Mathematisches Forschungsinstitut Oberwolfach

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Arithmetic Algebraic Geometry

July 30th – August 5th, 2000

The meeting was organized by Gerd Faltings (Bonn), Günter Harder (Bonn) and Nicholas M. Katz (Princeton). The talks given by the participants covered many areas of current interest in arithmetic algebraic geometry. The topics of the talks included

- Galois representations,
- the two Bloch-Kato conjectures for K-theory and L-values of Motives,
- stratifications of moduli spaces of abelian varieties,
- arithmetic theory of elliptic curves,
- Langlands correspondence for function fields of curves over finite fields,
- arithmetic cohomology theories, and more.

All the talks were at a very high level and many new and important results were presented.

The atmosphere of the conference was very ispiring and surely everybody is looking forward to the next meeting of that kind.

Abstracts

Laurent Lafforgue

La correspondance de Langlands sur les corps de fonctions: Preuve corrigée

L'an dernier, j'avais annoncé une démonstration de la correspondance de Langlands sur les corps de fonctions pour GL_r, généralisant la preuve de Drinfeld dans le cas $r = 2$. Elle consistait à réaliser cette correspondance dans la cohomologie ℓ-adique des champs Cht_N^r de chtoucas de Drinfeld de rang r avec structures de niveau N. La preuve utilisait en particulier la construction de résolutions des singularités pour les compactifications $\overline{Cht_N^r,^{\overline{p}\leq p}}$.

Or je me suis aperçu il ya deux mois que cette construction est incorrecte quand le niveau N a des multiplicités. On surmonte cette difficulté en montrant que l'ouvert $\overline{Cht_r^r,^{\overline{p}\leq p}}'$ de $\overline{Cht_r^r,^{\overline{p}\leq p}}$ défini en demandant que pôle, zéro et dégénérateurs évitent le niveau N est lisse et que, de façon remarquable, il est stabilisé par les correspondances de Hecke.

Karl Rubin

Kolyvagin Systems

(joint work with Barry Mazur)

Fix a prime p. Let R be a complete noetherian local ring with finite residue field of characteristic p, and let T be a free R-module of finite rank with a continuous action of $G_{\mathbb{Q}}$. A Selmer structure F on T is a collection of local conditions $H^1_F(\mathbb{Q}_\ell, T) \subset H^1(\mathbb{Q}_\ell, T)$ which we use to define a Selmer group $H^1_F(\mathbb{Q}, T) \subset H^1(\mathbb{Q}, T)$. F induces (by local duality) a Selmer structure F^* on $T^* = \mathrm{Hom}(T, \mu_{p^\infty})$.

A Kolyvagin system is a collection

$$\{\kappa_n \in H^1_{F(n)}(\mathbb{Q}, T/I_nT) : n \text{ squarefree + prime to a finite set } \Sigma_F\}$$

where: $F(n)$ is the Selmer structure F modified at ℓ dividing n,

$$I_n = \partial^{cd}_{\ell|n}(\ell - 1, \det(Fr_\ell - 1|T)) \subset R$$

and such that the localizations of κ_n and $\kappa_{n\ell}$ at ℓ are related in a certain precise way. Let $KS(T; F)$ denote the R-module of Kolyvagin systems for T and F.

For example, the "derivative classes" which Kolyvagin constructs from an Euler system form a Kolyvagin system.

Theorem: Suppose R is a field; T is irreducible; there is a $\tau \in G_{\mathbb{Q}}$ such that $\tau = 1$ on μ_{p^∞} and $\dim(T/(\tau - 1)T) = 1$; $H^1(\mathbb{Q}(T, \mu_{p^\infty})/\mathbb{Q}, T) = H^1(\mathbb{Q}(T, \mu_{p^\infty})/\mathbb{Q}, T^*) = 0$; and either $T \not\approx T^*$ or $p > 3$.

Then

(1) if $\dim H^1_F(\mathbb{Q}, T) \leq \dim H^1_{F^*}(\mathbb{Q}, T^*)$, then $KS(T, F) = 0$

(2) suppose $\dim H^1_F(\mathbb{Q}, T) = \dim H^1_{F^*}(\mathbb{Q}, T^*) + 1$. Then

- (a) $\dim KS(T, F) \leq 1$.
- (b) if $\kappa \in KS(T, F)$, $\kappa \neq 0$, then $\dim H^1_{F^*}(\mathbb{Q}, T^*) = \min\{\nu(n) | \kappa_n \neq 0\}$, ($\nu(n)$ = number of prime divisor of n),
- (c) if $\kappa \in KS(T, F)$, $\kappa \neq 0$, then $\kappa_n \neq 0 \Leftrightarrow H^1_{F(n)^*}(\mathbb{Q}, T^*) = 0$.

