MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Beliefs and their Impact on Teaching and Learning of Mathematics

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This conference was organized by Professors E. Pehkonen (Turku, Finland) and G. Törner (Duisburg, Germany). The participants presented their most recent work in the meeting. This and the marvelous surrounding created a lively scientific atmosphere with many stimulating discussions. These certainly will influence future directions and will contribute to further progress in the research field of mathematical beliefs. The lecture program covered a broad range of topics in this diverse area. In particular, many contributions dealt with the important relation between beliefs and knowledge.

VORTRAGSAUSZÜGE

B. BERGER: Computer world views of teachers. Habitualized conceptions of teachers in the context of computer science, mathematics, and computer culture

All over Europe, great efforts have been made to introduce computers and new media to schools. Teacher education and in-service training mainly focus on cognitive aspects, attaching importance to imparting up-to-date technical knowledge. However, a recent study by the author has revealed that especially affective factors have a major impact on the teachers' performance. As the study revealed, a teacher's individual computer world view or computer concept, i.e. her or his attitudes towards computers and the context in which they appear, represent a decisive factor in the teaching (and learning) process. The cognitive components constitute only a part of these computer concepts, the other parts being the affective and the operational components. The study showed that it is first and foremost the affective component which has a selective and directive impact on the teacher's performance. Even young teachers who on the surface seem to be well-equipped for their jobs as mathematics and computer science teachers did not altogether prove to be without an apprehension of computers, if unconsciously. Such observations gain even more importance, as the outcomes of the study indicate that a teacher's computer concept may constitute a hidden curriculum. Based on conceptions and outcomes of modern Sociology, Psychology, and Cognitive Linguistics, the study conceptualizes the multiple realities of individuals as specific socio-cultural frames (worlds: world of mathematics, world of computers, world of medicine, world of politics etc.). Belief systems, then, can be understood as habitualized conceptualizations of those worlds (world views) which are cognitive representations of frame-specific personal dispositions (habit: attitudes, self concept, implicit theories, tacit knowledge etc.) forming specific schemes of perception, assessment, and generation. Those world views emerge from a specific intra-personal context, i.e. a coherent complex of narrative elements (theme), which is inducing a specific context of life world factors (field: socio-cultural, individual, and objective factors). World views result in specific styles of behavior (practice: customs, rituals, artifacts) and thinking (diction: conceptions, metaphors, language). World views, in a fundamental way, are metaphorically organized. In order to contribute to an adequate training of mathematics and computer science teachers, an analysis of their computer concepts should be integrated into current research, and especially so on an intercultural level. Further studies should be undertaken to allow a deeper insight into the affective aspects of teaching and learning processes in the realm of mathematics and the new media. References Berger, P. (1998). Computer World Views of Teachers. Professional Beliefs, Attitudes, and Conceptions of German Mathematics and Computer Science Teachers. In: T.W. Chan, A. Collins & J. Lin (Hg.) Global Education on the Net. Proceedings of the 6th International Conference on Computers in Education (ICCE 98), Beijing, China, October 14-17, 1998. Berlin: Springer, Vol. 1, pp. 664-669. Berger, P. (1999). Computerweltbilder. Habitualisierte Konzeptionen von Lehrern im Kontext von Informatik, Mathematik und Computerkultur. [Computer World Views. Habitualized Conceptions of Teachers in the Contex of Computer Science, Mathematics, and Computer Culture]. Frankfurt: Lang.

H.H. BRUNGS: How to attract students into programs in mathematics and keep them there?

Various studies in recent years have documented the decline of students in programs in mathematics (e.g. S. A. Garfunkel, G. S. Young; The sky is falling; Notices of the AMS 45(2) (1998) 256-257). H. Bass (in Notices of the AMS 44(1) (1997) 18-21) analyses the transitional period in which we live and makes several suggestions how to cope, in particular he insists that University professors have to become more professional in their teaching. My own department has introduced new programs, in particular in Mathematical Finance, the number of graduate students has increased, however the number of students in the undergraduate programs has decreased, and our honors program, which in the past produced a number of highly trained and motivated students, will have to be reorganized in order to survive. Some staff members have conducted very creative Saturday afternoon sessions for Elementary and Junior High students, which , together with mathematical competitions, have attracted some students to our programs. After surveying several hundred students that graduated in mathematics, we also visited High Schools and provided information about mathematics and related careers. A fair number of students would be interested, but many nevertheless considered careers in engineering and computing. A mentor program for graduate students has been introduced in order to improve their teaching skills. However the fact that the administration takes the teaching evaluation, done by the students for every course, very seriously, has probably had the greatest impact on teaching. The students tell us that they want to see many examples, and I am not sure whether this information is sufficient to understand how our students actally think; maybe the evaluation could be used to help in the study of the mental representation of students (see: R. B. Davis in the Handbook of Research on Mathematics Teaching and Learning; D. A. Grouws, Editor, New York 1992, 724-734). To me it seems that teaching could be guided by what I. Kant (in: The critique of judgement) sees as central to aesthetic delight: The free play into which imagination and understanding enter.

