Mathematisches Forschungsinstitut Oberwolfach

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Global Invariant Manifolds in Dynamical Systems

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The conference, organized by Wolf-Jürgen Beyn (Bielefeld), Bernold Fiedler (Berlin) and John Guckenheimer (Ithaca), brought together mathematicians interested in various branches of Dynamical Systems. Topics ranged from interval maps to infinite dimensional dynamical systems, described by partial differential equations; from theory to applications. The discussed approaches were based both on analytical methods and numerical computational tools. Most talks were closely related to at least one of the following topics:

- bifurcation theory;
- infinite dimensional dynamical systems;
- numerical methods;
- singular perturbations.

In each of these areas the concept of invariant manifolds plays a quite important role and helps understanding the dynamics. The conference sessions were complemented by numerous informal discussions. Particular emphasis was to provide an opportunity for younger mathematicians to learn about the state of the art in the field.

[1]composed by Vassili Gelfreich

Abstracts

Convergence of center manifolds for PDEs under space discretization

Klaus Böhmer

After a short review of bifurcation numerics, we turn to center manifolds for PDEs. In most cases they require space and time discretization. The effects of time discretization have been studied for some cases, e.g., in Lubich/Ostermann. Space discretizations seem to be nearly untouched.
Our approach applies to the actual discretizations of nonlinear problems. Specifically, we have proved that some "simple" properties yield these results already for a large class of operators and discretizations. We require spaces with good approximation properties and use the well known fact that stability proofs can be reduced to the linear case. Stability is then reduced to a sequence of results: Monotone operators yield a stable discretization. Compact perturbations and bordered forms of operators with stable discretization are stable iff the (original) inverse exists and is bounded. Our method includes, e.g., finite difference, finite element, and spectral methods, finite volume and wavelet methods need to be worked out. Variational crimes are admitted as well. All these methods are linear and consistently differentiable. Exactly these properties are needed for a convergent computation of bifurcation and center manifolds. Operators, to which the above approach can be applied, include reaction-diffusion, Navier-Stokes and porous media systems. The theory allows to prove convergence of the coefficients of the discrete center manifolds to those of the exact center manifolds for PDEs under the above space discretizations. We have to require additionally that the center manifold is computed via a linear, invertible operator, combining the Frechet derivative of the differential operator with appropriate extensions and borderings. This is correct for all standard cases.

Finding zeros by multilevel subdivision techniques

Michael Dellnitz

We present a new set oriented numerical method for the detection of all the zeros of a given (smooth) function $g : \mathbb{R}^n \to \mathbb{R}^n$ within a compact region. The underlying idea is to view classical iteration schemes as specific dynamical systems and to apply adaptive multilevel subdivision techniques for the computation of their fixed points. These techniques have previously been developed for the identification of arbitrary invariant sets of dynamical systems. Based on this idea we construct a subdivision algorithm which allows to create close coverings of the set of zeros of g. We prove convergence of this algorithm in a general abstract context and propose three different realizations which are motivated by the theoretical results. Finally we discuss the numerical efficiency of these algorithms by a comparison with a standard zero finding procedure using a routine from the NAG library.

Computation of Periodic Orbits and their Stable and Unstable Manifolds in the N-Body Problem

Eusebius Doedel

It is shown how periodic solutions of systems of ordinary differential equations with one constant of motion can be continued numerically, using pseudo-arclength continuation. The method is applied to compute the family of Lyapunov periodic orbits that surround the Lagrange point L1 in the restricted 3-body problem. It is also shown how continuation can be used to compute the stable and unstable manifolds associated with these periodic orbits, using a boundary value approach. The method leads to easy detection of homoclinic and heteroclinic trajectories that play a role in space-mission design. Finally it is shown how one can compute families of periodic solutions and their bifurcations in the full 3-body problem by adding suitable integral constraints and unfolding parameters to take care of the various invariances. The method is applied to the recently found "figure-8 orbit" of Montgomery, Chenciner, and Simo. Varying the mass of one of the three bodies, connections are found, via bifurcations, to solutions of the restricted 3-body problem. The method used applies without change to the N-body problem for general N.

Takens-Bogdanov bifurcations without parameters

Bernold Fiedler

Joint work with Stefan Liebscher.

