MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 05 / 2001

Mixed Finite Element Methods and Applications

04.02. - 09.02.2001

The Oberwolfach conference witnessed the triumph of mixed methods and was devoted to plate elements, finite volume methods, edge elements in electro magnetics, enhanced strain elements in elasticity, discontinuous Galerkin schemes. As D.N. Arnold pointed out in the closing session, the variety of schemes and topics under the roof of mixed methods is remarkable. Algorithmical and mathematical issues ranged from a priori and a posteriori error control to adaptive mesh-refining algorithms and domain decomposition for parallel computations. Applications from solid and fluid mechanics and magnetics or acoustics proved superiority of mixed schemes in particular when a very small parameter (e.g., thickness of a plate) or a very large parameter (e.g., Lamé constant λ) causes trouble (e.g., locking) in displacement based discretizations.

The link to the engineering community has been an important aspect over the week for the distribution of the knowledge and the assessment of mixed finite element technology. The triumph of mixed methods may be seen as a master example for a mathematical analysis that improves ad hoc methods and/or clarifies their well- or misbehavior in practise.

The conference covered new understanding of mixed finite elements in elasticity (cf. the contributions of D.N. Arnold, D. Braess, B.D. Reddy), the related Stokes equations (cf. the contribution of C. Bernardi) or the Reissner-Mindlin-Plate (cf. the contribution of R.S. Falk) or flow problems (cf. the contribution of M.F. Wheeler, R. Winther).
Special attention was on improved discretizations by adaptive mesh-refining (cf. the contribution of S. Bartels) or hp-versions (cf. the contribution of C. Schwab) and fast solvers (cf. the contributions of L.F. Pavarino, J. Schöberl, O. Steinbach, L. Tobiska, W. Zulehner).
New trends for applications were marked on localized L^∞ estimates (cf. the presentation A. Demlow), finite volume methods (cf. the presentation J. Douglas, J.M. Thomas), Maxwell equations (cf. the presentation D. Boffi), superconvergence (cf. the presentation J. Brandts), as well as on new links to discontinuous

Galerkin methods (cf. the presentation D. Marini), discontinuous enriched methods (cf. the presentation L.P. Franca), least square methods (cf. the presentation G. Starke), sparse grids (cf. the presentation R. Hiptmair) or even wavelets (cf. the presentation W. Dahmen).

The fruitful discussions in small groups were equally important as the discussions with the very competent audience during or immediately after the presentations.

Collection of Abstracts

in alphabetical order of speakers

Mixed Finite Elements for Elasticity

Douglas N. Arnold (University Park, Pennsylvania, U.S.A.)
based on joint work with Ragnar Winther

There have been many efforts, dating back four decades, to develop stable mixed finite elements for elasticity in the stress-displacement formulation (i.e., based on the Hellinger-Reissner formulation). This entails devising a finite element discretization of $H(\mathrm{div}, \mathbf{R}^{n\times n}_{sym})$, the space of square integrable symmetric matrix fields with square integrable divergence, which enjoys a few properties. Although there are well-known finite element spaces for $H(\mathrm{div}, \mathbf{R}^n)$, i.e., vectorfields, the case of symmetric tensorfields has proven much harder. We present a new family of elements with the desired properties and prove stability and optimal order convergence.

Reliability of Averaging Techniques in A Posteriori Error Control for Lowest Order Nonconforming and Mixed Finite Element Methods

Sören Bartels (Kiel, Germany)
based on joint work with Carsten Carstensen

Averaging techniques are popular tools in finite element methods for the numerical treatment of second order partial differential equations, since they provide efficient a posteriori error estimates by a simple post-processing. In the talk we illustrate the reliability of such techniques for nonconforming and mixed finite element methods for the Laplace equation. It turns out that any averaging technique yields reliable a posteriori error control. Numerical experiments are presented and support the theoretical results.

More Pressure in the Finite Element Discretization of the Stokes Problem

Christine Bernardi (Paris, France)
based on joint work with Frédéric Hecht

We propose a new finite element discretization of the Stokes problem, relying on the Crouzeix-Raviart element for the velocity and a piecewise affine pressure space augmented by bubble functions. We prove different inf-sup conditions of Babuska and Brezzi type. However, the final one is not optimal. This leads to optimal a priori error estimates in the velocity, but the discrete pressure does not converge to the exact one. We finally propose an algorithm to solve the discrete problem.

