

Report No. 32/2001

## **Dynamical Systems**

July 15th – July 21st 2001

Die Tagung fand unter der Leitung von H. Hofer (New York), J. C. Yoccoz (Paris) und E. Zehnder (Zürich) statt.

Gegenstand der Tagung waren die neuen Resultate und Entwicklungen im Gebiet der klassischen dynamischen Systeme, insbesondere der endlich und unendlich dimensional Hamilton'schen Systeme. Themen waren unter anderem Stabilität und Instabilität von Hamilton'schen Systemen, Mather Theorie, Fragen der Ergodentheorie, Lyapunov Exponenten von volumenerhaltenden Diffeomorphismen, periodische Bahnen auf Energieflächen und neue interessante Lösungen des  $n$ -Körperproblems der Himmelsmechanik. Die Existenz der Arnold Diffusion, ein seit langem offenes Problem, wurde für eine grosse Klasse von Störungen angekündigt.

# Abstracts

## Morse homology for Lorentzian geodesics

ALBERTO ABBONDANDOLO

We describe how to build a Morse complex associated to the energy functional for geodesics on a class of Lorentzian manifolds. We deduce a classification of such geodesics in terms of their Maslov index and in terms of the topology of the path space. The main topics of this approach involve:

1. Fredholm index of ordinary differential operators in Hilbert spaces
2. Comparison results in Morse homology.

## Contact and Non-contact type hypersurfaces generated by $2^{nd}$ order Lagrangians

SIGURD ANGENENT

The Euler Lagrange equation for the variational problem

$$\delta \int \left\{ \frac{1}{2}(\ddot{u})^2 + \frac{\alpha}{2}(\dot{u})^2 + F(u) \right\} dt = 0,$$

i.e.,

$$\frac{d^4 u}{dt^4} - \alpha \frac{d^2 u}{dt^2} + F'(u) = 0,$$

can be put in Hamiltonian form, where

$$H(u, v, p_u, p_v) = p_u v + \frac{1}{2} p_v^2 - \frac{\alpha}{2} v^2 - F(u).$$

We ask “for which  $\alpha$  and  $F$  is the energy surface  $H^{-1}(0) \subset \mathbb{R}^4$  of contact type?” We found positive answers when  $\alpha \geq 0$ , or when  $F(u) > 0$  for all  $u$ . For  $\alpha < 0$  we found an example of an  $F$  (of the form  $u^2 - u^4 +$  perturbation) such that  $H^{-1}(0)$  is *not* of contact type for  $\alpha \rightarrow -\infty$ . For certain specific cases, such as  $F(u) = (1 - u^2)^2 - \frac{1}{2}$ , it is still unknown whether  $H^{-1}(0)$  is of contact type or not when  $\alpha < 0$ .

## Foliations by minimals for elliptic equations

MISHA BIALY

We consider the following elliptic equation for  $n \geq 3$ ,

$$\Delta u = -V'_u(u, x_1, \dots, x_n)$$

where the potential  $V$  is assumed to be periodic in  $u$  and  $V \equiv 0$  for  $|x|$  large enough. We show that for any slope  $\alpha = (\alpha_1, \dots, \alpha_n)$  there exists a continuous *foliation* of the space  $(u, x_1, \dots, x_n)$  by the graphs of solutions  $u_\lambda(x_1, \dots, x_n)$  having slope  $\alpha$ . The situation is quite opposite for the cases  $n = 2$  and  $n = 1$  where analogs of E. Hopf type rigidity occur.

## Invariant manifolds and applications

MARC CHAPERON

We state very general and simple non-perturbative theorems and sketch part of their proofs. They imply, among other things, results of Fenichel, Hirsch, Pugh and Shub on

stable or unstable manifolds of compact invariant manifolds, the pseudo-(un)stable theorem at a fixed point and Sternberg’s theorem on smooth conjugacy of hyperbolic germs of maps or vector fields. The Perron-Irwin approach via sequence spaces plays a crucial role in the proofs.

### **Action minimizing solutions of the $n$ -body problem**

ALAIN CHENCINER

I give a survey on the variational methods which led recently to new classes of periodic solutions of the equal-mass  $n$ -body problem (HipHop, Eight, Choreographics). The results alluded to came from collaborations with Nicole Desolneux, Joseph Gerver, Richard Montgomery, Carles Simó and Andrea Venturelli.