O. CHAPMAN: Beliefs as Generative Metaphors in Mathematics Teachers' Growth

Facilitating or accomplishing fundamental changes in mathematics teaching can be a challenging endeavour. But some teachers do make significant, positive shifts in their teaching that are self-motivated and self-determined. This paper reports on a study that investigated a sample of these teachers in terms of the role beliefs in the form of metaphors played in facilitating these shifts in their teaching. The participants were four experienced high school mathematics teachers whose teaching had evolved from being very teacher-centered, as they started their practice, to eventually being more student-centered. This shift did not occur as a result of any specific professional development program. Data collection involved extensive interviews and classroom observations. Data analysis involved identifying the relationship between the teachersa beliefs in the form of metaphors and the growth in their teaching of high school mathematics. The findings indicated a transformative connection between a particular metaphor held by each of the teachers and her or his teaching, i.e., the former played a significant role in influencing changes to the latter. These metaphors were stated explicitly as the nature of mathematics but embodied the teachersa beliefs about the learning and teaching of mathematics. The metaphors emerged in the teachersa talk without any prompting to provide one for mathematics. No similar metaphor emerged for learning or teaching mathematics. These metaphors of the participants were mathematics is experience, mathematics is play, and mathematics is language. Two of the teachers viewed mathematics as language, but had different interpretations of language. As the characteristics each teacher perceived for his or her metaphor changed, so did his or her views and actual teaching of mathematics. For example, the teacher who believed 1mathematics is play1, at different points in her practice, characterized play first as fun, then fun and strategy, then fun, strategy, and reflection. Her teaching changed significantly each time play was re-characterized. The metaphors seemed to represent a framework of beliefs held by the teachers that influenced their generation of new perspectives of teaching mathematics. The generative process was activated whenever there were significant conflicts for the teachers between their prevailing interpretations of the metaphors and their lived experiences in the classroom. The study highlights the importance of beliefs in the form of generative metaphors that may underlie mathematics teachersa growth and the possible significance of consciously attending to such metaphors to assist teachers in achieving desired changes in their teaching. Note: This paper is based on a study funded by the Social Sciences and Humanities Research Council of Canada.

T.J. COONEY: Examining what we believe about beliefs

Over the past 15 years, there has been a considerable amount of research that has focused on various aspects of teachersa beliefs. A question arises as to what beliefs are, that is, just what is being studied. A related question is how knowledge and belief can be differentiated. Scheffler (1965) claimed that X knows Q if and only if the following conditions hold: i. X believes Q ii. X has the right to be sure Q iii. Q The difficulty is that we can never be sure that Q exists+we can only surmize its existence based on the best available evidence. We can speak with confidence but not certainty that a certain condition Q holds. Accordingly, we can argue that X knows Q if and only if X believes Q and there is reasonable evidence to support the existence of Q.

This makes believing a weaker condition than knowing. But it still begs the question as to what constitutes a belief. Again, I refer to Scheffler who defines a belief as a 1cluster of dispositions to do various things under various associated circumstances1 (p. 85). One implication of this definition is that dispositions do not consist solely of statements made during interviews; dispositions also include actions in specific situations. This perspective leads us to question conclusions that suggest teachersa beliefs are at variance with their practice. What do we conclude, for example, when a teacher steadfastly maintains that the essence of mathematics is problem solving, yet we see only procedural knowledge being emphasized in the classroom? Typically, the researcher claims that there exists an inconsistency between the teacheras belief and his/her practice. But other interpretations exist. Consider the following possibilities: 1. We do not completely understand what the teacher means by problem-solving. 2. The teacher cannot act according to his/her belief because of practical or logistical circumstances. 3. The teacher holds the belief about problem solving subservient to the belief that the teaching of mathematics is about certainty and procedural knowledge. These interpretations raise questions about the conclusion that beliefs and practice are inconsistent and challenge us to develop a deeper understanding of how the teacher constructs meaning. The fundamental importance of teachersa beliefs lies in their explanatory power for understanding how teachers make sense of their world. One aspect of their sense making is how they come to know. Given that we can never know for sure about the existence of Q, we have to conclude that knowing is a relativistic construct dependent on context. Consequently, research on how teachers come to know and on the flexibility of their knowing can provide considerable insight into their potential to reform their teaching so as to accommodate more process oriented aspects of instruction, e.g., problem solving and reasoning. The development of a reform-oriented teacher, so defined, is rooted in the ability of the individual to doubt, to reflect, and to reconstruct. Teacher education then becomes a matter of focusing on reflection and on the inculcation of doubt in order to promote attention to context, that is, on promoting a more reflective and dynamic way of knowing. Indeed, our teacher education programs should model the kind of relativistic thinking that we would like to see exhibited by teachers in their classrooms. References Scheffler, I. (1965). Conditions of knowledge. Chicago: Scott Foresman and Company.