Finite Elements for the Time Harmonic Maxwell Equations

Daniele Boffi (Pavia, Italy)
based on joint work with R. Gastaldi

We consider the finite element approximation of the time harmonic Maxwell equations. It is well-known that the use of nodal elements (e.g., continuous piecewise linear elements for tetrahedrons) gives very poor results. In 2D some tricks provide reasonable approximations. However, dangerous difficulties are hidden, so that in general the use of nodal elements should be avoided.
Edge elements are known to be well suited for problems arising from electromagnetism.
In a joint work with R. Gastaldi (Brescia, Italy) we related the convergence of the resulting scheme to the good behaviour of the approximation of the eigenvalues of the Maxwell cavity problem. Most of the known families of edge elements fit into this framework. Some problems may arise on general quadrilateral meshes.

The Analogue of the Brezzi Condition for EAS-Elements

Dietrich Braess (Bochum, Germany)

The method of Enhanced Assumed Strains (EAS method) achieves a softening of the energy functional. The equivalence with a mixed method in the sense of Hellinger-Reissner was shown by Yeo and Lee - at least from the algebraic side. The softening is caused by a projection of the gradient. Whereas the target space is specified in the setting of the mixed method, the complement is specified in the EAS concept. We show that a strengthened Cauchy inequality is now required and that it is equivalent to the inf-sup condition.
The method is applied to nearly incompressible material. There is now a "discrete divergence" for which the kernel in the finite element spaces is large enough for achieving robust convergence.

Superconvergence in Mixed Finite Element Methods

Jan Brandts (Utrecht, The Netherlands)

We recall superconvergence of 2D uniformly triangulated RT elements for fluxes

$$k = 0 \ : \ \|p_h - \Pi_h p\|_0 \ \leq \ c h^2 \ | \ p \ |_2,$$
$$k = 1 \ : \ \|p_h - \Pi_h p\|_0 \ \leq \ c h^3 \ | \ p \ |_3 \ .$$

Then we extend the building block $| \ (\nabla u - \nabla L_h^{'} \nabla u, \nabla v_h) \ | \leq ch^2 \ | \ u \ |_3 | \ v_h \ |_1$ to uniform tetrahedral meshes in N dimensions. This leads to superconvergence of $-\mathrm{div}\, A \nabla u = f$ with standard linear FE in all dimensions, and to an attempt to tackle the curl-curl Maxwell equations in 3D.
The 3D RT superconvergence question will be discussed, but unfortunately, not solved.

Adaptive Wavelet Methods - Beyond the Coercive Case

Wolfgang Dahmen (Aachen, Germany)

This talk is concerned with adaptive schemes for a class of variational problems that are well-posed in the sense that the induced operator is an isomorphism from a certain Hilbert space $\mathcal{H}$ into its dual. The first conceptual step is to transform the original (continuous) problem into an equivalent infinite system of equations that is well-posed in ℓ_2. This transformation is based on suitable Riesz bases for $\mathcal{H}$ that are of wavelet type. One then aims at solving the ℓ_2-problem with the aid of an iterative scheme which is conceptually applied to the full infinite dimensional problem. It is shown that, under certain assumptions on the adaptive approximate application of the infinite matrix to a finitely supported vector, the scheme produces an approximate solution within any given target accuracy at a computational cost that stays (under certain circumstances) proportional to the best N-term approximation with respect to the underlying wavelet basis.
It is indicated how to realize the adaptive matrix-vector multiplication with the required properties for several examples of mixed formulations as well as problems involving boundary integral operators.

Sharply Localized L_∞ Estimates for Mixed Methods

Alan Demlow (Ithaca, New York)

Alfred Schatz has recently proven a form of sharply localzed or weighted, maximum norm estimates for Galerkin methods. These estimates, which are generalizations of previous maximum norm stability results, show that the higher the order of polynomial used in a Galerkin method, the more local the resulting approximation. We present analogous results for a mixed method for a general linear elliptic problem. We shall also make some comments concerning appropriate choices of mixed methods and elements for approximating solutions to such a problem and present a case in which a natural mixed method exhibits suboptimal convergence when the BDM elements are used.

