Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 26/2000

Geometric Analysis and Singular Spaces

25. Juni bis 1. Juli 2000

The international conference on *Geometric Analysis and Singular Spaces* was held from June 25th to July 1st, 2000, in Oberwolfach. The organizing committee consisted of J.-M. Bismut (Orsay), J. Brüning (Berlin), and R.-B. Melrose (Boston).

Again, this was a very interesting meeting with a number of excellent talks and lively discussions during the whole week. A characteristic feature of the meeting was the impression conferred by various talks that topics that had originated in former meetings have matured into complete theories like the ideas centering around the Verlinde formulas or the theory of the holomorphic torsion. On the other hand, various new ideas came up addressing new and promising global invariants like those presented by Fukaya. The main fields of interest and the people contributing to them could be summarized as follows:

1. *Index theory* (Jeffrey, Leichtnam, Ma, Meinrenken, Nestor, Singer)
2. *Spectral invariants* (Götte, Köhler, Lesch, Piazza, Szenes)
3. *Scattering theory and non-compact spectral theory* (Ballmann, Bunke, Lauter, Lott, Mazzeo, Vasy, Wolpert)
4. *Global invariants* (Carron, Farber, Fukaya, Lohkamp, Sjamaar, Youssin)

Name: Werner Ballmann
Title: *On the essential spectrum of the Dirac operator (joint work with J. Brüning)*
Abstract: Let M be a complete Riemannian manifold and $E \to M$ a Dirac bundle. The Dirac operator D on $L^2(E)$ is essentially selfadjoint. In the case of the exterior bundle $E = \bigwedge^* M$ and of a spinor bundle, the essential spectrum of D is tied to the asymptotic geometry of M at infinity. I discussed the case when the ends of M are cuspidal, that is, they look like the ends of a complete Riemannian manifold of finite volume with sectional curvature satisfying $-b^2 \leq K \leq -a^2 < 0$. In the case $E = \bigwedge^* M$, D is Fredholm if the pinching a/b is sufficiently close to one; the bound given in the talk is sharp, improving earlier work of Donnelly-Xavier. For $E =$ spinor bundle, the situation is different. There are explicit index formulas in the case when D is Fredholm. This work is related to recent work of John Lott.

Name: Ulrich Bunke
Title: *On scattering theory for geometrically finite subgroups of rank one Lie groups (joint work with M. Olbrich)*
Abstract: The objects of geometric scattering theory are the extension map, the scattering matrices, and the Eisenstein series. We construct these objects for geometrically finite subgroups along the lines prescribed in the convex cocompact case. Details, in particular the function and distribution spaces, are technically much more involved. Thus the ground for further applications as to Plancherel theorems, Poisson summation formulas etc. is settled.

Name: Gilles Carron
Title: *Some generalisations of Huber's theorem (joint work with M. Herzlich (U. Montpellier II))*
Abstract: The Huber theorem states that if a complete Riemannian surface S has Gaussian curvature in L^1 then S is conformally equivalent to a compact surface $\overline{S}$ with a finite number of points removed. It is known that naive generalization of this result in higher dimension is false. However, we obtain the following result: If $(\overline{M}, g_0)$ is a compact Riemannian manifold of dimension n, and if (M, g) is a domain of $\overline{M}$ endowed with a complete metric g which is conformal to g_0 and such that its Ricci tensor satisfies

$$\int_M |Ric_g|^{n/2} dvol_g < \infty$$

then $\overline{M} - M$ is a finite set.

We also obtain related results on complete, non-compact Riemannian manifolds which are locally conformally flat outside a compact set, and satisfy some Euclidean Sobolev inequality and integral bound on curvature.

Name: Mikhael Farber
Title: *Topology of billiard problems (joint work with S. Tabachnikov)*
Abstract: Let $T \subset \mathbb{R}^{m+1}$ be a strictly convex domain bounded by a smooth hypersurface $X = \partial T$. The billiard ball is a point, which moves in T in a straight line, except when it hits $X = \partial T$, where it rebounds making the angle of incidence equal the angle of reflection. Plane billiards ($m = 1$) were introduced into mathematics by G.D. Birkhoff in

1900's.