Bifurcation theory deals with vector fields, which depend on a parameter. In particular it studies changes of dynamics near trivial equilibria. It is often convenient to consider the parameter as an additional variable, which satisfies the equation $\dot{\lambda} = 0$. Then in this extended phase space the equilibria form an equilibrium manifold. In this lecture we drop the constant parameter condition, but keep the requirement of an equilibrium manifold. As a challenge to numerics, we present an example analogous to the Takens-Bogdanov bifurcation. We derive normal forms and describe the global dynamics of invariant manifolds in a neighborhood of the equilibrium manifold. Applications to non-Lax shocks in strictly hyperbolic stiffly nonlinear balance laws were skipped.

Finite Dimensional Dynamics in Reaction-Diffusion Systems: Equilibrium and Dynamics of Spikes.

Giorgio Fusco

In my talk I described some results (obtained in collaboration with G.Bellettini) on dynamics of spikes for the system:

$$(1) \qquad \begin{cases} u_{1,t} = \epsilon^2 \Delta u_1 + u_1^2 - u_1 + \sigma(u_1 - u_2), & \text{in } \Omega \\ \tau u_{2,t} = \epsilon^2 \Delta u_2 + u_2^2 - u_2 + \sigma(u_1 - u_2), & \text{in } \Omega. \end{cases}$$

where $0 < \tau < 1$, $\sigma > 0$ and $0 < \epsilon << 1$.

I discussed the following result:

Given $N \geq 1$, and points $\xi_1^0, \ldots, \xi_N^0 \in \Omega$, there is $\bar{\epsilon} > 0$ such that, for $\epsilon \in (0, \bar{\epsilon})$, there exists a solution $u^\epsilon = (u_1^\epsilon, u_2^\epsilon)$ of the form

$$u_i^\epsilon = \sum_{i=1}^{N} U(\frac{|x - \xi_i^\epsilon(t)|}{\epsilon}) + O(\exp(\frac{-k}{\epsilon})), i = 1, 2,$$

(where U is the unique positive radial solution of $\Delta U + U^2 - U = 0$ in the whole space) for some functions $\xi_i^\epsilon(t)$ such that

$$\xi_i^\epsilon(0) = \xi_i^0, i = 1, \ldots, N.$$

Moreover u^ϵ is globally defined on $[0, \infty)$ and $\dot{\xi}_i^\epsilon = O(\exp(\frac{-k}{\epsilon}))$.

The asymptotic behavior of $\dot{\xi}_i^\epsilon$ was also discussed.

Splitting of invariant manifolds

Vassili Gelfreich

Bifurcations near a strong resonance in a Hamiltonian system with 2 degrees of freedom were described by Arnold, Bryuno et al n 70'. The corresponding resonant normal forms are integrable. On the other hand the original system is usually nonintegrable. The nonintegrabvility is caused by the separatrices splitting, which is smaller than any power of parameters, and can not be detected by standard (e.g. Melnikov type) perturbation methods. In this talk a new asymptotic formula for this exponentially small separatrices splitting is presented.

Morse theory on the space of braids and Lagrangian dynamics

Robert Ghrist

Joint work with R. VanderVorst and J.B.VanderBerg [Leiden].

We consider parabolic recurrence relations on a one-dimensional integral lattice: think of these as discretization of a parabolic PDE. By interpreting solutions as discretized closed positive braids, we obtain flow on spaces of braid diagrams which exhibit a monotonicity with respect to the algebraic length of the braid. Then we construct a Conley index for braid classes which yields a topological invariant independent of the dynamics and the discretization. When this index is non-vanishing, the associated braid class possesses an invariant set for any parabolic recurrence relation.

We apply this machinery to second-order Lagrangians of the form $L(u, u', u'')$. Under a very general variational hypothesis [twist condition], the variational flow yields a parabolic recurrence relation. We obtain forcing theorems to prove existence of infinitely many periodic orbits.

Bursting and period doubling in neuron models

Willy Govaerts

We are interested in developing numerical methods for the computation of fold bifurcations, period -doubling bifurcations and torus bifurcations of periodic orbits and also of periodic orbits with many spikes. This is motivated by the study of neuron models, in particular by the presence of many islands of cascades of period-doubling bifurcations. Such islands are present and well documented in the literature in many models where no fast and slow dynamics are present and no spiking behavior is observed. However in neural models these islands are closely related to the birth of new spikes in a burst. Typically a new spike is added (or removed) after the passage of periodic orbits through an island. We illustrate this by extensive numerical tests in the case of the model of Plant (the R15 neuron) and the AB-neuron (Anterior burster neuron of the stomatogastric ganglion of the spiny lobster Panulirus Interruptus). It is clear that a computation of the islands is essential to a good understanding of dynamics in the interesting parameter region.