The proof goes by identifying $KS(T,F)$ with the global sections of a sheaf constructed over a graph whose vertices are the indices n.

Annette Werner

Arakelov intersections, buildings and symmetric spaces

We present a geometrical interpretation of Arakelov theory at infinity for projective spaces. This is motivated by Manin's results interpreting 3-dimensional hyperbolic geometry as ∞-adic Arakelov geometry in the case of curves.

We show how to interpret certain non-Archimedean Arakelov intersection numbers of homologically trivial cycles on $\mathbb{P}^{n-1}$ with the combinatorial geometry of the Bruhat-Tits building associated to $PGL(n)$. This geometric setting has an Archimedean analogue, namely the Riemannian symmetric space associated to $SL(n,\mathbb{C})$, which we use to interpret analogous Archimedean intersection numbers in a similar way. Our results suggest to regard this symmetric space as an "Archimedean model" of $\mathbb{P}^{n-1}_{\mathbb{C}}$, and half-geodesics in this space as "Archimedean reductions" of linear cycles in $\mathbb{P}^{n-1}_{\mathbb{C}}$.

Richard Pink

The Ekedahl-Oort stratification and a problem of algebraic groups

Ekedahl and Oort break up the closed fiber at p of the moduli space of principally polarized abelian varieties of dimension g by the isomorphism class of the p-kernel $A[p]$ over an algebraically closed field. They show that the number of strata is finite, determine their dimension and prove various other properties. Ben Moonen achieved similar results for Shimura varieties of PEL type. I propose to approach this problem by working inside the reductive group defining the Shimura variety, reduced modulo p, instead of using PEL-type information explicitly. The possible isomorphy classes of p-kernels with "G-structure" are conjecturally classified by the following orbits.

Let G be a connected reductive group over $\mathbb{F}_p$ and $\mu_0 : \mathbb{G}_{m,\mathbb{F}_p} \to G$ a cocharacter. (One can generalize this to μ_0 not necessarily defined over $\mathbb{F}_p$.) It determines two opposite parabolics $U^{\pm} \rtimes L$ of G where $\mathrm{Lie}G = \mathrm{Lie}U^{+} \oplus \mathrm{Lie}L \oplus \mathrm{Lie}U^{-}$ is the decomposition into weight $> 0, = 0, < 0$ spaces for μ_0. Let $(U^{+} \times U^{-}) \rtimes L$ act on G by $g \mapsto u^{+}\ell \cdot g \cdot {}^{\sigma}(u^{-}\ell)^{-1}$, where σ is Frobenius over $\mathbb{F}_p$. Let $T \subset L$ be an $\mathbb{F}_p$-rational maximal torus, W its Weylgroup, and $\Phi = \Phi^{(+)} \amalg \Phi^{(0)} \amalg \Phi^{(-)}$ its roots in U^{+}, L, U^{-} respectively.

Theorem 1: Every $(U^{+} \times U^{-}) \rtimes L$-orbit of G contains an element of W. Moreover, $\dot{w}, \dot{w}' \in \dot{W}$ lie in the same orbit iff $\exists \tau \in W \cap L$ and $\exists \rho \in \bigcap_{i \in \mathbb{Z}} {}^{w^i}(W \cap L)$ such that $w' = \tau \cdot w \cdot \rho \cdot {}^{\sigma}\tau^{-1}$.

Corollary: The number of orbits is finite. (It is actually $[W : W \cap L]$.)

Theorem 2: The codimension of the orbit of $w \in W$ is $|\Psi_w|$, where

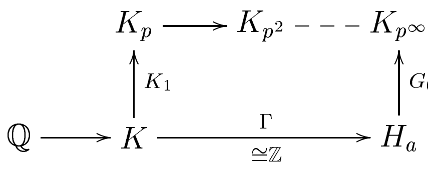
$$\Psi_w := \bigcup_{i>0} \left(\Phi^{(+)} \cap \bigcap_{0<j<i} {}^{w^j}\Phi^{(-)} \right).$$

Open problem: When does the orbit of $\dot{w}'$ lie in the closure of the orbit of $\dot{w}$? The analogue of this question in Ekedahl-Oort's original case is also open. Results of Oort show that the partial order cannot be simply the Bruhat order for suitable representatives.