In the talk I gave topological lower bounds on the number of billiard trajectories satisfying a variety of boundary conditions in convex billiards of arbitrary dimension. In particular I gave an estimate from below on the number of n-periodic billiard trajectories.

The last problem reduces to a topological problem of estimating the Lusternik - Schnirelman category of cyclic configuration spaces $G(S^m, n)$ of m-dimensional spheres. If X is a manifold, the cyclic configuration space $G(X, n)$ is (by the definition) the set of all configurations $(x_1, \ldots, x_n) \in X \times X \times X \times \ldots X$ (n times) such that $x_i \neq x_{i+1}$ for any $i = 1, 2, \ldots, n$. Here we understand $x_{n+1} = x_1$. Our main effort is in computing the cohomology algebra of $G(S^m, n)$. We also compute the cohomology algebra of the quotient $G(S^m, n)/D_n$, where D_n denotes the dihedral group. We apply the Leray spectral sequence, similar to work of B. Totaro.

Name: Kenji Fukaya
Title: *Floer homology for families of Lagrangian submanifolds (joint work with Yong Geun Oh, Hirosi Ohta and Kaoru Ono)*
Abstract: Mirror symmetry predicts the existence of a pair of manifolds (M, Ω) and (M', J). Here (M, ω) is a symplectic manifold (and $\Omega = \omega + \sqrt{-1}B$, B being closed two form), and (M', J) is a complex manifold.

Moreover it is expected that a Lagrangian submanifold L of M together with a flat line bundle $\mathfrak{L}$ on it corresponds to a sheaf $\mathfrak{E}(L, \mathfrak{L})$ on (M', J). (More precisely $\mathfrak{E}(K, \mathfrak{L})$ is an object derived category of the coherent sheaves on (M', J).)

Moreover it is conjectured to exist a functorial isomorphism

$$HF((L_1, \mathfrak{L}_1), (L_2, \mathfrak{L}_2)) \cong Ext(\mathfrak{E}(L_1, \mathfrak{L}_1), \mathfrak{E}(L_2, \mathfrak{L}_2)).$$

Here the left hand side is the Floer homology group.

We found that not all pair $(L, \mathfrak{L})$ of Lagrangian submanifold and a flat line bundle $\mathfrak{L}$ has a mirror $\mathfrak{E}(K, \mathfrak{L})$. In fact there is an obstruction theory for the existence of the Floer homology group.

Namely we found a formal map

$$s : H^{odd}(L) \to H^{even}(L), s = \sum s_i T^{\lambda_i},$$

where s_i is a polynomial and λ_i are positive real numbers converging to infinity, such that its zero set $\mathfrak{M}(L, \mathfrak{L})$ gives a moduli space of infinitesimal deformation of the pair $(L, \mathfrak{L})$ for which the Floer homology exists. ($\mathfrak{M}(L, \mathfrak{L})$ may be empty for some $L, \mathfrak{L}$.)

This map is expected to coincide with the Kuranishi map

$$Ext^{odd}(\mathfrak{E}(L, \mathfrak{L}), \mathfrak{E}(L, \mathfrak{L})) \to Ext^{even}(\mathfrak{E}(L, \mathfrak{L}), \mathfrak{E}(L, \mathfrak{L}))$$

determining the deformation theory of $\mathfrak{E}(L, \mathfrak{L})$ in the mirror.

For a pair of Lagrangian submaniflds L_1, L_2, we can construct a holomorphic family of chain complexes of bundles HF (family of Floer homologies) on $\mathfrak{M}(L_1, \mathfrak{L}_1) \times \mathfrak{M}(L_2, \mathfrak{L}_2)$. We expect it to coincide with the family of cohomology gropus $Ext(\mathfrak{E}(L_1, \mathfrak{L}_1), \mathfrak{E}(L_2, \mathfrak{L}_2))$ in the mirror.

