

Tagungsbericht 33/2000

## **Jordan-Algebren**

13.08.-19.08.2000

An der Tagung über Jordan-Algebren, die unter der Leitung von W. Kaup (Tübingen), K. McCrimmon (Charlottesville), H.P. Petersson (Hagen) und E.I. Zelmanov (New Haven) stattfand, nahmen 42 Mathematikerinnen und Mathematiker aus Brasilien, Deutschland, England, Frankreich, Hongkong, Italien, Kanada, Österreich, Russland, Schweden, Spanien, Ungarn und den USA teil.

Die während der Tagung gehaltenen Vorträge behandelten die Gebiete

- Jordan-Strukturen in Algebra, Arithmetik und Geometrie,
- Jordan-Strukturen in der Analysis,
- Jordan-Superstrukturen,
- Graduierte Lie-Algebren und Wurzelsysteme,
- Allgemeine nichtassoziative Strukturen.

Neben 10 Übersichtsvorträgen, in denen der aktuelle Stand der Forschung in Teildisziplinen zusammenhängend dargestellt wurde, wurden in 28 Spezialvorträgen kürzlich erreichte Resultate präsentiert und diskutiert.

# Vortragsauszüge

B. ALLISON

## Simple Kantor pairs

In this talk, we report on some work in progress with Oleg Smirnov on simple Kantor pairs. Kantor pairs generalize Jordan pairs and are related to 5-graded Lie algebras, just as Jordan pairs are related to 3-grades Lie algebras. This relationship was established (for triple systems rather than pairs) by I.L. Kantor in 1972-1973.

We show that any simple Kantor pair is isomorphic to one of the following: (a) A simple Jordan pair; (b) A Kantor pair associated to a nondegenerate sesquilinear form over an associative algebra with involution; (c) A Kantor pair associated with a nondegenerate bilinearform; or (d) A Kantor pair of exceptional Lie type. This theorem is proved using the work of E. Zelmanov and of Smirnov on simple Lie algebras with finite gradings.

J.A. ANQUELA

## Outer inheritance in Jordan systems

We show that outer ideals of Jordan algebras, pairs and triple systems inherit nondegeneracy, strong primeness and primitivity. When dealing with pairs and triple systems, our results are based on the use of local algebras and the results on the inheritance of regularity by ample outer ideals of Jordan algebras due to K. McCrimmon [*Outer Inheritance in Jordan Algebras*, Comm. Algebra (to appear)]. As a corollary, we manage to remove ampleness as a hypothesis when dealing with outer ideals of Jordan algebras.

G. BENKART

## Extended affine Lie algebras

The extended affine Lie algebras (EALAs) are natural generalizations of the finite-dimensional simple complex Lie algebras and the affine Lie algebras. We survey their recent classification. EALAs are closed related to the Lie algebras graded by finite root systems (what are often called  $\Delta$ -graded Lie algebras). The core of an EALA is a  $\Delta$ -graded Lie algebra, and so  $\Delta$ -graded Lie algebras play an essential role in the classification of EALAs.

W. BERTRAM

## The geometry of Jordan structures

The starting point of our investigations is the question whether there is a "Jordan-functor": can we associate to a Jordan structure (algebra, triple system or pair) a geometric object in a similar way as the "Lie functor" links Lie groups and Lie algebras, resp. symmetric spaces and Lie triple systems? In the real finite dimensional case we gave an affirmative answer by using differential geometric methods (cf. the authors Habilitationsschrift, to appear in Springer Lecture Notes).

In this talk we present a possible generalization to general base fields  $\mathbb{K}$  by introducing the concepts of "generalized projective geometry over  $\mathbb{K}$ " (corresponding to Jordan pairs)

and "generalized polar geometry" (corresponding to Jordan triple systems); to the latter we can (in the finite dimensional case) associate an "algebraic symmetric space over  $\mathbb{K}$ " [work in progress].

L.J. BUNCE

## Classification of sequentially weakly continuous $*$ -triples

(Joint work with C-H. Chu and B. Zalar)

Let  $Aut(D(A))$  denote the group of biholomorphic automorphisms of the open unit ball  $D(A)$  of a  $JB^*$ -triple  $A$ . The  $JB^*$ -triple  $A$  is said to be sequentially weakly continuous if all members of  $Aut(D(A))$  are sequentially weakly continuous mappings.

It can be shown that the following three conditions are equivalent:

- (i)  $A$  is sequentially weakly continuous.
- (ii) Every primitive ideal of  $A$  is maximal and  $A^{**}$  is a Type I  $JBW^*$ -triple with no infinite spin component.
- (iii) For each primitive ideal  $P$  of  $A$ ,  $A/P$  is an elementary  $JB^*$ -triple that is not an infinite dimensional spin factor.

One consequence is that every  $JB^*$ -triple  $A$  contains a smallest closed ideal  $J$  for which the only sequentially weakly continuous members of  $Aut(D(A/J))$  are the linear ones.

C.-H. CHU

## Jordan algebras and harmonic functions

We show the occurrence and some applications of Jordan algebras in the theory of harmonic functions on groups.

T. CORTÉS

## Local and subquotient inheritance of simplicity in Jordan systems

We prove that the local algebras of a simple Jordan pair are simple. Jordan pairs all of which local algebras are simple are also studied, showing that they have a nonzero simple heart, which is described in terms of powers of the original pair. Similar results are given for Jordan triple systems and algebras. Finally, we characterize the inner ideals of a simple pair which determine simple subquotients, answering the question posed by O. Loos and E. Neher in [*Complementation of Inner Ideals in Jordan pairs*, J. Algebra **166** (2), (1994), 255-295].

C.M. EDWARDS AND GOTTFRIED T. RÜTTIMANN

## Central structure of inner ideals in $JBW^*$ -triples

Let  $\mathcal{I}(A)$  be the complete lattice of weak\*-closed inner ideals in a  $JBW^*$ -triple  $A$  and let  $\mathcal{ZI}(A)$  be the complete Boolean lattice of weak\*-closed ideals in  $A$ . The annihilator  $L^\perp$  of a subset  $L$  of  $A$  consists of elements  $b$  of  $A$  for which  $\{L b A\}$  is equal to zero, and the kernel  $\text{Ker}(L)$  of  $L$  consists of those elements  $b$  in  $A$  for which  $\{L b L\}$  is equal to zero.