F. FURINGHETTI & E. PEHKONEN: A virtual panel evaluating characterizations of mathematical beliefs

In mathematics education the importance of beliefs/conceptions in the analysis of cognitive and metacognitive phenomena, as well as teachersa behavior and attitudes is widely recognized. The study presented in Oberwolfach meeting is set in the stream of research on beliefs. The feature we address to is the problem of the meaning of the terms used in this field. Generally, mathematicians define terms they use or, in the case of primitive terms, explicitly they declare that they are not defining these terms. Psychologists are less keen to define and this may be a very operative behavior, since their theories are usually based on experiments which are subject to changes and evolution. The problem of communicating ideas is the main problem we took into account when planning the present research. Our aim was not to arrive at a general agreement on a given definition (which could be ambitious and unrealistic), on the contrary we were aware that it would have been difficult to give any satisfying verbal definition. Simply we attempted to gather information on the subject in question and to point out the need that researchers make explicit their positions on a certain subject when dealing with this subject. In our case the issues at stake were beliefs - conception - knowledge. To pursue our aim we have worked out a questionnaire based on nine characterizations of the terms of the triad present in literature. The authors of the nine characterizations that we put in the questionnaire were not indicated. We invited a group of experts in the field to express their opinions about these characterizations. IN particular, we asked to the expert: - if they agree with the given characterization - possible improvements - reasons for disagreement - personal characterization. The outputs of the questionnaire were the background for the analysis presented in Oberwolfach about the terms of the triad beliefs - conception - knowledge and their mutual relationship.

G.A. GOLDIN: Affect, meta-affect, and mathematical belief structures

This talk, partly based on joint work with Valerie A. DeBellis, offers some theoretical perspectives on mathematical beliefs drawn from analysis of the affective domain, especially the interplay between meta-affect and belief structures in sustaining each other. I discuss affect (a) as a system of representation encoding information about the external physical and social environment, about mathematics, about cognitive and affective configurations of the self and others; (b) as a powerful, shared, essentially human evolutionary language for communication; (c) as intertwined with systems of cognitive representation (verbal/syntactic, imagistic, formal notational, strategic/ heuristic); (d) as pertaining to several domains [McLeod, DeBellis & Goldin]: emotions (rapidly changing, mild to intense, local, embedded), attitudes (moderately stable, balanced affect and cognition), beliefs (stable, highly cognitive, structured), and values, ethics, morals (stable, deep personal "truths,"! highly affective, structured); Then I examine the key construct of meta-affect [DeBellis & Goldin]: affect about affect, affect about and within cognition (which may again be about affect), monitoring of affect, and affect as monitoring. Profound changes in affect occur according to the prevailing meta-affect, which in turn hinges on cognitions and values. The meta-affect relating to a student's frustration during mathematical problem solving may be anxious and fearful (e.g., signaling anticipation of failure), or joyful (e.g., based on the "cognitive" belief in a high personal likelihood of success, that mathematical problems yield to certain insightful processes). Powerful affective representation inheres not so much in the affect, as in the meta-affect. The thesis here is that affect stabilizes beliefs, and beliefs establish meta-affective contexts. Stable beliefs are comfortable (though not necessarily pleasant), reinforcing defenses. Beliefs are defined to be multiply-encoded cognitive/affective configurations to which the holder attributes value (usually truth, validity, or applicability). Thus I distinguish among (1) working assumptions or conjectures, (2) weakly- or strongly-held beliefs, (3) warrants for beliefs, (4) psychological functions of beliefs, (5) individual and shared beliefs, (6) knowledge (beliefs that in some sense apart from the fact of belief, are true or valid), and (7) individual and shared values. Different notions of truth or validity may pertain in different domains. In contexts such as the physical world, or mathematical problems, belief does not affect truth; in others, e.g., an estimation of an individual's own mathematical ability, they may have a partial influence; in still others, e.g., personal values, belief can creates its own truth (self-referential). A preliminary typology of mathematically-related beliefs is offered, organized not by who holds them but by their content. Belief structures that can intersect several of these categories, may be about: (a) the physical world, and the correspondence of mathematics to it; (b) facts, rules, equations, theorems, of mathematics; (c) how mathematical truths are established (validity); (d) effective mathematical reasoning methods and strategies, heuristics; (e) the metaphysics or philosophy of mathematics; (f) mathematics as a social phenomenon; (g) aesthetics, beauty, meaningfulness, or power in mathematics; (h) individual people who do mathematics, their traits and characteristics; (i) the learning, teaching, and psychology of doing mathematics; or (j) oneself in relation to mathematics, including one's ability, emotions, history, integrity, motivations, self-concept, stature in the eyes of others, etc. Finally I discuss how belief structures, warrants for belief, and meta-affect can establish and sustain each other, and explore some warrants for mathematical beliefs in relation to affective structures: intuitions, diagrams, rational arguments, proofs; but also (sometimes) appeal to authority, feelings, personal history and education, values, or social acceptability.

A. GARDINER: Conflicts between mathematics graduates' proof behaviours and their stated beliefs about proof

It was described results of a questionnaire about studentsa beliefs on proof administrated in School of Mathematics at the University of Birmingham.