Christoph Cornut

(Heegner Points) A conjectures of Mazur (ICM 84)

Let $\pi : X_0(N) \to \mathbb{E}/\mathbb{Q}$ be a modular elliptic curve, K a imaginary quadratic field, $p|N$ a prime. Suppose: $q|N \Rightarrow q$ splits in K (Heegner Hypothesis). This implies: $\exists \mathcal{N} \mathcal{O}_k$-ideal such that $\mathcal{O}_n/\mathcal{N} \simeq \mathbb{Z}/N$. For $(c \wedge N) = 1$, put $\mathcal{N}_c = \mathcal{N} \cap \mathcal{O}_c \subset_N \mathcal{O}_c$. Let $c_n = [\mathbb{C}/\mathcal{O}_{p^n} \to \mathbb{C}/\mathcal{N}_{p^n}^{-1}] \in X_0(N)$. Consider the galois situation:



K_1: Hilbert class field.
K_{p^n}: Ring class field (p^n),
$H_\infty = \mathbb{Z}_p$-anticyclotomic extensions of K.
Then it is known that $c_n \in X_0(N)(K_{p^n})$, and Mazur conjectured that:

Theorem: $\mathrm{tr}_{G_0}(\pi(c_n)) \in \mathbb{E}_{\mathrm{tors}}$ if $n >> 0$.

The proof goes like this: Let $G_1 \subset G_0$ be $G_1 = \langle \mathrm{Frob}(Q, K_{p^\infty}/K) \mid Q|q|d_K, q \neq p \rangle$, $\sigma_1, \ldots, \sigma_s$ representatives of G_0/G_1. For $a \subset K$ such that $\mathcal{O}(a) = \mathcal{O}_{p^n}$ some n, define $H(a) = [\mathbb{C}/a \to \mathbb{C}/a\mathcal{N}_{p^n}^{-1}] \in X_0(N)(K_{p^n})$. Then:
• the action of G_1 on the $H(a)$'s can be realized "geometrically".
• the action of the σ_i's is "random", in the sens that: $\forall (n_i)_{i \in \mathbb{Z}^s} \neq 0$,
$\Sigma_{i=1}^s n_i \sigma_i \pi(H(a)) \neq 0$ for infinitely many a's.
This last fact is proven by reduction at P inert in K, where the key argument, due to N. Vatsal, is the use of Ratner's theorem (ICM 94).

Frans Oort

Foliations of moduli spaces

We work in $\mathcal{A} := \mathcal{A}_{g,1,n} \otimes \mathbb{F}_{p,n}$. (Here $\mathbb{F}_{p,n}$ is the smallest field containing $\mathbb{F}_p$ and a primitive n-th root of 1, $(n,p) = 1$.)

What is the Hecke orbit of a point $x \in \mathcal{A}$? Clearly you cannot come outside W_β (the NP stratum, $\beta = \mathcal{N}(X)$). NP = Newton Polygon.

Conjecture: $\forall x \in \mathcal{A}$, $x = [(X, J)]$, $\beta = \mathcal{N}(X)$, then $\overline{Hx} = W_\beta$. How many components do NP-strata have? The supersingular locus $S = W_\sigma$ is highly reducible (for $p >> 0$).

Conjecture: $\forall \beta \neq \sigma$, W_β is **irreducible**.

In this talk we give two foliations of $W_\beta \subset \mathcal{A}$, and we predict what $\langle H_\ell \cdot x$ should be.

Def. $x \in \mathcal{A}(k)$, $\mathcal{C}(x) := \{(Y, \mu) \in A(L) \mid L \hookrightarrow \Omega = \overline{\Omega} \hookleftarrow k,\ Y[p^\infty] \otimes \Omega \simeq X[p^\infty] \otimes \Omega\}$; this gives a "foliation" $\bigcup_x \mathcal{C}(x)$ of W_β, $\beta := \mathcal{N}(X)$.

Def. $\langle H_\alpha$: only isogenies involving iterated α_p-covers considered. $x \in \mathcal{A}(k)$, $k = \overline{k}$, $I(x) := \bigcup$ (irr.cpts. of $\langle H_\alpha \cdot x$ containing x).

Rk $I(x) \subset \mathcal{A}_k$ closed; $I(x)$ is complete; $\dim I(x) = 0 \Leftrightarrow p-$rank $(X) \geq g-1$. In general $\langle H_\alpha \cdot x$ is not of finite type (can have ∞ many components).

Rk $\mathcal{C}(X) \subset \mathcal{A} \otimes k$ closed, $\mathcal{C}(x)$ complete $\Leftrightarrow \#\mathcal{C}(x)(k) < \infty \Leftrightarrow X$ is supersingular.

We work this out in one example $g = 3$, $\beta = (2,1) + (1,2)$, $W_\beta = V_0 \hookrightarrow \mathcal{A}$.

Theorem: $g = 3$, $V_0 \hookrightarrow \mathcal{A}$ is irreducible.

