MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 34/1999

Complex Geometry: Varieties of Low Dimensions

22.08. - 28.08.1999

Die Tagung fand unter der Leitung von A. Beauville (Paris), F. Catanese (Göttingen), E. Looijenga (Utrecht) und Ch. Okonek (Zürich) statt.

Wie schon bei früheren Tagungen über komplexe Geometrie in Oberwolfach haben auch in diesem Jahr wieder viele bedeutende Mathematiker aus verschiedenen Ländern an der Tagung teilgenommen, und es war deshalb nicht schwer, ein interessantes Tagungsprogramm zusammenzustellen.

Viele Vorträge bezogen sich auf wichtige komplex-geometrische Themen wie z.B. Modulräume von Flächen, Hilbertschemata, birationale Automorphismen projektiver Räume, Abelsche Varietäten, Kurven auf Flächen. Behandelt wurden auch moderne Entwicklungen und neuste Resultate in der komplexen Geometrie, etwa: projektive Kontaktmannigfaltigkeiten, Mori-Theorie, Calabi-Yau Varietäten, holomorphe Blätterungen, Zopf-Monodromie, Fundamentalgruppen von Kähler Mannigfaltigkeiten. Darüber hinaus wurden auch Anwendungen von Methoden aus *symplektischer* und *fast komplexer Geometrie* dargestellt.

L. Ein

Effective Nullstellensatz

We discuss our recent joint work with R. Lazarsfeld.
Let X be a smooth complex projective variety of dimension n and L be an ample line bundle on X. Consider a linear system $|V| \subseteq |L|$. Let B be the base locus of $|V|$. Let $Z_1, \ldots, Z_r$ be the distinguished subvarieties of B (in the sense of Fulton's intersection theory). We show that there are positive integers $a_1, \ldots, a_r$ satisfying the Bezout inequality

$$c_1(L)^n \geq \sum_{i=1}^{r} a_i \, deg_L Z_i \ ,$$

such that

$$\mathcal{I}_{Z_1}^{(na_1)} \cap \ldots \cap \mathcal{I}_{Z_r}^{(na_r)} \subseteq \mathcal{I}_B \ .$$

We also discuss the relation between the Seshadri numbers and asymptotic regularity of ideal sheaves.

S. Mukai

Moduli of abelian surfaces and the regular polyhedral groups

Let (A, L) be an abelian surface of type $(1, d)$. A <u>bilevel structure</u> is the pair of a canonical level structure

$$(K(L), Weil) \xrightarrow{\sim} \left((\mathbb{Z}/5)^{\oplus 2}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right)$$

of (A, L) and that of the dual abelian surface $(\hat{A}, \hat{L})$. The moduli space and its Satake compactification $\overline{\mathcal{A}^{bl}(1,d)}$ has a natural action of the group $G_d \times G_d$, where $G_d = PSL(2, \mathbb{Z}/d)$.
Theorem 1: *For $d = 2, 3$ and 4, $\overline{\mathcal{A}^{bl}(1,d)}$ is smooth and $G_d \times G_d$-equivariantly isomorphic to $\mathbb{P}^3$.*
For $2 \leq d \leq 5$, G_d is a regular polyhedral group. $G_d \times G_d$ acts on $\mathbb{P}^1 \times \mathbb{P}^1$ and its ambient space $\mathbb{P}^3$. Note that the complement of $\mathbb{P}^1 \times \mathbb{P}^1 \subset \mathbb{P}^3$ is isomorphic to $PGL(2)$. $\overline{\mathcal{A}^{bl}(1,5)}$ is no more smooth. The singular locus is the set of 72 point cusps.

