Mathematisches Forschungsinstitut Oberwolfach

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Discrete Geometry

28.5.-3.6.2000

The meeting was organized by Ulrich Brehm (TU Dresden), Jacob E. Goodman (City College, City University of New York), Richard M. Pollack (New York University) and Jörg M. Wills (Universität Siegen). It was attended by 45 mathematicians from 10 countries. It featured 11 survey lectures as well as 21 shorter talks. The conference attempted to cover the whole breadth of the current developments in Discrete Geometry. Some of the main topics that became apparent were

- triangulated surfaces and the face structure of polytopes,
- finite packings, parametric density and Wulff-shapes,
- packings, coverings and tilings (periodic and aperiodic),
- combinatorial geometry problems,
- applications of discrete geometry in computer science, biology and biochemistry.

One of the highlights was the announcement of a proof of the three-dimensional Hadwiger-Gohberg-Marcus covering conjecture by V. Boltyanski. The conference resulted in fruitful discussions in small groups and productive collaborations among the participants. The extended plenary problem session on tuesday evening (problems listed at the end of this volume) stimulated a lot of further discussions and activities.

Abstracts

Group Actions, Polyhedral Fundamental Domains and Links of Singularities

Ludwig Balke

Consider the following two data. First, you are given a 3–manifold M in terms of a polyhedron P together with sidepairings. Secondly, there is a function $f : M \longrightarrow \mathbb{C}$ such that $K := f^{-1}(0)$ is a knot or link in M and $\phi : M \setminus K \longrightarrow S^1, m \mapsto \frac{f(m)}{|f(m)|}$ is an open book fibration.

Assume that each fibre $F_t = f^{-1}(t)$ intersects the boundary of P nicely, i.e. in a graph whose complement is connected. This gives a good combinatorial description of f and its monodromy. Vice versa, such a description allows to (re)construct the polyhedron P and the side pairings.

Examples for this situation arise naturally in the study of fundamental domains for group actions which give links of certain isolated singularities of complex surfaces.

$0-1$ Polytopes with Superexponentially Many Facets

Imre Bárány

Answering a question of K. Fukuda and G. M. Ziegler we show that there exist n-dimensional $0-1$ polytopes with superexponentially many facets. We give a random construction. Writing K_N for the random $0-1$ polytope with N vertices (in R^n) the following holds in the range $\exp\log^2 n < N < \exp n/\log n$. The expected number of facets of K_N is at least $(c\log N)^{n/4}$, where c is a universal constant. In the proof, extensive use is made of a beautiful result of Dyer, Füredi, and McDiarmid (Random Structures and Algorithms, 1992).

This is joint work with Attila Pór.

Discrete Isoperimetry

Ulrich Betke

For the finite subsets of a lattice Λ we introduce an energy which somewhat generalizes the inner energies considered in crystallography. We define a notion of convexity on these subsets and say that a convex finite subset with cardinality i is an i–crystal, if its energy is minimal among the subsets with exactly i elements. We show that we obtain the shape of a crystal known from crystallography by taking the limit of the shapes of i–crystals rather than considering only special polytopal subsets as usually done in crystallography. Moreover we obtain a stability result. We use our results to compare the surface energy of crystallography to a related function studied in the theory of finite packings and find a rather different behaviour.

This is joint work with Martin Henk.

Covering a Square Ring with Strips

Andras Bezdek

Call a closed region between two parallel lines a strip. Let $w(K)$ be the width of the narrowest strip which covers a given convex region K. $w(K)$ is called the width of K. The well known Bang's theorem says that if a convex domain K is covered by a collection of strips, then the sum of the widths of the strips is at least $w(K)$. As a special case we have that if the unit square is covered by a finite collection of strips, then the sum of the widths of the strips is at least 1. We show that the sum remains at least 1 even if one needs to cover the remainder of the square after cutting an edge-parallel square hole of edge length $1-\frac{1}{\sqrt{2}}=0.29\ldots$. The proof utilizes lemmas of T.Bang (see reference). We prove similar statements for the regular $2n$-sided polygon ($n>1$). Showing analogue statements for general convex regions (in particular for a circle) remains a challenge.

Reference:

T.Bang, A solution of the plank problem, *Proc. American Math. Soc.*, **2** (1951) 990–993.

The Danzer-Grünbaum Theorem Revisited

Károly Bezdek

The following result that has been conjectured by Erdős and Klee is due to Danzer and Grünbaum: In the d-dimensional Euclidean space $\mathbb{E}^d$ there can be at most 2^d points such that all angles determined by any triple of points are less than or equal to $\frac{\pi}{2}$ In the first part of the talk we prove the following stronger version of Danzer-Grünbaum theorem. Let $f(\alpha)$ be the maximum number of vertices of a convex polyhedron in $\mathbb{E}^3$ such that all angles between adjacent edges of the polyhedron are less than or equal to α, where $0<\alpha<\pi$.

Theorem *For any $\frac{\pi}{2}\leq\alpha<\frac{2\pi}{3}$ we have that $f(\alpha)\leq\lfloor\frac{4\pi}{2\pi-3\alpha}\rfloor$.*

The main tool of the proof is the following spherical geometry version of Pál's theorem.

Lemma *The spherical area of any spherically convex domain of width $\omega\leq\frac{\pi}{2}$ on S^2 is at least as large as the spherical area of an equilateral triangle of width ω.*

In the second part of the talk we give a sharpening of Danzer-Grünbaum theorem in $\mathbb{E}^d$ for very large d under some metric conditions.

Geometry of the Space of Phylogenetic Trees

Louis J. Billera

We consider a continuous space which models the set of all phylogenetic trees having a fixed set of leaves. This space has a natural metric of nonpositive curvature, giving a way of measuring distance between phylogenetic trees and providing some procedures for averaging or combining several trees whose leaves are identical. This geometry also shows which trees appear within a fixed distance of a given tree and enables construction of convex hulls of a set of trees.

This geometric model of tree space provides a setting in which questions that have been posed by biologists and statisticians over the last decade can be approached in a systematic fashion. For example, it provides a justification for disregarding portions of a collection of trees that agree, thus simplifying the space in which comparisons are to be made.

This is joint work with Susan Holmes (Stanford) and Karen Vogtmann (Cornell).

On Recent Contributions to the Theory of Oriented Matroids

Jürgen Bokowski

For uniform oriented matroids $\mathcal{M}$ with n elements, there is in the realizable case a sharp lower bound $L_r(n)$ for the number $mut(\mathcal{M})$ of mutations of $\mathcal{M}$: $L_r(n) = n \leq mut(\mathcal{M})$ due to Shannon. Finding a sharp lower bound $L(n) \leq mut(\mathcal{M})$ in the non-realizable case is an open problem for rank $d \geq 4$. Las Vergnas conjectured $1 \leq L(n)$. In the rank 4 case Richter-Gebert showed $L(4k) \leq 3k + 1$ for $k \geq 2$. We can show $1 \leq L(n)$ for $n < 13$ and $L(7k + c) \leq 5k + c$ for all integers $k \geq 0$ and $c \geq 4$ (joint work with H.Rohlfs).