Name: Sebastian Goette

Title: *Morse functions and torsion forms (joint work with J.-M. Bismut)*

Abstract: Assume that $M \to S$ is a fiber bundle with compact fiber X, and assume that there exists a function $f\colon M \to \mathbf{R}$ such that f is Morse-Smale on every fiber. Let $F \to M$ be a flat vector bundle. In this situation, we compare the analytic torsion form of Bismut and Lott with a combinatorial torsion form constructed from the bundle of fiber-wise Thom-Smale complexes. This extends previous work of Bismut and Zhang to the family case. The difference of the two torsion forms consists of several terms, one of which is an "exotic" genus J (related to the R-genus in Arakelov theory) applied to a graded version of the vertical tangent bundle at the fiber-wise critical points. All this is done in an equivariant setting.

As a negative result, we show that the non-equivariant analytic torsion forms vanish in degree > 0 if $M \to S$ admits a function f as above and the bundle F is unitarily flat and fiber-wise acyclic.

As a positive result, we show that the equivariant torsion form of a unit sphere bundle in an arbitrary Euclidean vector bundle $E \to S$ is a multiple of $J(E)$.

Name: Robin C. Graham

Title: *A Blow-up Approach to Edge-of-the-Wedge Theorems (joint work with M. Eastwood)*

Abstract: A method is described of proving theorems of edge-of-the-wedge type by blowing up the space along the edge and analyzing solutions of the resulting involutive structure on the blow-up. An elementary proof along the lines of Bogolyubov's original edge-of-the-wedge theorem is given, and a generalization to hypersurfaces in $\mathbb{C}^n$, and more generally, a microlocal result on general hypoanalytic manifolds, is discussed. The fundamental tool is the microlocal theory of hypoanalytic structures due to Baouendi-Chang-Treves.

Name: Lisa C. Jeffrey

Title: *The Verlinde formula for moduli spaces of parabolic bundles*

Abstract: The moduli space $M(n,d)$ is an algebraic variety parametrizing those representations of the fundamental group of a punctured Riemann surface into the Lie group $SU(n)$ for which a loop around the boundary is sent to the n-th root of unity $exp(2\pi id/n)$ multiplied by the identity matrix. If n and d are coprime it is in fact a Kähler manifold. One may relax the constraint and study moduli spaces $M(\lambda)$ parametrizing those representations for which the loop around the boundary is sent to an element conjugate to λ, if λ is some element in $SU(n)$, and these are also Kähler manifolds for a suitable class of λ.

The Verlinde formula calculates the dimension of the space of holomorphic sections of certain line bundles over the spaces $M(n,d)$ and $M(\lambda)$: these dimensions are in a sense the dimensions of the quantizations of these spaces. We recall how a new proof of the Verlinde formula for $M(n,d)$ given in joint work with Frances Kirwan (Ann. Math.) may be obtained, and show how to modify this proof to obtain a proof of the variant of the Verlinde formula which applies to $M(\lambda)$.

This work is presented in math.AG/0003150.

Name: Kai Köhler
Title: *Applications of equivariant holomorphic torsion (joint work with D. Roessler and with Ch. Kaiser)*
Abstract: In Arakelov K-theory introduced by Gillet and Soulé for schemes over Dedekind rings, holomorphic torsion provides a direct image construction. In joint work with D. Roessler we have shown a Lefschetz fixed point formula for this direct image, which can be regarded as a fixed point formula for equivariant holomorphic torsion. Among the applications are: - a submersion formula for equivariant holomorphic torsion (shown in larger generality by X. Ma) - torsion for circle action is (mainly) a rational function in the group parameter - a new proof of the Jantzen sum formula for lattice representations of Chevalley schemes - a topological formula for global heights of all generalized flag spaces - an explicit formula for Faltings heights of abelian varieties with complex multiplication - a Bott residue formula for arithmetic characteristic classes.

Name: Robert Lauter
Title: *Essential spectra for Laplace operators on manifolds with corners (joint work with V. Nistor)*
Abstract: We present a general approach to compute essential spectra for geometric operators on compact manifolds with corners using a pseudodifferential calculus on an associated differentiable groupoid. An important step is to understand the C^*-algebras generated by the operators of order 0 and $-\infty$, respectively. This is achieved by characterizing those pseudodifferential operators that are Fredholm or compact. Using the Cayley transform, we reduce the case of elliptic, self-adjoint operators of positive order to the case of operators of order 0.