### **Minimal measures in the resonant regions of nearly integrable Hamiltonian systems**

CHONG-QING CHENG

Consider a Hamiltonian system of KAM type  $H(p, q) = N(p) + P(p, q)$ . It has  $n$  degrees of freedom ( $n > 2$ ) and is positive definite in  $p$ . Let

$$\Omega = \{\omega \in \mathbb{R}^n \mid \text{there exists } k \in \mathbb{Z}^n \text{ such that } \langle k, \omega \rangle = 0\}.$$

We show that for most rotation vectors in  $\Omega$ , in the sense of Lebesgue measure, the minimal measures consist of  $n - 1$  dimensional invariant tori. The Lebesgue measure estimate is independent of the form of perturbations  $P$  and only depends on the  $C^m$ -norm of the perturbation, for some positive integer  $m$ .

### **Exponential stability in the D’Alembert spin-orbit problem**

LUIGI CHIERCHIA

The D’Alembert model for a rotational planet with polar radius slightly smaller than the equatorial radius, whose center of mass revolves on a Keplerian ellipse of small eccentricity around a fixed star occupying one of the foci, and subject to the gravitational attraction of the star is considered.

Such a model admits a Hamiltonian formulation, which, however, exhibits several serious degeneracies (proper degeneracy, lack of convexity in the “secular” term, strong anisotropy in the analytic properties).

The problem is to study the relative motion of the planet and in particular to study the stability of the perturbed integrals of motions (action variables)  $J_1 =$  absolute value of the angular momentum of the planet and  $J_2 =$  absolute value of the projection of the angular momentum of the planet into the direction orthogonal to the ecliptic plane.

Let  $p$  and  $q$  be positive coprime integers different from  $(1, 1)$  and  $(2, 1)$ . Then the following “Nekhoroshev-type” result holds.

If the flatness  $\varepsilon$  and the eccentricity  $\mu \leq \varepsilon^c$  (for some  $c > 1/2$ ) are small enough and if  $J_1|_{t=0}$  is “close” to a  $p : q$  spin-orbit resonance (i.e., if  $J_1|_{t=0}$  is close to  $(p/q)\omega$ ,  $2\pi/\omega$  being the period of revolution of the planet), then the evolution of the perturbed integrals  $J_1$  and  $J_2$  stay close to their initial data for exponentially (in  $1/\varepsilon$ ) long times (here “close” means  $\varepsilon^a$ -close, for some  $a > 0$ ).

## Resonant water waves

WALTER CRAIG

(joint work with Daniel Nicholls)

This work is concerned with the existence of traveling water waves, which is to say solutions of the free surface problem for Euler's equation for an ideal fluid. Solutions are asked to be periodic in the  $n - 1$  many horizontal variables, and to satisfy bottom and nonlinear free surface boundary conditions. The problem is considered nonresonant when the phase velocity of the linearized equations (the bifurcation point) has a null space of minimal nontrivial dimension – namely of four dimensions. The problem is resonant if the dimension of the null space, necessarily even, is bigger than four. Our main theorem is that solutions exist for the nonlinear problem, given any fundamental domain for the periodic horizontal variables. In case of nonresonant domains, the solution surface has the form of four surfaces in  $\mathbb{R}^4$ , intersecting mutually at one bifurcation point. This is in analogy with the Lyapunov center theorem. When the problem is resonant, and the null space is of dimension  $2m$ , then on every fixed action surface there exist at least  $m - 1$  many distinct traveling wave solutions. The latter result is analogous to the resonant Lyapunov center theorem of A. Weinstein and J. Moser.

## Almost reducibility of linear quasi-periodic systems

HAKAN ELIASSON

We discuss reducibility and almost reducibility of linear quasi-periodic systems. We prove a result showing that when such a system is analytic, close to constant coefficients, and has Diophantine frequencies, it is always almost reducible in the following sense: it can be analytically conjugated to a system arbitrarily close to constant coefficients. We discuss when this approach converges and gives full reducibility, and when it does not.