The Folkman-Lawrence topological representation theorem for oriented matroids uses methods that need two chapters in the oriented matroid book by Björner et al. We (joint work with I.Streinu and S.Mock) present an elementary proof of the Folkman-Lawrence topological representation theorem for oriented matroids of rank 3 which is promising also for the general case.

Solution to the Illumination Problem of Three-Dimensional Compact, Convex Bodies

Vladimir Boltyanski

Let $M \subset R^n$ be a compact, convex body. Denote by $c(M)$ the minimal number of nonzero vectors, whose directions illuminate the boundary of M. In 1957, I. Gohberg and A. Markus proved for $n = 2$ that $c(M) = 4$ as M is a parallelogram and $c(M) = 3$ otherwise. (By particular conditions in the USSR, their article was published only in 1960.) Evidently, $c(M) = 2^n$ for n-dimensional parallelotope. By this, Gohberg and Markus formulated (in other terms) the following

CONJECTURE. $c(M) \leq 2^n$ *for any compact, convex body* $M \subset R^n$*, the equality being hold only for parallelotopes.*

The same conjecture was independently formulated by H. Hadwiger in 1957.

M.Lassak proved $c(M) \leq 8$ for every *centrally symmetric* compact, convex body $M \subset R^3$. H. Martini proved $c(M) \leq \frac{3}{4} \cdot 2^n$ for every n-dimensional zonotope M distinct from a parallelotope. V. Boltyanski and P. Soltan proved that Martini's estimate holds for every n-dimensional zonoid. Later Boltyanski proved that Martini's estimate holds for every n-dimensional belt body.

Using the functional *md* introduced by Boltyanski in 1976, we formulate the following

CONJECTURE. $c(M) \leq 2^n - 2^{n-m}$ *for every compact, convex body* $M \subset R^n$ *with* $\mathrm{md}M = m \geq 2$.

Justifying this conjecture, we obtain positive confitmation of the Gohberg-Markus-Hadwiger conjecture. If $m = 2$, the above conjecture means that for every compact, convex body $M \subset R^n$ with md$M = 2$ Martini's estimate holds. This result is proved by the speaker. This implies that to solve the illumination problem for three-dimensional bodies, it is sufficient to establish $c(M) \leq 7$ for every compact, convex body $M \subset R^3$ with md$M = 3$. This assertion also is proved by the speaker.

Parametric Density — an Overview

Károly Böröczky, Jr

The notion of parametric density is relatively new, it was introduced by Jörg M. Wills at the early 90's. The notion connects classical problems (for example, László Fejes Tóth's Sausage Conjecture) with applications (for example, the Wulff shape for crystals and quasi–crystals), and gives a clear explanation for the shape of various optimal finite arrangements. The talk introduces the roots of the theory, summarizes the most important results, obtained by U. Betke, M. Henk, U. Schnell, J.M. Wills and myself. Many yet open problems are proposed, especially for finite coverings.

Combinatorial Geometry Problems with Pattern Matching Applications

Peter Braß

The question for the maximum number of unit distances among n points in the plane is a typical extremal problem of combinatorial geometry. Such problems have been much studied, since they are easy to explain, and for the small cases also easy to solve, but turn out to be quite difficult in general, and related to a number of questions in graph theory, geometry and number theory. It recently became apparent that such problems also have applications, some of them related to pattern matching. This connection appears, since the most difficult sets for those pattern matching tasks are such sets in which some simple fragments of the pattern (like a unit-distance pair or a triangle) occurs very often; thus for the analysis of pattern matching algorithms it becomes necessary to bound the number of occurences of these fragments. So the classical 'number of unit distances'-bound of $O(n^{\frac{4}{3}})$ is in an effective version the basis of the currently best algorithm for the detection of congruent subsets in a planar pointset. We present a new bound of $O(n^{\frac{7}{4}}\beta(n))$ for the number of congruent triangles among n points in three-dimensional space, which gives the currently best algorithm for three-dimensional congruent subset detection, and survey some similar extremal problems motivated by pattern matching applications, among the question for the maximum number of empty congruent triangles among n points in the plane, which we conjecture to be $O(n)$.

Minimal Simplicial Dissections and Triangulations of 3-Polytopes

Ulrich Brehm

When considering minimal triangulations of 3-polytopes the following natural questions occur:

- Can the minimal number of simplices be reduced if additional interior vertices are allowed?
- Can a simplicial dissection have fewer vertices than a minimal triangulation?
- Does the minimal number of simplices depend on the realization of the polytope?

All these questions have an affirmative answer (joint work with A. Below, J. De Loera, F. Richter-Gebert).

Moreover, when allowing additional vertices the minimal number of simplices for a triangulation is not even an invariant of the chirotope determined by the given vertices.

An Inflation-Species of Planar Triangular Tilings, which is not Repetitive

Ludwig W. Danzer

It has been conjectured that an inflation-species of tilings with the following properties

(D) the inflation posseses a *unique* inverse,

(M) the protoset of tiles is *minimal* (with respect to the inflation), and

(F) the protoset is *finite* with respect to translations (i.e. no tile occcurs in infinitely many orientations)

necessarily is *repetitive*, if not even linearly repetitive. The following counterexample will be presented and explained.

Let A be the rectangular triangle with edge lengths 1, η and η^4. With $x := \eta^2$ we have $x^4 - x - 1 = 0$.

Define $B := \eta A$, $C := \eta B$, $D := \eta C$, then $\eta D = \phi(A) \cup \psi(B)$, where ϕ and ψ are euclidean isometries. Thus we can define an inflation by $\text{infl}(A) := B, \ldots, \text{infl}(D) := \phi(A) \cup \psi(B)$. Then the species

$$\mathcal{S} := \mathcal{S}\Big(\{A, B, C, D\}, \text{infl}\Big)$$

satisfies (D), (M) and (F), but is not of locally finite complexity. The reason is: The circles of radius x^4 centered at the midpoints of the hypotenuses of the supertiles $\text{infl}^n(A)$ contain infinitely many pairwise incongruent clusters of tiles. This implies that $\mathcal{S}$ is not repetitive.

Nevertheless for every cluster $\mathcal{C}$, which occurs in $\mathcal{S}$, there is a radius r such that in every tiling belonging to $\mathcal{S}$ in every r-circle a translate of $\mathcal{C}$ can be found ("weak repetitiveness").