The techniques apply to Hodge-Laplace operators on complete Riemannian manifolds that embed densely into a compact manifold with corners such that the metric has a "controlled" behavior at the boundary faces. For instance, we show that the essential spectrum of the Hodge-Laplacian Δ_p of an asymptotically multi-cylindrical metric (corresponding to an exact b-metric on a manifold with corners), or, slightly more general, of a "multi-cusp" metric, is the union of rays $[\gamma_H, \infty)$ where $0 \leq \gamma_H$ is determined by the Hodge Laplace operators on the boundary hyperfaces H of a given manifold with corners. This form of the essential spectrum of the Laplace operator of an exact b-metric was expected by R.B. Melrose.

Name: Eric Leichtnam
Title: *A local formula for the index of a Fourier Integral Operator (joint work with R. Nest and B. Tsygan)*
Abstract: Let X and Y be two closed connected Riemannian manifolds of the same dimension and $\phi : S^*X \mapsto S^*Y$ a contact diffeomorphism. We show that the index of an elliptic Fourier integral operator Φ associated with ϕ is given by $\int_{B^*(X)} e^{\theta_0} \widehat{A}(T^*X) - \int_{B^*(Y)} e^{\theta_0} \widehat{A}(T^*Y)$ where θ_0 is a certain characteristic class depending on the principal symbol of Φ and, $B^*(X)$ and $B^*(Y)$ are the unit ball bundles of the manifolds X and Y. The proof uses the algebraic index theorem of Nest-Tsygan for symplectic Lie Algebroids and an idea of Paul Bressler to express the index of Φ as a trace of 1 in an appropriate deformed algebra.

In the special case when $X = Y$ we obtain a different proof of a theorem of Epstein-Melrose conjectured by Atiyah and Weinstein.

Name: Matthias Lesch
Title: *The glueing formula for the analytic torsion*
Abstract: Given a compact Riemannian manifold and a flat bundle, i.e. a representation of the fundamental group. The celebrated Cheeger-Müller theorem states that for unitary representations of the fundamental group the analytical torsion (an invariant of the spectrum of the Laplacian) equals the combinatorial torsion (a combinatorial invariant). There exist several proofs and generalizations of this result.

A very interesting approach, however only in the case of a trivial representation, is due to S. Vishik. The idea is to prove the glueing formula for the analytic torsion directly.

In my talk I presented a proof of this glueing formula which works without any restriction on the representation. The proof is based on methods from the theory of boundary value problems for Dirac type operators, which were developed jointly with J. Brüning.

As an application one obtains a Cheeger-Müller type theorem for manifolds with boundary.

Name: Joachim Lohkamp
Title: *Geometric analysis on 3-manifolds*
Abstract: We describe a semilocal approach to general existence results for negatively curved 3-manifolds. It exploits the analytical properties of sectional curvature and resembles an h-principle strategy. A typical result obtained by this method is the following one: Given an open 3-manifold M with $2k$ ends one may find k properly embedded curves L_i such that $M \setminus \bigcup_i L_i$ admits a complete metric with curvature K where $-a < K < -b$, $a, b > 0$. More generally, one gets approximation results and a lot of 'derived' results claiming the existence of various geometric properties.

Name: John Lott
Title: *Collapsing, Forms and Spinors*
Abstract: I discussed results about the behavior of the differential form Laplacian and geometric Dirac-type operators under a collapse with bounded sectional curvature. In the case of the differential form Laplacian, if a sequence of manifolds $\{M_i\}_{i=1}^{\infty}$ collapses with bounded curvature to a limit space X then I showed that after passing to a subsequence, there is a triple (E, A', h^E) on X so that the jth eigenvalue of the p-form Laplacians on $\{M_i\}_{i=1}^{\infty}$ converges to the jth eigenvalue of the Laplace-type operator $\triangle_p^E$ on X. Here E is a $\mathbf{Z}$-graded vector bundle on X, A' is a flat degree 1 superconnection on E and h^E is a Euclidean inner product on E. I gave applications to the question of small eigenvalues for the p-form Laplacian, to the question of upper bounds of the jth eigenvalue of the p-form Laplacian and to the L^2-cohomology of finite-volume negatively-curved manifolds.