Theorem 2: *Let $\widetilde{\mathbb{P}^3}$ be the blow-up of $\mathbb{P}^3$ at the 60 points $G_5 \subset PGL(2) \subset \mathbb{P}^3$. Then there exists a $G_5 \times G_5$-equivariant birational morphism $\widetilde{\mathbb{P}^3} \longrightarrow \overline{\mathcal{A}^{bl}(1,5)}$ which is an isomorphism outside the point cusps. There are 72 lines in $\mathbb{P}^3$ passing through 5 of G_5 and the strict transforms of these lines are contracted to the point cusps. (Normal bundle is $\mathcal{O}(-4) \oplus \mathcal{O}(-4)$.)*

J. Wiśniewski

(report on a joint work with S. Kebekus, Th. Peternell, A. Sommese)

Projective contact manifolds

Let X be a projective manifold of dimension $2n+1$ defined over $\mathbb{C}$. Suppose there exists a line bundle L over X and a twisted form $\theta \in H^0(X, \Omega_X \otimes L)$ such that

$$\theta \wedge (d\theta)^{\wedge n} \in H^0(X, K_X \otimes L^{\otimes(n+1)})$$

does not vanish anywhere. Such a form is called a contact structure on X. The only known examples of contact projective manifolds over $\mathbb{C}$ are:
(1) Wolf spaces-homogenous rational manifolds, each one defined for a simple algebraic group over $\mathbb{C}$, and
(2) projective bundles $\mathbb{P}(T_Y)$, where Y is an arbitrary smooth projective $(n+1)$-fold and $L = \mathcal{O}_{\mathbb{P}(T_Y)}(1)$.
The following theorem provides important information for the minimal model algorithm run for the classification of contact projective manifolds:
Theorem (Kebekus, Peternell, Sommese, Wiśniewski):
Let $\varphi : X \longrightarrow Y$ be an extremal ray contraction of a projective contact manifold (as defined above). If $dimY > 0$ then Y is smooth of dimension $n+1$, $X = \mathbb{P}(T_Y)$, φ is the projection, $L = \mathcal{O}_{\mathbb{P}(T_Y)}(1)$ and the contact structure comes from $Aut(T_Y)$.

M. Lehn

The cobordism class of Hilbert schemes of points on a surface

Let X be a smooth projective surface over the complex numbers. For each natural number n let $X^{[n]}$ denote the Hilbert scheme of zero-dimensional closed subschemes in X of length n. By a theorem of Fogarty $X^{[n]}$ is a projective manifold of dimension $2n$. For any manifold Y let $[Y]$ denote its class in the complex cobordism ring $\Omega_* = \Omega_*^U \otimes \mathbb{Q}$. We prove that $[X^{[n]}]$

depends only on $[X]$ and n. As a corollary, there is a well-defined linear map $\Omega_2 \to \Omega[[z]]$, $[X] \mapsto \log(\sum_n [X^{[n]}]z^n)$. In particular, the value of any complex genus $g : \Omega_* \longrightarrow R$ on a Hilbert scheme can be computed from its values on the Hilbert schemes of any two surfaces that rationally span Ω_2. Thus methods using the toroidal structure of $\mathbb{P}^2$ and $\mathbb{P}^1 \times \mathbb{P}^1$ become available. The main theorem is equivalent to the existence of universal polynomials such that $c_\lambda(X^{[n]}) = P_\lambda(c_1^2(X), c_2(X))$ for any Chern number c_λ, λ a partition of $2n$. The proof of this claim proceeds by induction on n using the incidence variety of all pairs $(\xi, \xi') \in X^{[n]} \times X^{[n+1]}$ with $\xi \subset \xi'$.

L. Caporaso

Uniformity of rational points over function fields

Let $g \geq 2$, B a smooth projective curve over $\mathbb{C}$, $S \subset B$ a finite set.
Shafarevich conjecture, proved by Parshin and Arakelov, states that there exists only a finite number of non-isotrivial families of smooth curves of genus g over $B \setminus S$.
– We prove that such a number is uniformly bounded by a function on g, the genus of the base B, the cardinality of S.
– We generalize this to higher dimensional base.
– We apply the above results to obtain uniform boundedness of rational points of curves over function fields.