The proof requires detailed considerations of the eigenvalues and eigenspaces in $\mathbb{C}^4$ of the inflation-matrix (whose characteristic polynomial is of course $x^4 - x - 1$). Presumably other equations of type $x^n - x^m - 1 = 0$ will lead to similar examples, provided the largest root is not a PV-number. In particular, this is the case for $n = 5$, $m = 3$, and for $n = 6, 7, 8$ with $m = n - 1$.

Folding and Unfolding Linkages, Paper, and Polyhedra

Erik Demaine

I will discuss several recent results in the theme of the title. For linkages, my focus will be on the latest result: any simple polygon can be convexified while maintaining all the edge lengths and never crossing any edges (joint work with Robert Connelly and Günter Rote). For paper, I will describe two results: (1) every polyhedron can be folded/wrapped out of a large enough piece of paper (joint work with Martin Demaine and Joseph Mitchell); and (2) folding a piece of paper flat and making one complete straight cut suffices to make any embedded planar graph of cuts (joint work with Martin Demaine and Anna Lubiw, and with Marshall Bern, David Eppstein, and Barry Hayes). Finally, for polyhedra, I will describe two bodies of research: (1) unfolding nonconvex polyhedra, in particular a simplicial polyhedron that cannot be cut along its edges and unfolded into a simple planar polygon (joint work with Marshall Bern, David Eppstein, Eric Kuo, Andrea Mantler, and Jack Snoeyink); and (2) folding a polygon and gluing together its boundary to form a convex polyhedron (joint work with Martin Demaine, Anna Lubiw, and Joseph O'Rourke).

Topological Persistence and Simplification

Herbert Edelsbrunner

We formalize a notion of topological persistence within the framework of filtrations, which are histories of growing simplicial complexes. We classify a topological change that happens during growth as either a feature or noise depending on its life-time or persistence within the filtration. We give fast algorithms for computing persistence and experimental evidence for their speed and utility.

(collaboration with David Letscher and Afra Zomorodian)

On the Moment Theorem

Gábor Fejes Tóth

For a domain D, a point p and a function f the integral

$$M_f(D, p) = \int_D f(px)dx$$

is called the *moment* of D with respect to p taken with the function f. Here px denotes the distance of $x$to p. The Moment Theorem of László Fejes Tóth states the following:

Let H be a convex polygon in $\mathbb{E}^2$ with at most six sides and f a non-increasing function defined for non negative reals. Let $p_1, \ldots, p_n$ be distinct points and let D_i be the Dirichlet cell of p_i relative to H. Then we have

$$\sum_{i=1}^{n} M_f(D_i, p_i) \leq n M_f(H_n, o),$$

where H_n is a regular hexagon of area $a(H_n) = a(H)/n$ centered at o.

We establish a stability criterion to this theorem and also extend it to the case when $n \geq 2$ and H is an arbitrary convex body. The problem of finding a stability version of the Moment Theorem was raised by Peter Gruber, who independently proved a theorem analogous to mine.

On Helly Numbers for Hyperplane Transversals to Convex Sets

Jacob E. Goodman

We present three recent results on Hadwiger's problem of finding the Helly number for line transversals to disjoint unit disks in the plane, and about its generalizations, both to higher dimensions and to arbitrary compact convex sets. One of these corrects a 40-year old error, while another constitutes the first Helly-type theorem known for hyperplane transversals to compact convex sets of arbitrary shape in dimension greater than two. This is joint work with Boris Aronov, Richard Pollack, and Rephael Wenger.

Let, as usual, $T(k)$ (resp. T) denote the assertion that every k (resp. all) of a family of given convex sets have a common transversal of a given dimension, and let the minimum k for which $T(k) \Rightarrow T$ be called the *Helly number* of such a family. We prove

(1) There exist arbitrarily large collections of disjoint unit disks in the plane for which $T(4)$ does not imply T for line transversals, contradicting a claim to the contrary going back to 1958;

(2) The Helly number for hyperplane transversals to collections of $d+3$ or more separated unit balls in $\mathbb{R}^d$ is at least $d+3$;

(3) The Helly number for hyperplane transversals to finite collections of sufficiently many ϵ-separated compact convex sets in $\mathbb{R}^d$ is at most $2d+2$.

Here, a finite collection of compact convex sets in $\mathbb{R}^d$ is "ϵ-separated" if any k of the sets can be separated from any other $d-k$ of them by a hyperplane whose distance from each is at least $\epsilon D/2$, where D is the maximum of the diameters of all the sets in the collection; the size of the collection needed depends on ϵ as well as on the dimension.

A Conjecture of J.M. Wills and View-Obstruction

Rajinder Hans-Gill

Wills (1968) considered the function

$$\chi(n) := \inf_{k_1,\ldots,k_n\in\mathbb{N}} \max_{x\in[0,1]} \min_{1\le i\le n} \|k_i x\|$$

where the infimum is taken over n-tuples of positive integers $(k_i)_{i=1}^n$, and $\|\cdot\|$ denotes the distance to the nearest integer.

Wills showed that $\chi(1) = \frac{1}{2}$, $\chi(2) = \frac{1}{3}$ and $\frac{1}{2n} \le \chi(n) \le \frac{1}{n+1}$ for all n. He conjectured that $\chi(n) = \frac{1}{n+1}$.

Several proofs are now known for $n = 2, 3, 4$. Markoff type chains of isolated extreme values have been obtained for $n = 2, 3$. Several authors have made contributions: Wills, Betke,Cusick, Pomerance, Chen, Dumir, Hans-Gill, Bennia, Goddyn, Gvozdjak, Sebö and Tarsi. The case $n = 5$ has recently been completed by Bohman, Holzman and Kleitman. The conjecture is still open for $n \ge 6$. Some equivalent formulations of the problem are

1. View-obstruction problem for Boxes.
2. Billiard ball problem for special paths
3. Lonely runner problem

The view-obstruction problem was formulated by T.W. Cusick. This formulation has the advantage that the problem can be formulated also for other sets. It is easy to solve for domains in two dimensions. For spheres it has been solved for dimension ≤ 5.

Tight Triangulations

Wolfgang Kühnel

A triangulation of a manifold M is called a *tight triangulation* if the span A of any subset of vertices is topologically essential in M, i.e., if the induced homomorphism $H_*(A) \to H_*(M)$ in homology is injective. Tightness is a generalization of convexity, and the tightness of a triangulation is a fairly restrictive property. For 2-manifolds (without boundary) it is equivalent to the completeness of the edge graph. We give a review on all known examples of tight triangulations in dimension $d \ge 3$. Altogether, six new examples of tight triangulations are presented, a 15-vertex triangulation of the non-orientable 4-manifold $(S^3 \underline{\times} S^1)\# 5(\mathbf{CP}^2)$ admitting a vertex-transitive group action, furthermore a 13-vertex triangulation of the simply connected homogeneous 5-manifold $\mathbf{SU(3)/SO(3)}$ with a vertex-transitive action, two non-symmetric 12-vertex triangulations of $S^3 \times S^2$, and two non-symmetric triangulations of $S^3 \times S^3$ on 13 vertices.

