MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 29 / 2000*

Mathematical Aspects of Gravitation

16.07.–22.07.2000

Introduction

General relativity (GR) is the central theoretical tool in the study of gravitation. In the past the mathematics applied in the study of GR was drawn mainly from the area of Lorentzian geometry. Recently this has changed, with the theory of partial differential equations and modern techniques from Riemannian geometry playing an increasingly important role. This conference, organized by Gerhard Huisken, Jim Isenberg and Alan Rendall, was intended to bring together workers in mathematical relativity with mathematicians expert in relevant areas of PDE theory and geometry. In particular, the aim was to make interesting mathematical problems posed by GR familiar to non-specialists and to introduce relativists to new mathematical techniques. Listening to the lively exchanges of ideas during the week indicated that the conference succeeded in achieving this.

Most of the talks were closely related to one of four major topics:

(1) Asymptotic structure of spacetime and radiation (R. Bartnik, H. Friedrich, F. Nicolò, N. Zipser).

(2) Wave maps and critical collapse (P. Bizon, C. Gundlach, M. Struwe).

(3) Structure of spacetime singularities (L. Andersson, H. Andréasson, B. Berger, H. Ringström, P. Tod, M. Weaver).

(4) Applications of Riemannian geometry (M. Anderson, H. Bray, J. Corvino, J. Lohkamp, D. Pollack, W. Simon.).

There were also talks on individual topics of interest by P. Chruściel, F. Finster and R. Wald.

*composed by Oliver Henkel

The above crude classification cannot do justice to the diversity of the talks and the relations between them; the talk of Corvino, for instance, although listed under point (4) is intimately connected to point (1). It should also be mentioned that although the use of numerical methods was not a subject focussed on in this conference, the importance of the interaction between numerical and rigorous work was a recurring theme and was an important element of several talks (Bartnik, Berger, Bizon, Gundlach).

The organisers chose to limit the number of talks and use the possibility of posting contributions by all participants during the week. This was apparently received favourably by the participants, who appreciated the time which was freed for discussions in this way.

Abstracts of the talks

LARS ANDERSON

Quiescent Singularities

(joint work with Alan Rendall)

The most detailed existing proposal for the structure of spacetime singularities originates in the work of Belinskii, Khalatnikov and Lifshitz. We show rigorously the correctness of this proposal in the case of analytic solutions of the Einstein equations coupled to a scalar field or stiff fluid. More specifically, we prove the existence of a family of spacetimes depending on the same number of free functions as the general solution which have the asymptotics suggested by the Belinskii-Khalatnikov-Lifshitz proposal near their singularities. In these spacetimes a neighbourhood of the singularity can be covered by a Gaussian coordinate system in which the singularity is simultaneous and the evolution at different spatial points decouples.

MICHAEL T. ANDERSON

On long-time vacuum evolution and geometrization of 3-manifolds

A general picture relating the long-time future asymptotic behavior of vacuum cosmological space-times wth the geometrization of 3-manifolds was outlined. Let (M, g) be a vacuum, globally hyperbolic space-time with compact CMC slice Σ. Suppose M is both time-like geodesically complete to the future of Σ and CMC time-complete to the future of Σ, i.e. the CMC foliation Σ_τ fills M to the future of Σ. Then provided certain curvature decay assumptions hold, the asymptotic behavior of the slices Σ_τ, after natural rescaling, induces a weak geometrization of the 3-manifold Σ. More precisely, for any sequence $\tau_i \to 0$, the rescaled metrics $(\Sigma, \bar{g}_{\tau_i})$ subconverge to a complete hyperbolic metric of finite volume $H \subset \Sigma$ while the complement G of H in Σ is a graph manifold which collapses along S^1 or T^2 fibers.

Details may be found in the corresponding paper, which may be viewed or downloaded at: www.math.sunysb.edu/$\sim$ anderson or gr-qc/0006042

HAKAN ANDRÉASSON

On global existence for the spherically symmetric Einstein-Vlasov equation

The classical problem of gravitational collapse in the spherical symmetric situation is studied using the Vlasov equation as matter model. Rein and Rendall proved in 1992 that solutions to the Einstein-Vlasov equation remain smooth for all times in Schwarzschild coordinates if the initial data is small enough. In this case the matter disperses and spacetime is geodesically complete. For large data spacetime singularities form. In Schwarzschild time the spacetime singularities are most likely avoided (supported by numerical simulations by Rein, Rendall and Schaeffer in 1998) and the problem is again to show that solutions remain smooth for all times in these coordinates (for large data). For initial data that vanishes inside an arbitrary small ball around the centre and which also vanishes if angular momentum is arbitrary small (note that in the kinetic description the system has angular momentum also in the spherical symmetric case) it is proved that if the matter initially falls inward and if it continues to do so no singularities will form in finite time. The assumption of "falling inward" (which can be weakened by instead assuming bounds on the outgoing matter components) was first chosen as a "worst case scenario" but from a mathematical point of view it seems to be harder to control the solution having both ingoing and outgoing matter in the system. The aim is of course to get around this assumption in future work and also study the relation to weak cosmic censorship if a global existence theorem is established.