Development of Higher Order Finite Volume Methods via Mixed Finite Element Methods

Jim Douglas, Jr. (West Lafayette, U.S.A.)

A derivation of a higher order FVM from an MFEM, $BDFM_2$, by use of an appropriate quadrature rule was presented. The flux function for the FVM coincides with that for $BDFM_2$, implying the known superconvergence results for $BDFM_2$ as well as the global estimates.

An Overview of Finite Element Methods for the Reissner-Mindlin Plate Problem

Richard S. Falk (Piscataway, U.S.A.)

A unified error analysis is presented for a class of finite element approximations for the Reissner-Mindlin plate problems. These approximations introduce the shear stress to write a mixed method. The use of an interpolation operator into the shear stress approximation space helps avoid the usual "locking problem". A historical perspective on the development of locking free methods is also presented.

Mixed Method Features of the Discontinuous Enrichment Method (DEM)

Leo P. Franca (Denver, U.S.A.)
based on joint work with T.G. Huntley, C. Farhat, and I. Harari

We consider a finite element method by enriching the standard piecewise polynomials with discontinuous functions that solve homogeneous differential equations. Continuity is enforced weakly by introducing Lagrange multipliers defined on element boundaries. The resulting formulation is of a mixed method form and Brezzi's theorem sets the framework for analysis. Numerical results are performed for the Helmholtz equation that indicate better approximability characteristics compared to the standard finite element method with piecewise polynomials.

Mixed Finite Elements on Sparse Grids

Ralf Hiptmair (Tübingen, Germany)

Sparse grids offer very efficient approximations of smooth functions by using finite element spaces that are obtained by merging the spaces defined on certain anisotropic tensor product meshes. A lot of research has been devoted to H^1-conforming finite elements on sparse grids. It is known that H(div)- and H(curl)-conforming finite elements are closely related to the H^1-conforming schemes. They all can be regarded as discrete differential forms. This suggests that sparse grid schemes can be generalized to arbitary discrete differential forms.

In my presentation, I explained how discrete differential forms on sparse grids can be constructed. Parallel to the H^1-conforming case I established hierarchical bases and estimates for the interpolation error. New aspects emerge: First, the evaluation of the interpolation operator can no longer be done exactly. Thus, I introduced an approximate interpolation operator that does not involve a loss in accuracy. Second, relationships between spaces of discrete forms have to be studied in order to confirm the stability of discrete variational problems. In this respect, a key role is played by the explicit construction of discrete potentials.

Discontinuous Galerkin Methods for Diffusive Problems

Donatella Marini (Pavia, Italy)

In the last decade DG-methods have been used successfully for the numerical solution of pure hyperbolic and nearly hyperbolic problems. Recently they have been applied also to elliptic problems in order to use them for a wider class of problems. We present a unified framework that includes all the DG-methods present in the literature til now. Thus, we establish the basic ingredients necessary to carry out the analysis in a simple way and to produce optimal error estimates.

Balancing Neumann-Neumann Methods for Incompressible Stokes Equations

Luca F. Pavarino (Milano, Italy)
based on joint work with Olof B. Widlund

Balancing Neumann-Neumann methods are introduced and studied for incompressible Stokes equations discretized with mixed finite or spectral elements with discontinuous pressure. After decomposing the original domain of the problem into nonoverlapping subdomains, the interior unknowns (interior velocities and all except the constant pressure) of each subdomain are implicitly eliminated. The resulting saddle point Schur complement is solved with a Krylov space method with a balancing Neumann-Neumann preconditioner based on the solution of a coarse Stokes problem with a few degrees of freedom per subdomain and on the solution of local Stokes problems with natural and essential boundary conditions on the subdomains. This preconditioner is of hybrid form in which the coarse problem is treated multiplicatively while the local problems are treated additively. The condition number of the preconditioned operator is independent of the number of subdomains and is bounded from above by the product of the square of the logarithm of the local number of unknowns in each subdomain and the inverse of the inf-sup constant of the discrete problem and that of the coarse subproblem. Numerical results show that the method is quite fast and are fully consistent with the theory.