For each element  $J$  of  $\mathcal{I}(A)$ ,  $J^\perp$  also lies in  $\mathcal{I}(A)$ , and  $A$  enjoys the generalized Peirce decomposition

$$A = J_2 \oplus J_1 \oplus J_0,$$

where  $J_2 = J$ ,  $J_0 = J^\perp$ , and  $J_1$  is the intersection of the kernels of  $J$  and  $J^\perp$ . There exist unique structural projection  $P_2(J)$  and  $P_0(J)$  with ranges  $J_2$  and  $J_0$ , respectively, and a projection  $P_1(J)$  onto  $J_1$  such that

$$\text{id}_A = P_2(J) + P_1(J) + P_0(J).$$

Furthermore,

$$\{A J_0 J_2\} = \{0\}, \quad \{A J_2 J_0\} = \{0\}.$$

and, for  $j, k$  and  $l$  equal to 0, 1 or 2, the Peirce relations

$$\{J_j J_k J_l\} \subseteq J_{j+l-k},$$

when  $j + l - k$  is equal to 0, 1 or 2, and

$$\{J_j J_k J_l\} = \{0\},$$

otherwise, hold, except in the cases when  $(j, k, l)$  is equal to  $(0, 1, 1)$ ,  $(1, 1, 0)$ ,  $(1, 0, 1)$ ,  $(2, 1, 1)$ ,  $(1, 1, 2)$ ,  $(1, 2, 1)$ , or  $(1, 1, 1)$ . A weak  $*$ -closed inner ideal  $J$  for which the Peirce relations hold in all cases is said to be a Peirce inner ideal. Two elements  $J$  and  $K$  of  $\mathcal{I}(A)$  are said to be compatible if, for  $j$  and  $k$  equal to 0, 1 or 2,

$$[P_j(J), P_k(K)] = 0.$$

An element  $I$  of  $\mathcal{I}(A)$  is compatible with all elements of  $\mathcal{I}(A)$  if and only if  $I$  lies in  $\mathcal{ZI}(A)$ , which is a Boolean sub-complete lattice of  $\mathcal{I}(A)$ . The central kernel  $k(L)$  of a subset  $L$  of  $A$  is the largest element of  $\mathcal{ZI}(A)$  contained in  $L$ . It is shown that

$$k(J) = (J_1)^\perp \cap J, \quad k(J^\perp) = (J_1)^\perp \cap J^\perp,$$

and, when  $J$  is a Peirce inner ideal,

$$(J_1)^\perp = k(J) \oplus k(J^\perp).$$

An inner ideal  $J$  in  $A$  is said to be faithful if, for every non-zero ideal  $I$  in  $A$ ,  $I \cap \text{Ker}(J)$  is not equal to  $\{0\}$ . It is shown that every element  $J$  of  $\mathcal{I}(A)$  has a unique orthogonal decomposition

$$J = I \oplus K,$$

where  $I$  lies in  $\mathcal{ZI}(A)$  and  $K$  is a faithful element of  $\mathcal{I}(A)$ . In this case  $I$  is the central kernel  $k(J)$ . When applied to the weak $*$ -closed inner ideal  $A_2(u)$ , the Peirce two-space corresponding to a tripotent  $u$ , this reduces to a result of Horn and Neher.

A. ELDUQUE

## On algebras satisfying the adjoint identity

The commutative algebras satisfying the adjoint identity:  $x^2x^2 = N(x)x$ , where  $N$  is a cubic form, are shown to be related to a class of generically algebraic Jordan algebras of degree at most 4 and to the pseudo-composition algebras. They are classified under a nondegeneracy condition.

As byproducts, the associativity of the norm of any pseudo-composition algebra is proven and the unital commutative and power-associative algebras of degree  $\leq 3$  are shown to be Jordan algebras.

J. FARAUT

## Analysis on symmetric spaces associated to Jordan algebras

Since the work of Koecher we know that Jordan algebras provide a powerful method for studying the geometry of Hermitian symmetric spaces. As we have seen more recently, this method is also suited for studying geometry and analysis of a large class of symmetric spaces. As an illustration we consider in this talk the cross ratio of four points in a simple complex Jordan algebra  $V$ :

$$\{x_1, y_1, x_2, y_2\} = \frac{\Delta(x_1 - x_2)}{\Delta(x_1 - y_1)} \cdot \frac{\Delta(y_1 - y_2)}{\Delta(x_2 - y_2)},$$

where  $\Delta$  is the determinant polynomial (or reduced norm), which has been introduced by Kantor (1967). It is a conformal invariant, and satisfies a remarkable Bernstein identity:

$$D_\alpha \{x, y, x_0, y_0\}^\alpha = b(\alpha)^2 \{x, y, x_0, y_0\}^{\alpha-1},$$

where  $D_\alpha$  is an invariant differential operator, depending polynomially on  $\alpha$ , and

$$b(\alpha) = \alpha \left( \alpha + \frac{d}{2} \right) \cdots \left( \alpha + (r-1) \frac{d}{2} \right).$$

This identity is proved by using harmonic analysis (Unterberger-Upmeyer, 1994). In fact the kernel of the Berezin transform on a Hermitian tube domain is essentially a power of the cross ratio, and its Fourier transform has been computed explicitly. Then the Bernstein identity for the cross ratio follows by using the Harish-Chandra isomorphism from the algebra of invariant differential operators onto the algebra of polynomials which are invariant under the Weyl group. Several recent results are applications of this Bernstein identity: - A mean value theorem (Engliš, 1997), - Formula for the Fourier transform of the Berezin kernel on a compact Hermitian symmetric space (Zhang, 1997), - Analytic continuation of Riesz integrals on ordered symmetric spaces (Khelif, 2000).