## Groups acting on one-manifolds

JOHN FRANKS

This talk represents joint work with Benson Farb. We consider groups of homeomorphisms and diffeomorphisms of  $\mathbb{R}$ ,  $[0, 1]$  and  $S^1$ . The objective is to relate the dynamics (and smoothness) of a group action to algebraic properties of the group, and more generally to understand groups of homeomorphisms and diffeomorphisms in analogy with the theory of Lie groups and their discrete subgroups. For example, Plante–Thurston proved that every nilpotent subgroup of  $\text{Diff}^2(S^1)$  is abelian. One of our main results is that, by contrast,  $\text{Diff}^1(S^1)$  contains every finitely generated torsion-free nilpotent group.

## A $C^2$ -smooth counterexample to the Hamiltonian Seifert conjecture in $\mathbb{R}^4$

VICTOR GINZBURG

(joint work with Bařak Gürel)

We construct a proper  $C^2$ -smooth function on  $\mathbb{R}^4$  such that its Hamiltonian flow has no periodic orbits on at least one regular level set. This result can be viewed as a  $C^2$ -smooth counterexample to the Hamiltonian Seifert conjecture in dimension four.

## Integrability of vector fields and the Poincaré center problem

HIDEKAZU ITO

Let  $X$  be a  $2n$ -dimensional real analytic vector field near an equilibrium point  $z = 0$ . Assume that the eigenvalues of the linear part  $DX(0)$  are  $\pm i\alpha_1, \dots, \pm i\alpha_n$  ( $\alpha_k \in \mathbb{R}$ ,  $i = \sqrt{-1}$ ) and that  $\alpha_1, \dots, \alpha_n$  are rationally independent. Then we have:

**Theorem.** In addition to the vector field  $X = X_1$  satisfying the condition above, assume that there exist  $n - 1$  real analytic vector fields  $X_2, \dots, X_n$  such that

- 1)  $X_1, \dots, X_n$  are pairwise commuting and their lowest order part vectors are (functionally) linearly independent.
- 2) There exist (functionally independent) real analytic functions  $G_1, \dots, G_n$  which are invariant under the flows of  $X_1, \dots, X_n$ .

Then there exists a real analytic coordinate transformation  $z = \varphi(\zeta)$  ( $\zeta = (\xi, \eta)$ ) which takes  $X_k$  ( $k = 1, \dots, n$ ) into the form

$$\dot{\xi}_j = P_{k_j}(\tau)\eta_j, \quad \dot{\eta}_j = -P_{k_j}(\tau)\xi_j \quad (j = 1, \dots, n).$$

Here,  $\tau = (\tau_1, \dots, \tau_n)$ ,  $\tau_j = \frac{1}{2}(\xi_j^2 + \eta_j^2)$ , and  $P_{k_j}$  are convergent power series in  $\tau_1, \dots, \tau_n$ . Moreover, the integrals  $G_k$  are transformed into functions in  $\tau_1, \dots, \tau_n$  alone. The conditions 1) and 2) are considered as the definition of integrability of  $X$ , and this result gives an answer to the Poincaré center problem.

## Rigidity of actions of higher rank abelian groups

ANATOLE KATOK

Differentiable actions of groups  $\mathbb{Z}^k$  and  $\mathbb{R}^k$  for  $k \geq 2$  on compact manifolds exhibit a variety of rigidity properties if they satisfy certain conditions of irreducibility and hyperbolicity. Among these properties are local and global differentiable orbit rigidity, scarcity of invariant measures and rigidity of measurable orbit structures.

We discuss in detail the joint work with Svetlana Katok and Klaus Schmidt concerning the last type of rigidity. We consider actions of  $\mathbb{Z}^d$ ,  $d \geq 2$ , by automorphisms of a torus  $\mathbb{T}^k$ . The condition which provides both hyperbolicity and irreducibility is that  $\mathbb{Z}^d$  contains a  $\mathbb{Z}^2$  subgroup consisting of ergodic automorphisms. Under this condition any measurable isomorphism or factor map is affine. In the special case of Cartan actions ( $\mathbb{Z}^{k-1}$  actions by ergodic automorphisms of  $\mathbb{T}^k$ ) we develop a number theoretic apparatus which allows to distinguish actions which are isomorphic over  $\mathbb{Q}$  but not over  $\mathbb{Z}$ . This leads to a variety of interesting new examples where the isomorphism type is not determined by natural invariants: entropy function, measurable centralizers, etc.