Name: Xiaonan Ma
Title: *An equivariant family index theorem and its applications (joint work with K. Liu and W. Zhang)*
Abstract: Let $\pi : M \to B$ be a fibration of compact manifolds with compact fiber X. Let W be a complex vector bundle on M.

Assume that S^1 acts fiberwise on M, and TX has a S^1-equivariant spin structure, and the S^1 action lifts on W. For the family Dirac operator $D^X \otimes W$, we know the index bundle $\mathrm{Ind}(D^X \otimes W) \in K_{S^1}(B)$.

Our first result says that $\mathrm{Ind}(D^X \otimes W)$ can be expressed by the index bundles of certain Dirac operators on the fixed point set of the S^1-action on M. This kind of formula is new and can't be directly obtained by Atiyah-Singer's original method, we prove it in analytic way.

As its applications, we explain how to prove rigidity and vanishing theorems in K-theory. Especially, we generalize Atiyah and Hirzebruch's $\widehat{A}$ vanishing theorem and Witten rigidity theorem for ellliptic genus on K-theory level. Finally, we discuss vanishing theorems for foliations.

Name: Rafe R. Mazzeo
Title: *Resolvents, Martin boundaries and Riemannian products (joint work with A. Vasy)*
Abstract: This talk surveyed some ongoing work which concerns continuing efforts to extend the scope of geometric scattering theory to spaces which are asymptotically locally symmetric. Basic questions include the nature of the resolvent of the Laplace operator near, or the possible continuation beyond, the spectrum, the existence and structure of plane wave solutions and the scattering operator, etc. A brief review was given of two well-established cases: asymptotically flat manifolds (scattering metrics), which is work of Melrose and Melrose-Zworski, and asymptotically hyperbolic manifolds (conformally compact metrics), which is work of Mazzeo-Melrose and Mazzeo. The rest of the lecture contained an examination of the simplest rank 2 case: the product of two hyperbolic spaces, or more generally, the product of two conformally compact manifolds. A useful formula for the resolvent of a product space was established, and this was then applied to the determination of the structure of the Martin compactification of these spaces. A brief indication was given at the end of further directions. These include the construction of the 'resolvent compactification', of which the Martin compactification is just a slice and on which the resolvent naturally lives. This sets the stage for the definition of a calculus of pseudodifferential operators associated to these asymptotic structures.

Name: Eckhard Meinrenken
Title: *Verlinde formulas for non-simply connected groups (joint work with A. Alekseev and C. Woodward)*
Abstract: In this talk we describe formulas for the index of prequantum line bundles over moduli spaces of flat G connections over surfaces with one boundary component. Here, G is a compact, simple, connected Lie group. If G is simply connected, the indices are given by the usual Verlinde numbers. (This includes singular cases, where the index may be defined by desingularization.) For the non-simply connected case, we describe a sufficient condition for pre-quantizability and give a generalized Verlinde formula computing the indices. Similar formulas (for dimensions of holomorphic sections rather than indices) had been found by Pantev (for $SO(3)$) and Beauville (for $PSU(p)$, $p \geq 3$ prime). Our method of proof relies on a fixed point formula for loop group actions.

Name: Victor Nistor
Title: *A family index theorem for equivariant elliptic operators (joint work with R. Lauter)*

Abstract: I will discuss the connections between 'boundary fibration structures,' (as introduced by Melrose) and families of operators invariant with respect to a bundle of Lie groups (called 'equivariant families' in what follows). Such equivariant families were used before by Mazzeo, Melrose, and Bismut, among others; they are useful in establishing the Fredholmness of elliptic operators on certain open manifolds. I also establish an index theorem for these families, when the fibers of the bundle of Lie groups are simply-connected, solvable Lie groups.