J.R. FAULKNER

## Jordan pairs, Hopf algebras and algebraic groups

If  $V$  is a vector space over  $\mathbb{K}$  and  $\mathcal{A}$  is a  $\mathbb{K}$ -algebra  $\rho_n : V \rightarrow \mathcal{A}$  with  $\rho_n(v) = v^{(n)}$  is a sequence of *binomial divided power maps* if

1.  $v^{(n)} = 1$

2.  $(\lambda v)^{(n)} = \lambda^n v^{(n)}$
3.  $(v + w)^{(n)} = \sum_{i+j=n} v^{(i)} w^{(j)}$

for all extensions of  $\mathbb{K}$ .

A *devided power specialization* of a Jordan pair  $(V^+, V^-)$  is a pair of b.d.p maps with

$$ad_x^{(k)} y^{(l)} = \begin{cases} (Q_x(y))^{(l)} & \text{for } k = 2l \\ 0 & \text{for } k > 2l \end{cases}$$

for all extensions where  $x \in V^\sigma, y \in V_{-\sigma}$  and  $ad_x^{(k)} u = \sum_{i+j=k} x^{(i)} u(-x)^{(j)}$ .

Let  $U(V)$  be the universal devided power representation.

**Theorem 1:**  $U(V)$  is  $\mathbb{Z}$ -graded cocommutative Hopf algebra with primitive elements

$$\mathcal{P} = (V^-)^{(1)} \oplus \mathcal{P}_0 \oplus (V^+)^{(1)}.$$

**Theorem 2:** If  $\mathcal{H}$  is a  $\mathbb{Z}$ -graded Hopf algebra with primitive elements  $\mathcal{P} = \mathcal{P}_{-1} \oplus \mathcal{P}_0 \oplus \mathcal{P}_1$  and there is a homogeneous devided power sequence (in the sense of Hopf algebras) over each  $x \in \mathcal{P}_{-1} \cup \mathcal{P}_1$ , then the sequence is unique and  $(\mathcal{P}_{-1}, \mathcal{P}_1)$  is a Jordan pair with  $Q_x(y) = x^{(2)}y - xyx + yx^{(2)}$ .

Let  $\rho_n^\sigma : V^\sigma \rightarrow \mathcal{A}$  be a devided power specialization which is homogeneous and assume  $\mathcal{A}$  is finite dimensional. Let  $I$  be the kernel of the homomorphism  $U(V) \rightarrow \mathcal{A}$  given by the universal property. Let  $F(I)$  be the smallest family of ideals containing  $I$ ,  $\ker \epsilon$  and closed under  $J \wedge K, J \cap K, S^{-1}(J)$ , where  $J \wedge K$  is the kernel of  $U(J) \xrightarrow{s} U(V) \otimes U(V) \rightarrow U(V)/_J \otimes U(V)_K$ . Let  $\mathcal{H} = U(V)^{F(I)} = \{f \in U(V)^* : f(s) = 0 \text{ for some } J \in F(I)\}$ .

**Theorem 3:**  $G = Alg(\mathcal{H}, -)$  is an affine algebraic group scheme with  $G(K)$  a subgroup of units in  $\mathcal{A} \otimes K$ .

A. FERNANDEZ LÓPEZ

## Local techniques in Banach Jordan pairs

We comment in this talk on three works which illustrate the importance of local techniques in the theory of Banach Jordan pairs:

1. Characterizations of the socle of a semiprimitive Banach Jordan pair.
2. Noetherian Banach Jordan pairs.
3. Derivations on Banach Jordan pairs.

W.T. GAN

## Arithmetic of Jordan algebras and exceptional groups

This is a survey talk on some recent results on the arithmetic of octonion and Jordan algebras, as well as their automorphism groups. In particular, four directions and areas of investigations are highlighted, and references provided for the relevant papers:

- Classification and study of maximal orders in octonion or Jordan algebra  $V$  over  $\mathbb{Q}$  and  $\mathbb{Q}_p$ ; c.f. [EG1] and [Gr].
- Relating arithmetic of the algebra with that of the automorphism group  $G$ , and in particular, over  $\mathbb{Q}_p$ , giving a concrete description of the Bruhat-Tits building of  $G$  in terms of orders in the algebra; c.f. [GY1] and [GY2].
- Studying morphisms between orders in different octonion or Jordan algebras. In particular, counting the number of embeddings of one order into another; c.f. [EG2], [EG3], [GG1] and [GG2].
- Relations with the theory of modular forms, just as the arithmetic of quadratic forms gives rise to the theory of theta functions; c.f. [G] and [GGS].

## References

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E. GARCÍA

## On Herstein's theorems relating Jordan and associative pairs and triple systems

We obtain pair versions of Herstein's constructions by McCrimmon [*On Herstein's Theorems Relating Jordan and Associative Pairs*, Journal of Algebra **13**, (1969), 382-392], relating ideals of  $R$  and  $R^{(+)}$  (respectively,  $R$  and  $H(R, *)$ ) when  $R$  is an associative pair (respectively, an associative pair with involution  $*$ ). The proof is based on the use of homotope algebras, which allows to extend for pairs the results of McCrimmon, skipping the combinatorial work. As a consequence, we manage to relate the simplicity of  $R$  and

$R^{(+)}$  (respectively,  $R$  and  $H(R, *)$ ) for an associative pair  $R$  (respectively, an associative pair with involution  $*$ ), also obtaining a version for triple systems by using tight double pairs.

U. HAGENBACH

## Jordan algebras and analysis on non-convex cones

(Joint work with H. Upmeyer)

The classical Toeplitz  $C^*$ -algebra  $\mathcal{T}(D)$  acting on the Hardy space  $H^2(D)$  over the unit disc  $D$  was a central topic of research throughout decades, not only because of being one of the rare examples of good understandable operator algebras, but also because of their importance in complex analysis, geometry and engineering.