## Reducibility of linear quasi-periodic cocycles

RAPHAËL KRIKORIAN

We will give some local and global results concerning the reducibility of linear quasi-periodic cocycles of the form

$$(\alpha, A): \mathbb{T}^d \times G \rightarrow \mathbb{T}^d \times G, \quad (\theta, y) \mapsto (\theta + \alpha, A(\theta)y)$$

where  $A: \mathbb{T}^d \rightarrow G$  is  $C^\infty$  smooth and  $G = \text{SU}(2)$  or  $\text{SL}(2, \mathbb{R})$ . Reducibility means the possibility of writing

$$A(\theta) = B(\theta + \alpha) A_0 B(\theta)^{-1}$$

where  $B: \mathbb{T}^d \rightarrow G$  is  $C^\infty$  and  $A_0$  is constant.

## Étale homotopy in dynamics

KRYSZYNA KUPERBERG

Étale homotopy (or shape theory) was introduced by K. Borsuk in topology and by A. Grothendieck in algebra in the late 1960's and 1970's. The theory relates to the classical homotopy theory in the same fashion as Čech homology to singular homology. It provides useful tools for investigating the properties of sets that often appear in dynamics as attractors, minimal sets, etc. It is known (Günter and Segal) that a strong attractor of an  $\mathbb{R}$ -action on a manifold has the shape of a polyhedron. Borsuk's notion of movability, resembling some of the conditions of stability, seems to be applicable to the study of invariant sets. Solenoids are non-movable, but an invariant stable solenoid in a flow on  $\mathbb{R}^3$  is approximated by circular orbits (Bell and Meyer) whose union with the solenoid is movable. A stable invariant set need not be approximated by circular orbits, but, at least in some cases, it is approximated by movable invariant sets.

## Generalized Aubry–Mather sets for diffeomorphisms of the annulus and the torus

PATRICE LECALVEZ

If  $f$  is an area-preserving diffeomorphism of the annulus, there exists for every  $\rho$  in the rotation set of  $f$  an invariant compact set of rotation number  $\rho$ . It is obtained as a limit of “good” periodic orbits. There is a dissipative version of this result, even on the 2-torus.

## Closed characteristics on convex and starshaped hypersurfaces

YIMING LONG

We give a survey on the recent progress on the multiplicity and stability of closed characteristics on smooth compact convex and starshaped hypersurfaces in  $\mathbb{R}^{2n}$ . These results are applications of the iteration theory of Maslov-type index theory for symplectic matrix paths established in recent years.

## Arnold diffusion

JOHN MATHER

Consider a small perturbation of an integrable Hamiltonian:

$$H_\epsilon = h(I) + \epsilon P(\theta, I, t, \epsilon)$$

where  $I = (I_1, \dots, I_n) \in B^n$ , a ball of radius 2 in  $\mathbb{R}^n$ ;  $\theta = (\theta_1, \dots, \theta_n) \in \mathbb{T}^n = \mathbb{R}^n/\mathbb{Z}^n$ , the  $n$ -torus;  $t \in \mathbb{T} = \mathbb{R}/\mathbb{Z}$  (i.e., the system is time-periodic of period 1); and  $0 < |\epsilon| < \epsilon_0$  (i.e.  $\epsilon$  is a small parameter). We study solutions of Hamilton's equations

$$\dot{\theta} = \frac{\partial H}{\partial I}, \quad \dot{I} = -\frac{\partial H}{\partial \theta}.$$

The oscillation  $\text{osc}I$  along a trajectory is defined by

$$\text{osc}I = \sup \|I(t_1) - I(t_0)\|.$$

A particular perturbation  $H_\epsilon$  is said to exhibit Arnold diffusion if there exists a trajectory for which  $\text{osc}I \geq 1$ .

In this talk, we announced the existence of a large class of perturbations which exhibit Arnold diffusion, in the case that  $\frac{\partial^2 h}{\partial I^2} > 0$ .