Proof:
$a(X) = 2 \Leftrightarrow X[p^\infty] \cong_u G_{2,1} + G_{i,2}$
$a(X) = 1 \Rightarrow X = Z/\alpha_p, X[p^\infty] \cong_u G_{2,1} + G_{1,2}$
$a(X) = 1 \Rightarrow \exists \psi : \mathbb{P}^1 \to W_\beta, \psi(\mathbb{P}^1) = I(x)$
Hence ($W_\beta(a = 2) =: \mathcal{C}$is irreducible) $\Rightarrow W_\beta$is irreducible.
Construction (EO): $\overline{\mathcal{C}} \supset L \supset \Sigma$, $L = \{(X,J) | X \cong_\mu E \times (E \times E/\alpha_p)\}$,
Σ (super special locus) $= \{(K,J) \mid X \cong_\mu E^3\}$; L is connected. (EO)
$\overline{\mathcal{C}}^{\wedge y} = Spf(k[[t_{ij}]]/(t_{ij} = t_{ji}, AA^p = 0)$ $1 \leq i,j \leq g$, $A = (t_{ij})$, $y \in \Sigma$.
Lemma: $\overline{\mathcal{C}}^{\wedge y}$ is irreducible (easy) $\Rightarrow \overline{\mathcal{C}}$ irreducible. q.e.d.

Afterthought: EO (see talk Pink) by $\cong \mathcal{C}$ of BT_1's.
These foliations: by $\cong \mathcal{C}$ of BT_m's for $m >> 0$.

Bjorn Poonen

Néron-Tate projection of algebraic points

Let X be a geometrically irreducible closed subvariety of an abelian variety A over a number field k. Let $\pi : A(\overline{k}) \to A(k) \otimes \mathbb{R}$ be the orthogonal projection relative to a Néron-Tate height pairing $\langle\ ,\ \rangle : A(\overline{k}) \times A(\overline{k}) \to \mathbb{R}$. We prove that $\pi(X(\overline{k})) = A(k) \otimes \mathbb{Q}$. Moreover, there exist $c_1, c_2 > 0$ such that for all $a \in A(k) \otimes \mathbb{Q}$,

$$\{x \in X(\overline{k}) : \pi(x) = a \quad \text{and} \quad h(x) < c_1 h(a) + c_2\}$$

is Zariski dense in X. (Here h is the canonical height function, $h(x) = \langle x, x\rangle$.) Finally, on the other hand, we remark that the following is a formal consequence of the Mordell-Lang conjecture: if $X_{\overline{k}}$ does not contain the translate of any abelian subvariety of $A_{\overline{k}}$ of positive dimension, then there exists $\Phi \in \mathrm{Hom}(A(\overline{k}) \otimes \mathbb{Q}, \mathbb{Q})$ such that $\Phi(X(\overline{k}))$ is a discrete subset of $\mathbb{Q}$ in the archimedean topology, and such that $\{x \in X(\overline{k}) : \Phi(x) = a\}$ is finite for each $a \in \mathbb{Q}$.

Richard Taylor

Some remarks on a conjecture of Fontaine and Mazur

Suppose that ℓ is an odd prime and that $\rho : G_{\mathbb{Q}} \to GL_2(\overline{\mathbb{Q}}_\ell)$ is a continuous irreducible representation such that

- ρ is unramified almost everywhere.

- $\det\rho(c) = -1$ ($c =$ complex conjugation).
- $\rho\mid_{G_\ell}\sim\begin{pmatrix}\chi_1\varepsilon^n & * \\ 0 & \chi_2\end{pmatrix}$ where G_ℓ is the decomposition group at ℓ; ε the ℓ-adic cyclotomic character; χ_1, χ_2 are finitely ramified; $n \in \mathbb{Z}_{>0}$ and $\varepsilon^n\chi_1\chi_2^{-1}(I_\ell)$ is not pro $-\ell$.

Then we show the following:

1) For almost every prime p, every eigenvalue α of $\rho(\text{Frob } p)$ is in $\overline{\mathbb{Q}}$ and for every embedding $\overline{\mathbb{Q}} \hookrightarrow \mathbb{C}$ we have $|\alpha|^2 = p^n$. In particular if $\iota : \overline{\mathbb{Q}}_\ell \tilde{\to} \mathbb{C}$, then the L-function

$$L(\iota\rho, s) = \prod_{p\neq\ell} \det\left(1 - \frac{\iota\rho_{I_p}(\text{Frob } p)}{p^s}\right)^{-1}$$

(explicit Euler factor at ℓ) converges to a holomorphic function in some right half plane.

2) If $N(\rho)$ denotes the conductor of ρ, then $L(\iota\rho, s)$ extends to a meromorphic function on all of $\mathbb{C}$ and satisfies a functional equation:

$$N(\rho)^{s/2}(2\pi)^{-s}\Gamma(s)L(\iota\rho, s) = WN(p)^{\frac{n+1-s}{2}}(2\pi)^{s-1-n}$$

$$\Gamma(n+1-s)L(\iota(\rho\otimes\epsilon^n(\det\rho)^{-1}), n+1-s),$$

for some $W \in \mathbb{C}$ with $|W| = 1$.