P. KLOOSTERMAN: Mathematical beliefs and motivation of high school students in the United States

This paper focuses on the extent to which United States high school students (ages 14 to 18) think mathematics is conceptual as opposed to procedural, and how important memorization is in the learning process. The data come from a larger study in which 56 United States high school student volunteers were interviewed using an extensive interview protocol. Three major themes emerged with respect to the nature of mathematics and the importance of memorization. The first theme was that the nature of mathematics as a discipline is not an issue that United States high school students think about. The second was that when students are pressed to talk about the nature of mathematics, they mention that mathematics can be used to solve a variety of problems and that it involves numbers. They often mention the procedural nature of mathematics and they sometimes mention the logical nature of mathematics but almost never mention deduction or proof. Data to support these two themes include the fact that a number of students talked about whether they liked mathematics, why one needed to learn mathematics, and the daily procedures in mathematics classes (grading homework, listening to an explanation, etc.). Fifteen of the 56 students mentioned specifically that mathematics involved steps, procedures, or formulas. Seven students spoke of mathematics as a way of thinking or a logical system. The third theme was that students tend to feel that memorization, and the ability to memorize procedures, is an important part of being successful in mathematics. On the other hand, they also feel that students who are not good at memorizing can still learn mathematics if they work hard enough. From a motivation perspective, those students who have doubts about their ability to memorize formulas and procedures have good reason to question whether they have ability to do mathematics and thus whether they should bother to try. Like the question about the nature of mathematics, the question about memorization in mathematics caught a number of students by surprise. They understood what they were being asked and many had very strong opinions. On the other hand, many of the comments they had seemed contradictory in that they would claim that memorization was very important and then they would also claim that students who were not very good at memorizing could still do well in mathematics. The implication of this study for instruction is that students need to be exposed to an issue they are not exposed to now. That issue is what it means to know and do mathematics. When students are asked to think about what the discipline of mathematics entails, they likely to put some of their efforts into learning to reason and think mathematically as opposed to being motivated solely to memorize procedures.

G.C. LEDER: Measuring mathematical beliefs and their impact on the learning of mathematics: a new approach

It is now widely accepted that cognitive as well as affective factors - such as attitudes, beliefs, feelings, and moods - must be explored if our understanding of the nature of mathematics learning is to be enhanced. How studentsa beliefs and attitudes about mathematics influence their learning of this subject has attracted considerable research attention. Yet, finding ways to infer beliefs and attitudes from behaviours has continued to be a challenge to researchers. In an influential article, Schoenfeld (1992) argued: 1The older measurement tools and concepts found in the affective literature are simply inadequate; they are not at a level of mechanism and most often tell us that something happens without offering good suggestions as to how or why1 (p. 364). In the full paper a novel and rich approach for capturing affect, including beliefs, is described. This technique, the Experience Sampling Method (ESM), has been discussed in some detail by Csikszentmihalyi, Rathunde, and Whalen (1993) in their study of talented teenagers, but seems not to have been used before in mathematics education research. Briefly, on receipt of a signal, participants are requested to chart their daily activities, and reactions to those activities, through completion of a specially designed form, the Experience Sampling Form or ESF. Respondents have the opportunity to describe and comment, over an extended period of time, on the activities being undertaken as well as the attitudes, beliefs, emotions, and moods elicited by those activities. The sample comprised twenty mature age students, i.e., students aged 21 or over at the beginning of the academic year in which entry into the University was sought. As part of a more extensive set of data collected, participants were asked to carry an electronic beeper for six consecutive days. Six signals were sent between the hours of 7am and 10pm on week days and between 10am and 10pm on weekend days, with respondents expected to complete the ESFs within 30 minutes of receipt of the signal. The response rate of 81request of completion of at least four out of the six sheets each day and was assumed indicative of the groupas strong commitment to the research project. The activities of the students when beeped could be divided into eight major categories (study, paid work, relaxation, family, chores, transit, sleeping, eating) and reinforced information gained from other data we had gathered through interviews, observations and regular e-mail interactions. The ESF entries offered unique insights into the 1how and why1 of the participantsa activities, their beliefs about study, mathematical thinking and the intrinsic value of mathematics. They also revealed the extent to which the students valued mathematics and related studies compared to other interests and daily activities. References Csikszentmihalyi, M., Rathunde, K., & Whalen, S. (1993) Talented teenagers. Cambridge: University of Cambridge Press Schoenfeld, A H (1992) Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D A Grouws (Ed.) Handbook of Research on Mathematics Teaching and Learning (pp. 334-370) New York: MacMillan

S. LERMAN: Research on mathematics teachers' beliefs: a situated perspective

Much work has gone into the analysis of teachers' beliefs about mathematics, about mathematics education and the possible connections between them and it remains of great interest today. It has been argued that teachersa beliefs are critical factors determining how they teach. So-called mismatches between theories and practices have been discussed in the literature, although it is sometimes referred to as espoused and enacted theories of mathematics teaching. In her review of the research in this field, Thompson (1992, p. 138)

suggested that the relationship between teachersa conceptions of mathematics and their practice is complex and argued for viewing the relationship as a dialectic one. Teacher educators will be familiar with the problem that "Inservice teachers' resistance to change and preservice and beginning teachers' reversion to teaching styles similar to those their own teachers used are legendary" (Brown, Cooney & Jones, 1990, p. 649). It appears that courses do not provoke students to confront their naive notions of teaching mathematics. At the heart of the research on teachers' beliefs is the argument that teachersa or student teachers' beliefs and conceptions need to change for their teaching to change. Beliefs are taken to be an internal mental landscape that can be charted by suitable research instruments. They are assumed to be stable across the range of teaching sites and across the researcheras data collection sites but they are assumed to be amenable to change over time as a result of interventions or activities. I have some concerns about the first two of these assumptions and, as a consequence, wish to re-interpret the notion of 'change' as more elaborate than arising from a change in beliefs.
In my paper I propose that it is not the internal map of beliefs and awarenesses which changes but the identities of teachers, as models of mastery within communities are provided within the activities. These situations can be seen as productive of new elements of identities, or perhaps new identities, for the teachers. Some situations are particularly fruitful in teacher development, such as setting up on-going communities of teachers or teachers engaged in research together. Where they are not successful we may talk of students' identities as teachers not having developed. Similarly, so many preservice teacher education courses engage with students' identities as students on courses, and the same with the teachers in other situations to which Cooney and his colleagues refer, and do not impinge on their identities as teachers in classrooms. The expression of knowledge, beliefs or conceptions, however they are distinguished and by whatever research method they are elicited, are an expression of the person-in-context. References Brown, C., Cooney, T. A. & Jones, D. (1990) Mathematics Teacher Education. In W. R. Houston (Ed.) Handbook of Research on Teacher Education (639-656) New York: Macmillan. Thompson, A. G. (1992) Teachers' Beliefs and Conceptions: A Synthesis of the Research. In D. A. Grouws (Ed.) Handbook of Research on Mathematics Teaching and Learning. (pp. 127-146). New York: Macmillan.