ROBERT BARTNIK

Null Infinity in the NQS gauge

The NQS (null quasi-spherical) numerical Einstein solver [1,2] can be used to test conjectures about the asymptotic structure of gravitational field, near future null infinity. These numerical results can be compared against the predictions of formal expansion techniques. Starting only with the assumptions of existence of NQS coordinates near $\mathcal{I}$, and the first two terms of the asymptotic expansion of the outgoing shear, $\sigma_{NP} = \sigma_2 r^{-2} + \sigma_3 r^{-3} + o(r^{-3})$, which in terms of the NQS potential $r^2\sigma = \eth b$ becomes

$$b = b_0 + b_1 r^{-1} + o(r^{-1}),$$

we find that the NP Weyl component Ψ_0 satisfies

$$\Psi_0 = -\psi r^{-4} + O(r^{-5}\log r).$$

The coefficient ψ depends only on b_0, b_1:

$$\psi = \eth b_1 + \bar{b}_0 \eth^2 b_0 + b_0 \bar{\eth}\eth b_0 + 2\eth b_0 \eth \bar{b}_0.$$

Since $b_0, b_1 \neq 0$ even if $b(0)$ is compactly supported, we conclude that $\psi \neq 0$ generically in the natural evolution space for gravitational fields near $\mathcal{I}$. Note however that $\dot{\psi} = 0$ always under the above asymptotics, so full peeling is preserved in time, if it holds on the initial null hypersurface. This is consistent with results obtained from formal expansions in the Bondi gauge [3].

References:

[1] R. Bartnik, CQG 1997, gr-qc/9611045

[2] — and A. Norton, SIAM J Sci. Comp. 2000, gr-qc/9904045

[3] P. Chrusciel, M. MacCallum and D. Singleton, PRSL 1994, gr-qc/9305021

Beverly K. Berger

The Role of Numerical Simulations in the Study of Spacetime Singularities

Numerical simulations have proven to be a valuable tool in the study of spacetime singularities especially in collaboration with mathematical analysis of the same spacetimes. While computers cannot handle infinite or undefined values, their ability to evolve complicated nonlinear equations allows them to yield insight into the approach to pathalogical behavior in Einstein's equations. Examples which demonstrate this synergy include the numerical discovery of critical phenomena in gravitational collapse and the nature of the approach to the singularity in spatially inhomogeneous cosmologies.

Piotr Bizon

Formation of singularities for wave maps

I report on a recent joint work with T. Chmaj and Z. Tabor on the Cauchy problem for equivariant wave maps from $3+1$ Minkowski spacetime into the 3-sphere

$$u_{tt} = u_{rr} + \frac{2}{r}u_r - \frac{\sin(2u)}{r^2}, \quad u(0,r) = \phi(r), \quad u_t(0,r) = \psi(r). \tag{1}$$

The work was motivated by an attempt to get an analytic insight into some aspects of critical behaviour at the threshold for black hole formation. We first show that the equation (1) has a countable family of self-similar solutions $f_n(\rho)$ where $\rho = r/T - t$ and the index n is equal to the number of unstable modes. On the basis of numerical evidence combined with stability analysis of self-similar solutions we put forward two conjectures. The first conjecture states that singularities which are produced in the evolution of sufficiently large initial data are approached in a universal manner given by the profile of a stable self-similar solution f_0. In this sense the blowup can be considered as local convergence to the solution f_0. The second conjecture states that the self-similar solution f_1 plays the role of a critical solution, that is, its stable manifold determines the threshold for singularity formation. This provides a toy-model of Type II critical gravitational collapse. At the end we mention that an analogous problem for wave maps from $2+1$ Minkowski spacetime into the 2-sphere exhibits a completely different behaviour at the threshold for singularity formation which does not seem to fit into a standard dynamical system picture of critical phenomena.

Hubert Bray

Survey of the Penrose Conjecture

The Penrose Conjecture states that the ADM mass of a space-like slice of a space-time should be at least the mass contributed by the black holes in the space-time, defined to be the square root of the total area of their horizons divided by 16π. In the seventies, Geroch, Jang, and Wald observed that the Hawking mass of a surface was monotone under inverse mean curvature flow and proposed using this fact to prove the Penrose Conjecture (for space-like slices of a space-time with zero second fundamental form). However, existence of such a flow was far from clear since the mean curvature of the surfaces could presumably go to zero or even be negative. Then in 1997, Huisken and Ilmanen modified inverse mean curvature flow in such a way that the flow always existed, thereby proving this case of the Penrose Conjecture for a single horizon. Later, in 1999, the speaker developed another approach to the problem by defining a new conformal flow of 3-metrics which has the property that ADM mass decreases and the area of apparent horizons stays constant and eventually flows to a Schwarzschild 3-metric, thereby proving the inequality for any number of black holes.