The talk presents the first general treatment of Toeplitz operators over all non-convex cones  $C$ , arising as connected components of the regular set in a general semisimple real Jordan algebra  $X$ . We determine the whole spectrum of the Toeplitz  $C^*$ -algebra  $\mathcal{T}(\Xi)$  by constructing facial limit representations, leading to a complete description of the  $C^*$ -algebraic structure of  $\mathcal{T}(\Xi)$  in terms of the facial boundary structure of the underlying stratified tube domain  $\Xi$ . In the sense of "Quantization philosophy" according to which the algebra of observables and its spectrum represents a noncommutative deformation of the underlying geometry we prove the main result:

**Theorem:** There is a filtration

$$\{0\} = I_0 \triangleleft I_1 = \mathcal{K}(H^2(\Xi)) \triangleleft I_2 \triangleleft \dots \triangleleft I_r \triangleleft I_{r+1} := \mathcal{T}(\Xi)$$

of  $\mathcal{T}(\Xi)$  into  $C^*$ -ideals such that there are  $C^*$ -isomorphisms

$$I_{j+1}/I_j \cong \mathcal{C}_0(\mathcal{F}_j) \otimes \mathcal{K}(\mathcal{H}_j) ,$$

where  $\mathcal{F}_j$  is a locally compact Hausdorff space and  $\mathcal{H}_j$  a separable Hilbert space. Thus  $\mathcal{T}(\Xi)$  is solvable of length  $r = \text{rank}(X)$ .

In particular,  $\mathcal{H}_r$  is one-dimensional,

$$\mathcal{T}(\Xi)/I_r \cong \mathcal{C}_0(X)$$

and the  $C^*$ -ideal of the compact operators on  $H^2(\Xi)$  is just the common kernel of all constructed representations.

I.R. KANTOR

## On a vector fields formula for the Lie algebra of a homogeneous space

Let  $G$  be a group Lie and  $\mathcal{G}$  be the corresponding Lie algebra. Consider  $G$  as a homogeneous space acting on itself by left translations. The following well known formula gives the expression for the infinitesimal transformations (vectorfields) of this action in exponential coordinates

$$a \longrightarrow a \frac{ad(x) \cdot e^{ad(x)}}{e^{ad(x)} - 1} \quad \forall a, x \in \mathcal{G} ,$$

where  $a \cdot ad(x) = [a, x]$ .

The goal of the talk is to present and to prove the generalization of this formula for arbitrary homogeneous space given (locally) by a Lie algebra  $\mathcal{G}$  and a stationary subalgebra  $\mathcal{H}$ .

Let  $\mathcal{E}$  be a supplementary subspace to  $\mathcal{H} : \mathcal{G} = \mathcal{H} \oplus \mathcal{E}$ . Then the formula for vectorfields is as follows:

$$a \longrightarrow (a \cdot e^{ad(x)})_{\mathcal{E}} \left( \left( \frac{e^{ad(x)} - 1}{ad(x)} \right)_{\mathcal{E}} \right)^{-1} \quad \forall a \in \mathcal{G}, x \in \mathcal{E},$$

where  $(b)_{\mathcal{E}}$  denotes the projection of  $b \in \mathcal{G}$  on  $\mathcal{E}$  along  $\mathcal{H}$  and the expression  $(A)_{\mathcal{E}}$ , where  $A$  is a linear operator on  $\mathcal{G}$  acting from the right, denotes a linear operator on the subspace  $\mathcal{E}$  which acts on  $b \in \mathcal{E}$  as  $(b \cdot A)_{\mathcal{E}}$ .

W. KAUP

## Continuous Peirce decompositions and the perturbation of triple functional calculus

For the space  $\mathcal{H}$  of all hermitian operators on a separable complex Hilbert space many authors have studied the problem: When does a given  $\mathcal{C}^1$ -function  $f : \mathbb{R} \rightarrow \mathbb{R}$  induce (via the classical  $L^\infty$ -functional calculus) a differentiable map  $f = f_{\mathcal{H}} : \mathcal{H} \rightarrow \mathcal{H}$  and what is the derivative  $df_{\mathcal{H}}(a)$  at the point  $a \in \mathcal{H}$ ?

We discuss this problem for the odd continuous triple functional calculus on JB\*-triples (which are a certain Jordan theoretic generalization of operator algebras occurring in connection with bounded symmetric domains in complex Banach spaces). For every fixed element  $a$  in a JB\*-triple  $E$  we define (in terms of the Jordan triple product of  $E$ ) a certain commutative real Banach algebra  $\mathcal{A}$  of bounded  $\mathbb{R}$ -linear operators on  $E$  together with a continuous homomorphism  $\varphi$  from  $\mathcal{A}$  into the Banach algebra of all continuous real-valued functions on a certain compact subset  $\Sigma \subset \mathbb{R}^2$  (called the Peirce spectrum of  $A$ ). Every  $L \in \mathcal{A}$  can be recovered from its  $\varphi$ -image (called the symbol of  $L$ ) and may be considered as a Schur multiplier in a certain sense. For a large class of odd  $\mathcal{C}^1$ -functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  the induced map  $f = f_E : E \rightarrow E$  has derivative  $df_E(a) \in \mathcal{A}$  with symbol the divided difference of  $f$  on  $\Sigma$ . Details will appear in a joint paper with J. Arazy.

O. LOOS

## Locally finite root systems

(Joint work with E. Neher)

These are infinite root systems in infinite-dimensional vector spaces whose intersection with each finite-dimensional subspace is finite. We discuss a number of new phenomena as compared to the finite theory:

- the big Weyl group  $\overline{W}$ ,
- (non-) existence of bases,
- conjugacy classes of positive systems under  $\overline{W}$ ,
- extremal rays of chambers,
- representation of dominant weights as series of fundamental weights.

C. MARTINEZ

## Jordan superalgebras

This is a broad survey of the structure theory of Jordan superalgebras with the emphasis on classification problems.

First we discuss the classification of simple finite dimensional Jordan superalgebras of zero characteristic due to V. Kac and Racine-Zelmanov classification of modular Jordan superalgebras with semisimple even part.

The theory of modular superalgebras with non semisimple even part is parallel to the theory of superconformal algebras that has a rich physical content. In both cases the main objects are Kantor doubles of brackets and Cheng-Kac superalgebras.

We discussed speciality problems for these superalgebras and in particular showed that Cheng-Kac superalgebras are special.