I. Dolgachev

Birational automorphisms of finite order of $\mathbb{P}^2$

We discuss the following equivalent problems:
1) Classify the conjugacy classes of elements of finite order in the group of birational transformations of $\mathbb{P}^2$.
2) Classify rational surfaces with an action of a cyclic group up to equivariant birational isomorphisms.
3) Classify plane curves $f(x, y) = 0$ up to birational isomorphism such that the affine surface $z^n = f(x, y)$ is rational.
These problems were solved in XIX-century in the work of several mathematicians (E. Bertini, S. Kantor, E. Noether, A. Bottari are among them). In this talk we reconstruct this work by using modern techniques (e.g. the classification of conjugacy classes in the Weil group of root lattices of type

A_4, D_5, E_6, E_7, E_8). We also mention another approach to this classification suggested by De-Qi Zhang. It is based on the theory of log-terminal Del Pezzo surfaces.

E. Sernesi

Nodal curves on surfaces of general type

On a given projective irreducible nonsingular algebraic surface S defined over $\mathbb{C}$, let $|C|$ be a linear system whose general element is smooth irreducible. For a given integer $\delta \geq 0$ let $\mathcal{V}_{C,\delta} \subset |C|$ be a locally closed subscheme parametrizing the universal family of irreducible curves in $|C|$ having δ nodes and no other singularities. An irreducible component $W \subset \mathcal{V}_{C,\delta}$ is called REGULAR if it is nonsingular of codimension δ in $|C|$. The following result, due to F. Flamini, has been discussed:

Theorem (Flamini): Let S and C as above and assume:

1. $(C-2K_S)^2 > 0, \quad C(C-2K_S) > 0$
2. (i) $K_S^2 > -4$ if $C(C-2K_S) \geq 8$
   (ii) $K_S^2 \geq 0$ if $0 < C(C-2K_S) < 8$
3. $CK_S \geq 0$
4. $H(C,K_S) < 4[C(C-2K_S)-4]$ where $H(C,K_S) = (CK_S)^2 - C^2K_S^2$
5. (i) $\delta \leq \frac{1}{4}C(C-2K_S) - 1$ if $C(C-2K_S) \geq 8$
   (ii) $\delta < \frac{1}{8}\Big(C(C-K_S) + \sqrt{C^2(C-2K_S)^2}\Big)$ if $0 < C(C-2K_S) < 8$.

Then every irreducible component of $\mathcal{V}_{C,\delta}$ is regular.

H. Kaji

Projective geometry of adjoint varieties

An *adjoint variety* $X(\mathfrak{g})$ associated to a complex simple Lie algebra $\mathfrak{g}$ is by definition the homogeneous projective variety in $\mathbb{P}_*(\mathfrak{g})$ obtained from the adjoint action of *Int* $\mathfrak{g}$ on $\mathfrak{g}$. The main results here (a joint work with O. Yasukura) are as follows:

Theorem 1: *For general points $x, y \in X(\mathfrak{g})$, the tangent locus of $[x, y]$ is equal* *to $\{x,y\}$, where $[x,y]$ denotes the point in $\mathbb{P}_*(\mathfrak{g})$ defined by the Lie bracket of x and y.*

Theorem 2: *The secant variety of $X(\mathfrak{g})$ is decomposed as a disjoint union of projectivizations of a finite number of nilpotent orbits in $\mathfrak{g}$ and the orbit in $\mathfrak{g}$ through the semi-simple element associated to the minimal nilpotent orbit. Moreover there is a unique maximal orbit in those nilpotent orbits with respect to the closure ordering, which is equal to $\mathcal{O}_{[2]}$ for $\mathfrak{sl}_2$, $\mathcal{O}_{[31^{n-3}]}$ for $\mathfrak{sl}_{n\geq 3}$, $\mathcal{O}_{[2^2 1^{2n-4}]}$ for $\mathfrak{sp}_{2n}$, $\mathcal{O}_{[3^2 1^{n-6}]}$ for $\mathfrak{so}_{n\geq 6}$, $\mathcal{O}_{A_2}$ for $E_{6,7,8}, F_4$ and $\mathcal{O}_{G_2(a_1)}$ for G_2.*

Theorem 3: *The variety of k-secants of $X(\mathfrak{sl}_n)$ is given by the projectivization of the locus in the space $\mathfrak{sl}_n$ of traceless matrices with rank at most $k+1$, for $0 \leq k \leq n$.*