Reference:
W.Kühnel and F.H.Lutz: A census of tight triangulations. *Periodica Math. Hung.* **39** (2000), to appear

(joint work with Frank H. Lutz)

Knots in Minimal Trees

Wlodzimierz Kuperberg

Given a finite set of points, S, a minimal tree spanned by S is a connected graph containing S whose total length (i.e. the sum of lengths of its edges) is smallest possible. If S lies on the boundary of the unit ball in $\mathbb{R}^3$, and if T is a minimal tree spanned by S, then $T-S$ lies in the interior of the ball. We say that T is unknotted if there exists a topological disk properly embedded in the ball and containing T, otherwise T is knotted. Answering a question of Michael Freedman (Can a minimal tree spanned by a set lying on the boundary of the ball in $\mathbb{R}^3$ be knotted?), Krystyna Kuperberg constructed a spanning set whose minimal tree contains a trefoil-knotted arc (see `http://front.math.ucdavis.edu/math.MG/9806080` for a preprint). We investigate the problem of knotted minimal trees further, and discover that, besides the trefoil, certain other types of knots are possible in such minimal trees. Also, we consider similar problems for minimal trees spanned by a subset of a convex surface other than a sphere.

(joint work with Krystyna Kuperberg)

Apollonian Circle Packings

Jeffrey C. Lagarias

An Apollonian circle packing is an infinite packing of circles in the plane generated from a Descartes configuration (four mutually touching circles) by a group of Möbius transformations consisting of inversions in the circles passing through each triple of tangent points. Some Apollonian packings have the special property that all circle curvatures are integers, and the curvature $\times$ center of each circle has integer coordinates. We call these strongly integral packings. The existence of integer curvatures is explained by the Descartes circle theorem (first stated in 1638) which says that the curvatures $c_i = \frac{1}{r_i}$ of the circles in a Descartes configuration satisfy $(c_1^2+c_2^2+c_3^2+c_4^2) = \frac{1}{2}(c_1+c_2+c_3+c_4)^2$. If one starts with an integral solution to this equation, integrality of curvatures is preserved under the action of the group above. We show that Descartes' theorem can be generalized to similar equations involving curvature $\times$ center, which explains strong integrality. We characterize Descartes configurations in a coordinate system involving the isochronous Lorentz group $O^{\uparrow}(3,1,\mathbb{R})$, and give generalizations valid for n-dimensional Euclidean, spherical and hyperbolic space. In this coordinate system Apollonian packings are expressed in terms of the action of a discrete subroup of integer matrices, called the Apollonian group. We give a geometric interpretation of the generators of this group, and extend it to a super-Apollonian group, which is a Coxeter group, and of finite index in $O^{\uparrow}(3,1,\mathbb{Z})$. These results also have n-dimensional analogues.

(This is joint work with R. L. Graham, C. Mallows, A. Wilks (AT&T Labs) and C. Yan (Texas A&M.)

Squeezed 2- and 3-Spheres are Hamiltonian

Carl W. Lee

We consider the dual graphs to Kalai's squeezed spheres, where vertices of the graph correspond to facets of the sphere, and edges of the graph correspond to adjacent facets. We show that squeezed 2- and 3-spheres yield Hamiltonian dual graphs. As one consequence, no obstacle to the Hamiltonicity of a simple 4-polytope can be deduced from its f-vector alone.

(joint work with Robert Hebble, Morehead State University)

Simplicial decompositions of convex polytopes

Jesus De Loera

A **cover** of a convex d-polytope is a finite collection of d-simplices whose union is the whole polytope. The vertices of each member of the collection are vertices of the polytope. A cover is **irreducible** if after the removal of any simplex from the collection it is not a cover anymore. A **dissection** of d-polytope is an irreducible cover with the additional conditional that the interiors are disjoint. A **triangulation** is a dissection the additional condition that any pair of faces intersect in a common face, i.e. it is a simplicial complex. The **size** of a cover (dissection, triangulation) is the number of members it has. A cover is **maximal** if it is largest possible. A cover is **minimal** if it is smallest possible. Maximal and minimal decompositions are not unique.

There exist families of convex 3-polytopes such that

1. Minimal triangulations (or dissection) larger than minimal covers. The difference of size can be linear on the number of vertices.

2. Maximal triangulations (or dissection) smaller than maximal covers. The difference of size can be quadratic on the number of vertices.

3. The above statements can be repeated for triangulations and dissections.

4. For some Lattice polytopes the properties (1) and (2) happen simultaneously.

5. When fixing a combinatorial type of polytope with n vertices, the sizes of triangulations vary greatly. An important example.
   regular d-cube, max triangulation: $d!$
   Klee-Minty d-cube, max triangulation $>$ $c^d d!$, for $c > 1$.

These results are joint with Below, Brehm, Richter-Gebert, Santos and Takeuchi.

Some recent results on geometric graphs

Horst Martini

A geometric graph any two edges of which intersect is said to be an *intersector*. A useful generalization of intersectors is given by the notion of *successors* (i.e. by geometric graphs without isolated vertices where the neighboring edges of every vertex form a convex star with edges giving a succession regarding angle ordering, cf. [KM1]). This generalization keeps essential properties and thus gives better insight in the structure of intersectors on the other hand, but allows also to obtain new results (not true in the restricted category of intersectors) with applications in different fields of planar geometry. These applications are combinatorial and metrical in nature, and they refer to

- *isoperimetric problems* related to planar sets of constant width (unified approach to the theorems of Blaschke-Lebesgue and Firey-Sallee, cf. [KM2]),
- planar sets S of diameter h having the *weak circular intersection property* (i.e. , sets with the property that the intersection of all unit discs with centers from S is a set of constant width h, see [KM3]),
- time optimal constructions of *Releaux polygons*, cf. [KMW]

References:
[KM1] Y. Kupitz, H. Martini: From intersectors to successors. *Graphs and Combinatorics*, to appear.
[KM2] Y. Kupitz, H. Martini: On the isoperimetric inequalities for Releaux polygons. *Journal of Geometry*, to appear.
[KM3] Y. Kupitz, H. Martini: On the weak circular intersection property. *Studia Sci. Math. Hungar.*, to appear.
[KMW] Y. Kupitz, H. Martini, B. Wegner: A linear-time construction of Releaux polygons. *Beiträge zur Algebra und Geometrie* **37** (1996), 415–427.