Piotr Chrusciel

The area theorems

In my talk I will describe joint work with E.Delay, G.Galloway and R.Howard, in which we prove that the area of sections of future event horizons in space–times satisfying the null energy condition is non–decreasing towards the future under any one of the following circumstances: 1) the horizon is future geodesically complete; 2) the horizon is a black hole event horizon in a globally hyperbolic space–time and there exists a conformal completion with a "H–regular" Scri plus; 3) the horizon is a black hole event horizon in a space–time which has a globally hyperbolic conformal completion. This extends a theorem of Hawking, in which piecewise smoothness of the event horizon seems to have been assumed. No assumptions about the cosmological constant or its sign are made. We prove smoothness or analyticity of the relevant part of the event horizon when equality in the area inequality is attained — this has applications to the theory of stationary black holes, as well as to the structure of compact Cauchy horizons. In the course of the proof we establish several new results concerning the differentiability properties of horizons.

Justin Corvino

Constructing Vacuum Spacetimes Identically Symmetric near Spatial Infinity

We establish the existence of asymptotically flat, scalar-flat metrics on $\mathbb{R}^n$ ($n \geq 3$) which are spherically symmetric, hence Schwarzschild, outside a compact set [1]. Such metrics provide time-symmetric Cauchy data for the Einstein vacuum equations which evolve into nontrivial vacuum *spacetimes* that are identically Schwarzschild near spatial infinity.

The proof uses a local deformation result for the scalar curvature operator, whose linearization L_g has adjoint L_g^* with injective symbol. The obstruction to local deformation is the presence of kernel of the overdetermined-elliptic operator L_g^*. We generalize work of Fischer-Marsden [2] by studying the deformation problem on a domain in a Riemannian manifold, where the deformation tensor should vanish outside the domain in question. We achieve this by using suitably weighted spaces of functions and tensors. In the case when g is the flat metric on $\mathbb{R}^3$, the kernel is the span of $\{1, x^1, x^2, x^3\}$. We glue a Schwarzschild metric to a given asymptotically-flat, scalar-flat metric at a large radius R. In the annular gluing region, the scalar curvature is close to zero. To deform it to zero we use both the local deformation result, as well as the freedom to adjust the mass and center-of-mass of the outer Schwarzschild region to account for the kernel of L_δ^*, which becomes an issue as the metric in the annular gluing region is approaching the flat metric.

In a forthcoming work (joint with Rick Schoen) we treat the non-time-symmetric case. The previous method extends to the full set of constraints, but the corresponding kernel is bigger in this case, and in fact can be accounted for by also considering the linear and angular momentum, and so the basic model at infinity is a suitable slice in Kerr.

References:

[1] Corvino, J.: Scalar Curvature Deformation and a Gluing Construction for the Einstein Constraint Equations. Comm. Math. Phys. *To appear.*

[2] Fischer, A.E., Marsden, J.E.: Deformations of the Scalar Curvature. Duke Math. J. **42** 519-547 (1975)

Felix Finster

The Long-Time Dynamics of Dirac Particles in the Kerr-Newman Black Hole Geometry

We consider the Cauchy problem for the massive Dirac equation in the non-extreme Kerr-Newman geometry outside the event horizon. We derive an integral representation for the Dirac propagator involving the solutions of the ODEs which arise in Chandrasekhar's separation of variables. It is proved that for initial data with compact support, the probability of the Dirac particle to be in any compact region of space tends to zero as t goes to infinity. This means that the Dirac particle must either disappear in the black hole or escape to infinity.

This is joint work with Niky Kamran, Joel Smoller, and Shing-Tung Yau.

Helmut Friedrich

Einstein Equations and Conformal Structure

The conformal regularity of Einstein's vacuum field equations with cosmological constant allows us to show that under suitable assumptions on the data the solutions admit a smooth conformal structure at null infinity. Thus we have precise control on the asymptotic behaviour of the solutions, which cannot be strengthened. In the remaining open case, which is concerned with the asymptotic behaviour of asymptotically flat solutions near space-like infinity, is is shown that under certain assumptions on the data a "regular finite initial value problem near space-like infinity" can be formulated. This problem implies certain "regularity conditions" which allow us for the first time to formulate a reasonable conjecture under which conditions on the initial data the solutions will admit a smooth structure at null infinity. Moreover, the new initial value problem allows us to express the values of the NP constants - absolutely preserved quantities which are defined by certain integrals over cuts of null infinity - in terms of the initial data. For further information and references till 1998 we refer to the article: H. Friedrich, "Einstein's equation and geometric asymptotics", gr-qc/9804009.

Carsten Gundlach

Critical phenomena in gravitational collapse – ten years on

The boundary in the space of initial data for GR between those data that form a black hole and those that disperse is mathematically analogous to a critical phase transition. In particular, data near the threshold evolve through an intermediate attractor (critical solution) that is self-similar and is the same for all initial data for a given type of matter coupled to GR. The black hole mass scales as an irrational power of distance from the threshold (on the collapse side of the threshold). This critical exponent is again universal.