Theorem 4: *A homogeneous projective variety has one apparent double point if and only if it is projectively equivalent to one of irreducible Freudenthal manifolds in $\mathbb{P}_*(\mathfrak{g}_1)$ defined by $\{Y \in \mathfrak{g}_1 | (adY)^2 \mathfrak{g}_{-2} = 0\}$ for simple Lie algebras $\mathfrak{g}$, where $\mathfrak{g} = \oplus_{k=-2}^{2} \mathfrak{g}_k$ is a graded decomposition of contact type.*

Theorem 5: *The intersection of $X(\mathfrak{g})$ and $\mathbb{P}_*(\mathfrak{g}_1)$ coincides with the Freudenthal manifold associated to $\mathfrak{g}$.*

G. Pareschi

Syzygies of abelian varieties

Let X be a projective variety and let L be a (usually very ample) line bundle on X. According to M. Green, the pair (X,L) is said to satisfy the property N_p if the first p steps of a minimal resolution of R_L over S_L are as follows:

$$\oplus S_L(-p-1) \longrightarrow \ldots \longrightarrow \oplus S_L(-3) \longrightarrow S_L(-2) \longrightarrow S_L \longrightarrow R_L \longrightarrow 0$$

where $R_L := \oplus_{n\geq 0} H^0(L^{\otimes n})$ is the graded algebra associated to L and $S_L := \oplus_{n\geq 0} Sym^n(H^0(L))$ is the symmetric algebra over $H^0(L)$.

I prove the following result conjectured by Rob Lazarsfeld:

Theorem: *Let X be an abelian variety over an algebraically closed field of characteristic zero. Let A be an ample line bundle on X and let us denote $L = A^{\otimes n}$. Let $p \in \mathbb{N}$. If $k \geq 3+p$ then the pair (X,L) satisfies the property N_p.*

The proof uses methods of vector bundles on abelian varieties, which can be of independent interest.

K. Konno

Relative canonical algebra for pencils of curves

Let $f : S \longrightarrow B$ be a relatively minimal fibration of curves of genus ≥ 2, where S (resp. B) is a non-singular projective surface (resp. curve). One of the naive objects associated with f is the relative canonical algebra:

$$R(f) = \oplus_{n \geq 0} R_n, \qquad R_n := f_*(\omega_{S/B}^{\otimes n}).$$

But not much is known so far about the structure of $R(f)$. Some years ago, Miles Reid conjectured that $R(f)$ is generated by elements of degrees ≤ 3 and related in degrees ≤ 6 (1-2-3 conjecture). By Nakayama's lemma, studying this problem for $R(f)$ is equivalent to studying the same problem for the canonical ring $R(F, K_F)$ of any fibre F of f.

Theorem 1: *$R(F, K_F)$ is generated in degrees ≤ 3 and related in degrees ≤ 6 except in the case:*

> *F is a multiple fiber which contains a cycle E with $p_a(E) = 1$, $E^2 = -1$ such that $E \subseteq Bs|K_F|$.*

In the exceptional case, $R(F, K_F)$ is generated in degrees ≤ 4 and related in degrees ≤ 8.

A main part of Theorem 1 is a corollary to the following:

Theorem 2: *Let D be a 1-connected curve on a smooth surface, $p_a(D) \geq 2$ and K_D nef. Let L, M be line bundles on D such that $L - 2K_D, M - 2K_D$ are both nef. Then the multiplication*

$$H^0(D, L) \otimes H^0(D, M) \longrightarrow H^0(D, L + M)$$

is surjective except in the following cases:
(1) $p_a(D) = 2$, $L = M \equiv 2K_D$
(2) D contains a curve E, $p_a(E) = E \cdot (D - E) = 1$ and $L = M \equiv 2K_D$ on E.