Topological Aspects in the Theory of Transversals

Luis Montejano Peimbert

We would like to emphasize the idea that transversals, as a subset of a Grassmanian manifold, should be studied topologically.

Let $F = \{A^0, ..., A^r\}$ be a family of convex sets in $\mathbb{R}^n$ and let $T_m(F)$, the space of transversals of F, be the subspace of the Grassmannian $G(n,m)$ of m-planes in $\mathbb{R}^n$ that intersect all members of F.

Our first purpose is to study the homotopy type of $T_m(F)$ through C_r^m, the polyhedron of configurations of $(r+1)$ points in $\mathbb{R}^m$. In particular, if $r = m+1$ and $T_{m-1}(F) = \phi$, it is possible to calculate the homotopy type of $T_m(F)$ through the finite set of all possible order types achieved by the m-transversals when they intersect F. It is also possible to prove that the set of all m-transversals of $T_m(F)$ that intersect F with a prescribed order type is a contractible space.

Of course these theorems are false when $T_{m-1}(F) \neq \phi$ or if we consider m-transversals of a family of $(r+1)$ convex sets with $r > m+1$.

Our next purpose is to study Helly type theorems for transversals in the following spirit:

It is natural to expect generalizations of Helly's Theorem about families of convex sets, replacing the concept of intersection points (0-transversals) by the concept of k-planes that intersect all the convex sets (k-transversals). Let us consider a family F of compact convex sets, our philosophy is that if subfamilies of F with few members have enough transversals of small dimension, then the whole family F has many transversals of a fixed dimension.

We shall measure the size of the set of transversals according with their topological complexity inside the corresponding Grassmannian manifold. That is, if X is a set of n-planes in $\mathbb{R}^{n+k}$, we say that $\mu(X) \geq \lambda$ if X has "topologically" as many n-planes as the set of all n-planes through the origin in $\mathbb{R}^{n+\lambda}$. Our main result states that the set of n-transversals of a family F of compact convex sets in $\mathbb{R}^{n+k}$ is greater or equal than k if and only if for every subfamily F' of F with $k+2$ members, the set of λ-transversals of F' is also greather or equal than k.

Four degrees of separation

János Pach

J. Urrutia asked the following question. Given a family of pairwise disjoint compact convex sets in the plane (on a sheet of glass), is it true that one can always separate from one another a constant fraction of them using edge-to-edge straight-line cuts? We answer this question in the negative, and establish lower and upper bounds for the number of separable sets. In particular, we show:

Theorem 1: Any family $\mathcal{F}$ of n pairwise disjoint convex polygons in the plane has at least $n^{\frac{1}{3}}$ separable members, and a subfamily with this property can be constructed in $O(N + n\log n)$ time, where N is the total number of sides of the members of $\mathcal{F}$.

A set is calles ε-fat if the ratio of its inradius and circumradius is at least ε. The variance of a family is the ratio of the circumradius of the largest and smallest member.

Theorem 2: For any $\varepsilon > 0$ there is a constant c_ε with the following property: Any family of n pairwise disjoint compact convex ε-fat sets in the plane contains at least $c_\varepsilon n^{\frac{\log\log V}{\log V}}$ separable members, where V denotes the variance of that family.

This is joint work with Gábor Tardos.

New results about circles in the plane

Rom Pinchasi

Let $\mathcal{C}$ be a family of (at least 5) unit circles in the plane. In Ascona Conference (1999) A. Bezdek conjectured that if the circles in $\mathcal{C}$ are pairwise intersecting then there must exist an intersection point through which exactly two circles pass. We present a prove to bezdek's conjecture. Together with Noga Alon, Hagit Last, and Micha Sharir, we consider families of pairwise intersecting circles of arbitrary radii and show that if $\mathcal{C}$ is large enough then there always exists an intersection point through which at most 3 circles pass, unless $\mathcal{C}$ is a pencil, i.e., all the circles in $\mathcal{C}$ pass through two given points.

We also derive improved upper bounds on the number of cells with two edges in the planar arrangement of the circles in $\mathcal{C}$. In particular we show that if $\mathcal{C}$ is a family of pairwise intersecting circles then the number of those cells is linear in the size of $\mathcal{C}$.

Incidence theorems on manifolds

Jürgen Richter-Gebert

We focus on structural aspects of algebraic proofs for incidence theorems. Several classical approaches to proving incidence theorems by algebraic methods are compared and it is shown that all of them are essentially equivalent. A special role is played by proofs that are generated by joining together many distinct copies of Ceva's or Menelaus' Theorem. These proofs can be dircetly associated to an underlying cycle structure. The fact that one "indeed has a proof" corresponds to the fact that the cycle has no boundary. In many cases this point of view provides additional structural insight. The correspondence between cycles and other types of algebraic proofs (like the "Area Method", or "binomial final polynomials") makes use of essential facts that occur in the theory of *Tutte Groups for matroids* that was introduced by Dress and Wenzel in 1984. As applications of these new structural insights one can, for instance, generate a complete classification of liftable rhombic tilings with three directions.

Infinitesimal Unfoldings and Sibson's Area-Stealing Formula

Günter Rote

It is an outstanding open problem whether every three-dimensional convex polytope can be unfolded into the plane without overlap by cutting along some edges while keeping the rest of the surface connected. Together with Eric Demaine, we have proposed a simpler model, where we want to unfold an "infinitesimally flat" patch of a polytope surface, by introducing a scaling parameter that makes the surface flatter and flatter as it approaches zero. This model is easier to analyze, and we can give a characterization of the cut graphs which lead to overlapping unfoldings when the flatness parameter is arbitrarily close to 0.

While this approach has so far not lead to a resolution of the unfolding question, the computations that were necessary to work out how a polytope would unfold have produced a new proof of *Sibson's area-stealing formula* [1980] for Voronoi diagrams, or more generally, for power diagrams (regular polyhedral subdivisions). The area-stealing formula states that a point P within the convex hull of some other points is the weighted average of those points, where the weight of point Q is proportional to the area of P's region in the Voronoi diagram that is "stolen" from Q's region when P is added to the point set. This formula has applications in surface interpolation.

After establishing the correspondence with infinitesimal unfoldings, the area-stealing formula becomes immediately obvious, without any further computation. In this correspondence, there is some similarity to the definition of the *Steiner point* of a convex polytope.

We wonder if this relation points to generalizations of the area-stealing formula to different settings.