I gave a by now standard review talk on the underlying mechanism and sketched the calculation of the critical exponents. More details can be found in my review paper on www.livingreviews.org, article 1999-4. Then I discussed the structure of spherically symmetric critical solutions – they have a naked singularity – and my current effort to find the most general way in which they can be (non-uniquely) continued beyond the Cauchy horizon of the singularity. Fuchsian methods may help to show that these continuations exist.

Joachim Lohkamp

Positive Scalar Curvature and Energy Theorems

There is a notable relation between positive energy theorems and and nonexistence results for positive scalar curvature on large manifolds. In the talk we explained this geometric correspondence as well as how to approach this kind of problem without dimensional or spin assumptions by goemetric means.

Francesco Nicolò

A new proof of the stability of the Minkowski space

In this work we present a modified approach to the proof of the stability of the Minkowski space, based only on null outgoing and incoming hypersurfaces. The introduction of this double null "canonical"[1] foliation is a significant technical simplification and avoids completely the introduction of the maximal spacelike hypersurfaces[2]. Moreover this new approach allows us to prove also a global existence result[3] which holds outside the domain of dependence of a sufficiently large compact set of arbitrary, strong asymptotically flat, initial data set, without having to prove the full stability of Minkowski spacetime[4].

1 We call a foliation made by null outgoing hypersurfaces "canonical", if these hypersurfaces are level surfaces of a solution of the eikonal equation with a specific choice of the initial data.

2 The elliptic estimates on the spacelike hypersurfaces are substituted by estimates associated to the evolution equations along the null geodesics generating the null hypersurfaces, which are ordinary differential equations and by elliptic estimates on the two dimensional surfaces, diffeomorphic to S^2, intersections of the outgoing and incoming null hypersurfaces.

3 The outgoing leaves of the double null foliation are complete, that is the null geodesics generating them can be indefinitely extended toward the future.

4 In the proof of stability the regions internal and external to a "cone" with vertex at the origin are completely decoupled.

Daniel Pollack

A gluing construction for the Einstein constraint equations

We address the following general question: given two set of initial data $(\Sigma_i, \gamma_i, K_i)$, for $i = 1, 2$, each satisfying the vacuum Einstein constraint equations, together with specified points $p_i \in \Sigma_i$; can we find a family of solutions to the constraint equations $(\Sigma_s, \gamma_s, K_s)$, for $s \in \Lambda$ (a set of space of free parameters) with the following properties? For each $s \in \Lambda$

- Σ_s is the topological connected sum of original two 3-manifolds, $\Sigma_s = \Sigma_1 \# \Sigma_2$.
- there exist small geodesic balls $B_{r_i(s)}(p_i)$ of radius $r_i(s) > 0$ (possibly depending on s) about each p_i such that on $\Sigma_i \setminus B_{r_i(s)}(p_i) \subset \Sigma_s$ both γ_s and K_s are small perturbations of γ_i and K_i for $i = 1, 2$ respectively.

Here a "small perturbation" is one which vanishes as the parameter $s \to \infty$ in Λ. In joint work with Jim Isenberg and Rafe Mazzeo we give an affirmative answer to this question in the case that the mean curvature $\tau = \mathrm{Tr}_{\gamma_i} K_i$ is constant (with the same value) and the 3-manifolds Σ_i are closed. The solutions we construct have perturbations which are exponentially small of order $\exp(-\alpha\, s)$ for an explicit $\alpha > 0$, where $s \in \mathbb{R}_+ \subset \Lambda$. Moreover the distance in Σ_s between $\Sigma_1 \setminus B_{r_1(s)}(p_1)$ and $\Sigma_2 \setminus B_{r_2(s)}(p_2)$ tends to zero as $s \to \infty$. The construction makes extensive use of the conformal method as developed by Choquet-Bruhat, Lichnerowicz and York. Similar results are expected to hold in the cases of asymptotically flat or asymptotically hyperboloidal initial data.

Hans Ringström

Bianchi IX orthogonal perfect fluids

The talk concerned the asymptotic behaviour of Bianchi IX spacetimes close to the singularities. The matter model was assumed to be an orthogonal perfect fluid with linear equation of state $p = (\gamma - 1)\rho$, where $1 \leq \gamma \leq 2$. First, results concerning curvature blow up were mentioned. The class of Bianchi IX spacetimes contains the subclass of Taub-NUT solutions. These solutions are locally rotationally symmetric vacuum solutions, and as one approaches a singularity, the curvature remains bounded. In fact, these solutions can be extended beyond the singularities in inequivalent ways. The result stated was that all Bianchi IX solutions considered, except for the Taub-NUT solutions, exhibit curvature blow up. According to the BKL conjecture, Bianchi IX solutions are the prototypes for the local behaviour of generic gravitational collapse. According to this conjecture, the behaviour of solutions should be oscillatory and the matter should be unimportant as one approaches a singularity, with the exception of stiff fluid solutions. In terms of the variables of Wainwright and Hsu, we mentioned the result that, except for the stiff fluid case, the solutions generically oscillate indefinitely, and the density parameter generically converges to zero. In fact, the solutions generically converges to an attractor in the non-stiff fluid case. In the stiff fluid case, all solutions converge and the density parameter converges to a non-zero value in that case.