Bifurcation in slow-fast systems

John Guckenheimer

This lecture described joint work with Kathleen Hoffman and Warren Weckesser. We used the program AUTO to compute families of periodic orbits in a slow fast system with two fast and two slow variables and one varying parameter. We were surprised to find a periodic orbit three separate canards, trajectory segments that follow unstable slow manifolds. Our discovery prompted us to investigate the mechanisms associated with the formation of canards in this example, and in generic slow-fast systems. The types of bifurcations that occur in the family of periodic orbits we studied are still a mystery. We have begun a program to classify the bifurcations that occur near systems in which the slow-fast decomposition of the trajectories is degenerate.

Stability and instability of solitary waves

Mariana Haragus

We consider the Euler equations describing nonlinear waves on the free surface of a three-dimensional inviscid, irrotational fluid layer of finite depth. For large surface tension, Bond number larger than 1/3, and Froude number close to 1, the system possesses a one-parameter family of two-dimensional, small-amplitude, traveling solitary wave solutions. First we show that these solitary waves are spectrally stable with respect to two-dimensional perturbations of finite wave-number (joint work with Arnd Scheel). In particular, we exclude possible unstable eigenvalues in the long-wavelength regime, where a Boussinesq-equation governs the dynamics, and unstable eigenvalues arising from non-adiabatic interaction of the infinite-wavelength soliton with finite-wavelength perturbations. We then consider three-dimensional perturbations (joint work with M. Groves and S.-M. Sun). Using bifurcation theory we prove the existence of a branch of periodically modulated solitary waves. As a consequence, we obtain that the solitary waves are unstable under transverse periodic perturbations.

Birkhoff averages and bifurcations

Joint work with Todd Young.

I will discuss the dependence of ergodic properties, in families of dynamical systems, on a parameter. This will be worked out for interval maps. Specifically, let $\{f_\gamma\}$ be a family of unimodal maps on an interval, unfolding a boundary crisis bifurcation. Consider Birkhoff averages

$$\Phi_\gamma(x) = \lim_{n\to\infty} \sum_{i=0}^{n-1} \varphi(f_\gamma^i(x))$$

of a continuous function φ. The behavior of $(x, \gamma) \mapsto \Phi_\gamma(x)$ near the boundary crisis bifurcation will be treated.

Generalized true slow serfaces and canards for slow-fast systems on the torus without auxiliary parameters

Yulij Ilyashenko

Geometric theory of slow-fast systems was developed in 79 by Fenichel. He proved that near the hyperbolic part M of the slow surface of the fast system, the slow-fast system has the invariant manifold that depends smoothly on ϵ (the time scaling parameter.) It was shown in the talk that this result is an easy consequence of the theorem on the persistence of a normally hyperbolic smooth invariant manifold under perturbation. The generalization of the Fenichel's theorem is obtained for the case, when M is an invariant manifold of the fast system, not necessarily consisting of singular points only. This result is due to O. Anosova. Another result shows that on a two torus there exist a slow-fast system without an auxiliary parameter that has attractive canard cycles for arbitrary small values of ϵ (joint result with J. Guckenheimer.) The system is $\dot{x} = a - \cos x - \cos y, \dot{y} = \epsilon, a \in (1, 2)$.

Numerical computation of families of invariant curves for n-dimensional maps and applications

Angel Jorba

The talk focuses on a numerical study of a 6-D symplectic map that models the dynamics near the triangular points of the Earth-Moon system. The map is obtained from a discretization of a continuous model that includes the effect of the Sun as a periodic forcing. The study is based on the computation of some 1 parametric families of invariant curves, plus a numerical method to compute their linear normal behaviour (including normal frequencies). From these calculations, we can easily derive the existence of a quasi-stable region for the model considered. This region is at some distance of the triangular points. The results show a good agreement with a numerical simulation using the full solar system.