Mixed Finite Element Methods for Displacement-Stress-Strain Formulations in Elasticity

Batmanathan Dayanand Reddy (Capetown, South Africa)

This talk provides an overview of a range of mixed and related methods, in which combinations of displacement, stress, enhanced strain and displacement are treated as the primary unknown variables. The key properties of Enhanced Assumed Strain (EAS) and Hellinger-Reissner formulations are discussed, and their connection made clear. Next, a three-field formulation based on ideas from Fortin, Guénette, and Pierre is presented, in the context of elasticity. Convergent finite element approximations are described, in the context of compressible elasticity. The transition to the incompressible limit awaits attention.

Robust Multigrid Preconditioning for Parameter Dependent Problems

Joachim Schöberl (College Station, U.S.A.)

We consider a class of parameter dependent problems including nearly incompressible mode Timoshenko beam and Reissner-Mindlin plate models. Robust discretizations of the symmetric and positive definite problems are based on equivalent mixed finite element methods. The objective is the design of preconditioners which are robust with respect to the small parameter and the mesh size.

Key components are robust grid transfer operators based on the solution of local stable Dirichlet problems, and block smoothers operating on the kernel of the high energy term.

Mixed hp-Finite Elements on Anisotropic Meshes in $\mathbf{R}^3$

Christoph Schwab (Zurich, Switzerland)
based on joint work with A. Toselli

We consider the divergence-stabilty of mixed FEMs for the Stokes problem. We consider meshes which are geometrically and anisotropically refined towards faces (boundary layer meshes) and edges (edge meshes), consisting of stretched hexahedral elements of arbitrary large aspect ratio. We prove on such meshes $\mathcal{T}^{n,\sigma}$ with n layers and grading factor $\sigma \in (0,1)$ that for all $k \geq 2$, $0 < \sigma < 1$, $n \in \mathbf{N}$

$$(*) \qquad \inf_{0 \neq p \in S_0^{k-2,0}(Q,\mathcal{T}^{n,\sigma})} \quad \sup_{0 \neq \underline{u} \in S_0^{k,1}(Q,\mathcal{T}^{n,\sigma})^3} \quad \frac{(p, \nabla \underline{u})_Q}{\|p\|_{0,Q} \mid \underline{u} \mid_{1,Q}} \;\geq\; C(\sigma)$$

where $C(\sigma)$ is independent of n and the aspect ratio of the elements in $\mathcal{T}^{n,\sigma}$. Here, the velocities are in Q_k^3, continuous, and the pressures are in Q_{k-1}, discontinuous, and $Q := (-1,+1)^3$.

Extensions of (*) to a) curvilinear, mapped meshes, b) anisotropic, polynomial degrees in the elements $K \in \mathcal{T}^{n,\sigma}$, c) elements with continuous pressures in Q'_{k-1},

are also presented.
(*) implies exponential convergence for $(\underline{u}^{FE}, p^{FE})$ for Stokes in polyhedra in $\mathbf{R}^3$ if the data is piecewise analytic, and if $n = k$ in $\mathcal{T}^{n,\sigma}$.

Least-Squares Mixed Finite Element Methods for Parabolic Problems

Gerhard Starke (Hanover, Germany)

We present an approach for the numerical solution of parabolic initial-boundary value problems which is completely based on least-squares principles - for the discretization in space as well as in time. To this end, a least-squares functional for an equivalent first-order system - introducing flux as a new variable - is discretized with respect to suitable (time- and space-) discrete spaces resulting in a family of one-step methods. For the scalar variable we use continuous, piecewise polynomial functions in time and space. The flux is approximated by piecewise polynomial, not necessarily continuous functions in time and Raviart-Thomas elements in space. The least-squares functional is shown to be equivalent to the consistency error associated with a time-step, measured in an appropriate norm. In this talk, we concentrate on the study of a hierarchical basis a posteriori error estimator for the elliptic problem arising in each time-step. A strengthened Cauch-Schwarz inequality between the coarse and the hierarchical surplus space and a bound on the condition number of the auxiliary problem is shown, both uniform in the mesh size h and in the time-step τ implying efficiency and reliability of the estimator.