F. Bogomolov

Szpiro inequality for elliptic surfaces and mapping class groups

We consider an elliptic smooth nonisotrivial fibration V over $\mathbb{P}^1$ with multiplicative degenerate fibers only and without multiple fibers. In this case we

have two invariants $D = \#$ number of singular fibers and $N = \#\sum_{s\in S} n_s$, where S is a set of singular fibers and n_s is a number of components in the singular fiber over s. It is clear that $D \leq N$. Szpiro inequality says that $N \leq 6D$ and the lecture contains a proof of this result which uses only the monodromy of the fibration. Local monodromies are powers $T_s^{n_s}$ of local positive Dehn twists and we can write the natural relation for them as $\prod T_s^{n_s} = 1$. T_s are elements of $Map(1) = SL(2,\mathbb{Z})$ but it is more natural to consider them in $Map(1,1)$ which is the $\mathbb{Z}$ central extension of $Map(1)$. This extension is induced from the universal covering $\widetilde{SL(2,\mathbb{R})}$ of $SL(2,\mathbb{R})$ and positivity of T_s is interpreted as the property of it's natural lifting $\widetilde{T_s} \subset Map(1,1)$ to move all the points of $\mathbb{R}$= universal covering of $\mathbb{P}^1(\mathbb{R}) = \mathbb{S}^1$ to the right.
The group $Map(1,1)$ has a natural character $\chi : Map(1,1) \longrightarrow \mathbb{Z}$, $\chi(\tilde{T}_s) = 1$ and $\chi(c) = 12$, where c generates the kernel $Map(1,1) \longrightarrow Map(1)$. Now $\prod \tilde{T}_i^{n_s} = Mc$ where $M = \frac{N}{12}$. On the other hand, the displacement angle for $\tilde{T}_i^{n_s}$ is bounded by π independently on n_s. Thus we obtain $2\pi M \leq D\pi$, i.e. $N \leq 6D$.
We extend this result to the case of elliptic fibrations over any curve, with any fibers and also to the families of curves of any genus (see Alg-geom preprint of Amoros, Bogomolov, Katzarkov, Pantev).

L. Bonavero

Mori theory on non projective toric manifolds

In this talk we are interested in the following:
let X be a non projective toric manifold such that there exists an invariant curve $C \subset X$ such that the blow-up $B_C(X)$ of X along C is projective. Then, we classify extremal Mori contractions on $\tilde{X} = B_C(X)$ for which the exceptional locus intersects the exceptional divisor E of $\tilde{X} \longrightarrow X$. We show that there are 3 types only, all being smooth blow-down on a smooth center. For this we use Mori theory for toric variety due to M. Reid. As a simple corollary, we show that there exists no such toric 3-fold X such that $\tilde{X}$ is Fano, without using any classification.

L. Katzarkov

Algebraic geometry methods in symplectic geometry

Let V be a 4-dimensional symplectic manifold. We say that it has a structure

of a topological Lefschetz pencil (TLP) iff:
-there exists a subset of points A
-there exists a subset of points $\{x_\alpha\}$
-a $\mathcal{C}^\infty$ map $f : \{V \setminus A\} \setminus \{x_\alpha\}$ such that
1) near A there are complex coordinates $z_1 = z_2 = 0$ such that $f : (z_1, z_2) \longrightarrow z_1/z_2$
2) near $\{x_\alpha\}$ $f : (z_1, z_2) \longrightarrow z_1^2 + z_2^2$.

Theorem (Donaldson): *Every (V, ω) with $[\omega]$ integral has a structure of a TLP. Moreover the fibers are symplectic submanifolds PD to $k[\omega]$ for some $k >> 0$.*

Using the above theorem one can define a series of invariants:

$$\rho_k : \pi_1(\mathbb{P}^1 \setminus \{p_1, \ldots, p_{n_k}\}) \longrightarrow Sp(2g_k, \mathbb{Z}/d\mathbb{Z}).$$

Conjecture: *For H_1, H_2 Horikawa surfaces the sequences of representations:*

$$\rho'_k : \pi_1(\mathbb{P}^1 \setminus \{p_1, \ldots, p_{n_k}\}) \longrightarrow Sp(2g_k, \mathbb{Z}/3\mathbb{Z}).$$

$$\rho'_k : \pi_1(\mathbb{P}^1 \setminus \{p_1, \ldots, p_{n_k}\}) \longrightarrow Sp(2g_k, \mathbb{Z}/3\mathbb{Z}).$$

corresponding to $m_{k H_1}$ and $m_{k H_2}$ are not Hurwitz and and conjugacy equivalent.