Let  $E$  be a topological vector space. We will say that a subset  $W$  of  $E$  is *cuspidal-residual* if the following holds:

- 1) There exists an open dense subset  $U$  of  $E$  such that  $v \in U$  and  $\lambda > 0$  imply  $\lambda v \in U$ .
- 2) There exists an open subset  $V$  of  $U$  such that if  $\gamma: [0, \delta_0] \rightarrow E$  is a  $C^1$ -curve,  $\gamma(0) = 0$  and  $\gamma'(0) \in U$ , then there exists  $0 < \delta \leq \delta_0$  such that  $\gamma((0, \delta)) \subset V$ .
- 3)  $W$  is an open dense subset of  $V$ .

In the following we assume  $r$  is a large integer,  $\infty$ , or  $\omega$ , and  $\frac{\partial^2 h}{\partial I^2} > 0$ . We assume  $h$  is  $C^r$ .

**Theorem** *There exists a cuspidal-residual set  $W$  in the space of  $C^r$  perturbations of  $h$  such that any perturbation in  $W$  exhibits Arnold diffusion.*

## Total convergence or general divergence in small divisors

RICARDO PEREZ-MARCO

Refining an old argument due to Y. Ilyashenko, we prove that for fairly generic holomorphic families of dynamical systems presenting problems of small divisors with fixed arithmetic, the formal series are all convergent or they are all divergent except maybe for a pluri-polar exceptional set of parameter values. Related results are obtained in Hamiltonian dynamics (Birkhoff normal forms) and analytic centralizers.

## An example of a diffeomorphism with non-zero Lyapunov exponents and countably many ergodic components

YAKOV PESIN

We describe a construction of a diffeomorphism on the torus  $\mathbb{T}^n$  which preserves the volume, has non-zero Lyapunov exponents and countably (not finitely) many ergodic components. This provides an affirmative solution of an old problem that systems with non-zero Lyapunov exponents may not be finitely ergodic. The construction uses some recent advances in stable ergodic theory. The resulting diffeomorphism has countably many invariant tori on which it is identity, and it is ergodic (indeed, Bernoulli) in the areas between them.

## Kick stability in groups and dynamical systems

LEONID POLTEROVICH

“How far can a flow be kicked?” More precisely, consider the behaviour of a dynamical system under the influence of a sequence of kicks arriving periodically in time. We are interested in the following stability type question: Does the kicked system inherit some recurrence properties of the original flow? It turns out that in some situations (linear flows on tori, “cat” maps of the 2-torus, Hamiltonian flows on symplectic manifolds) such a stability indeed takes place even when the kicks are quite large. The talk is based on a joint work with Zeev Rudnick.

# Stability and instability for Gevrey quasi-convex near-integrable Hamiltonian systems

DAVID SAUZIN

(joint work with Jean-Pierre Marco and the late Michael Herman)

We prove a theorem about the stability of action variables for Gevrey quasi-convex near-integrable Hamiltonian systems and construct in that context an example of Arnold diffusion whose speed allows us to check the optimality of the stability theorem.

Our stability result generalizes the one by Lochak–Neishtadt and Pöschel, which gives precise exponents of stability in the Nekhoroshev Theorem for the quasi-convex case, to the situation in which the Hamiltonian function is only assumed to belong to some Gevrey class instead of being real-analytic. For  $n$  degrees of freedom and Gevrey- $\alpha$  Hamiltonians,  $\alpha \geq 1$ , we prove that one can take  $a = 1/2n\alpha$  as exponent for the time of stability and  $b = 1/2n$  as exponent for the radius of confinement of the action variables, with refinements for the orbits which start close to a resonant surface (we thus recover the result for the real-analytic case by setting  $\alpha = 1$ ).

On the other hand, for  $\alpha > 1$ , the existence of compactly supported Gevrey functions allows us to exhibit for each  $n \geq 3$  a sequence of Hamiltonian systems with wandering points, whose limit is a quasi-convex integrable system, and where the mean speed of drift is characterized by the exponent  $1/2(n-2)\alpha$ . This exponent is optimal for the kind of wandering points we consider, inasmuch as the initial condition is located close to a doubly-resonant surface and the stability result holds with precisely that exponent for such an initial condition. We also discuss the relationship between our example of instability, which relies on a specific construction of a perturbation of a discrete integrable system, and Arnold’s mechanism of instability, whose main features (partially hyperbolic tori, heteroclinic connections) are indeed present in our system.