Some applications of the volume formula for polyhedra

Idjad Kh. Sabitov

For the triangle a formula expressing the area in terms of the sidelengths was already found by Heron:

$$A^2 = \frac{1}{16}(2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4).$$

A similar, but much more complicated, formula for the volume of a threedimensional simplex was found by Tartaglia (1561) and Euler (1752), which expressed the squared volume as a polynomial of the squared edge-lengths of the simplex. This is a special case of the following theorem (S. 1996):

Theorem: For any orientable polyhedron P in $\mathbb{R}^3$ with triangular faces and given lengths of edges there is a polynomial Q with the properties

- the coefficients of Q are polynomials (over rational numbers) in the edge-lengths of P, the polynomials being determined by the combinatorial structure of P, and
- the squared volume V^2 is a zero of the polynomial: $Q(V^2) = 0$.

We further discuss some corollaries of this result, its algebraic meaning, the canonical volume polynomial, examples, isometric realizations and the special cases of octahedra with symmetries.

Small protosets and matching conditions (and their complexity)

Peter Schmitt

A basic question in the theory of tilings is the following:

Given a set of bodies (called *prototiles*). How can they (more precisely, congruent copies) be used to tile space?

The set of all distinct (i.e., incongruent) tilings admitted by a set of prototiles is often called the *species* (determined by the *protoset*). It usually will be empty, or contain uncountably many tilings. However, it also may be finite, or countably infinite. Moreover, a more detailed classification of the species, e.g., according to symmetry properties of its members, is of interest. I call this, i.e., the structure of the species, the *versatility* of the protoset (or the species).

A special case (related to aperiodicity) is the classification according to periodicity properties: I am looking for small protosets which realize a given versatility vector (v_k) where $v_k(P)$ is the number of all tilings which have translations in k independent directions among their symmetries.

It seems that, for a single prototile, only sporadic examples exist. Pairs of prototiles can be quite versatile, and three prototiles – two large and one small shape – are sufficient to construct examples of many (most) types of versatility. In space, it is possible to reduce this to one large and one small prototile for many cases.

These general examples use tiles of complex shape. Using other types of matching rules (like an atlas of vertex figures) it is possible to use simpler shapes, but only in exchange for complicated rules.

On Finite Lattice Coverings

Uwe Schnell

Optimal finite lattice packings measured by the parametric density lead to the socalled Wulff–shape which is known from crystallography ([BB], [W], [S1], [S2]). A new approach to coverings based on the parametric density was given in [BHW]. The classical density corresponds to the case when the parameter ϱ is 0. This was investigated under various aspects by Bambah, Rogers, Woods, Zassenhaus, Gritzmann and G. Fejes Tóth. Here our question is whether one can give detailed information on the (asymptotic) shape of thinnest lattice coverings. We present results from [MS].

We consider finite lattice coverings of strictly convex bodies K. For planar centrally symmetric K we characterize the finite arrangements C_n such that $\operatorname{conv} C_n \subset C_n + K$, where C_n is a subset of a covering lattice for K (which satisfies certain conditions). We prove that for a fixed lattice the optimal arrangement (measured by the parametric density) is either a sausage, or a double sausage or a Wulff–shape (asymptotically) depending on the parameter. This shows that the Wulff–shape plays an important role for packings as well as for coverings. Further we give a version of this result for variable lattices. We characterize the set of covering lattices for the Euclidean d–ball with the property that the optimal arrangement is a sausage (for sufficiently large parameter).

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Locally unitary groups generated by involutory reflections

Egon Schulte

We discuss complex groups G generated by n involutory reflections that preserve a hermitian form. Such a group G is called *locally unitary* if each subgroup generated by $n-1$ of the generators is a finite unitary reflection group. These groups naturally arise in the enumeration of abstract regular polytopes whose facets or vertex-figures are toroidal. This is joint work with Peter McMullen.

The k-set problem

Micha Sharir

Let S be a set of n points in $\mathbb{R}^d$ in general position, and let $0 < k < n$ be an integer. A subset $S' \subset S$ is called a k-set if $|S'| = k$ and S' can be separated from $S \setminus S'$ by a hyperplane. Let $f_k^{(d)}(n)$ denote the maximum number of k-sets for any set S of n points in d-space. The problem of obtaining sharp bounds for these quantities have been posed by Erdős, Lovász and others around 1970, and is still far from being solved, even in the plane. In the talk I will discuss the problem, show its connection to various problems in computational and combinatorial geometry, involving arrangements of lines, hyperplanes, and other surfaces, and review the recent progress that has been made. The main topic in the talk is a very recent improvement of the upper bound on $F_k^{(3)}(n)$ to $O(nk^{3/2})$, improving the previous bound of $O(nk^{5/3})$. (Joint work with Shakhar Smorodinsky and Gábor Tardos.)

Finite edge-to-edge tilings by convex polygons

Geoffrey C. Shephard

A polygon Q is said to be tiled by a finite number r of polygons $P_1 \ldots, P_r$ if the interiors of these polygons P_i are pairwise disjoint, and the union of these polygons, together with their interiors, is the polygon Q and its interior. (A polygon is a piecewise closed simple curve in the plane.) A tiling of a convex m-gon Q by convex n-gons P_i is said to be edge-to-edge if

(a) for any two polygons of the polygons P_i, if their intersection is a line segment, then this segment is a side of each, and

(b) each side of Q is a side of one of the polygons P_i.

An edge-to-edge tiling of a convex m-gon by r convex n-gons is said to be of type $< m, n, r >$ and the problem is to determine all possible types of tilings. This is completely solved by:
Main Theorem: Tilings of types $< m, n, r >$ exist for all $m \geq 3, n \geq 3$ and $r \geq 1$, if and only if these integers satisfy the relations $m \equiv nr (\bmod 2)$, $3 \leq m \leq (n-2)r + 2$, and

$$m^2 \geq r(n-2)((n-6)r + 12) - 4(n-3) \tag{1}$$

except there is no tiling of type $< 3, 5, 13 >$.

It is remarkable that the same inequality (1) holds for all n in spite of the fact that the problems of tiling by 3-gons, 4-gons, and 5-gons are very different from tilings by n-gons ($n \geq 6$). It is also very remarkable that there is an anomalous case: there is no tiling of type $< 3, 5, 13 >$ in spite of the fact that this satisfies the conditions of the theorem.

This work was done jointly with Roswitha Blind.

Realizations of regular toroidal maps

Asia Ivić Weiss

We determine and completely describe all pure realizations of the finite toroidal maps. For type $\{4,4\}$ maps most such realizations are 8-dimensional. For type $\{3,6\}$ and $\{6,3\}$ maps most such realizations are 12-dimensional.

This is joint work with Barry Monson.