J. MARTINEZ

## Separate weak\*-continuity of the triple product in dual real JB\*-triples

(Joint work with A. Peralta)

In 1995, J. M. Isidro, W. Kaup and A. Rodriguez introduced real JB\*-triples as closed real subtriples of complex JB\*-triples and they showed that given a real JB\*-triple,  $A$ , there exists a unique complex JB\*-triple,  $B$ , and a unique conjugation,  $j$ , such that  $A$  is the set of all  $j$ -symmetric elements of  $B$ . So  $A$  is a real form of  $B$ . Clearly the class of real JB\*-triples includes all real C\*-algebras, all JB-algebras and obviously all complex JB\*-triples. Recently, a theory of real JB\*-triples has been developed, extending to the real context many results in complex JB\*-triples. However the extension, to the real case, of the important result proved by Barton and Timoney, in 1986, assuring that: If  $B$  is a complex JB\*-triple which is a dual Banach space, then  $B$  has a unique predual and the triple product is separately weak\*-continuous, was an open problem which appears in several papers. We solve this problem. Our proof does not depend on Barton–Timoney’s Theorem. So we have a new proof of the separate weak\*-continuity of the triple product in dual complex JB\*-triples. From our theorem we also deduce some classical results in JB-algebras and JB\*-algebras.

K. MCCRIMMON

## Nathan Jacobson’s legacy for Jordan algebras

I will concentrate on new insights, new concepts, and new tools which Jake brought to Jordan algebras, focusing on 6 concepts:

1. Universal Gadgets
2. Triple Products
3. U-Operators
4. Isotopes
5. Generic Norms
6. Inner Ideals.

(1) Jake introduced and popularized the use of the universal special envelope to study the structure and representations of Jordan algebras. (2) He introduced triple products into Lie and Jordan theory, and (after some uncertainty) the triple product  $\{xyz\} \approx xyz + zyx$  was adopted as the fundamental one, especially because of its close connection to 3-graded Lie algebras. (3) The U-operator  $U_x y = \frac{1}{2}\{x, y, z\} \approx xyz$  and the Fundamental Formula  $U_{U_x y} = U_x U_y U_x$  have recast our view of the Jordan landscape (so that a general quadratic theory of Jordan rings is based on the product  $U_x y$ ). These operators arise naturally in differential geometry from the inversion  $j(x) = -x^{-1}$  via  $U_x = (\partial|_x)^{-1}$ . (4) Inserting a fixed element  $u$  into the Jordan triple product gives a new Jordan algebra  $J^{(u)}$  with bilinear product  $\{x, y\}^{(u)} = \{x, u, y\}$ ; if  $u$  is invertible the new algebra has unit  $1^{(u)} = u^{-1}$ , so we can view isotopy as change-of-unit. It can also be viewed as change-of-involution: if  $A$  is an associative algebra with involution,  $H(A, *)^{(u)} \cong H(A, *^{(u)})$  where the new involution is  $x^{*(u)} = ux^*u^{-1}$ . (5) Jake introduces the generic norm for any finite-dimensional power-associative algebra, generalizing the determinant on matrix algebras. This norm plays a crucial role in studying the automorphisms and structure groups of a Jordan algebra, and in differential geometry. (6) Inner ideals (spaces  $B$  invariant under *inner* multiplication by  $J, U_B J \subset B$ ) play the role of one-sided ideals. Jacobson first obtained an Artin-Wedderburn theory for Jordan rings with d.c.c. on inner ideals, and finally developed a structure theory for rings with capacity, which was just the formulation needed when Efim Zelmanov ushered in the New Age of Jordan theory with his classification of the simple algebras of arbitrary dimension.

K. MEYBERG

## Traces

An elementary very effective method is presented to deal with traces of linear operators on finite dimensional algebras with a non degenerate associative bilinear form. From the wide range of applications we present only two:

At first we derive trace formulas in Lie algebras of linear mappings which allow the computation of  $\text{trace}(ad(x))^k$  in terms of  $\text{trace}(x^l)$ . And secondly we discuss - using our trace formalism - M. Rost's proof of the dimension relation  $d(d-1)(d-3)(d-7) = 0$  for composition algebras of dimension  $d+1$ .

F. MONTANER

## Maximal modular inner ideals in Jordan systems

(Joint work with E. García)

The Jordan notion of primitivity adapts the intrinsic characterization of primitive associative algebras through one-sided ideals, and therefore it is based on a notion of modular inner ideal.

We investigate that notion of modularity and the related notion of weak modularity, and show that maximal primitizers of Jordan systems  $J$  are either maximal inner ideals in Jordan systems with finite capacity, or the system  $J$  is special and in any  $*$ -envelope  $R$  of  $J$  there is a maximal modular right ideal  $M$  such that the primitizer has the form  $M \cap J$ . We also prove that maximal modular inner ideals are maximal among all inner ideals (conjectured by Hogben and McCrimmon) and that maximal-weakly modular inner ideals are modular (conjectured by Anquela and Cortéz).

## Zelmanovian classification of prime $JB^*$ - and $JBW^*$ -triples

The classical structure theory for complex  $JB^*$ -triples consists of a precise classification of certain prime complex  $JB^*$ -triples (the so-called complex Cartan factors) and the fact that every complex  $JB^*$ -triple has a faithful family of Cartan factor representations. We applied the techniques of E. Zel'manov to obtain classification theorems for real and complex prime  $JB^*$ -triples [MoRo1, MoRo2].

For a  $C^*$ -algebra  $A$ , we denote as usual by  $A_{sa}$  the self-adjoint part of  $A$ , and  $M(A)$  will stand for the multipliers of  $A$ .

**Theorem 1** [MoRo2, Theorem 8.2]. *If  $J$  is a prime complex  $JB^*$ -triple, then one of the following assertions hold for  $J$ :*

- (i)  $J$  is either the type **V** or the type **VI** complex Cartan factor.
- (ii)  $J$  is a complex spin factor.
- (iii) There exist a prime complex  $C^*$ -algebra  $A$  and a projection  $e$  in  $M(A)$  such that  $J$  can be regarded as a  $JB^*$ -subtriple of the complex  $C^*$ -algebra  $M(A)$  contained in  $eM(A)(1 - e)$  and containing  $eA(1 - e)$ .
- (iv) There exist a prime complex  $C^*$ -algebra  $A$ , a projection  $e$  in  $M(A)$ , and a  $*$ -involution  $\tau$  on  $A$  with  $e + e^\tau = 1$  such that  $J$  can be regarded as a  $JB^*$ -subtriple of the complex  $C^*$ -algebra  $M(A)$  contained in  $H(eM(A)e^\tau, \tau)$  and containing  $H(eAe^\tau, \tau)$ .