3) If $n > 1$ then ρ occurs in the ℓ-adic cohomology of some smooth projective variety over $\mathbb{Q}$. If $n = 1$ and for some prime $p \neq \ell$ we have $\rho\mid_{G_p}\sim\begin{pmatrix}\epsilon\chi & * \\ 0 & \chi\end{pmatrix}$, then ρ occurs in the Tate module of some abelian variety $A \mid \mathbb{Q}$ (which we may assume has "GL_2-type").

As a consequence we deduce that the L-function of any abelian variety $A \mid \mathbb{Q}$ of GL_2-type (i.e. for which 3 a number field $M/\mathbb{Q}$ such that $M \hookrightarrow \text{End}(A/\mathbb{Q}) \otimes \mathbb{Q}$ and $[M : \mathbb{Q}] = \dim A$) has meromorphic continuation to the whole complex plane and satisfies the expected functional equation.

Jean-Benoît Bost

Algebraic leaves of foliations over number fields

Consider a number field $K \subset \mathbb{C}$, a smooth algebraic variety X over K, equipped with a K-rational point P, and F an algebraic subvector bundle of the tangent bundle T_X, defined over K. Assume moreover that F is closed under Lie bracket, and let $\mathcal{T}$ be leaf through P of the $\mathbb{C}$-analytic foliation of $X(\mathbb{C})$ defined by $F(\mathbb{C})$. We have that $\mathcal{T}$ is algebraic if:

1. for almost every prime $\mathcal{P}$ of the ring of integers $\mathcal{O}_K$, the reduction $\mod \mathcal{P}$ of F is stable by p-th power ($p :=$ characteristic of $\mathbb{F}_\mathcal{P} := \mathcal{O}_K/\mathcal{P}$).

2. the analytic manifold $\mathcal{T}$ satisfies the Liouville property (i.e., any bounded p.s.h. function on $\mathcal{T}$ is constant).

Applied to translation invariant foliations on an algebraic group G over K, this criterion shows that a Lie subalgebra $\underline{h}$ of $\underline{g} := LieG$ (over K) is algebraic, iff, for almost every $\mathcal{P}$, its reduction $\mod \mathcal{P}$, $\underline{h}_{\mathbb{F}_\mathcal{P}}$, is a restricted Lie subalgebra of $\underline{g}_{\mathbb{F}_\mathcal{P}}$.

Our algebraicity criterion for leaves follow from a more basic algebraicity criterion concerning smooth formal germs of subvarieties inside an algebraic variety over a number field, proved by means of transcendence techniques "à la Chudnovsky", combined with the formalism of Arekelov geometry.

Markus Rost

On norm varieties and characteristic numbers

The Bloch-Kato conjecture stats the bijectivity of the norm residue homomorphism

$$K_n^M k/p \longrightarrow H^n(k, \mu_p^{\otimes n}).$$

We reported about two topics in the current approach to this conjecture. One is **Hilbert's 90 for symbols:** Let $u = \{a_1, ..., a_n\} \mod p \in K_n^M k/p$ be a symbol and let

$$\varphi(u) := \bigoplus_{\substack{E|k \text{ finite} \\ u_E = 0, E \subset \overline{k}}} E^* \xrightarrow{\mathcal{N}} k^*$$

be the norm map. Put

$$\mathcal{A}(u) := \varphi(u)/R - \text{trivial elements in } \ker \mathcal{N}.$$

Then **Hilbert's Theorem 90 for** u states that the induced map

$$\mathcal{A}(u) \xrightarrow{\overline{\mathcal{N}}} k^*$$

is injective.

Voevodsky has announced a theorem which essentially says that Hilbert's 90 for symbols implies the Bloch-Kato conjecture. One of the tools in proving Hilbert's 90 in the so called **Degree Formula**: Let $f : Y \to X$ be a morphism of projective smooth irreducible varieties (both of dimension d) over $k \subset \mathbb{C}$. Then the degree formula says the following:

$$[Y] = (\deg f) \cdot [X] \mod I_{d-1}(X).$$

Here $[X]$ denotes the complex cobordism class of $X(\mathbb{C})$, and $I_r(X) \subset MU_*$ is the ideal generated by all $[Z]$ with $Z \to X$ (defined over k) and $\dim Z \leq r$. Currently the proof of this formula relies on Voevodsky's stable homotopy theory.

J.-L. Colliot-Thélène

Rationale Punkte auf Varietäten, welche eine Schar Torseure abelscher Varietäten besitzen

In den letzten Jahren hat Swinnerton-Dyer, teilweise mit Hilfe anderer Autoren, eine Methode entwickelt, um das Bestehen (vieler) rationaler Punkte auf bestimmten Varietäten obiger Gestalt zu beweisen, allerdings unter zwei schweren Annahmen: Erstens, es gilt die sogenannte Schinzelsche Hypothese; zweitens, die Tate-Shafarevich Gruppen sind endlich. Vor kurzem hat Swinnerton-Dyer einen interessanten Fall entdeckt, wie man die Schinzelsche Hypothese gar nicht braucht. Es handelt sich um das Studium der $\mathbb{Q}$-rationalen Punkte auf diagonalen kubischen Flächen $ax^3 + by^3 + cz^3 + dr^3 = 0$.