A corollary from this conjecture will be that H_1 is not symplectomorphic to H_2.

C. Voisin

On the Hilbert scheme of points of an almost complex fourfold

Recently it was proved by Ellingsrud, Göttsche and Lehn that the complex cobordism class of $Hilb^k(X)$, where X is a complex compact surface, depends only on the complex cobordism class of X. Here $Hilb^k(X)$ is the Hilbert scheme parametrizing subschemes of length k of X. One natural question raised by this result is whether or not the assignment $X \longmapsto Hilb^k(X)$ can be extended to the context of almost complex varieties. We prove essentially this

Theorem: *Let (X, J) be an almost complex fourfold. Then there exists a differentiable variety $Hilb^k(X)$, with a stable almost complex structure and a differentiable map*

$$c : Hilb^k(X) \longrightarrow X^{(k)},$$

such that $c^{-1}(Z)$ is diffeomorphic to $c_I^{-1}(Z)$ for any complex structure I on X defined near Z and c_I the corresponding Hilbert Chow morphism. This $Hilb^k(X)$ is well defined up to deformation.

K. Oguiso

Finiteness of $c_2 = 0$ contractions on a Calabi-Yau 3-fold

By a Calabi-Yau 3-fold we mean a minimal projective 3-fold $/\mathbb{C}$ such that $\mathcal{O}_X(K_X) \cong \mathcal{O}_X$ and $h^1(\mathcal{O}_X) = 0$. By a contraction $\Phi : X \longrightarrow W$, we mean a surjective morphism on a normal projective variety W with connected fibers. Φ is called $c_2 = 0$ contraction if $\Phi = \Phi_{|D|}$ such that $(D.c_2(X)) = 0$. As a corollary of the classification of $c_2 = 0$ contractions, we have shown the following:

Theorem:

(1) $\#(\{c_2 = 0$ *contractions on* $X\}/isom) < +\infty$

(2) Assume $c_2(X) \equiv 0$ in $Pic(X)$. Then the Picard number $\rho(X) = 2$ or 3 and

   (a) the nef cone $\bar{A}(X)$ is rational simplicial cone, and

   (b) every nef rational divisor on X is semi-ample.

As a corollary we also find the following:

Corollary: *If $\rho(X) = 1$, then $\pi_1(X)$ is finite.*

J. Wahl

Hyperplane sections of Calabi-Yau varieties

A Calabi-Yau variety Z^n will mean a normal complex projective variety with isolated singularities, so that $K_Z \cong \mathcal{O}_Z$, $h^1(\mathcal{O}_Z) = 0$, and the singularities are canonical (=rational Gorenstein). If $n = 2$, this means a $K3$ surface with RDP's.

One expects from "boundedness of families of Calabi-Yaus" that there are few of these, hence few hyperplane sections $X = Z \cap H$. Note $K_X = K_Z + H|_X = \mathcal{O}_X(1)$ is very ample. So we ask which canonically polarized (X, K_X) are hyperplane sections of a C-Y. We explained the proof of the

Theorem: *Suppose W is a smooth variety with $h^1(\mathcal{O}_W) = 0$. Then a sufficiently ample smooth divisor X on W cannot be a hyperplane section of a Calabi-Yau variety Z, unless $W = Z$.*

Corollary: *A non-singular hypersurface of degree d in $\mathbb{P}^n$ ($n \geq 2$) cannot be a hyperplane section of a Calabi-Yau once $d > 2n + 2$.*

M. McQuillan

Non-commutative minimal models

The object of this talk was to study in the spirit of Mori's programme one of the basic examples of non-commutative geometry, namely foliations. Precisely we study pairs (X, F) consisting of an integrable foliation F on a normal algebraic space X. Restricting to the case of surfaces, we obtain an almost complete foliated analogue of Enriques-Kodaira classification. Various hyperbolicity results, already established by the author, would modulo considerations from diophantine geometry approximation, follow immediately or having a complete rather than almost complete classification, i.e. finishing off the remaining case.

