups N. The tangent space TN contains a sub-bundle HN spanned by the left invariant vector fields from the subspace   generating the Lie algebra   =   +   of N. A contact mapping $f : N \to N$ is a differentiable mapping which preserves HN : $f_*HN \subseteq HN$.

We consider the vector fields which generate local one-parameter groups of contact mappings. It is shown that these vector fields make up a finite dimensional Lie algebra in all cases when the dimension of the center   is at least 3.

The Heisenberg group is the special case $\dim$ $= 1$ and here the vector fields generating contact mappings are well known. The case $\dim$ $= 2$ is the complexified Heisenberg group. For this case it is shown that the contact mappings are holomorphic.

FRANCIS RIBAUD
Self-similar solutions of the nonlinear wave equations   (*joint work with A. Youssfi*)

We prove the existence of a class of 'special solutions' of the nonlinear wave equations

$$(\text{NLW}) \quad \begin{cases} \partial_t^2 u - \Delta u &= -\lambda |u|^{\alpha-1} u \qquad (t,x) \in \mathbb{R}^+ \times \mathbb{R}^n, \quad \lambda \in \mathbb{R} \\ u_{|t=0} &= f \\ \partial_t u_{|t=0} &= g \ . \end{cases}$$

Also, we show that those special solutions allow sometimes to describe the asymptotic behaviour of some finite energy solutions of (NLW). More precisely, we prove the existence of self-similar solutions of (NLW), i.e. solutions such that $u(t,x) = a^{\frac{2}{\alpha-1}} u(at, ax) \ \forall \ a > 0$. One can prove that u is a self-similar solution if, and only if, the initial data (f, g) are of the particular form

$$f(x) = \frac{\Omega_1\left(x\|x\|^{-1}\right)}{\|x\|^{\frac{2}{\alpha-1}}} \quad , \quad g(x) = \frac{\Omega_2\left(x\|x\|^{-1}\right)}{\|x\|^{\frac{\alpha+1}{\alpha-1}}} \ ,$$

where the Ω_i are functions defined on the unit sphere S^{n-1}. So, to prove the existence of self-similar solutions we study the global Cauchy problem (NLW) with homogeneous initial data. The main problem is now that such initial data never belong to the usual spaces of resolution of (NLW) (Lebesgue or Sobolev spaces). To overcome this problem we introduce some non-standard resolution spaces which allow us to prove the existence of not necessarily radially symmetric self-similar solutions when $\|\Omega_1\|_{C^n} + \|\Omega_2\|_{C^{n-1}} \leq \varepsilon$ and when $\alpha > \alpha_0(n)$ ($n = 2, 3, 4, 5$) and when $\alpha \in]\alpha_0(n), \frac{n+3}{n-1}] \cup]\alpha_1(n), +\infty[$, ($n \geq 6$), where $\alpha_0(n)$ is the lowest bound for the scattering theory and $\alpha_1(n) \sim \frac{n}{2} - 1$. Next we prove that initial data of the form

$$\tilde{f}(x) = (1 - \varphi(x)) f(x) \quad , \quad \tilde{g}(x) = (1 - \varphi(x)) g(x)$$

(φ is a cut-off function in a neighbourhood of 0) give raise to finite energy solutions of (NLW) which behave asymptotically like the self-similar solution with initial data (f, g) when $n = 3, 4, 5, 6$ and when $\alpha_0(n) < \alpha < \alpha^*(n) = \frac{n+2}{n-2}$.

WILHELM SCHLAG
On minima of the absolute value of certain random exponential sums

Let $T_n(x) = \sum_{j=1}^{n} \pm e^{2\pi i j^2 x}$ where $\pm$ stands for a random choice of sign with equal probability. It is shown in this talk that with high probability $\min_{x \in [0,1]} |T_n(x)| < n^{-\sigma}$ provided n is large and $\sigma < \frac{1}{12}$. Similar results are proved for other powers than squares. The problem of determining the optimal σ is open.

For the case $T_n(x) = \sum_{j=1}^{n} r_j \, e^{2\pi i j^d x}$, where $d = 2, 3, \ldots$, is fixed and with standard normal r_j we show that the minima are typically on the order of $n^{-d+\frac{1}{2}}$ with high probability and for large n.