Um eine grobe Idee der Methode zu geben, sei $X \to \mathbb{P}^1_k$ eine 1-parametrige Familie von Kurven vom Geschlecht Eins. Hier ist k ein Zahlkörper. Sei $K = k(\mathbb{P}^1)$ der Funktionenkpörper von $\mathbb{P}^1$. Sei J_K die Jacobische Varietät der geometrischen Faser X_K. Nehmen wir an, daß die Klasse von X_K in $H^1(K, J_K)$ die Ordnung zwei ist. Unter einer Annahme, die das Nichtbestehen eines Brauer-Maninschen Hindernisses gewährleistet $(X(\mathbb{A}_k)^{Br} \neq \emptyset)$ versucht man einen rationalen Punkt $P \in \mathbb{P}^1(k)$ zu finden, so daß die Faser X_P (glatt ist und) überall lokal rationale Punkte besitzt $(X_p(\mathbb{A}_k) \neq \emptyset)$. Hier wird die Schinzelsche Hypothese benutzt. Man verlangt aber mehr: man will P so wählen, daß gleichzeitig die 2-Torsionsgruppe ${}_2\omega(J_P)$ (hier ist J_P die Jacobische Varietät von X_P) der Ordnung höchstens 2 ist (dafür benutzt man noch mal die Schinzelsche Hypothese). An diesem Punkt verlangt die Methode, daß J_K wenigstens ein konstantes $(\mathbb{Z}/2)_K$ enthält.

Wenn $ш(J_P)$ **endlich** ist, folgt jetzt aus den Eigenschaften der Cassels-Tate Paarung, daß ${}_2ш(J_P) = 0$. Also ist $[X_P] = 0 \in ш(J_P)$, d. h. X_P besitzt einen rationalen Punkt. Was man eigentlich kontrolliert, ist die Selmergruppe. Dafür wurde eine neue Nethode für die Berechnung von Selmergruppen entwickelt, die ermöglicht, die Gruppe in einer Familie von elliptischen Kurven zu kontrollieren – wenn die generische Faser eine nicht-triviale konstante Torsion besitzt.

Bei diagonalen kubischen Flächen betrachtet man die affine Gleichung

$$ax^3 + by^3 = \lambda = cz^3 + dt^3.$$

Der Parameter ist λ. Da hat man ein Faserprodukt von zwei elliptischen Kurven. Da es nur zwei entartete Fasern gibt, kann man hier die Schinzelsche Hypothese durch Dirichlets Satz über Primzahlen ersetzen.

Guido Kings

The Bloch-Kato conjecture for Dirichlet L-functions

(joint work with A. Huber)

Consider the motive $M(\chi)$ associated to a Dirichlet character $\chi : G(\overline{\mathbb{Q}}/\mathbb{Q})^{\mathrm{ab}} \to \mathbb{C}^*$. Fix a lattice $T_B \subset M(\chi)_B$ in the Betti-realization, such that $T_p := T_B \otimes \mathbb{Z}_p$ is stable under $G(\overline{\mathbb{Q}}/\mathbb{Q})$ for all p. Fix also a lattice $\Omega \subset H^1_{\mathcal{M}}(M(\chi), r) = K_{2r-1}(M(\chi))^{(r)}_{\mathbb{Q}}$ in the $2r-1$ Quillen K-theory. Then the Bloch-Kato conjecture for (the leading coeff.) of the L-value $L(M^\vee, 1-r)^*$ is:

$$L(M(\chi^{-1}), 1-r)^* = \mathrm{vol}\left(\frac{M_B \otimes (2\pi i)^{r-1} \cdot \mathbb{R})^+}{r_{\mathcal{D}}(\Omega)}\right) \cdot \prod_p \frac{\# H^2(\mathbb{Z}[\frac{1}{p}], T_p(r))}{\#\left(\frac{H^1(\mathbb{Z}[\frac{1}{p}], T_p(r))}{r_p(\Omega \otimes \mathbb{Z}_p)}\right)}$$

where $r_{\mathcal{D}}$ and r_p are the regulators of Beilinson and Soulé respectively. Note that there is a formulation with coefficients and also one for the L-value $L(M, r)^*$, $r \geq 1$ which we call $BK(M, r)$. The aim of the talk was to explain the proof of the following theorem:

Theorem (A. Huber, G. Kings): a) $BK(M(\chi^{-1}), 1-r)$ holds for $\chi(-1) = (-1)^{r-1}$ and $BK(M(\chi), r)$ for $\chi(-1) = (-1)^r$.

b) If $BK(M(\chi^{-1}), 1-r)$ and $BK(M(\chi), r)$ are compactible under the functional equation, then $BK(M(\chi), r)$ and $BK(M(\chi^{-1}), 1-r)$ hold for all χ.