Set oriented numerical methods: from global zero finding to mission design, Part II

Oliver Junge

Continuing part I of the talk we showed how to use set oriented numerical methods to rigorously compute invariant sets in an infinite dimensional discrete dynamical system. The method is based on a Galerkin ansatz in order to reduce the system to a finite dimensional one and the concept of topologically self consistent apriori bounds in order to estimate the errors made by neglecting the higher order modes. The information on the resulting finite dimensional multivalued map (obtained by the set oriented methods and the Conley index theory) is then lifted to the infinite dimensional system. We continued the survey by introducing the continuation algorithm for the computation of global invariant manifolds of (in principle) arbitrary dimension. We showed how to adapt this algorithm to the Hamiltonian context and its application in the computation of a global invariant manifold of a periodic orbit in the circular restricted three body problem. We presented a new approach for finding energy efficient trajectories for spacecraft, the "shotgun shooting" method. The algorithm computes pseudo-trajectories joining a given initial condition to a prescribed target region in phase space. Finally it was demonstrated how one can detect nondegenerate codimension 1 heteroclinic bifurcations in parameter dependent vector fields by set oriented methods. The idea of the underlying hat algorithm is to compute intersections of coverings of stable and unstable manifolds for different parameter values and on different levels of the approximation.

Homoclinic Branch Switching (near Homoclinic Flip Points)

Bernd Krauskopf

Joint work with Bart Oldeman and Alan R. Champneys.

We present a new numerical method for switching from a branch of k-homoclinic orbits to a branch of lk-homoclinic orbits for any k and l. To this end we take l copies of the original k-homoclinic orbit and set up Lin's method in appropriate cross sections. By continuing in suitable combinations of times between peaks, Lin gaps and system parameters we construct the final lk-homoclinic orbit, which can then be continued in the system parameters alone. This method is now fully available in the HomCont part of the package AUTO. It is illustrated with the computation of all m-homoclinic orbits for all $m \in \{1, 5\}$ occurring near codimension-two homoclinic flip bifurcations. K and N.

Generalized Hopf bifurcation for nonsmooth planar systems

Tassilo Küpper and Susanne Moritz

Hopf bifurcation for smooth systems is characterized by a crossing of a pair of complex conjugate eigenvalues of the linearized problem through the imaginary axis. Since this approach is not at hand for nonsmooth systems we use the geometrical characterization given

by the change from an unstable to a stable focus through a center for a basic (piecewise) linear system.
In that way we find two mechanisms for the destabilizing of the basic stationary solution and for the generation of bifurcating periodic orbits:
a generation switch of the stability properties or the influence of the unstable subsystem measured by the time of duration spent in the subsystem. The switch between stable and unstable subsystems seems to be a general source of destabilization observed in several mechanical systems. We expect that the features analyzed for planar systems will help to understand higher-dimensional systems as well.

Lyapunov–Type Numbers and Invariant Curves

Jens Lorenz

Consider smooth maps $f(x, \lambda), f : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$, which have smooth closed invariant curves $\Gamma(\lambda)$ in $\mathbb{R}^2$ for $\lambda < \lambda_1$. As $\lambda \to \lambda_1$, will the curves persist or break? To study this question, four Lyapunov–type numbers $\nu(\lambda, p)$ etc. are defined for $\lambda < \lambda_1$ and $p \in \Gamma(\lambda)$. Their behaviour is related to the question of possible break–up of the curves. Numerical studies for the delayed logistic map illustrate the results.

Invariant manifolds under numerical discretization

Christian Lubich

The talk is concerned with the possible persistence of periodic orbits and invariant tori in situations where hyperbolicity gets lost or is not present at all. Three cases are considered:

- Hopf bifurcation
- Dissipatively perturbed Hamiltonian systems
- Integrable Hamiltonian systems, KAM systems

For the latter two cases, symplectic numerical methods show good behaviour. The question is considered as to how geometric properties (such as symplecticity, reversibility) imply favourable long-time dynamics of numerical methods.

Traveling Waves in Lattice Dynamical Systems with Imperfections and Patterns

John Mallet-Paret

We study a class of bistable lattice differential equations typified by

$$\dot{u}_i = \alpha_i(u_{i+1} - u_i) + \beta_i(u_{i-1} - u_i) - f(u_i), \qquad i \in \mathbf{Z},$$

with $\alpha_i, \beta_i > 0$ suitably bounded and f taken from a class of bistable nonlinearities which include the cubic $f(u) = (u^2 - 1)(u - a)$ with $-1 < a < 1$. There are three principal results of our studies.

(1) In the case $\alpha_{i+p} = \alpha_i$ and $\beta_{i+p} = \beta_i$ of spatial p-periodicity, existence and other properties of traveling wave (front) solutions joining certain spatially periodic equilibria $\{v^{\pm}\}$ as $i \to \pm\infty$ are obtained, that is,

$$\lim_{i \to \pm\infty} (u_i(t) - v_i^{\pm}) = 0. \qquad (*)$$

The results here extend to traveling waves in higher dimensional lattices, such as $\mathbf{Z}^d$.