## On the dynamics of magnetic flows for energies above Mañés value

KARL FRIEDRICH SIBURG

(joint work with Norbert Peyerimhoff)

Our aim is to study the dynamics of a magnetic flow, in particular in its relation to the underlying geodesic flow. We assume that the magnetic field  $\Omega$  on  $M$  pulls back to an exact 2-form on the universal cover  $\widetilde{M}$ . Let  $c \in [0, \infty]$  be Mañés’s critical value (which is finite iff  $\pi^*\Omega$  has a bounded primitive). Then our results are as follows:

- 1) For energies  $E > c$ , minimal magnetic geodesics are Riemannian  $A(E)$ -quasigeodesics; moreover,  $A(E) \rightarrow 1$  as  $E \rightarrow \infty$ .
- 2) Minimal magnetic geodesics of energy  $E > c$  lie in tubes around Riemannian geodesics; moreover, the tube width goes to 0 as  $E \rightarrow \infty$ . This follows from a new sharpened version of the Morse-Lemma.
- 3) The length associated to Mañés’s metric (again,  $E > c$ ) is just the Riemannian length – it forgets every information about the magnetic field.



## Lyapunov exponents of volume preserving maps

MARCELO VIANA

I report some very recent results about abundance of zero respectively non-zero Lyapunov exponents for volume-preserving diffeomorphisms and linear cocycles:

**Theorem 1** (joint with J. Bochi) *For a dense subset of  $C^1$  volume-preserving diffeomorphisms on any compact manifold  $M$ , there exists a partition*

$$M = Z \cup D \pmod{0}, \quad Z \text{ and } D \text{ invariant,}$$

*such that every  $z \in Z$  has all Lyapunov exponents equal to zero, and  $D$  is a countable increasing union of compact sets with a dominated splitting (continuous invariant cone field).*

We also get a version of Theorem 1 for symplectic diffeomorphisms. The latter had been announced by Mañé in the early eighties. In the 2-dimensional case he provided a sketch of a proof containing some fundamental gaps. A complete proof was recently given by Bochi.

Moreover, Theorem 1 has a counterpart for *continuous* linear cocycles. In fact, in this setting the conclusion is stronger: for a residual subset of cocycles the Oseledec decomposition is dominated or trivial (one single subspace).

With only a slight increase in regularity, the conclusion may change drastically:

**Theorem 2** (joint with C. Bonatti and X. Gomez-Mont) *Assume  $f: (M, \mu) \rightarrow (M, \mu)$  is hyperbolic (e.g. Anosov) and  $\mu$  is a Gibbs state. Then, for any  $0 < \nu \leq \infty$  there exists an open dense subset of  $C^\nu$  dominated cocycles, whose complement has infinite codimension, for which all Lyapunov exponents have multiplicity 1 (i.e. the Oseledec subspaces have dimension 1).*

The domination condition in the statement means that the action (expansion and contraction) of the cocycle on the projective space is everywhere weaker than the action of the hyperbolic transformation  $f$  along stable and unstable directions, respectively.

## Hamiltonian dynamics and elastic materials

EUGENE WAYNE

In this joint work with Lee DeVille we study a model for the behaviour of laminated materials (introduced by Schwab and Babuska) in thin three dimensional domains (i.e. plates). Using ideas related to the Nekhoroshev theory of classical mechanics we derive “reduced equations”, which are partial differential equations posed on two-dimensional domains, and we prove that if the thickness of the plate is sufficiently small, the solution of the reduced equation can approximate the solution of the original equation for an arbitrarily long time.

## KAM theory for lower dimensional tori

JIANGONG YOU

(joint work with Junxiang Xu)

We prove the persistence of lower dimensional invariant tori of integrable equations after Hamiltonian perturbations under the first Melnikov non-resonance condition. The proof is based on an improved KAM machinery which works for the angle variable dependent

normal form. By an example we also show the necessity of the first Melnikov non-resonance condition for the persistence of lower dimensional tori.

### **From invariant curves to strange attractors**

LAI-SANG YOUNG

A limit cycle subjected to periodic forcing is shown to exhibit a wide array of dynamical behaviours, ranging from an invariant curve to a strange attractor with an SRB measure and fully stochastic behaviour.

*Edited by Felix Schlenk*

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