The idea of the proof relies on Kato's observation that the main conjecture in Iwasawa theory implies the Bloch-Kato conjecture. Unfortunately the main conjecture as proven by Mazur-Wiles has problems with the primes $p|\Phi(\mathrm{cond}\ \chi)$. Our main idea to circumvent this is to use the classical line of proof as invented by Kolyvagin and Rubin using the theory of Euler systems developed by Rubin, Perrin-Riou and Kato. Decisive is the use of the Bloch-Kato formulation which allows to change the lattices involved.

C. Deninger

Γ-Factors of motives

For a smooth projective variety $X/\mathbb{Q}$ Serre defined the Γ-factor of $M = H^w(X)$ by the formulas

$$L_\infty(M,s) = \begin{cases} \prod_{p<q} \Gamma_\mathbb{C}(s-p)^{h^{pq}} & \text{if } 2 \nmid w \\ \prod_{p<q} \Gamma_\mathbb{C}(s-p)^{h^{pq}} \Gamma_\mathbb{R}(s-n)^{h^{n,+}} \Gamma_\mathbb{R}(s-n+1)^{h^{n,-}} & \text{if } w = 2n. \end{cases}$$

Here h^{pq}, $h^{n,t}$ are certain Hodge numbers and $\Gamma_\mathbb{R}(r) = 2^{-\frac{1}{2}}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})$ and $\Gamma_\mathbb{C}(s) = (2\pi)^{-s}\Gamma(s)$.

In the talk we explained a geometric construction of a complex $\mathcal{DR}_X$ on $X \times_{\mu_2} \mathbb{R}$ such that for the coherent $\mathcal{A}_{\mathbb{R}/\mu_2}$-sheaf $\mathcal{E} = \underline{\mathrm{Hom}}_{\mathcal{A}_{\mathbb{R}/\mu_2}}(R^w\pi_*\mathcal{DR}_X, \mathcal{A}_{\mathbb{R}/\mu_2})$ the formula

$$L_\infty(M,s) = \det{}_\infty \left(\frac{1}{2\pi}(s-\Theta) \mid \Gamma(\mathbb{R}/\mu_2, \mathcal{E})\right)^{-1}$$

holds. Here μ_2 acts on $X \times \mathbb{R}$ by $F_\infty \times (-\mathrm{id})$, $\mathcal{A}$ is the sheaf of real C^ω-functions on $\mathbb{R}/\mu_2$ and Θ is the infinitesimal generator of a canonical flow on $\Gamma(\mathbb{R}/\mu_2, \mathcal{E})$; $\pi : X \times_{\mu_2} \mathbb{R} \to \mathbb{R}/\mu_2$ projection. Essentially we have

$$\mathcal{DR}_X = \mathrm{coker}\ \left(\pi^{-1}\mathcal{A}_\mathbb{R} \longrightarrow \left(\Omega^\bullet_{X\times\mathbb{R}}, rd\right)\right)^{\mu_2}.$$

We also calculated the torsion of $R^n\pi_*\mathcal{DR}_X$ and related it to the ε-factor at infinity. Finally we explained an application to the "explicit formula" of analytic number theory of $L(M,s)$ and related $(\Omega^\bullet_{X\times\mathbb{A}^1/\mathbb{A}^1}, zd)$ to $\Omega^\bullet_{V/\mathbb{A}^1}$ where V is the deformation of X to the normal bundle of a point.

Jan Nekovar

Skew-symmetric pairings on Selmer groups

Let $E/\mathbb{Q}$ be an elliptic curve, p a prime number. We have Selmer groups associated to $T_p(E) = \varprojlim_n E[p^n]$ and $V_p(E) = T_p(E) \otimes \mathbb{Q}$:

$$0 \longrightarrow E(\mathbb{Q}) \otimes \mathbb{Z}_p \longrightarrow \mathrm{Sel}(\mathbb{Q}, T_p(E)) \longrightarrow T_p\omega(E/\mathbb{Q}) \longrightarrow 0,$$

$\mathrm{Sel}(\mathbb{Q}, V_p(E)) = \mathrm{Sel}(\mathbb{Q}, T_p(E)) \otimes \mathbb{Q}$.

Theorem. Assume that E has good ordinary reduction at $p > 2$. Then

(1) $\exists$ canonical filtration (by $\mathbb{Q}_p$-vector spaces) on $S = \mathrm{Sel}(\mathbb{Q}, V_p(E))$:

$$S = S^1 \supseteq S^2 \supseteq \dots$$

and alternating pairings (depending on a topological generator $\gamma \in 1 + p\mathbb{Z}_p$)

$$\langle\ ,\ \rangle_i : S^i \times S^i \longrightarrow \mathbb{Q}_p$$

with kernel S^{i+1}.