(At most j)-facets in 3-space

Emo Welzl

Let S be a set of n points in generic position in $\mathbb{R}^3$. An oriented triangle spanned by three of the points is called a j-facet if there are exactly j points from S on the positive side of its affine hull; e.g. 0-facets are the facets of the convex hull of S. We show that for $j \leq (n-4)/2$ the number of $(\leq j)$-facets (i.e. i-facets with $0 \leq i \leq j$) is maximized when S is in convex position. The proof proceeds by showing that this statement is equivalent to the Generalized Lower Bound Theorem for d-polytopes with at most $d+4$ vertices (employing the Gale transform).

Rigidity of Frameworks and Polyhedra: Euclidean, Spherical, Hyperbolic and Projective

Walter Whiteley

The first-order rigidity (and equivalent static rigidity) of a discrete configuration (framework, polyhedron etc.) in Euclidean space of any dimensions is projectively invariant. There is a consistent projective presentation of this rigidity which specializes to the first-order rigidity in each of the Eucldiean, elliptic and hyperbolic metrics. This 'explains' why and how the first-order motions of a configuration in the underlying projective space translates among these geometries, with the same projective points and abstract structure of constraints. In each of the metrics, an 'averaging principle' takes two structure with the same indexing, P and Q, into their average $\frac{1}{2}(P+Q)$ with infinitesimal motion $P-Q$ preserving a distance in the average if and only if this distance was equal in the two initial models. This and its converse (pushing an infinitesimal motion back and forth to generate P and Q) transform a connected pair of structures in one metric to another pair in the other metric (no longer the same projective coordinates) with corresponding pairs of lengths equal if and only if they were equal in the original pair (a construction implicit in the work of Pogorelov). This explicit projective approach, applied to points inside and outside the absolute, translates theorems such as Cauchy's rigidity theorem for triangulated convex polyhedra with fixed edge lengths in Euclidean space into both the rigidity theorems in hyperbolic space and to an extension of Andreev's Theorem for uniqueness of finite polyhedra with fixed dihedral angles in hyperbolic space.

This is joint work with Franco Saliola.

A discrete form of the Beckman-Quarles theorem for rational spaces

Joseph Zaks

The Beckman-Quarles theorem [Proc. AMS **4** (1953) 810–815] states that every unit-preserving mapping of $\mathbb{R}^d$ into $\mathbb{R}^d$ is an isometry ($d \geq 2$).

A. Tyszka [Math. Mag. 2000] proved the following two results:

Theorem 1: For $d = 8$, for every two rational points x, y in $\mathbb{Q}^d$ there exists a finite set $S_{x,y}$ of points in $\mathbb{Q}^d$ that contains x and y such that every unit-preserving mapping f of the set $S_{x,y}$ into $\mathbb{Q}^d$ preserves also the distance from x to y ($|f(x) - f(y)| = |x - y|$).

This implies the following:

Theorem 2: For $d = 8$, every unit-preserving map of $\mathbb{Q}^d$ into itself is an isometry.

Based on similar arguments, we can establish the following:

Theorem 3: If d is of the form $d = 4k(k+1)$ then every unit-preserving map of $\mathbb{Q}^d$ into itself is an isometry.

Theorem 4: If $d \geq 2$ is a complete square and if it is of the form $d = 2k^2 - 1$ then every unit-preserving map of $\mathbb{Q}^d$ into itself is an isometry.

Facet Subgraphs of Simple Polytopes

Günter M. Ziegler

The combinatorial structure of a d-dimensional simple convex polytope – as given, for example, by the set of the $(d-1)$-regular subgraphs of facets – can be reconstructed from its abstract graph [Blind & Mani 1988, Kalai 1988]. However, no polynomial/efficient algorithm is known for this task, although a polynomially checkable certificate for the correct reconstruction was found by [Kaibel & Körner 2000].

A much stronger certificate would be given by the following characterization of the facet subgraphs, conjectured by Perles: "The facet subgraphs of a simple d-polytope are exactly all the $(d-1)$-regular, connected, induced, non-separating subgraphs." We first observe that for any counterexample, the boundary of the (simplicial) dual polytope P^* contains a 2-complex without a free edge, and without 2-dimensional homology. One example of such a complex, a modification of "Bing's house," is then used to construct explicit 4-dimensional counterexamples to Perles' conjecture.

This is joint work with Christian Haase, TU Berlin

Edited by Peter Braß

Problems

Problem Session, Discrete Geometry 2000, Oberwolfach, Tuesday, May 30
Edited by Schnell/Wills

1. Problem (G. C. Shephard)

If a polygon P is divided into triangles by lines joining the vertices of P, and if T_i has area $a(T_i)$ and centroid g_i, then $g = \sum a(T_i)g_i / \sum a(T_i)$ is a point of P (its centroid) which is independent of the particular triangulation that is chosen.
It is not so well–known that if c_i is the circumcentre of T_i then $c = \sum ac_i / \sum a(T_i)$ is also independent of the choice of triangulation. (Note: the corresponding property is **not** true for incentres, orthocentres, etc).
The problem is to find a geometric interpretation for the point c (for example as the point where some function on P attains ist max/min). Note that if all the vertices of P lie on a circle, then c is the centre of this circle.

2. Problem (Luis Montejano Peimbert)

The False Plane of Symmetry.

Definition: Let K be a convex body and let H be a hyperplane in euclidean n–space. We say that H is a hyperplane of symmetry if there is a direction with the property that the middle points of all chords of K in this direction lies in H.

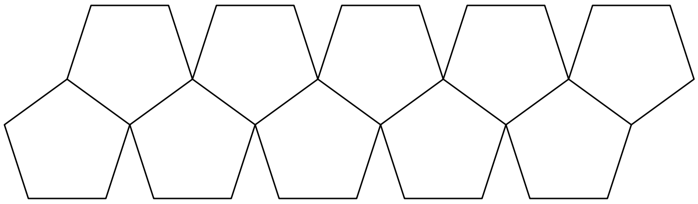
Conjecture: Let K be a convex body and let H be a hyperplane in euclidean n–space. Suppose that for every hyperplane N orthogonal to H, the intersection of K with N has an $(n-2)$–plane of symmetry parallel to H. Then, either K has a hyperplane of symmetry parallel to H, in the direction orthogonal to H, or K is an ellipsoid.

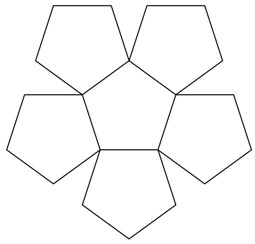
3. Problem (Heiko Harborth)

Can every planar graph be drawn as a plane graph with all edges being straight line segments which are of integer lengths?
A positive answer to the following question would help: Does every plane pentagon with integer sides and two integer diagonals incident to one vertexpoint contain a point with rational distances to all five vertexpoints?