Name: PAOLO PIAZZA
Title: *Two geometric applications of higher eta invariants (joint work with E. Leichtnam and J. Lott)*
Abstract: In this talk I have reported on two geometric situations in which Lott's higher eta invariant plays a fundamental role.

The first one (joint work with Eric Leichtnam and John Lott) concerns the proof of the homotopy invariance of the higher signatures defined by Lott on a manifold with boundary. Let M be an oriented manifold with boundary, Γ a finitely generated discrete group and $\nu : M \to B\Gamma$ a continuous map. Let $\widetilde{M} := \nu^*(E\Gamma)$ be the induced Galois coverings. We consider the flat bundle of C^*-algebras $\mathcal{V} = \widetilde{M} \times_\Gamma C_r^*(\Gamma)$ and $H^*(\partial M, \mathcal{V})$, the cohomology of ∂M with coefficient in the local system $\mathcal{V}$. Let $m = [\dim M/2]$. We assume that the natural map

$$H^m(\partial M, \mathcal{V}) \longrightarrow \overline{H}^m(\partial M, \mathcal{V}) := \operatorname{Ker} d_{\mathcal{V}}/\overline{\operatorname{Im} d_{\mathcal{V}}}$$

is an isomorphism. This assumption can be reinterpreted as a gap condition at 0 for the L^2 spectrum of the differential form Laplacian on $\partial\widetilde{M}$ *in degree m*. Under the above assumption the higher signature of Lott

$$\int_M L(M, \nabla^M) \wedge \omega - \widetilde{\eta}_{\partial M}$$

is well defined as a noncommutative de Rham class in $\overline{H}_*(\mathcal{B}_\Gamma^\infty)$ with $\mathcal{B}_\Gamma^\infty$ a smooth subalgebra of $C_r^*(\Gamma)$. We prove that it is an oriented homotopy invariant of the pair $(M, \partial M)$. If Γ is Gromov hyperbolic or virtually nilpotent then we can pair $\overline{H}_*(\mathcal{B}_\Gamma^\infty)$ with the cohomology groups $H^*(\Gamma, \mathbf{C})$, thus obtaining *numerical* oriented homotopy invariants; moreover, as a Corollary, we show that the higher signatures of a *closed* oriented manifold X are cut-and-paste invariants if the disconnecting hypersurface satisfies the above assumption.

In the second part of the talk (joint work with Eric Leichtnam) I explained how, in some cases, it is possible to use the higher eta invariant associated to a Dirac operator on a closed spin manifold in order to distinguish metrics of positive scalar curvature. More precisely, let N be a closed spin manifold admitting one metric of positive scalar curvature; we give sufficient conditions on $\pi_1(N)$ and dim$N$ ensuring the existence of an infinite number of non-concordant or even non-bordant metrics of positive scalar curvature on N.

Name: MICHAEL A. SINGER
Title: *An L^2 index theorem for Dirac operators on $R^3 \times S^1$ (joint work with Tom Nye)*
Abstract: The Dirac operator, coupled to a unitary connection in a bundle over $R^3 \times$

S^1 was considered. Suitable boundary conditions were imposed to ensure that such an operator is Fredholm in L^2 and the index was computed in this case. The index is expressed as a sum of two topological invariants that arise naturally in this situation.

The Φ-calculus of Mazzeo and Melrose is used to understand when these Dirac operators are Fredholm in L^2. The index formula is derived in two stages. First when the data are homotopic to data pulled back from R^3, one uses Fourier analysis in S^1 and the index theorem of Callias. Second one uses an excision argument to reduce the general case to this one.

The index formula can also be written as an integral over $R^3 \times S^1$ of the usual 4-dimensional characteristic class plus the integral over the boundary of the adiabatic limit of the η-invariant. This formulation suggests a natural conjecture for the index of Dirac operators over manifolds of the form $R^n \times K$, where K is a compact manifold without boundary.

Name: Reyer Sjamaar
Title: *Moment maps and Riemannian symmetric pairs (joint work with L. O'Shea)*
Abstract: Our main result is a "real form" of Kirwan's convexity theorem. We apply our result to flag varieties of real semisimple groups and thus obtain eigenvalue inequalities, which generalize inequalities found by Weyl, Ky Fan, Kostant, Klyachko, and many others. For instance, we find an answer to the following question: given the singular values of two rectangular matrices (of the same size), what are the possible singular values of their sum?