S. LLINARES: Elementary teacher students' beliefs and learning to teach mathematics

For some time now it has been assumed that learning to teach is not a passive activity. The constructivist postulate states that the elementary teacher student constructs his/her new knowledge by taking as a reference his/her prior knowledge. On the other hand, situated learning perspectives argue that the context and the nature of the activities that the individual carries out form part of what is learnt. Learning to teach seen in this way is linked to the characteristics of the learner's ways of participation in learning environments and the development of identity becoming a membership of elementary teachers community. From this perspective, knowing is viewed as practices of communities and the abilities of individuals to participate in those practices. Lave & Wenger (1991) characterized learning of practices as processes of participation in which beginners are relatively peripheral in the activities of a community and as they become more experienced, their participation becomes more central. Fron this poitn of view, elementary teacher studentas learning to teach can be seen as movement from peripheral to fuller forms of praticipation in different aspects of mathematics teaching, and the learning to knowledgeable skills can be seen as coming to use knowledge base to teach (it would coincide with a involvement in a new practice of the community of elementary teachers). Since the activities, tasks and understanding do not exist in isolation, but that they are part of broader systems of relations in which they have meaning (included beliefs, ways of participation, ...) we looked at three types of activities in which teacher studentsa learning experiences are focused on meaningful setting of activity in which the contents of knowledge base for teaching mathematics are embeded: doing mathematics, analysing learning difficulties (pupilsa ways of thinking); and planning and mannaging the instruction. We used the theoretical constructs ways of participation and development of identity as two thinking tools to analysis the learning to teach in the above learning sites. In some aspects, the analysis help to illustrate the role played by the system of beliefs and understanding about it means to do mathematics, the role of representation modes in mathematics learning, the teacheras role in teaching and in the developing of pedagogical reasoning and instructional skills. Thus, systems beliefs and understandig of elementary teacher students appear to condition the ways of participation in learning environments and the development of identity of elementary teacher student as a elementary teacher.

G.M. LLOYG: Learning with and about mathematics curriculum: the role of teachersa conceptions

Ongoing reform efforts in the United States are based on views of mathematics, learning, and teaching that depart significantly from school mathematics traditions. Reforms aim to revise the conventional view of mathematics learning as the mastery of a fixed set of facts and procedures to more centrally locate the processes of investigation, sense-making, and communication in classroom activities. Bringing about such dramatic changes in mathematics instruction can be a daunting task for teachers. Perhaps the greatest obstacles for teachers is a lack of personal familiarity with mathematical problem-solving and sense-making - processes that most have never experienced themselves, as students or teachers. Even when teachersa efforts to change are supported by curriculum materials designed to aid in the enactment of reform visions, teachers struggle to bring about significant changes in classroom practice. Instructional reform is unlikely to take hold unless we can identify viable ways to encourage and enable teachers to make significant shifts in their conceptions. This paper draws attention to the role of teachersa conceptions in their experiences with reform-oriented curriculum materials. Of particular interest is what and how teachers learn from their experiences with reform-oriented curricula. There is a great diversity of possible experiences that teachers may have with innovative K-12 mathematics curricula. For instance, during inservice workshops or preservice courses at the university, teachers may work collaboratively 1as students1 on the mathematical lessons outlined in the materials. Doing so can offer teachers critical opportunities as learners because they can personally experience unfamiliar mathematics in novel ways. Another rich context for educative experiences with curricula is teachersa own classrooms. As teachers implement curriculum materials in their classrooms (or as student teachers), they may develop new mathematical and pedagogical skills on the basis of their design of lessons, interactions with students, use of technology, and so on. This inquiry about the nature of teachersa learning from experiences with innovative curriculum materials relies upon analysis of teachersa mathematical and pedagogical conceptions. Entwined within these conceptions are teachersa conceptions about mathematics curriculum. A better understanding of teachersa conceptions of mathematics curriculum is vital to the success of current reform efforts. Although textbooks have long held prominent roles in guiding practice in classrooms, we know surprisingly little about how teachersa conceptions of curriculum materials relate to their conceptions of mathematics, teaching, and learning, and how they develop during teacher education and school-based experiences.