(2) In the case of pinning in (1), that is wave speed $c = 0$, the existence of an "invariant monotone continuum," that is a totally ordered connected invariant set $\Gamma \subseteq l^\infty$ in the phase space l^∞ extending from v^- to v^+, with each $\{u\} \in \Gamma$ satisfying the "regularity condition" (*), is shown.

(3) An appropriate extension of a traveling wave solution to the case where α_i, β_i are not periodic in i is obtained. This extension entails replacing these coefficients with p-periodic ones $\widetilde{\alpha}_i, \widetilde{\beta}_i$, applying the results (1) and (2), and then letting $p \to \infty$. The main technical difficulty in taking this limit is ensuring the regularity condition (*) is maintained in the limit, and to this end certain spectral conditions on an unstable equilibrium v^0 between v^- and v^+ must be assumed.

Exponential Averaging for PDE and the Splitting of homoclinic orbits

Karsten Matthies

An averaging method for parabolic PDE and also for elliptic PDE in infinite cylinders is presented. The method reduces the influence of nonautonomous rapid forcing to exponential small remainder terms. This is applied to upper estimates for the size of the splitting of homoclinic orbits.

NLS as a Prototype Dynamical System in Infinite Dimension

David W. McLaughlin

The nonlinear Schrödinger equation is an excellent example of a near-conservative nonlinear wave equation, which should be viewed as an infinite dimensional dynamical system. This viewpoint displays properties such as spatial coherence, temporal chaos, spatio-temporal chaos, and dispersive wave turbulence.

A geometrical approach to spatio-temporal chaos for the Gray-Scott model

Yasumasa Nishiura

A new geometrical criterion for the transition to spatio-temporal chaos (STC) arising in the Gray-Scott model is presented. This is based on the interrelationship of global bifurcating branches of ordered patterns with respect to supply and removal rates contained in the model. This viewpoint not only gives us a new criterion for the onset of spatio-temporal chaos but also clarifies how the orbit itinerates among several ordered patterns in infinite dimensional space. Moreover the geometrical characterization gives us a universal viewpoint about the onset and termination of STC. There are at least two different mechanisms that cause re-injection dynamics and drive the spatio-temporal chaos: one is a generalized heteroclinic cycle consisting of self-replication and self-destruction processes, and the other involves annihilation of colliding waves instead of self-destruction.

Invariant Manifolds and Chaotic Dynamics in Random Dynamical Systems

Gunter Ochs

We discuss the notion of stable and unstable manifolds in the setup of random dynamical systems, which provide a "pathwise" interpretation of dynamics influenced or perturbed by probabilistic noise. The manifolds are random sets, which are moving under time evolution. By a random version of the Smale-Birlhoff theorem due to Gundlach transversal intersection of stable and unstable manifolds causes chaotic dynamics. This type of behavior is observed in numerical simulations of the Duffing – van der Pol oscillator perturbed by multiplicative white noise in a certain parameter regime. We briefly describe our set-oriented numerical methods for the approximation of random unstable manifolds. These include a box covering method based on an algorithm of Dellnitz and Hohmann for deterministic systems. In the case of one-dimensional unstable manifolds we also calculate polygonal approximations.

Non-orientable manifolds in three-dimensional vector fields

Hinke Osinga

Non-orientable manifolds in three-dimensional vector fields are topologically equivalent to a Möbius strip. Such manifolds typically arise as stable or unstable manifolds of a saddle periodic orbit. In this case, they can be studied in a Poincaré section as one-dimensional manifolds of a saddle point of the corresponding Poincaré first return map. The two-dimensional manifold of the vector field is non-orientable if the one-dimensional manifold in the Poincaré section corresponds to a negative eigenvalue of the Jacobian at the saddle point. We characterise the flow on non-orientable invariant manifolds and argue that it is useful to study two-dimensional manifolds in the full state space as opposed to one-dimensional manifolds of planar diffeomorphisms.

Stability and instability of spiral waves

Björn Sandstede

We investigate instability mechanisms of spiral waves on large unbounded domains. Specifically, we are interested in far-filed and core breakup of spiral waves as well as in the transition to meandering and drifting waves. Using spectral properties of spiral waves on bounded and unbounded domain, we predict the onset to instability caused by absolute instabilities. We also predict the super-spiral structure of meandering and drifting waves that bifurcate at Hopf bifurcations. This is joint work with Arnd Scheel.