(2) $S^\infty := \bigcap_{i\geq 1} S^i$ is equal to the "generic subspace" $S^{\mathrm{gen}} \subseteq S$, defined using a suitable deformation of $T_p(E)$.

(3) Certain non-vanishing conjectures about p-adic L-functions imply

$$\dim_{\mathbb{Q}_p}(S^{\mathrm{gen}}) = \begin{cases} 0 & \text{if } \mathrm{ord}_{s=1} L(E,s) \text{ is even} \\ 1 & \longleftarrow\!\text{''}\!\longrightarrow \text{ odd} \end{cases}$$

The parts (1) – (2) of the Theorem can be ganeralized to self-dual Galois representations associated to ordinary Hilbert modular forms over a totally real number field satisfying Leopoldt's conjecture.

Gerd Faltings

Eliminating Chow's Lemma

We prove coherence of direct images for proper *fppf* stacks. The usual method for schemes and algebraic spaces uses Chow's lemma, which does not work here. Instead we use methods from rigid geometry, using two affine coverings where one is the shrinking of the other. Then both can be used to compute cohomology, the identity map is compact, and we are done.

To introduce a rigid structure we extend the constants. Instead of R we use $R[t]$. For a t-torsion free $R[t]$-algebra A, finitely generated, we define the "enlargement" as follows: Write $A = R[t][x_1, \cdots, x_r] = R[t][T_1, \cdots, T_r]/I$, and let $A^{b,\nu} = R[t][t^\nu x_1, \cdots, t^\nu x_r]$. Then $\mathrm{Spec}(A) \to \mathrm{Spec}(A^{b,\nu})$ is an enlargement. If $\mathrm{Spec}(A)$ covers a proper *fppf*-stack X, one shows that the covering extends to a rigid covering $\mathrm{Spec}(A^{b,\nu}) \to X$. This uses the valuative criterion for properness. After that the proof proceeds as the one for complex spaces.

Bas Edixhoven

On the André-Oort conjecture

The conjecture in question asserts that the irreducible components of Zariski closures of subsets of special points in Shimura varieties are subvarieties of Hodge type, i.e., up to a Hecke correspondence, the image of another Shimura variety under the morphism induced by a morphism of Shimura data.

In the talk the proof is sketched for the case of Hilbert modular surfaces, under the generalized Riemann hypothesis. We also explain a variant (where the set of special points is contained in one Hecke orbit) where we do **not** need the generalized Riemann hypothesis. This variant has an interesting application to the theory of transcendance of special values of hypergeometric functions, by work of Wolfart, Cohen and Wüstholz.

For details, see the author's homepage, or the article (to appear in the proceedings of the conference on arithmetic geometry and abelian varieties, May 1999, Texel Island, The Netherlands).

S. Lichtenbaum

On the Weil topology for varieties over finite fields

We observe that a Grothendieck topology introduced many years ago by Deligne may prove very useful in the study of special values of zeta-functions. This topology, whose cohomology groups we denote by $H^*_\omega(X, \cdot)$, bears the same relation to the étale topology as the Weil group does to the Galois group. It may essentially be characterized by the following fact:

Let X be a variety of dimension d over a finite field k. Then the usual Hochschild-Serre spectral sequence is replaced by the spectral sequence

$$H^p(\mathbb{Z}, H^q_{\mathrm{et}}(\overline{X}, F)) \Rightarrow H^{p+q}_\omega(X, F),$$

where "$\mathbb{Z}$" denotes the subgroup of $\mathrm{Gal}(\overline{k}/k)$ generated by Frobenius. It seems likely that if we take F to be a "motivic complex" $\mathbb{Z}(r)$, then all the groups $H^i_\omega(X, \mathbb{Z}(r))$ are finitely generated. There is a canonical element θ is $H^1_\omega(X,\mathbb{Z})$. For any sheaf (or complex) F, cupping with θ turns the cohomology groups $H^i_\omega(X,F)$ into a complex. If F is $\mathbb{Z}(r)$, and X is smooth of dimension d, we conjecture the homology groups $h^i_\omega(X,\mathbb{Z}(r))$ of this complex are finite, and that its Euler characteristic is closely related to the leading terms $\zeta^*(X,d-r)$ of $\zeta(X,s)$ at $s=d-r$.

It is possible that a similar topology should exist in the number field case, with θ being replaced by the canonical element $\log\|\ \|$ in $H^1_\omega(\mathrm{Spec}O_F,\mathbb{R})\subseteq \mathrm{Hom}(W_F^{\mathrm{ab}},\mathbb{R})$.

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