4. **Problem** (Heiko Harborth)

Prove that the minimum area of the convex hull of an edge–to–edge packing of n congruent regular pentagons is attained by the following "double sausage" with one exception for $n = 6$.





5. **Problem** (Günter M. Ziegler)

Can every (simple) convex 3–polytope P be represented so that a dual polytope $Q \cong P^*$ can be constructed as the convex hull of vertices that are chosen in the relative interiors of the corresponding facets of P? That is, can one represent every 3–polytope with points on its facets, such that adjacent facets of P correspond to adjacent vertices on the convex hull P of the extra points?
In particular, can one do this for the 3–polytope obtained by cutting off the vertices of a tetrahedron?
(This was Problem 3 in B. GRÜNBAUM & G.C. SHEPHARD: *Some problems on polyhedra, J. Geometry* **29** (1987), 182–190.)

6. **Problem** (Joseph Zaks)

Let $\mathbb{Q}^d$ denote the rational d-space, and let $A(d)$ and $B(d)$ be defined by:
$A(d)$: For every two points x and y of $\mathbb{Q}^d$, there exists a **finite** set $S_{x,y}$ that contains x and y, such that every unit-preserving mapping (u.p.m.) $f : s_{x,y} \to \mathbb{Q}^d$ preserves also the distance from x to y, i.e.
$\text{dist}(x, y)(f(x), f(y)) = \text{dist}(x, y)$.
$B(d)$: Every u.p.m. $f : \mathbb{Q}^d \longrightarrow \mathbb{Q}^d$ is an isometry.

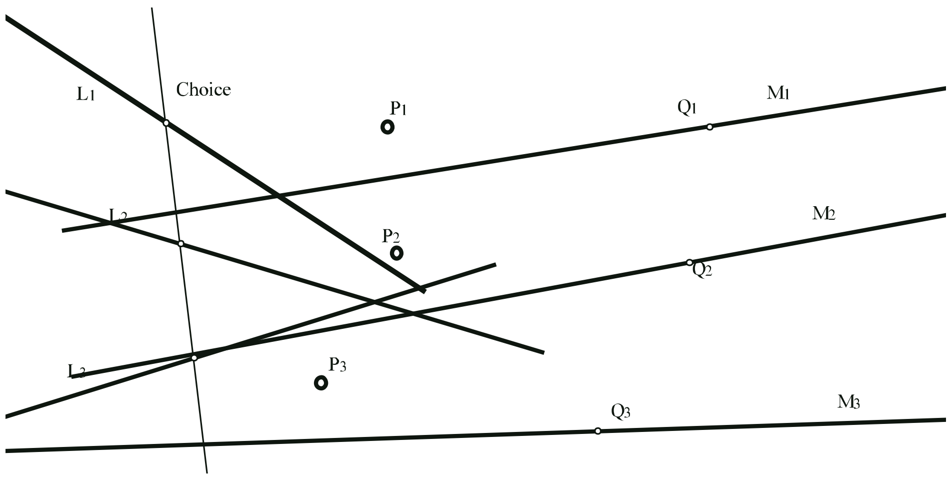
Open Problems:
1) Is it true that $A(d) \Rightarrow B(d)$ for all $d \geq 5$?
2) Are $A(d)$ and $B(d)$ true for all $d \geq 5$?
Obviously, $A(d) \Rightarrow B(d)$.
Known results:
1) $A(d)$ and $B(d)$ are false for $d = 1, 2, 3$ and 4.
2) $A(8)$ and $B(8)$ are true [A. Tyszka, Math. Mag (T.A.). see also Aequ.Math. 59 (2000), 124-133.]
3) $A(d)$ and $B(d)$ are true for all even d of the form $d = 4k(k + 1), k = 1, 2, \ldots$ and for all odd d which are complete squares, $d = x^2$, and are of the form $d = 2y^2 - 1$. (J.Z., in Prep.)

7. Problem (Peter Brass)

Is it true that if a convex polygon $x_1, \dots, x_n$ has the property for some a, b that for each order-a diagonal $\overline{x_i x_{i+a}}$ the point x_{i+b} is the farthest point to $\overline{x_i x_{i+a}}$ among the points cut off by that diagonal $\{x_{i+1} \dots x_{i+a-1}\}$ then $2b = a$?

8. Problem (Janos Baracs, submitted by Walter Whiteley)

The following problem was posed by Janos Baracs (Montreal) and is related to possible configurations in 4–space.
Given two indexed sets of 3–lines each: $L_1, L_2, L_3; M_1, M_2, M_3$; and 3 indexed points: P_1, P_2, P_3, we choose a line (Choice) which intersects each L_i in a point which is joined to P_i then intersected with M_i to create the point Q_i. The problem is to choose (construct) a line Choice so that the points $Q_1 Q_2 Q_3$ are collinear. For very degenerate situations, such as each P_i lies on its L_i, this may not be possible. However, experimentation with geometer's sketchpad indicates that, in general, there is such a line and it is not unique.



9. Problem (Wlodzimierz Kuperberg)

Determine the pairs of integers (k, d) with $3 \leq k \leq d$ for which the d–cube contains a regular k–simplex as a concentric 0–1 subpolytope.

10. Problem (Jürgen Richter-Gebert)

Does each convex 3–polytope have a realization with convex faces and integer edges?

11. Problem (Wolfgang Kühnel)

It is well known that for any $n \not\equiv 2(3), n \geq 4$ there is a triangular embedding of the complete graph K_n into some closed surface. In other words: There is a triangulation of some closed surface such that the edge graph is a K_n.
Is it true that for any $n = 3m$ with an integer $m \geq 3$ one can choose this triangulation in such a way that it contains m disjoint triangles? In terms of the graph K_n: Is there a partition into m triples such that any triple forms a triangle in the surface?
Warning: This is not true for $n = 6$, but it is true for $n = 9$ and $n = 12$. For certain cases a solution can be found in the book "Map color Theorem" by G. Ringel.

12. Problem (Wolfgang Kühnel)

Is there a 3–neighborly triangulation of any 4–manifold with $n = 14$ vertices? "3–neighborly" means that the number of triangles is $\binom{n}{3}$. There is none with a vertex–transitive automorphism group, so an example would presumably be rather irregular. The case $n = 14$ is the only case for $n < 20$ vertices which is undecided.

13. Problem (Imre Bárány)

Let $A(n)$ denote the minimal area that a convex lattice polygon with n vertices can have. It is known that the order of magnitude of $A(n)$ is n^3:

$$\frac{1}{54} \geq \frac{A(n)}{n^3} \geq \frac{1}{16\pi^2}$$

The lower bound here follows from a result of Rényi and Sulanke. A simple example shows the upper bound. Question: find

$$\lim \frac{A(n)}{n^3}.$$

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