Walter Simon

On Uniqueness of static vacuum spacetimes with negative cosmological constant

A well-known family of solutions of Einstein's equations with cosmological constant Λ are the "generalized Kottler solutions" (GKS) [1]

$$(1) \qquad ds^2 = -(k - \frac{2m}{r} - \frac{\Lambda}{3}r^2)dt^2 + (k - \frac{2m}{r} - \frac{\Lambda}{3}r^2)^{-1}dr^2 + r^2 d\Omega_k^2 \ , \quad k = 0, \pm 1.$$

where $d\Omega_k^2$ denotes a metric of constant Gauss curvature k on a 2-dim. manifold (which we assume to be compact), and m is a constant.

P. Chruściel and myself have recently carried out a rather systematic study of static solutions with $\Lambda < 0$ which asymptotically approach the GKS at a suitable rate [2]. If we assume, in addition, that the genus of infinity g_∞ is ≥ 2, that the horizon is connected, and that its surface gravity satisfies $\kappa \leq \sqrt{-\Lambda/3}$, we show the **"inverse Penrose inequality"** (IPI)

$2m + r_{\partial\Sigma} + \Lambda r^3_{\partial\Sigma}/3 \leq 0$, where m is the (suitably normalized) mass, $r_{\partial_\mathcal{H}\Sigma}$ is defined by $4\pi(g_{\partial_\mathcal{H}\Sigma} - 1)r^2_{\partial_\mathcal{H}\Sigma} = A_{\partial_\mathcal{H}\Sigma}$, and $g_{\partial_\mathcal{H}\Sigma}$ and $A_{\partial_\mathcal{H}\Sigma}$ are the genus and the area of the horizon, respectively.

We expect that the **Penrose inequality** (PI, which goes precisely the opposite way as the IPI) holds under the same conditions (and in fact under much less restrictive ones) as above. In fact the PI has been shown under the additional assumption of a smooth "inverse mean curvature flow". If both the PI and the IPI hold, it follows that the corresponding GKS are unique.

Our proof of the IPI combines the strategies under which similar inequalities have been obtained before, in the special cases of vanishing Λ on the one hand, and in the case of the absence of the black hole on the other hand (where the IPI becomes the "negative mass theorem" of Boucher et al. [3]).

In the case $\Lambda = 0$, the PI is known and leads, in combination with the IPI, to a uniqueness proof, (which is, in essence, Israel's proof [4]). On the other hand, in the case $\Lambda < 0$ without black hole a positive mass theorem (and a uniqueness result for the anti-de Sitter solutions in the static case) are likely to hold but have not yet been established rigorously.

References:

[1] Annalen der Physik **56** (1918), 401–462

[2] gr-qc/0004032

[3] Phys. Rev. D **30** (1984), 2447–2451

[4] Phys. Rev. **164** (1967), 1776–1779

MICHAEL STRUWE

Wave maps

Wave maps $u = (u^1, \ldots, u^n) : \mathbb{R} \times \mathbb{R}^m \longrightarrow N \hookrightarrow \mathbb{R}^n$ from (1+m)-dimensional Minkowski space to a smooth, compact target manifold $N \subset \mathbb{R}^n$ without boundary by definition are solutions to the equation

$$(1) \qquad \Box u = u_{tt} - \triangle u = A(u)(Du, Du) \perp T_u N \quad ,$$

where $Du = (u_t, \nabla u)$ denotes the space-time derivatives and with A denoting the second fundamental form of N.

It is conjectured that the Cauchy problem for (1) is (locally) well-posed for initial data

$$(u, u_t)_{|t=0} = (u_0, u_1) \in H^s \times H^{s-1}(\mathbb{R}^m; TN)$$

whenever $s \geq \frac{m}{2}$. I survey recent progress towards a proof of this conjecture using the special geometric, analytic, and algebraic structure of equation (1).

PAUL TOD

Isotropic cosmological singularities

Penrose has suggested that the initial singularity of the universe must have finite Weyl curvature, while this should not be true of any singularity formed in gravitational collapse. It is not known how to characterise a singularity with finite Weyl tensor but *isotropic cosmological singularities* are a class of singularities which are easy to define and which manifestly have this property. An isotropic cosmological singularity (ICS) is one which can be removed by conformally rescaling the physical metric: in the rescaled, unphysical metric the physical singularity occurs at a smooth space-like hypersurface on which the conformal factor vanishes (though it need not be smooth there). Since the Weyl tensor is conformally invariant and it is finite in the unphysical space-time, it will be finite at the ICS in the physical space-time.