Name: Andras Szenes
Title: *Non-commutative deformations of the moduli space of flat connections*
Abstract: We prove the existence of a non-commutative deformation of the algebraic functions on the moduli space of flat connections on a punctured Riemann surface. The deformed algebra has a trace functional with values in meromorphic functions on the unit disc in the deformation parameter q. The meromorphic functions which arise have an asymptotic expansion at $q = 1$ which recovers the Verlinde formula. This suggests a conjecture on the characteristic class of the deformation.

Name: Andras Vasy
Title: *Propagation of singularities and asymptotic completeness in many-body scattering*
Abstract: In this talk I extend the familiar example that singularities of solutions of the wave equation propagate along (generalized) broken bicharacteristics, to many-body scattering. This establishes a connection between the classical and quantum systems. Unlike in the absence of bound states, when quantum propagation is described by classical motion along (broken) bicharacteristics, in the general setting there is a mixing of quantum (bound states) and classical effects even at the level of propagation of singularities. In particular, the characteristic set of the Hamiltonian must be modified to describe the bound states properly. I also explain the implication of these results for the structure of the wave front relation of the scattering matrices, and the relationship of these results to the proof of asymptotic completeness.

Name: Scott A. Wolpert

Title: *Semi-classical limits for the hyperbolic plane*

Abstract: A discussion of the high-energy limit for eigenfunctions of the Laplace-Beltrami operator for the hyperbolic plane is presented. Motivation is provided by the 'random wave' model for high-energy oscillations and by the conjecture that for a finite area hyperbolic quotient $\Gamma\backslash\mathbb{H}$ the unit-norm eigenpairs $\{(\phi_j,\lambda_j)\}$ should have weak*-limit $\lim_{\lambda_j\to\infty}\phi_j^2=(Area(\Gamma\backslash\mathbb{H}))^{-1}$. Motivation is also provided by the consideration from analytic-number theory of *square-root* cancelation in coefficient partial sums.

S. Zelditch's microlocal lift of a (eigen)function u on $\mathbb{H}$ with $\Delta u+(\frac{1}{4}+r^2)\,u=0$ is given by first considering the lift u_0 of u to the root unit-cotangent bundle $T_1(\mathbb{H})^{1/2}\approx SL(2;\mathbb{R})$. The 'raisings' and 'lowerings' of u_0 are defined for $E^{\pm}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\pm i\begin{pmatrix}0&1\\1&0\end{pmatrix}$ and $m\in\mathbb{Z}$ by

$$\begin{aligned}(2ir+2m+1)u_{2m+2}&=E^+u_{2m}\\(2ir-2m+1)u_{2m-2}&=E^-u_{2m}\end{aligned}$$

The distribution $Q(u)=u_0\sum_m\overline{u_{2m}}$ is the microlocal lift of u (the Schwartz kernel for the matrix element associated to the pair (u,u)). We observe for $\{(u_j,\lambda_j)\}$ with u_j normalized in a weighted L^2-space that Fejér summation can be used to show that each accumulation distribution of a subsequence of $\{Q(u_j)\}$ for $\lambda_j\to\infty$ is a positive measure. Zelditch showed that $Q(u)$ satisfies an exact differential equation and in consequence that the accumulation measures are geodesic flow invariant (are right-invariant by $A=\{\begin{pmatrix}a&0\\0&a^{-1}\end{pmatrix}\}$).

For the elementary eigenfunctions $y^s, s=\frac{1}{2}+ir, y=\operatorname{Im}z, z\in\mathbb{H}$ the limit $\lim_{r\to\infty}Q(y^s)$ is the indicator measure for the pencil of vertical geodesics on $\mathbb{H}$. A discussion is presented of the microlocal lift of the Macdonald-Bessel functions K_{ir}. A formula is presented giving the r-limit of the microlocal lift of $K_{ir}(y)e^{ix}, x+iy\in\mathbb{H}$, in terms of the one-dimensional Dirac measure for an orbit of $A=\{\begin{pmatrix}a&0\\0&a^{-1}\end{pmatrix}\}$ in $SL(2;\mathbb{R})$.