**Theorem 2** [MoRo1, Theorem 8.4]. *If  $J$  is a prime real  $JB^*$ -triple, then one of the following assertions hold for  $J$ :*

- (i)  $J$  is the type **V**, **VI**, **V<sup>0</sup>**, **V<sup>0<sub>0</sub></sup>**, **VI<sup>0</sup>**, or **VI<sup>0<sub>0</sub></sup>** generalized real Cartan factor.
- (ii)  $J$  is the type **IV<sub>n</sub>** or **IV<sub>n</sub><sup>r,s</sup>** generalized real spin factor.
- (iii) There exists a prime real  $C^*$ -algebra  $A$  such that  $J$  can be regarded as a  $JB^*$ -subtriple of the real  $C^*$ -algebra  $M(A)$  contained in  $M(A)_{sa}$  and containing  $A_{sa}$ .
- (iv) There exists a prime real  $C^*$ -algebra  $A$  with  $*$ -involution  $\tau$  such that  $J$  can be regarded as a  $JB^*$ -subtriple of the real  $C^*$ -algebra  $M(A)$  contained in  $S(M(A), \tau) \cap M(A)_{sa}$  and containing  $S(A, \tau) \cap A_{sa}$ .

Refining slightly the tools necessary for the above classifications, we obtain in [MoRo2, Theorems 27 and 23] the corresponding classification of  $JBW^*$ -factors, that is, prime  $JB^*$ -triples which are Banach dual spaces.

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M. NEAL

## Contractive projections and operator spaces

(Joint work with B. Russo)

Parallel to the study of finite dimensional Banach spaces, there is a growing interest in the corresponding local theory of operator spaces. We introduce a family of Hilbertian operator spaces  $H_n^k$ ,  $1 \leq k \leq n$ , generalizing the row and column Hilbert spaces  $R_n, C_n$  and show that an atomic subspace  $X \subset B(H)$  which is the range of a contractive projection on  $B(H)$  is isometrically completely contractive to an  $\ell^\infty$ -sum of the  $H_n^k$  and Cartan factors of types 1 to 4. In particular, for finite dimensional  $X$ , this answers a question posed by Oikhberg and Rosenthal. Since the range of a contractive projection is completely contractive and isometric to a  $JC^*$ -triple, Jordan algebraic techniques are used in the proof. In addition to the above result, we classify up to complete isometry all  $w^*$ -closed atomic  $JW^*$ -triples without an infinite dimensional rank 1 summand.

E. NEHER

## Quadratic Jordan superpairs

In this talk, the new concept of a quadratic Jordan superpair over arbitrary superrings, not necessarily containing  $\frac{1}{2}$ , was introduced. It was shown that examples of Jordan superstructures, previously only studied in characteristic  $\neq 2$ , can in fact be defined over arbitrary superrings. The classification of Jordan superpairs covered by a grid was announced. Finally, Lie superalgebras graded by root systems were introduced.

**Theorem.** *A central extension of the Tits-Kantor-Koecher algebra of a Jordan superpair over a superring  $S$  and covered by a grid is a Lie superalgebra over  $S$  graded by a root system without any irreducible factor of type  $E_8, F_4$  or  $G_2$ . Conversely, if  $\frac{1}{2} \in S$  any Lie superalgebra over  $S$ , graded by such a root system arises in this way.*

**Application.** Since Jordan superpairs covered by a grid are known one simply has to determine their Tits-Kantor-Koecher algebras in order to get a classification, modulo central extensions, of root-graded Lie superalgebras for the root systems mentioned in the theorem.

J.M. OSBORN

## $\mathbb{Z} \times \mathbb{Z}$ -graded Lie algebras

In this talk I discuss some first steps toward a classification theory for Lie algebras graded by a torsion-free abelian group. Examples of such algebras include the Lie algebras of Cartan type, and my generalizations of these algebras. A possible model for a classification theory for graded Lie algebras is the classification of the finite-dimensional Lie algebras of characteristic  $> 5$ . The structure of Lie algebras graded by the integers  $\mathbb{Z}$  where the graded components are finite-dimensional has been formed by Olivier Matthieu. In this talk I described the results which have been obtained by Kaiming Zhao and myself on the structure of Lie algebras graded by  $\mathbb{Z} \times \mathbb{Z}$  where the graded components have dimension  $\leq 1$ .

A.M. PERALTA

## Grothendieck's inequalities for real and complex JBW\*-triples

(Joint work with A. Rodríguez-Palacios)

It is a well known result of Grothendieck that there exists a universal constant  $G > 0$  such that if  $\Omega_1$  and  $\Omega_2$  are compact Hausdorff spaces and  $U$  is a bounded bilinear form on  $C(\Omega_1) \times C(\Omega_2)$  then there are probability measures  $\mu_i$  on  $\Omega_i$  such that

$$|U(f, g)| \leq G \|U\| \left( \int_{\Omega_1} |f|^2 d\mu_1 \right)^{\frac{1}{2}} \left( \int_{\Omega_2} |g|^2 d\mu_2 \right)^{\frac{1}{2}}$$

for all  $(f, g) \in C(\Omega_1) \times C(\Omega_2)$ .

For non-commutative  $C^*$ -algebras Pisier and Haagerup proved that there exists a universal constant  $G > 0$  such that if  $A$  and  $B$  are two  $C^*$ -algebras and  $U$  is a bounded bilinear form on  $A \times B$  then there are states  $\varphi$  on  $A$  and  $\psi$  on  $B$  such that

$$|U(x, y)| \leq G \|U\| \left( \varphi\left(\frac{xx^* + x^*x}{2}\right) \right)^{\frac{1}{2}} \left( \psi\left(\frac{yy^* + y^*y}{2}\right) \right)^{\frac{1}{2}}$$

for all  $(x, y) \in A \times B$ .