D.B. MCLEOD: Mathematical beliefs and curriculum reform

During the last decade, efforts to change and improve the mathematics curriculum in the United States have been heavily influenced by the Curriculum and Evaluation Standards for School Mathematics, a 1989 publication of the National Council of Teachers of Mathematics. In this paper the views of various groups are examined, including especially the beliefs of parents and mathematicians who have opposed the NCTMas efforts to change the curriculum. The purpose of this paper is to investigate how beliefs about mathematics can influence the reactions of these groups to the NCTM Standards. Many parents hold the belief that mathematics is primarily about computation, especially the memorization of rules and procedures in arithmetic. For a sample of these traditional views of mathematics, see the web site of the group Mathematically Correct at http://ourworld.compuserve.com/homepages/mathman/. The material on the web site focuses on the importance of basic computational skills and emphasizes memorization of facts and procedures. Given their beliefs about mathematics, it is not surprising that these parents oppose NCTMas effort to emphasize mathematical problem solving, reasoning, and applications.
Mathematicians hold a variety of different beliefs about mathematics, and these differences appear to influence their opinions about the NCTM Standards. In the Standards 2000 project, an effort to revise the 1989 Standards, NCTM has gathered the opinions of leaders of the major mathematical organizations. The advice of these Association Review Groups is available through the NCTM Standards 2000 web site at
http://www.nctm.org/standards2000/. The differences among the mathematiciansa beliefs about school mathematics are quite pronounced. For example, the comments by the representatives of the Mathematical Association of America ranged from those who supported an emphasis on problem solving to those who wanted to maintain an emphasis on rote learning of skills. The use of calculators in school mathematics was also a very controversial topic. The mathematiciansa beliefs about mathematics are a useful guide to

understanding the sources of their opinions about efforts to change school mathematics. The comments from the MAA representatives often indicated some agreement with the NCTM Standards (e.g., the emphasis on applied mathematics), but would then point out how the NCTM documents did not handle the idea properly. As one leading critic has pointed out, some mathematicians believe that only research mathematicians are qualified to change the school mathematics curriculum.

P. OP T' EYNDE, E. DE CORTE & L. VERSCHAFFEL: Balancing between cognition and affect: Students' mathematics-related beliefs and their emotions during problem solving

Student's mathematics-related beliefs are situated at the intersection of the cognitive and the motivational, or better affective, domain. On the one hand, students' beliefs determine how one chooses to approach a problem and which techniques and strategies will be used (Schoenfeld, 1985a). On the other hand, it is argued that they provide an important part of the context within which emotional responses to mathematics develop (McLeod, 1992). Rarely scholars have addressed in their research this relation between students' mathematics-related beliefs and emotions experienced during problem solving in the classroom. We need to develop a better understanding of how mathematics-related beliefs determine the emotions they experience and the influence they have on students' problem solving behavior. This analysis of the relations between students' mathematics-related beliefs, their emotions, and their problem-solving behavior has become the focus of our research.
Taken into account the complexity of the phenomenon under study and the specific nature of our variables, we opted for a multiple case study, using questionnaires, documents, observations and interviews to gather data. We selected three different junior high classes in three different schools. Students of these classes (age 14) were presented a self-developed questionnaire on mathematics-related beliefs. Starting from existing questionnaires who usually measure only one kind of beliefs (e.g., Or beliefs about math, or motivational beliefs), we developed a more integrated instrument that asked students about their beliefs on mathematics education, on their self-related beliefs in relation to math, and on their beliefs about the social context in their specific class. Then we made a selection of two students out of each class based on their mathematics ability as evaluated by the teacher (one high achiever and one low achiever). They were asked to solve a complex realistic mathematical problem in class and had to fill in the first part of the 1On-line Motivation Questionnaire (OMQ)1 after they had skimmed it and before they actually started to work. Every student was asked to think aloud during the whole problem-solving process that was also videotaped. Immediately after finishing, the student accompanied the researcher to a room adjoining the classroom where a 3Video Based Stimulated Recall Interviewa took place. The results indicate that students experience different emotions during mathematical problem solving, negative and positive ones. The experience of a negative emotion always triggered students to redirect their behavior using alternative cognitive strategies or heuristics to find a way out of the problem. However, there are big differences in the

effectiveness and efficiency of the cognitive strategies used. The kind of emotions students experience during problem solving and their intensity appear to be determined by students' beliefs, and more specifically their beliefs about the self. Students' perceptions of this specific task in this specific context are closely related to these more general beliefs. Nevertheless they sometimes differ, due to the specific context. Although research shows that the influence of beliefs on students' problem solving is mediated through task-specific perceptions, it turns out that the more general beliefs also have a direct influence through the experienced emotions during problem solving. Students' beliefs about the self, and specifically their self-efficacy beliefs, seem to determine students' problem solving not only in an indirect, but also in a more direct way. Students' emotions during problem solving and their impact on their further behavior are a least partly a function of students' general beliefs about the self, independently of their task-specific perceptions.

E. PEHKONEN: Beliefs as obstacles for implementing an educational change in problem solving
¿From an international need for change, a framework of teacher change is discussed, emphasizing the complexity of change. The basic concepts in teachersa and pupilsa beliefs are described. The theoretical statements on beliefs as obstacles for an educational change are illustrated within the research program in Helsinki (1989-98) which aims for improvement in mathematics teaching through problem solving in the Finnish lower secondary school.