The problem is to show that there exist reasonable cosmological models with an ICS. The idea is to do this by finding (and solving) a well-posed initial value problem with data given at the singularity surface. In two recent papers with Keith Anguige (1999, below), I considered this problem for perfect fluids with polytropic equation of state and, with spatial homogeneity, for mass-less Einstein-Vlasov. In a third paper, Anguige (2000) solved the mass-less Einstein-Vlasov case without the assumption of spatial homogeneity. In each case, the Einstein equations can be reduced to a symmetric hyperbolic system with a singularity in the time, of the appropriate form for the existence and uniqueness theorem of Claudel and Newman (1998) to be used. Thus there are many cosmological models with an ICS.

Because the system is Fuchsian, less data can be freely specified: for the perfect fluid case one can give the 3-metric of the singularity arbitrarily (the second fundamental form must vanish, and there are no constraints and no extra data for the matter); for the Einstein-Vlasov case one can give the initial distribution function, subject to a vanishing first moment; the initial metric and second fundamental form are then determined by the distribution function. For the perfect fluid, if the initial Weyl tensor is zero rather than just finite, it will always be zero and the metric will be Robertson-Walker. For the Einstein-Vlasov this is not the case; the Weyl tensor can be zero at the singularity and nonzero later.

References:

[1] K Anguige and K P Tod Ann.Phys. **276** (1999) 257-293, 294-320, gr-qc 9903008, 9903009

[2] K Anguige Ann.Phys. **282** (2000) 395-419, gr-qc 9903018 C M Claudel and K P Newman Proc. Roy. Soc. Lond. **A 454** (1998) 1073-1107

Robert Wald

Conserved Quantities

In general relativity, at spatial infinity conserved quantities can be defined in a natural way via the Hamiltonian framework: Each conserved quantity is associated with an asymptotic symmetry and the value of the conserved quantity is defined to be the value of the Hamiltonian which generates the canonical transformation on phase space corresponding to this symmetry. However, such an approach cannot be employed to define "conserved quantities" in a situation where symplectic current can be radiated away because there does not, in general, exist a Hamiltonian which generates the given asymptotic symmetry. (This fact is closely related to the fact that the desired "conserved quantities" are not, in general, conserved!) A. Zoupas and I have proposed a prescription for defining "conserved quantities" in such situations by postulating a modification of the equation that must be satisfied by a Hamiltonian. Our prescription is a very general one, and is applicable to a very general class of asymptotic conditions in arbitrary diffeomorphism covariant theories of gravity derivable from a Lagrangian, although we have not investigated existence and uniqueness issues in the most general contexts. In the case of general relativity with the standard asymptotic conditions at null infinity, our prescription agrees with the one proposed by Dray and Streubel from entirely different considerations.

Marsha Weaver

Gowdy spacetimes with spikes

(joint work with Alan Rendall)

We construct C^∞ solutions to Einstein's equation which belong to the T^3 Gowdy class with a transformation which takes a Gowdy solution to a Gowdy solution. The seed solutions we use are a large class (having the correct number of arbitrary functions) which are known explicitly up to a remainder term which converges to zero in the singular time direction. Thus the arbitrary functions are called "data on the singularity." Solutions in this previously known class do not reproduce the features (spikes) which have been seen in numerical simulations. The new solutions again have the correct number of arbitrary functions, and they do reproduce the features seen in numerical simulations. In addition, now we can make rigorous various statements about the asymptotic properties of the spikes that were indicated by previous work. The spikes do persist to the singularity. That is, the "data on the singularity" is discontinuous at the asymptotic position of the spike, and smooth except for a set of measure zero. All of the solutions obtained here are convergent. That is, the evolution at each spatial point converges to that of a Kasner solution. None of the solutions obtained here are extendible past the singularity, since the Kretschmann scalar blows up there. The rate of curvature blowup is discontinuously greater at a "true" spike than at any given nearby spatial point, close enough to the singularity, in terms of a geometrically defined time coordinate, the area of the two-dimensional orbits of the isometry group.

NINA ZIPSER

Solutions of the Maxwell-Einstein Equations

In **The Global Nonlinear Stability of the Minkowski Space**, Christodoulou and Klainerman show that given asymptotically flat initial data which satisfy a smallness condition, there exist global, smooth nontrivial solutions to the Einstein-Vacuum equations. This result can be generalized to show the global nonlinear stability of the trivial solution of the Einstein-Maxwell equations. In particular instead of solving the Einstein field equations $\mathbf{G}_{\mu\nu} = 8\pi\mathbf{T}_{\mu\nu}$ with the energy momentum tensor $\mathbf{T}_{\mu\nu}$ equal to zero, $\mathbf{T}_{\mu\nu}$ is set to equal the stress-energy tensor of an electromagnetic field F such that F satisfies the Maxwell equations.

As in **The Global Nonlinear Stability of the Minkowski Space**, the stability of the Minkowski space for the Einstein-Maxwell system is proven by showing the existence of unique, globally hyperbolic, smooth, and geodesically complete solutions which are close to Minkowski Space. The proof of existence starts with a maximally foliated space-time slab obtained by proving short time existence for the Maxwell-Einstein equations and requiring that all curvature remain bounded by some ε_0. A null structure is defined on the space-time by constructing an optical function which is the solution of the Eikonal equation. The optical function is used to define "almost" conformal killing fields which are analogous to the conformal killing fields in Minkowski space-time. These vector fields are used in conjunction with the stress-energy tensor and the homogeneous field equations for the electromagnetic field and the Bel-Robinson tensor and the inhomogeneous field equations for the Weyl tensor to obtain estimates for the space-time Riemann curvature tensor.