References:

1) McQuillan M., *"Diophantine approximation and foliations"*,
Publ.Math I.H.E.S., vol.87, 1998
2) McQuillan M., *Holomorphic curves on hyperplane sections of 3-folds"*,
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R. Piene (joint work with S. Kleiman)

Enumerating singular curves on surfaces

Given a family of smooth projective surfaces $\pi : F \longrightarrow Y$ and $D \subset F$ a relative divisor, T a topological type (= Enriques diagram), let
$Y(T) := \{y \in Y | D_y$ has singularities of type $T\}$. We show that, under certain genericity assumptions, and for $cod(T) \leq 8$, the class $u(D, T) := \left[\overline{Y(T)}\right]$ can be expressed as a polynomial of degree $r := \#\{$roots of the diagram $T\}$ in the classes $\mu(a, b, c) := \pi_* c_1(\mathcal{O}(D))^a c_1(\Omega^1_{F/Y})^b c_2(\Omega^1_{F/Y})^c$. We conjecture this holds for any T, at least for the case that $T = rA_1$ ($= r$ nodes). In this latter case, write $u(D, r) := u(D, rA_1)$ and (inspired by Göttsche's work) the generating function as $\sum u(D, r)t^r = \exp(\sum a_q t^q / q!)$; we show that the a_q are linear and we give an algorithm for computing them – this computes $u(D, r)$

for $r \leq 8$.
Applied to $F = S \times Y$, S surface, $Y = |\mathcal{L}|$, $\mathcal{L} = \mathcal{M}^{\otimes m} \otimes \mathcal{N}$ we prove that the above formula is valid provided $m \geq 3r + g^2 + g + 4 - (s + x)/12$ where $g := 1 + c_1(\mathcal{M})(c_1(\Omega^1_S) + c_1(\mathcal{M}))/2, s = c_1(\Omega^1_S)^2, x = c_2(\Omega^1_S)$.

We also recover two examples where Y is non-linear: Vainsencher's enumeration of 6-nodal quintic plane curves on a general quintic hypersurface in $\mathbb{P}^4$, and Bryan-Leung's enumeration of the number $N_{g,n}$ of genus g curves algebraically equivalent to a given curve C, with $C^2 = 2g - 2 + 2n$, $n \leq 8$, on an Abelian surface.

M. Teicher

New invariants of braid monodromy

§1. Introduction: Braid Monodromy Type (BMT) characterize curves and distinguish among families of curves. When applied to branch curves of generic projections of surfaces of general type, it characterize surfaces and it is an invariant of deformation families. It was proven in 1998 by the author and Viktor Kulikov that it two plane curves S_1 and S_2 are of the same BMT, then they are isotopic. Moreover, if two surfaces of general type have branch curves which are of the same BMT, then the surfaces are diffeomorphic. Thus BMT is between Def and $Diff$: $Def \Rightarrow BMT \Rightarrow Diff$. There is a work in process to adapt this notion of BMT to the symplectic situation (following thesis of Auroux, who produced "generic" projection to $\mathbb{CP}^2$ of symplectic 4-manifolds).
We want to provide discrete invariants of braid monodromy factorizations. The situation is as follows:

§2. Defining Braid Monodromy Type of curves: Let S be a plane curve of degree m. Let φ be the braid monodromy related to S ($\varphi : \pi_1 \longrightarrow B_m$). Let $\{\delta_i\}_{i=1}^N$ be a free geometric base of π_1. Let Δ^2 be the generators of $center(B_m)$. Then by Artin: $\Delta^2 = \prod_{i=1}^N \varphi(\delta_i)$. Such a presentation is called a Braid Monodromy Factorization (BMF). In the talk I defined Hurwitz equivalence of BMF, which induces the notion of BMT (S_1 and S_2 are of the same BMT if they have factorization which are Hurwitz and conjugacy equivalent).