G. PHILIPPOU & C. CHRISTOU: Efficacy beliefs with respect to mathematics teaching

Several dimensions of the construct "mathematical beliefs" have recently been studied. Focusing on teachers' beliefs, apart from beliefs concerning the nature, the teaching and learning of mathematics, teaching efficacy beliefs constitute a decisive factor influencing the selection, organization and management of learning environments. Perceived self-efficacy beliefs with respect to teaching any subject can be defined as one's confidence in his/her capabilities to organize and orchestrate effective learning activities. As a part of an ongoing project we examined primary teachers efficacy beliefs with respect to teaching mathematics, through analyzing questionnaire data from 157 subjects and 18 semi-structured interviews. The questionnaire comprised of 28 five-point items, dealing with personal efficacy along the internal and external interpretation of learning control, the anxiety from mathematics teaching, the enjoyment, the school climate, the effectiveness of the preservice program, and beliefs about general teaching efficacy. In the interviews we encouraged teachers to describe their feelings concerning mathematics teaching and their views on the pre-service program they passed through. The subjects were primary school teachers, with a length of service non-exceeding 14 years, who graduated from the Pedagogical Academy (PA), the University of Cyprus (UC), and various Greek Universities (GU). Specifically 58.6graduates and 13.4The results showed a high level of teacher self-confidence in teaching mathematics. However, most teachers did not feel capable to control pupil's learning. The lowest efficacy level was found on the "pre-service mathematics program" and on the "school climate" dimensions. More than 80mathematically", "consult experienced colleagues", and they wouldn't "give away mathematics", if they could. On the other side, less than 50teachers felt efficient "to cover the subject matter", "to help the weak students", considered that "all students are teachable" and that "the pre-service program" was effective. The analysis showed significant differences among the sample groups on the general teaching efficacy dimension, on the pre-service program, and on the total scale with respect to years of experience. Specifically, UC graduates, more than the rest of participant groups, hold the belief that students are teachable (F = 3.150, 3, p = .027), showed higher level of efficacy beliefs on the general teaching efficacy, and were more satisfied with their pre-service program (F = 8.992, 3, p = .000). Significant differences were found among "age groups" on the total scale, indicating variation of efficacy beliefs by years of teaching experience. The means show that teachers' beliefs tend to get worst during the first period of professional life and improve subsequently through experience ('X1 = 3.59 e 'X2 = 3.37 e'X3 = 3.65). In the interviews teachers expressed views that affirm the results of the questionnaire analysis. In general, teachers feel quite competent to facilitate students' mathematical learning, though not so much the non-motivated students, they tend to be critical about the pre-service program, and they don't seem to approve the school climate. The differences among UC graduates and the graduates from other institutions were affirmed. Particularly highlighted were found the differences on the pre-service program. The results provide some support to the hypothesis that the pre-service program at the University of Cyprus is relatively succeeding its goals.

N.C. PRESMEG: Effects of beliefs about the nature of mathematics on semiosic chaining linking everyday and school mathematical practices.

This paper documents the role of graduate teachers' and high school students' ontological mathematical beliefs in a research project that investigated ways in which semiosic chaining might link mathematics in and out of school. A discrepancy was observed between high school students who believed that mathematics is "a bunch of numbers" and who could not see clear connections between school mathematics and their daily lives or cultural practices, and graduate students with broader ontological conceptions of mathematics that enabled them to construct such connections. This discrepancy caused the writer to search for theoretical frameworks to illuminate issues and enable construction of connections. Semiotics and the constraints and affordances of situated cognition theory were explored. Conclusions were that individual beliefs about the nature of mathematics and about constraints and affordances in learning were interrelated, and that these beliefs either facilitated or hindered the construction of connections between school mathematics and everyday practices.

D. TIROSH: Intuitive Beliefs, Formal Definitions and Undefined Operations: The Cases of Division by Zero

This paper describes a study that explores secondary school students' adherence to the numeric-answer belief, in two specific contexts: division of a non-zero number by zero, and division of zero by zero. Our aims were: (1) to examine whether secondary school students identify expressions involving division by zero as undefined, or tend to assign numerical values to them, (2) to study their justifications, and (3) to analyze the effects of age (grade) and level of achievement in mathematics on responses. A substantial number of the participating secondary school students argued, in line with the numeric-answer belief, that division by zero results in a number. Moreover, performance on division-by-zero tasks did not improve with age. Level of achievement in mathematics, however, was highly related to performance on tasks. Possible causes and the educational implications of these findings are discussed.

G. TARNER: Domain-specific beliefs and calculus - some theoretical remarks and phenomenological observations

It is central questions in the theory of beliefs whether there are internal structures within beliefs systems and by which elements they are organized. A paper by Green (1971) proposed to distinguish beliefs with respect to the categories 'logical' (primary versus derivative) as well 'psycho-logical' (central versus pheripheral). On the other hand, content-related aspects provide a different approach. One may speak of global beliefs, e.g. on mathematics, its learning and teaching, where on the other side numerous authors are interested in subject matter orientated beliefs. On the basis of six triple-sectioned experience reports on calculus lessons in school and at university the problem is researched as to which related domain-specific beliefs are articulated by post-undergraduate teacher students. It seems to be helpful to extend the just mentioned levels 'global beliefs' versus 'subject matter beliefs' to a third level, 'domain-specific beliefs'. Besides a content identification of students' beliefs, the question arises as to which quasi-logical relationship these global beliefs on mathematics stand using the terminology of Green. There are hints that global beliefs are of high dominance.