Once good estimates are obtained for the Riemann curvature tensor, all the parameters of the time foliation are determined purely by solving an elliptic system The proof of global existence follows from a continuation argument which shows that the weighted L^2-norms of the curvature and the parameters of the foliation must remain small.

The asymptotic behavior of the components of the Weyl tensor and the electromagnetic field with respect to a null frame are seen to be the same as in the linear case shown by Christodoulou and Klainerman in "Asymptotic properties of linear field equations in Minkowski space".

References:

[1] C. Bruhat Recent Results on the Cauchy Problem for Gravitation and Yang-Mills Fields. *Proceedings of the second marcel Grossmann meeting on General Relativity*, 1982.

[2] C. Bruhat and J. York. Geometrical well posed systems for the Einstein equations. *Acad. Sci. Paris, t. 321, Série I, p. 1089-1095, 1995.*

[3] D. Christodoulou-S. Klainerman. Asymptotic properties of linear field equations in Minkowski space. *Comm. Pure Appl. Math.* 43,1990, 137-199.

[4] D. Christodoulou-S. Klainerman. *The Global Nonlinear Stability of the Minkowski Space.* Princeton University Press 1993.

[5] J. Leray. *Hyperbolic differential equations.* IAS, Princeton, 1952.

[6] M. Taylor. *Partial Differential Equations III: Nonlinear Equations.* Springer, 1996.

List of participants

Prof. Dr. Stephen Anco
Mathematics Department
Brock University
St. Catherines, Ontario L2S 3A1
CANADA

Prof. Dr. Michael Anderson
Department of Mathematics
State University of New York
at Stony Brook
Stony Brook , NY 11794-3651
USA

Prof. Dr. Lars Andersson
Dept. of Mathematics
Royal Institute of Technology
Lindstedtsvägen 25
S-100 44 Stockholm

Dr. Hakan Andréasson
Department of Mathematics
Chalmers University of Technology
at Göteborg
S-412 96 Göteborg

Prof. Dr. Robert Bartnik
School of Mathematics and Statistic
University of Canberra
Canberra ACT 2601
AUSTRALIA

Prof. Dr. Helga Baum
Institut für Reine Mathematik
Fachbereich Mathematik
Humboldt-Universität Berlin
10099 Berlin

Prof. Dr. Robert Beig
Institut für Theoretische Physik
Universität Wien
Boltzmanngasse 5
A-1090 Wien

Prof. Dr. Beverly Berger
Physics Department
Oakland University
Rochester , MI 48309
USA

Prof. Dr. Piotr Bizon
Institute of Physics
Jagiellonian University
ul. Reymonta 4
30-059 Krakow
POLAND

Prof. Dr. Hubert Bray
Department of Mathematics
Massachusetts Institute of
Technology
Cambridge , MA 02139-4307
USA

Prof. Dr. David Brown
Dept. of Mathematics
North Carolina State University
P.O.Box 8205
Raleigh , NC 27695-8205
USA

Prof. Dr. Piotr T. Chrusciel
Departement de Mathematiques
Faculte des Sciences de Tours
Universite de Tours
Parc de Grandmont
F-37200 Tours

Prof. Dr. Clarissa Claudel
Perrydene
Pear Tree Lane
Shorne
Nr. Gravesend
GB-Kent DA12 3JS

Prof. Dr. Justin Corvino
Department of Mathematics
Brown University
151 Thayer Street/ Box 1917
Providence, RI 02912
USA

Dr. Sergio Dain
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Klaus Ecker
Dept. of Mathematics
University of Melbourne
Parkville, Victoria 3052
AUSTRALIA

Prof. Dr. Jürgen Ehlers
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Felix Finster
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig

Dr. Helmut Friedrich
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Gregory Galloway
Dept. of Mathematics and Computer
Science
University of Miami
P.O. Box 248011
Coral Gables , FL 33124
USA

Prof. Dr. David Garfinkle
Physics Department
Oakland University
Rochester , MI 48309
USA

Prof. Dr. Carsten Gundlach
Faculty of Mathematical Studies
University of Southampton
GB-Southampton , SO17 1BJ

Oliver Henkel
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Gerhard Huisken
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
72076 Tübingen

Prof. Dr. James Isenberg
Dept. of Mathematics
University of Oregon
Eugene , OR 97403-1222
USA

Thomas Jurke
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Satyanad Kichenassamy
Departement de Mathematiques
Universite de Reims
Moulin de la Housse
BP 1039
F-51687 Reims Cedex 2

Prof. Dr. Hans Lindblad
Dept. of Mathematics
University of California, San Diego
9500 Gilman Drive
La Jolla , CA 92093-0112
USA