Coexistence in spatially extended systems

Arnd Scheel

We study bifurcations from spatially homogeneous equilibria in reaction-diffusion systems in R^n. We first review results in one spatial dimension, $n = 1$. Emphasis is laid on degenerate bifurcations, where different, stable states exist. We show that interfaces between homogeneous or long-wavelength states typically move, whereas for example Turing patterns or standing waves typically can coexist with a homogeneous state. We then show bifurcation of radially symmetric, temporally stationary or periodic, patterns in R^n. We interpret these patterns as coexistence patterns, where front propagation is balanced by interfacial energy — in the nonvariational context of reaction-diffusion systems. In case of a Turing instability, we show existence of focus defects, consisting of concentric rings. Proves are based on a non-autonomous center-manifold reduction and normal form theory for radial dynamics in the far-field, combined with a matching procedure in the center of the pattern.

Modulating pulse solutions for a class of nonlinear wave equations

Guido Schneider

Joint work with Mark Groves.

We consider modulating pulse solutions for a nonlinear wave equation on the infinite line. Such a solution consists of a permanent pulse-like envelope steadily advancing in the laboratory frame and modulating an underlying wave-train. The problem is formulated as an infinite-dimensional dynamical system with one stable, one unstable and infinitely many neutral directions. Using a partial normal form and invariant-manifold theory we establish the existence of modulating pulse solutions which decay to small-amplitude disturbances at large distances.

Qualitative Properties of Discretizations for Index 2 Differential Algebraic Equations

JOHANNES SCHROPP

We analyze Runge-Kutta discretizations applied to index 2 differential algebraic equations (DAE's). We compare the asymptotic features of the numerical and the exact solutions. It is shown that discrete methods satisfying the first order constraint condition of the DAE exactly reproduce the geometric properties of the smooth system correctly. The proof combines embedding techniques for DAE's with an invariant manifold result of Nipp and Stoffer.

Smoothness of Invariant Manifolds and Foliations for Nonautonomous Dynamical Systems

STEFAN SIEGMUND

Nonautonomous dynamical systems are skew-products where the base could be a probability space, a topological space, a compact group, etc. This notion extends dynamical systems theory to nonautonomous problems, e.g.

- Nonautonomous ODEs / PDEs
- Stochastic ODEs / PDEs
- Control systems
- Discretization problems
- Flows with symmetry
- Delay equations

We present new results on the smoothness of invariant manifolds and foliations.

Invariant manifolds in geometric singular perturbation theory

PETER SZMOLYAN

We present a geometric singular perturbation analysis of a chemical oscillator. In this problem interesting dynamical phenomena like relaxation oscillations, canard solutions, and mixed mode oscillations occur. Thus, the problem is interesting in itself, however, the main theme of the talk is to explain recent developments in the dynamical systems approach to singular perturbation problems in the context of a specific example.
Geometric singular perturbation theory has been very successful in the case of normally hyperbolic critical manifolds, i.e. in singular perturbation problems with exponential layer behaviour. However, at points where normal hyperbolicity fails, e.g. at fold points in van der Pol type relaxation oscillators, the well developed geometric theory has difficulties. In the chemical oscillator problem various prototypical types of loss of normal hyperbolicity occur. We show how blow-up techniques can be used to understand the dynamics and asymptotics of such problems.

Dimension of non-local bifurcational problem

Dmitrii Turaev

Some general scheme for a development of non-local bifurcations is proposed. The approach is as follows: before starting to study a particular bifurcation problem, we are looking to a particular hyperbolic structure which presents here. Plus, consideration related to contraction of volumes in the center-unstable direction are important. We also discuss, why there is seemingly no other restrictions to the richness of the dynamics which could be generated by homoclinic bifurcations.

Existence and stability of modulating pulse solutions for Maxwell's equations describing nonlinear optics

Hannes Uecker

Joint work with Guido Schneider.

We show the existence and stability of bifurcating modulating pulse solutions for Maxwell's quasilinear integro–differential equations describing nonlinear optics. These solutions consist of a traveling pulse–like envelope modulating a traveling wavetrain. They can be described using a generalized Ginzburg–Landau equation that has exponentially stable pulses. The existence proof for the modulating pulses for Maxwell's equation uses a center manifold reduction for the spatial dynamics formulation. For the stability of these pulses we use Floquet–theory and combine the validity of the modulation equation with the exponential stability of the pulses in the modulation equation. The analysis is worked out in detail for bulk media, and we discuss how the results extend to optical fibers.

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