L. VERSCHAFFEL, B. GREEN & E. DE CORTE: Pupils' beliefs about the role of real-world knowledge in mathematical modelling of school arithmetic word problems

Some years ago Greer (1993) and Verschaffel, De Corte and Lasure (1994) provided evidence that after several years of traditional mathematics instruction children have developed a tendency to reduce mathematical modeling to selecting the correct formal-arithmetic operation with the numbers given in the problem, without seriously taking into account their common-sense knowledge and realistic considerations about the problem context. This evidence was obtained by means of a series of especially designed word problems with problematic modeling assumptions from a realistic point of view, administered in the context of a mathematical lesson. After having summarized these two initial studies, we briefly review a series of replication studies executed in different countries showing the omnipresence of this tendency among pupils. Then two related but different lines of follow-up studies are presented. While the first line of research investigated the effects of different forms of scaffolds added to the testing setting aimed at enhancing the mindfulness of studentsa approach when solving these problematic items, the second one looked at the effectiveness of attempts to increase the authenticity of the testing setting. After having discussed these empirical studies, the results are interpreted against the background of schooling in general, and the mathematics classroom in particular. The notion of "the game of word problems" is introduced to refer to the "hidden" rules and assumptions that need to be known and respected to make the game of word problems function efficiently. In this respect a study is reported which reveals that the strong tendency toward non-realistic mathematical modeling is found among (student-)teachers too. Afterwards two studies aimed at changing students' perceptions of word problem solving by taking a radical modelling perspective, are reported. The paper ends with some theoretical, methodological and instructional implications of the work reviewed.

S. VINNER: Beliefs we live by and quite often are even not aware of - their possible impact on teaching and learning mathematics

An attempt is made to deal with beliefs from a general perspective, namely, to analyze beliefs in human behavior and to view mathematical beliefs as a special case. The best domain to understand beliefs is religion. In religion beliefs are associated with rituals. A characterization of rituals in religious behavior is suggested. Then some activities in mathematical behavior are identified as mathematical rituals. It is claimed that in our cognition a ritual schema has been formed and it makes us behave accordingly in many situations. Among rituals which are common in mathematical behavior thee is the "invent a story for a given numerical behavior" - ritual, the proof ritual and more. It is claimed that not only students are involved in (meaningless) rituals in their mathematical behavior, also the academic community is involved in rituals. It is suggested that everybody will examines his actions and will try to find out whether they are (meaningless) rituals or similar concepts related to beliefs as dogmas, models or rhetoric.

S. WILSON: Mathematical and Pedagogical Conceptions of Secondary Teachers

The conceptions secondary teachers hold about mathematics and the teaching and learning of mathematics are important contextual factors influencing their practices. One prominent idea in discussions about teachersa conceptions is the role of authority in mathematics teaching and learning. Data from four studies of secondary teachers in the US illustrate the categories of Mathematical Authority and Pedagogical Authority. Mathematical Authority involves teachersa conceptions of mathematics. It is important for teachers to focus on mathematical content and how to make that content more accessible to students. An important component of teacher development often includes the realization of the power of moving away from an emphasis on procedures toward a greater emphasis on relational or conceptual understanding. Pedagogical Authority deals with teachersa conceptions of the teaching and learning of mathematics. To acknowledge and honor diverse ways of knowing and to allow students to build their own mathematical understandings through cooperative exploration also require substantial changes for many teachers. The difficulty of this kind of change may be related to teachersa underlying pedagogical orientations: many mathematics teachers see themselves as the ultimate arbiters of mathematical correctness and find it extremely difficult to share responsibility with their students.

E. YACKEL & C. RASMUSSEN: Beliefs and norms in the mathematics classroom//
The position that we set forth in this paper is that it is possible to develop ways to explain how changes in beliefs might be initiated and fostered in mathematics classrooms by coordinating sociological and psychological perspectives. In doing so, we are drawing the interpretive framework for analyzing classrooms that Cobb and Yackel have developed as a result of their work in elementary school mathematics classrooms. The framework coordinates both individual (psychological) and collective (sociological) perspectives. In outlining the interpretive framework, Cobb and Yackel refer to beliefs as individual constructs that are correlates of social constructs. For example, they take individual studentsa beliefs about their role and othersa roles and the general nature of mathematics in school (and at the university) as psychological correlates of classroom social norms and individual studentsa mathematical beliefs and values as psychological correlates of sociomathematical norms. Social and sociomathematical norms are sociological constructs. An important aspect of the framework is the interrelationship between beliefs and norms. Cobb and Yackel take norms and beliefs to be reflexively related. This means more than that each contributes to the constitution of the other. It literally means that one does not exist without the other. This interrelationship between beliefs and norms is critical because it provides a means for talking about changes in beliefs. Changes in beliefs occur concomitantly with the constitution of norms. In this paper we use examples from a university level differential equations class to clarify and illustrate the constructs within the framework. The examples demonstrate both the normative aspects of the classroom and the corresponding studentsa beliefs. In the differential equations classroom studied, studentsa mathematical beliefs changed dramatically over the course of the semester. The theoretical constructs of the interpretive framework are used to explain this change.
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