For the first application consider a cofinite group $\Gamma\subset SL(2;\mathbb{R})$ containing the 'cusp' subgroup $\{\begin{pmatrix}1&k\\0&1\end{pmatrix}|k\in\mathbb{Z}\}$ and a sequence of automorphic eigenpairs $\{(\phi_j,\lambda_j)\}$, ϕ_j with 'Fourier coefficients at infinity' $\{a_{j,n}\}$ and $\lim_j Q(\phi_j)$ convergent. The limit $\lim_j Q(\phi_j)$ is explicitly related to the limit of the coefficient sum $\sum_{|n|\le\lambda_j^{1/2}t}a_{j,n}e^{in\theta}$. The coefficient sums are of independent interest in analytic-number theory. For the case of congruence subgroups $\Gamma_0(\ell)\subset SL(2;\mathbb{Z})$ and Hecke eigenforms the sums are related to the behavior of the associated L-functions on their critical line. For the second application the first modulus of continuity result is presented for a limit $\tau_\psi=\lim_j Q(\psi_j)$ of Hecke eigenforms $\{\psi_j\}$ for a congruence subgroup.

Theorem: Provided the totally singular support of τ_ψ is compact on $\Gamma\backslash SL(2;\mathbb{R})$ there is a positive constant C such that the mass of a metric ball satisfies $\tau_\psi(\mathbb{B}(\gamma;\epsilon))\le C\,\epsilon\,(\log\log\epsilon^{-1})^{-2}$ for $\epsilon<e^{-1}$.

Name: Boris Youssin

Title: *Resolving algebraic group actions (joint work with Zinovy Reichstein)*

Abstract: Let X be an algebraic variety with a generically free action of an algebraic group G; we shall assume that the base field k is algebraically closed and is of characteristic zero. The singularities of X are of two kinds: points where X is not smooth, and points where G acts with a nontrivial stabilizer. It follows from canonical resolution of singularities (due to Villamayor and to Bierstone—Milman) that there exists a finite sequence of blowups with smooth G-invariant centers which removes the singularities of the first kind (the points where X is not smooth).

Singularities of the second kind (the points with nontrivial stabilizer) are more stubborn and more interesting.

Recall that any algebraic group H over k has Levi decomposition $H = U \rtimes L$ where U is the unipotent radical of H and L is reductive. If L is commutative, then, being also reductive, it must be diagonalizable, i.e., a product of a finite abelian group and a few copies of k^*. We shall call such H *Levi-commutative.*

Theorem 1. There exists a finite sequence of blowups $X' \to X$ with smooth G-invariant centers such that the stabilizer of any point of X' is Levi-commutative.

Note that in case G is finite, Theorem 1 states that the stabilizers can be all made abelian.

Theorem 1 cannot be improved; any point fixed by a Levi-commutative subgroup cannot be removed by blowups, and moreover, is a birational invariant of the G-action:

Theorem 2. If $X \to Y$ is a G-equivariant *rational* map, X has a point fixed by a Levi-commutative subgroup H of G and Y is complete, then Y also has a H-fixed point.

(Kollár and Szabó gave a simple proof of Theorem 2 in any characteristic.)

Most of the applications of these results this far have been in algebra, via the correspondence between birational isomorphism classes of G-varieties and various algebraic objects (such as division algebras and quadratic forms) related to G.

Still, there is one application to geometry, a new proof of the following "Key Lemma" (due to Parusiński) which plays an important role in his work on the existence of Lipschitz stratifications in the class of semianalytic sets:

For any positive integer n, there is a finite set of homogeneous symmetric polynomials $W_1, ..., W_N \in \mathbb{Z}[x_1, ..., x_n]$ and a constant $M > 0$ such that

$$|dx_i/x_i| < M \max(|dW_1/W_1|, ..., |dW_N/W_N|)$$

as densely defined functions on the tangent bundle of $\mathbb{C}^n$.

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