ANDREAS SEEGER
Failure of weak amenability and a family of singular oscillatory integrals (*joint work with Michael Cowling, Brian Dorofaeff and James Wright*)

Let $A(G)$ be the Fourier algebra of a locally compact group G. We say that $\{\phi_i\}$ is c-completely bounded approximative unit if $\|\phi_i \phi - \phi\|_{A(G)} \to 0$ for all $\phi \in A(G)$ and $\|\phi_i\|_{M_0 A(G)} \leq c$ uniformly in i; here $M_0 A(G)$ denotes the algebra of Herz-Schur multipliers. The approximation number $\Lambda(G)$ is defined as the infimum over all c so that there is a c-completely bounded approximative unit.

The object of this study is to complete the project of determining $\Lambda(G)$ for all matrix groups. Using structure theory of Lie groups and various previously known results one is led to the case of $G_n = SL(2, \mathbb{R}) \ltimes H_n$ where $n \geq 2$, H_n is the $2n + 1$ dimensional Heisenberg group and $SL(2, \mathbb{R})$ acts via the irreducible representation of dimension $2n$ fixing the center of H_n. Following an idea of Haagerup for the case $n = 1$ we identify a family of distributions $\mathcal{D}_\mathcal{R}$ which act on the Fourier algebra $A(H)$ of a certain nilpotent subgroup H of G and have the following two properties, for large R.

(i) If ϕ_i is an $SO(2)$-biinvariant approximative unit in G_n then
$$\limsup_i \langle \mathcal{D}_R, \phi_i|_H \rangle \geq C_0 \log R. \tag{$*$}$$

(ii) For all $g \in A(H)$
$$|\langle \mathcal{D}_R, g \rangle| \leq C_1 \log \log R \|g\|_{A(H)}. \tag{$**$}$$

One can use these facts to show that $\Lambda(G_n) = \infty$; in fact there is no multiplier bounded approximative unit on $A(G_n)$.

In order to show the crucial bound $(**)$ one uses a Fourier transform argument and is led to proving bounds for a family of singular oscillatory integral operators T^R acting on functions in $L^2(\mathbb{R}^2)$. Let $n = 1, 2, \ldots$ and let p be a polynomial of degree $\leq n$. Define $\beta(s) = (1 + s^2/4)$ and

$$\begin{aligned}
\Psi(x, y) &= (x_1 - y_1)(x_2^2 + y_2^2) - (x_2 - y_2)p(x_1 + y_1) \\
\theta(x, y) &= \beta(x_1 - y_1)|x_2 + y_2 + p'(x_1 + y_1)|(x_2 - y_2).
\end{aligned}$$

For $f \in C_0^\infty(\mathbb{R}^2)$ let
$$T^R f(x) = \int\int \frac{e^{i\frac{n}{2}\Psi(x,y)}\sin\theta(x,y)}{\beta(x_1-y_1)(x_2-y_2)}\chi_{[-R,R]}(x_1-y_1)f(y_1,y_2)\,\mathrm{d}y_1\,\mathrm{d}y_2.$$

Theorem. *T^R extends to a bounded operator on $L^2(\mathbb{R}^2)$ and for large R the operator norm is $\leq C\log\log R$; here C depends on n but not on the particular polynomial of degree $\leq n$.*

The proof relies on a crucial cancellation property for the affine case $p(x) = ax + b$, for which one obtains the bound $\|T^R\| = O(1)$. The general case involves an approximation by operators which share the properties of the affine case; for various remainder terms one uses the oscillatory properties of the phase function and Hilbert integral arguments.

PETER SJÖGREN
A maximal estimate for the real and the complexified Ornstein-Uhlenbeck semigroup
(joint work with J. García-Cuerva, G. Mauceri, S. Meda and J.L. Torrea)