We prove that, if  $M > 4(1+2\sqrt{3})$  and  $\varepsilon > 0$ , if  $\mathcal{V}$  and  $\mathcal{W}$  are complex JBW\*-triples (with preduals  $\mathcal{V}_*$  and  $\mathcal{W}_*$ , respectively), and if  $U$  is a separately weak\*-continuous bilinear form on  $\mathcal{V} \times \mathcal{W}$ , then there exist norm-one functionals  $\varphi_1, \varphi_2 \in \mathcal{V}_*$  and  $\psi_1, \psi_2 \in \mathcal{W}_*$  satisfying

$$|U(x, y)| \leq M \|U\| \left( \|x\|_{\varphi_2}^2 + \varepsilon^2 \|x\|_{\varphi_1}^2 \right)^{\frac{1}{2}} \left( \|y\|_{\psi_2}^2 + \varepsilon^2 \|y\|_{\psi_1}^2 \right)^{\frac{1}{2}}$$

for all  $(x, y) \in \mathcal{V} \times \mathcal{W}$ . Here, for a norm-one functional  $\varphi$  on a complex JB\*-triple  $\mathcal{V}$ ,  $\|\cdot\|_{\varphi}$  stands for the prehilbertian seminorm on  $\mathcal{V}$  associated to  $\varphi$  in [BF1]. We arrive in this ‘‘Grothendieck’s inequality’’ through results of C-H. Chu, B. Iochum, and G. Loupias [CIL], and a corrected version of the ‘‘Little Grothendieck’s inequality’’ for complex JB\*-triples due to T. Barton and Y. Friedman [BF1]. We also obtain extensions of these results to the setting of real JB\*-triples.

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A. RODRÍGUEZ PALACIOS

## Non-associative $C^*$ -algebras revisited

After the non-associative versions of the Gelfand-Naimark and Vidav-Palmer theorems (see [10] and [11], respectively) both alternative  $C^*$ -algebras and non-commutative  $JB^*$ -algebras become reasonable non-associative generalizations (the second containing the former) of classical  $C^*$ -algebras.

The basic structure theory for non-commutative  $JB^*$ -algebras is concluded about 1984 (see [1], [2], [8], and [9]). Alternative  $C^*$ -algebras are specifically considered in [3] and [8].

In recent years A. Kaidi, A. Morales, and the author have revisited the theory of non-commutative  $JB^*$ -algebras and alternative  $C^*$ -algebras with the aim of refining some previously known facts, as well as of developing some previously unexplored aspects. In this lecture we review the main results got in this goal.

The lecture is divided in five sections as follows.

1. Geometric properties of the products of alternative  $C^*$ -algebras [4].
2. Prime non-commutative  $JB^*$ -algebras [5].
3. Holomorphic characterization of non-commutative  $JB^*$ -algebras [6].
4. Multipliers on non-commutative  $JB^*$ -algebras [7].
5. Isometries of non-commutative  $JB^*$ -algebras [7].

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G. ROOS

## Compactification of Jordan triple systems, volume of bounded symmetric domains and polynomial morphisms of JTS

For an irreducible complex circled homogeneous domain, there is a natural normalization of the Euclidean volume, such that this volume is an *integer*, which is equal to the *degree* of some projective realization of its compact dual.

We give an explanation of this phenomenon in the language of Jordan triple systems. We first introduce a slightly simplified version of the projective imbedding of a compactification introduced by O. Loos. We then compute the pullback of the invariant projective volume form by this imbedding. Finally, we prove the equality between the volume of the domain and the degree of its compact dual, using some special real analytic isomorphism (defined *via* the Jordan structure) between the bounded domain and its ambient vector space.

I. SHESTAKOV

## Jordan superalgebras defined by brackets

(Joint work with C. Martínéz and E. Zelmanov)

Jordan superalgebras defined by brackets on associative and commutative superalgebras are studied. It is proved that any such a superalgebra is imbedded into a superalgebra defined by Poisson brackets. In particular, all Jordan superalgebras of brackets are *i-special*. The speciality of these superalgebras is also examined and it is proved, in particular, that the Cheng-Kac superalgebra is special.

L. STACHO

## Weighted grids in Jordan\* triples over commutative rings with involution

Weighted grids are linearly independent systems  $\{g_w : w \in W\}$  of signed tripotents in Jordan\* triples over commutative rings with involution indexed by figures  $W$  in modules over the real part of the ring of scalars such that  $\{g_u g_v g_w\} \in \text{Span} g_{u-v+w}$ ,  $u, v, w \in W$  with the convention  $g_z := 0$  for all  $z \notin W$ . Weighted grids arise naturally as systems of weight vectors of certain abelian families of Jordan\* derivations. A classification of the possible underlying weight systems is given on the basis of Neher's grid theory. In contrast with the classical semisimple case, infinite strings may occur in the weight figure and the Peirce matrix does not determine the triple up to isomorphisms in general. As a first step toward the general structure theory of Jordan\* triples spanned by weighted grids, we classify the complex Jordan\* triples spanned by non-nil weighted grids with the weight figure  $\mathbb{Z}^2$ . Finally we give an example for a complex Jordan\* triple with non-vanishing triple product which is spanned by weighted grid of nil tripotents.

M.L. THAKUR

## Kummer elements in Albert algebras

One understands the mod-2 invariants of Albert algebras fairly well, for example, the invariant  $f_3$  "divides" the invariant  $f_5$ . We want to understand what symbols occur in

the decomposition of the invariant  $g_3$  of a given Albert algebra  $J$ . It suffices to do this for Tits' first construction Albert division algebras, since  $g_3$  is a decomposable element in  $H^3(k, Z/3)$ , i.e.,  $g_3(J) = (a) \cup (b) \cup (c)$  for some  $a, b, c \in k^*$ ,  $k$  being the base field. Assume now on that  $k$  contains third roots of unity. We call an element  $x \in J$  a *Kummer element* if  $x^3 = \lambda$  for some  $\lambda \in k^*$ . We then have the equality of sets

$$\{\mu \in k^* | J \simeq J(A, \mu)\} = \{\mu \in k^* | x^3 = \mu, x \in J\}.$$

In other words, the scalar symbols occurring in the decomposition of  $g_3(J)$  are precisely the norms of Kummer elements in  $J$ . This also proves that for a given Kummer element  $x \in J$  with norm  $\lambda$ , there exists a central division algebra of degree 3 over  $k$  such that  $J \simeq J(D, \lambda)$ .