Variational Methods with Lagrange Multipliers

Olaf Steinbach (Stuttgart, Germany)

We consider a generalized mixed formulation with Lagrange multipliers to incorporate Dirichlet boundary conditions. This approach is based on (local) Dirichlet-Neumann maps. By using finite elements, appropriate approximations of the Steklov-Poincaré operators are defined. Since this corresponds to a solution of a Dirichlet problem, no matching conditions on the underlying meshes are required. To ensure the inf-sup condition of the mixed problem, we investigate the stability of a related projection operator in $H^{1/2}(\Gamma)$.

Finite Element Analysis of Finite Volume Methods

Jean-Marie Thomas (Pau, France)

Let p be the solution of the homogeneous Dirichlet problem: $-\operatorname{div}(\mathcal{A}\operatorname{grad} p) = f$ on Ω. A mixed formulation à la Petrov-Galerkin leads to consider a finite dimensional problem of the form: find $p_h \in P_h \subset H_0^1(\Omega)$, $\mathbf{u_h} \in U_h \subset L^2(\Omega)^2$ such that

$$\begin{aligned} \int_\Omega \operatorname{div} \mathbf{u_h}\, q_h\, dx &= -\int_\Omega f\, q_h\, dx \quad , \quad q_h \in Q_h \subset L^2(\Omega) \ , \\ \int_\Omega (\mathbf{u_h} - \mathcal{A}\operatorname{grad} \mathbf{u_h}) \cdot \mathbf{v_h}\, dx &= 0 \quad , \quad \mathbf{v_h} \in V_h \subset L^2(\Omega)^2 \ . \end{aligned}$$

For adequate choices of the spaces P_h, Q_h, U_h, and V_h, we prove that the problem has a unique solution. We establish the link with the class of node centered finite volume methods.
Error estimates for $p-p_h$ in $H^1(\Omega)$ and in $L^2(\Omega)$, as well as for $\mathbf{u}-\mathbf{u_h}$ in $H(\operatorname{div}, \Omega)$ are given.

Non-Nested Multi-Level Solvers for Mixed Problems

Lutz Tobiska (Magdeburg, Germany)

A general framework for analyzing the convergence of multi-level solvers applied to finite element discretizations of mixed problems is considered. As a basic new feature our approach also allows to use different finite element discetizations on each level of the discretization. Thus, in our multi-level approach accurate higher order finite element discretizations can be combined with fast multi-level solvers based on lower order (nonconforming) finite element discretizations, which leads to the design of efficient multi-level solvers for higher order finite element discretizations.

Mixed Finite Element Methods for Multiphase Flow in Porous Media

Mary F. Wheeler (Austin, U.S.A.)

In this presentation the governing equations for multiphase flow in porous media were just described and a multiblock methodology developed. Multiblock, multimodel, and multinumeric mixed finite elements using a mortar space as a dual interface were next formulated for the multiphase equations. Computational results for models coupling one, two and three phase problems as well as implicit and sequential time stepping for different physical models were presented. Theoretical results to single phase problems were also discussed. Finally, examples exhibiting linear speedup on PC clusters and examples of upscaling were presented.

A Robust Finite Element Method for Darcy-Stokes Flow

Ragnar Winther (Oslo, Norway)
based on joint work with K. Mardel and X.-Ch. Tai

Finite element methods for a class of singular perturbation problems are discussed. When the perturbation parameter is large, the system corresponds to the Stokes equation, while it degenerates to a mixed formulation of a second order elliptic equation, when the parameter is zero. A method is constructed with convergence properties that are uniform with respect to the perturbation parameter.

On Additive Schwarz-Typr Smoothers for Saddle Point Problems

Walter Zulehner (Linz, Austria)
based on joint work with J. Schöberl

A class of iterative methods for saddle point problems, introduced by Bank, Welfert, and Yserentant [1], is discussed. This class includes the Braess-Sarrazin smoother, inexact variants of the Braess-Sarrazin smoother, and additive Vanka-type iterations. Sufficient conditions for the convergence of the iterative methods are derived. In the context of multigrid methods the smoothing property of the iterative methods is analyzed.

[1] R.E. Bank, B.D. Welfert, and H. Yserentant; A class of iterative methods for solving saddle point problems. Numer. Math. **56** (1990), 645-666.

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