Let $\mathrm{d}\gamma(x) = e^{-|x|^2}\,\mathrm{d}x$ in $\mathbb{R}^d$ and $L = -\frac{1}{2}\Delta + x\cdot\mathrm{grad}$, which is a self-adjoint operator in $L^2(\gamma)$. Then $H_t = e^{-tL}$, $t > 0$, is the Ornstein-Uhlenbeck semigroup. Here t can be replaced by a complex parameter z with $\Re ez > 0$ and H_z has a region $E_p \subset \mathbb{C}$ of holomorphy in $L^p(\gamma)$ for each $1 < p < \infty$. This extension is important in particular for spectral multipliers. It is natural to form the maximal operator given by $\sup_{z\in E_p}|H_z f(x)|$, for $f \in L^p(\gamma)$. We prove that for any $\delta > 0$, the smaller operator

$$\sup_{z\in E_p, |z|<\delta}|H_z f(x)|$$

is of weak type (p,p) with respect to γ, for $1 < p < 2$. The proof applies also to the case $p = 1$ and then gives a new proof of the known weak type $(1,1)$ estimate for $\sup_{t>0}|H_t f(x)|$. In the proof, the operator is split into a local and a global part, where 'local' refers to the length scale $\dfrac{1}{1+|x|}$ near x.

LESZEK SKRZYPCZAK
Atomic decomposition on symmetric spaces and applications

Let X be a Riemannian manifold with bounded geometry. For the Triebel-Lizorkin spaces $F^s_{p,q}(X)$ and Besov spaces $B^s_{p,q}(X)$ defined on X ($s \in \mathbb{R}$, $1 \leq p,q \leq \infty$) an atomic decomposition is introduced. The atomic decomposition theorem can be used for investigation of boundedness of pseudodifferential operators as well as for characterization of the above spaces in terms of heat and harmonic extensions.

If X is a Riemannian symmetric manifold of the noncompact type, then the following facts can be proved:

1. the Bernstein type theorem about absolute integrability of the Helgason-Fourier transform $\mathcal{H}f$ of $f \in B^n_{11}(X)$, $n = \dim X$,

2. the generalized Riemann-Lebesgue lemma for functions belonging to $B_{p,\infty}^{\gamma(X)}(X)$, $1 \leq p < 2$, $\gamma(X) > 0$ being the constant depending on X,

3. convolution properties of Besov spaces related to the Kunze-Stein phenomenon.

HART SMITH
Global Strichartz estimates for nontrapping metrics   (*joint work with C. Sogge*)

We discuss joint work with C. Sogge establishing mixed-norm (Strichartz) estimates globally in space-time for solutions to certain variable coefficient wave equations. Precisely, let g be a (smooth) Riemannian metric in $\mathbb{R}^n$ such that $g^{ij}(x) = \delta^{ij}$ for $|x| \geq R$, and let $\mathcal{K} \subseteq \{x : |x| \leq R\}$ be an obstacle which is strictly convex relative to g. Consider the wave equation

$$
\begin{cases}
\partial_t^2 u & = & \Delta_g u + F & (t,x) \in \mathbb{R} \times \mathbb{R}^n \setminus \mathcal{K} \\
u(0,x) & = & f(x) \\
\partial_t u(0,x) & = & g(x) \\
u(t,x) & = & 0 & x \in \partial \mathcal{K} \, .
\end{cases}
$$

Assuming that the metric g is nontrapping (all geodesics go to ∞) we show that, for n odd, $n \geq 3$, the global estimate

$$
\|u\|_{L_t^p L_x^q(\mathbb{R} \times \mathbb{R}^n \setminus \mathcal{K})} \leq c \left(\|f\|_{\dot{H}^\gamma} + \|g\|_{\dot{H}^{\gamma-1}} + \|f\|_{L_t^r L_x^s} \right)
$$

holds, provided that this estimate holds on Euclidean space, and provided it holds locally in space time for the obstacle problem (the local estimates were established by the authors in previous work). The key idea in the proof is to use local energy decay estimates of Morawetz-Taylor to reduce the problem to the known estimates.

DANIEL TATARU
Strichartz estimates for hyperbolic operators with nonsmooth coefficients