Prof. Dr. Istvan Racz
Theoretical Dept.
MTA KFKI Res. Inst. for Particle
and Nuclear Physics
Konkoly Thege ut 29-33
H-1525 Budapest

Joachim Lohkamp
Institut für Mathematik
Universität Augsburg
Universitätsstr. 14
86159 Augsburg

Prof. Dr. Alan Rendall
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Francesco Nicolò
Dipartimento di Matematica
Universita degli Studi di Roma
Tor Vergata
Via della Ricerca Scientifica
I-00133 Roma

Prof. Dr. Hans Ringström
Dept. of Mathematics
Royal Institute of Technology
Lindstedtsvägen 25
S-100 44 Stockholm

Prof. Dr. Niall O'Murchadha
Department of Physics
University College Cork
Cork
IRELAND

Dr. Bernd Schmidt
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Fabrice Planchon
Laboratoire d'analyse numerique
Universite Paris VI
175 rue du chevaleret
F-75013 Paris

Dr. Walter Simon
Institut für Theoretische Physik
Universität Wien
Boltzmanngasse 5
A-1090 Wien

Prof. Dr. Daniel Pollack
Dept. of Mathematics
Box 354350
University of Washington
C138 Padelford Hall
Seattle , WA 98195-4350
USA

Prof. Dr. Michael Struwe
Mathematik Departement
ETH Zürich
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. A. Shadi Tahvildar-Zadeh
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton , NJ 08544-1000
USA

Prof. Dr. Paul Tod
Mathematical Institute
Oxford University
24 - 29, St. Giles
GB-Oxford OX1 3LB

Prof. Dr. Charles Torre
Department of Physics
Utah State University
Logan , UT 84322-4415
USA

Prof. Dr. Claes Uggla
Department of Mathematics & Physics
University of Karlstad
S-65188 Karlstad

Prof. Dr. Robert Wald
Department of Physics
The University of Chicago
5640 South Ellis Avenue
Chicago , IL 60637-1433
USA

Prof. Dr. Marsha Weaver
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14424 Golm

Prof. Dr. Nina Zipser
Dept. of Mathematics
Harvard University
1 Oxford Street
Cambridge , MA 02138
USA

List of E-mail addresses

Stephen Anco	sanco@spartan.ac.brocku.ca
Michael Anderson	anderson@math.sunysb.edu
Lars Andersson	larsa@math.kth.se
Hakan Andréasson	hand@math.chalmers.se
Robert Bartnik	bartnik@ise.canberra.edu.au
Helga Baum	baum@mathematik.hu-berlin.de
Robert Beig	beig@merlin.ap.univie.ac.at
Beverly Berger	berger@oakland.edu
Piotr Bizon	bizon@thrisc.if.uj.edu.pl
Hubert Bray	bray@math.mit.edu
David Brown	david_brown@ncsu.edu
Piotr T. Chrusciel	chrusciel@univ-tours.fr
Clarissa Claudel	clarissa@ukgateway.net
Justin Corvino	corvino@math.stanford.edu
Sergio Dain	dain@aei-potsdam.mpg.de
Klaus Ecker	klaus.ecker@sci.monash.edu.au
Jürgen Ehlers	ehlers@aei-potsdam.mpg.de
Felix Finster	felix.finster@mis.mpg.de
Helmut Friedrich	hef@aei-potsdam.mpg.de
Gregory Galloway	galloway@math.miami.edu
David Garfinkle	garfinkl@oakland.edu
Carsten Gundlach	cg@maths.soton.ac.uk
Oliver Henkel	henkel@aei-potsdam.mpg.de
Gerhard Huisken	gerhard.huisken@uni-tuebingen.de
James Isenberg	isenberg@bright.math.uoregon.edu
Thomas Jurke	jurke@odin.aei-potsdam.mpg.de
Satyanad Kichenassamy	satyand.kichenassamy@univ-reims.fr
Hans Lindblad	hans@math.ucsd.edu
Joachim Lohkamp	lohkamp@math.uni-augsburg.de
Francesco Nicolò	nicolo@mat.uniroma2.it
Niall O'Murchadha	niall@ucc.ie
Fabrice Planchon	fab@ann.jussieu.fr
Daniel Pollack	pollack@math.washington.edu
Istvan Racz	istvan@sunserv.kfkr.hu
Alan Rendall	rendall@aei-potsdam.mpg.de
Hans Ringström	hansr@math.kth.se
Bernd Schmidt	bernd@aei-potsdam.mpg.de
Walter Simon	simon@ap.univie.ac.at
Michael Struwe	michael.struwe@math.ethz.ch
A. Shadi Tahvildar-Zadeh	shadi@math.princeton.edu
Paul Tod	tod@maths.ox.ac.uk
Charles Torre	torre@cc.usu.edu
Claes Uggla	uggla@physto.se, claess.uggla@kau.se
Robert Wald	rmwa@midway.uchicago.edu
Marsha Weaver	weaver@aei-potsdam.mpg.de
Nina Zipser	zipser@math.harvard.edu