§3 The Hecke Representation Invariants: On each BMF we define a new invariant derived from the Hecke algebra. Let $Hecke(B_m)$ be the Hecke algebra of B_m, $dimHecke(B_m) = m!$. Let HR be the representation of B_m in $Hecke(B_m)$ defined by multiplication from the left. For each BMF, $\prod_{i=1}^{N} b_i$ we look at the $m! \times Nm!$ matrix $HR(b_1) - I$, $HR(b_N) - I$ and at the g.c.d. of its maximal order minors. We compute it for several examples and currently we try to apply it on branch curves.

N. Shepherd-Barron

Moduli and pencils of Enriques surfaces

This describes a joint work with T. Ekedahl.
There is a coarse moduli space M over $\mathbb{Z}$ for appropriately polarized Enriques surfaces. The geometric fibres are 10-dimensional and irreducible (this is well known) if $char \neq 2$, while in $char = 2$ there are two 10-dimensional components, meeting in an irreducible 9-dimensional variety. This stratification reflects the possible values of Pic^τ. The proofs depend upon an analysis of the period map (in the sense of Ogus) defined in the case where $Pic^\tau \neq \mu_2$. In contrast to the K3-case, the fibres of the pencil map are open subsets of $\mathbb{P}^1$ (this comes from a construction similar to that of Moret-Bailly), but there is a generic Torelli theorem where $Pic^\tau = \alpha_2$.
From the viewpoint of general moduli problems, the quotient map from the stack to M has the property that the local rings of M are not always invariant subrings (with respect to the automorphism group scheme) of the local rings of the universal deformation spaces. In fact, there can be 1-dimensional fibres, while the group schemes are finite.

F. Campana

Green-Lazarsfeld sets and solvable quotients of Kähler groups

Theorem 1: *Let X be a compact Kähler manifold. Let G be a linear (i.e. embeddable in $Gl(m, \mathbb{Q}), m \gg 0$) solvable group which is a quotient of $\pi_1(X)$. Then either G is almost nilpotent, or there exists a finite etale abelian cover $\tilde{X}$ of X and a surjective holomorphic map $f : \tilde{X} \longrightarrow C$ to a curve C of genus $g \geq 2$.*

When X is projective, this result is due to Arapura-Nori, who obtained it by means of arithmetic geometry.

It is here derived from:

Theorem 2: *Let X be as above. Let $\widehat{\pi_1(X)} = Hom(\pi_1(X), \mathbb{C}^*)$ be the complex* *algebraic group of characters of* $\pi_1(X)$*, and*

$$\Sigma^1(X) := \{\chi \in \widehat{\pi_1(X)} \ s.t.\ H^1(\pi_1(X), \mathbb{C}_\chi) \neq 0\}$$

be the Green-Lazarsfeld set of X*, where* $\mathbb{C}_\chi$ *is* $\mathbb{C}$ *considered as a* $\pi_1(X)$*-module via* χ*. Then* $\Sigma^1(X)$ *is a finite union of* <u>*torsion*</u> *translates of subtori of* $\widehat{\pi_1(X)}$*.*

When X is projective, theorem 2 was proved by C. Simpson (in a more general form). The Kähler case is reduced to the projective case, using arguments of A. Beauville, who established (among other things) the special case where the derived group $D\pi_1(X)$ of $\pi_1(X)$ is finitely generated.

G.P. Pirola

On subvarieties of a generic abelian variety

We show that the higher Abel-Jacobi mapping gives some geometrical informations on subvarieties of a generic abelian variety.
We prove:

Theorem: *If* X *is a* n*-dimensional smooth variety with a nontrivial morphism* $f : X \longrightarrow A$ *where* A *is a generic abelian variety of dimension* a*,* $a > 3$*,* $dim f(X) = n$*, then*

$$p_g(X) \geq \frac{1}{2}(a-n)(a-n-1) + h^{n,0}(A).$$

The main tools are:

1) Nori Theorem which shows that the Abel-Jacobi image of homologically trivial cycles is torsion modulo the largest abelian subvariety of the suitable intermediate Jacobian.
2) An explicit computation of the differential of the normal function associated to the cycle $f(X)$.

These methods give bounds on the number of moduli of irregular varieties.

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