The Strichartz estimates are $L^p(L^q)$ estimates for solutions to the wave equation which are related to the restriction theorem for the cone and thus to the (number of) nonvanishing curvatures of the characteristic cone. Such estimates have been proved quite useful in the study of various semilinear hyperbolic equations.
The aim of the current work is to study whether similar results hold for operators with low regularity coefficients.
The main result is that the full Strichartz estimates hold for a second order hyperbolic operator $P(x,\partial) = g^{ij}(x)\partial_i\partial_j$ provided that $\partial_{x,t}^2 g \in L_t^1(L_x^\infty)$. Some weaker results are obtained under the assumption that $\partial^s g \in L^1(L^2)$, $0 \leq s \leq 2$.
An essential tool in the analysis is the FBI transform, which provides a very convenient way of localizing simultaneously in the physical space and in the frequency. The analysis leads to

a nice expression for an approximate parametrix for the wave equation of the form

$$K(y, \tilde{y}) = \int\limits_{t \geq 0} \iint\limits_{\text{cone}} e^{i\lambda\xi(x-y)} \, e^{-i\lambda\xi_t(x_t-\tilde{y})} \, G_1(x, y, \xi) \, G_2(x_t, y, \xi_t) \, \mathrm{d}x \, \mathrm{d}\xi \, \mathrm{d}t$$

where $(x, \xi) \mapsto (x_t, \xi_t)$ is the null bicharacteristic flow and G_1, G_2 behave essentially like the Gaussians

$$G(x, y, \xi) \simeq e^{-c\lambda(x-y)^2}.$$

This seems to be related to the recent construction of parametrices for the smooth coefficient case which involves complex phase functions.

One application of these estimates is to improve the local theory for quasilinear hyperbolic equations by $\frac{1}{3}$ derivative $(n \geq 3)$ respectively $\frac{1}{6}$ derivative $(n = 2)$ below the classical results. It is not known whether these new results are sharp or not. They go only $\frac{2}{3}$ of the way to the known counterexamples.

CHRISTOPH THIELE
On multilinear singular integrals

We discuss n-linear forms of the type

$$T(f_1, \ldots, f_n) = \int_{\Gamma} \int_{\mathbb{R}} \int_{\Gamma'} \Big(\bigotimes_i f_i \Big)(\xi) \, D_{2^{-t}}^{\infty} \, L_{-\eta-\beta} \, \varphi^{n\otimes}(\xi) \, \mathrm{d}\eta \, \mathrm{d}t \, \mathrm{d}\xi \; .$$

Here Γ is the hyperplane in $\mathbb{R}^n$ perpendicular to $(1, 1, \ldots, 1)$, Γ' is a k-dimensional subspace of Γ with $0 \leq k \leq n - 2$, β is a vector in Γ perpendicular to Γ' with sufficiently large norm, φ is a Schwartz function with $\operatorname{supp} \widehat{\varphi} \subset [-1, 1]$, and we have $L_y f(x) = f(x - y)$ and $D_{2^{-t}}^{\infty} f(x) = f(2^t x)$. We have the following result.

Theorem [C. Mascalu, T. Tao, C. Thiele]
For $0 \leq k < \frac{n}{2} - 1$, $\sum\limits_{i=1}^{n} \frac{1}{p_i} = 1$, $1 < p_i < \infty$, *we have*

$$T(f_1, f_2, \ldots, f_n) \leq C \prod_{i=1}^{n} \|f_i\|_{p_i} \; .$$

WALTER TREBELS
On Laguerre multipliers *(joint work with G. Gasper)*

Let the Lebesgue space $L_{w(\gamma)}^p$ be defined by its norm

$$\|f\|_{L_{w(\gamma)}^p} := \left(\int\limits_0^{\infty} \left| f(x) \, e^{-x/2} \right|^p \, x^{\gamma} \, \mathrm{d}x \right)^{1/p} < \infty \, ,$$

$p \geq 1$, $\gamma > -1$. The polynomials are dense in $L^p_{w(\gamma)}$ and we restrict ourselves in the following to a polynomial f. Develop f into a Laguerre series of order $\alpha > -1 : f = \sum a_k L^\alpha_k$.

1. Motivated by the approach of Hardy and Littlewood we consider the fractional integral $I^\sigma(t) = \sum (k+1)^{-\sigma} a_k L^\alpha_k$. With the use of projection formulae, the problem

$$I^\sigma: \ L^p_{w(\gamma)} \to L^r_{w(\delta)}, \quad 1 < p \leq r < \infty,$$

   for appropriate γ and δ, is reduced to the well-known behaviour of fractional integrals on the half-line; in particular, the natural weight case $\gamma = \delta = \alpha$ leads to $\frac{1}{r} = \frac{1}{p} - \frac{\sigma}{\alpha+1}$. The result contains and improves previous ones due to Kanjin, Sato '95 and Gasper, Stempak, Trebels '95. Spezialisation of the parameters in combination with quadratic transformations yields a corresponding fractional integration theorem for Hermite expansions.