H. UPMEIER

## Spectral analysis on real and complex Jordan algebras

A complex hermitian Jordan triple  $Z$  with open unit ball  $B$ , endowed with a Jordan involution  $z \mapsto \bar{z}$ , gives rise to an real bounded symmetric domain  $B_{\mathbb{R}} := \{z \in B : \bar{z} = z\}$  which can be realized as  $B_{\mathbb{R}} \approx G_{\mathbb{R}}/K_{\mathbb{R}}$ . Here  $G_{\mathbb{R}} := \{g \in G : \overline{g(z)} = g(\bar{z}) \forall z \in B\}$  is a reductive subgroup of  $G := \text{Aut}(B)$ . Let  $H_{\nu}(B)$  denote the  $\nu$ -th Bergman space of holomorphic functions on  $B$ . Generalizing the well-known Toeplitz-Berezin operator quantization  $\mathcal{C}^{\infty}(B) \rightarrow \mathcal{L}(H_{\nu}^2(B))$ , we define "vector" deformations of Toeplitz type

$$\mathcal{T} : \mathcal{C}^{\infty}(B_{\mathbb{R}}) \longrightarrow H_{\nu}^2(B)$$

by

$$(\mathcal{T}f)(z) := \int_{B_{\mathbb{R}}} f(\xi) K_{\xi}(z) K_{\xi}(\xi)^{\frac{1}{2}} d\mu_0(\xi)$$

for all  $f \in \mathcal{C}^{\infty}(B_{\mathbb{R}})$ . Here  $K$  is the reproducing kernel of  $H_{\nu}^2(B)$  and  $\mu_0$  is invariant measure. Generalizing the Unterberger-Upmeier spectral analysis of the complex hermitian case, we find (in joint work with Jonathan Arazy, University of Haifa) the spectral decomposition of the "real" Berezin transform  $\mathcal{T}^* \mathcal{T}$  on  $\mathcal{C}^{\infty}(B_{\mathbb{R}})$ . This is related to work by G. Zhang, van Dijk-Perzner and Neretin. We also discuss other functional calculi such as Weyl and Wick deformation.

X. XU

## Simple conformal algebras generated by Jordan algebras

Conformal algebras are local structures of certain infinite-dimensional Lie algebras with one-variable structure, whose representations are main algebraic structures in quantum field theory. Simple conformal algebras of finite type were classified by Kac. In this talk, I will give a brief introduction to conformal algebras. Then I will present my constructions of three families of simple conformal algebras of infinite Type generated by simple Jordan algebras of types A, B, and C, respectively.

Y. YOSHII

## Division $(\Delta, G)$ -graded Lie algebras

Let  $F$  be a field of characteristic 0,  $\Delta$  a finite irreducible root system,

$$\Delta' = \begin{cases} \Delta & \text{if } \Delta \text{ is reduced} \\ \text{type } B & \text{otherwise, i.e., } \Delta \text{ has type } BC, \end{cases}$$

$\mathcal{G} = \mathcal{F} \oplus \bigoplus_{\mu \in \Delta'} \mathcal{G}_\mu$  a split simple Lie algebra over  $F$  of type  $\Delta'$ , where  $\mathcal{F}$  is a split Cartan subalgebra of  $\mathcal{G}$ ,  $\Delta^V \subset \mathcal{F}$  the set of coroots and  $G$  an abelian group.

A  $\Delta$ -graded Lie algebra  $L$  over  $F$  is called a  $(\Delta, G)$ -graded Lie algebra if  $L = \bigoplus_{g \in G} L^g$  is a  $G$ -graded Lie algebra such that  $\mathcal{G} \subset L^0$ . Then we have the double grading

$$L = \bigoplus_{\mu \in \Delta \cup \{0\}} \bigoplus_{g \in G} L_\mu^g$$

where  $L_\mu^g = L_\mu \cap L^g$ . Moreover, a  $(\Delta, G)$ -graded Lie algebra  $L$  is called a *division  $(\Delta, G)$ -graded Lie algebra* if for any  $0 \neq x \in L_\mu^g$  whenever  $L_\mu^g \neq (0)$  there exists  $y \in L_{-\mu}^{-g}$  such that  $[x, y] \equiv \mu^V \in \Delta^V \text{ mod } Z(L)$ , where  $Z(L)$  is the center of  $L$ . Also, we assume that  $\{g \in G \mid L^g \neq (0)\}$  generates  $G$ . Then the core of an extended affine Lie algebra with nullity  $n$  is a division  $(\Delta, \mathbb{Z}^n)$ -graded Lie algebra over  $\mathbb{C}$  with 1-dimensionality, that is,

$$\dim_{\mathbb{C}} L_\mu^g \leq 1 \quad \forall \mu \in \Delta, g \in G.$$

In particular, an affine Kac-Moody algebra (without derivation) is a division  $(\Delta, \mathbb{Z})$ -graded Lie algebra with 1-dimensionality. In one direction of generalized extended affine Lie algebras, we have classified division  $(A_2, \mathbb{Z}^n)$ -graded Lie algebras. The coordinate algebras of such Lie algebras are either division  $\mathbb{Z}^n$ -graded associative algebras or one of four types of division  $\mathbb{Z}^n$ -graded octonion rings.

G. ZHANG

## Tensor products of minimal holomorphic representations on bounded symmetric domains

Let  $D = G/K$  be a bounded symmetric domain with genus  $p$  and  $H^\nu(D)$  the weighted Bergman space for  $\nu > p - 1$ . It has analytic continuation in the parameter  $\nu$  and gives also unitary representations for  $\nu$  in the Wallach set, the last non-trivial point  $\nu = \frac{a}{2}$  being the minimal representation. We study the tensor product decomposition of  $H^{\frac{a}{2}} \otimes \overline{H^{\frac{a}{2}}}$ , and discover some new unitary spherical representations and find the expansion of the spherical functions in terms of Jack symmetric polynomials.

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