2. An equivalence between Laguerre multipliers $\left(\alpha = -\frac{1}{2}\right)$ and Hermite multipliers (in suitable L^p-spaces, $1 < p < \infty$) is established.

3. An (improved) analogue of a result of Coifman, Weiss '77 (relating radial Fourier multipliers on $\mathbb{R}^n$ with those on $\mathbb{R}^{n+2}$) is obtained : Increasing the smoothness of the Laguerre multiplier (with respect to β) by 1 leads to Laguerre multipliers with respect to $\alpha = \beta + \frac{p}{2-p}$, $1 \leq p < 2$.

STEPHEN WAINGER
Discrete Analogues of Spherical Maximal Functions

We are concerned with functions f defined on $\mathbb{Z}^d$ the lattice points in $\mathbb{R}^d$, that is points m in $\mathbb{R}^d$ with $m = (m_1, \ldots, m_d)$ and m_j integers. For λ a positive integer set $r^{(k)}_d(\lambda)$ to be the number of solutions in integers of the equation

$$\lambda = |n_1|^k + \cdots + |n_d|^k, \quad k = 2, 3, \ldots,$$

and for $m \in \mathbb{Z}^d$, let

$$S^k_\lambda f(m) = \frac{1}{r^{(k)}_d(\lambda} \sum_{\substack{n \in \mathbb{Z}^d \\ |n_1|^k + \cdots + |n_d|^k = \lambda}} f(m-n).$$

Theorem 1 [Magyar, Stein, and Wainger].
For $k = 2$,

$$\left\| \sup_\lambda \left| S^2_\lambda f(m) \right| \right\|_{\ell^p(\mathbb{Z}^d)} \leq A(p,d) \, \|f\|_{\ell^p(\mathbb{Z}^d)}$$

provided $d \geq 5$ and $p > \frac{d}{d-2}$.

19

Theorem 2 [Magyar, Stein, and Wainger].

For general k, there are numbers $a(k)$ and $b(k)$ so that

$$\left\| \sup_{\lambda} \left| S_{\lambda}^{k} f(m) \right| \right\|_{\ell^{p}(\mathbb{Z}^{d})} \leq A(p,d) \, \|f\|_{\ell^{p}(\mathbb{Z}^{d})}$$

provided $d > a(k)$ and $p > \frac{d}{d-b(k)}$.

JAMES WRIGHT
Multiple Hilbert transforms along polynomial surfaces *(joint work with A. Carbery and S. Wainger)*

We are interested in studying certain singular integrals in $\mathbb{R}^{n}$ whose distributional kernel is supported along a polynomial surface and possesses a product type singularity. There is a general theory due to F. Ricci and E.M. Stein which examines the situation of convolution operators where the convolution kernel is homogeneous with respect to a k parameter family of dilations on $\mathbb{R}^{1}$. A model case of such a convolution operator whose kernel is not homogeneous is given by convolution on $\mathbb{R}^{3}$ with the distribution

$$\Lambda(\varphi) = \underset{|s|,|t| \leq 1}{\int \int} \varphi\left(s, t, P(s,t)\right) \, \frac{1}{st} \, \mathrm{d}s \, \mathrm{d}t \ ,$$

where P is a polynomial on $\mathbb{R}^{2}$ with real coefficients. However, if $P(s,t) = s^{k}t^{\ell}$, then the Ricci-Stein theory applies and the corresponding convolution operator is bounded on $L^{p}(\mathbb{R}^{3})$, $1 < p < \infty$, if, and only if, either k or ℓ is even. For a general polynomial $P(s,t)$ not all monomials influence matters and we have for a general P :

Theorem. *$f \mapsto f * \Lambda$ is bounded on $L^{p}(\mathbb{R}^{3})$, $1 < p < \infty$, if, and only if, every vertex (k,ℓ) of the Newton diagram of P has at least one even entry.*

We remark that, if convolution on $\mathbb{R}^{3}$ is replaced with convolution on $\mathbb{H}^{1}$, then the corresponding operator is bounded on $L^{2}(\mathbb{H}^{1})$ for every polynomial P.

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