Mathematisches Forschungsinstitut Oberwolfach

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"Stochastic evolution equations and applications"

September 30th – October 6th, 2001

The conference was devoted to the latest developments in stochastic evolution equations and their applications. One part of the talks were devoted to the special linear case, i.e. Ornstein-Uhlenbeck processes. They focused on fine support properties, advanced results on characterizations of the domains of the corresponding generators and progress in the study of potential theory. Another portion of the talks dealt with stochastic evolution equations from the pure stochastic PDE point of view, based both on the so-called martingale approach and the semigroup approach. Among the covered topics new existence and uniqueness results, new approximations results, smoothness of densities, triviality of solutions and small noise limits were presented, as well as. Furthermore, a substantial number of lectures were devoted to latest progresses in the theoretical background for stochastic evolution equations, i.e. in infinite dimensional analysis and geometry. New results on invariant measures, infinite dimensional flows, quasi-invariance under group actions, integration by parts with explicit boundary terms and Poincaré inequalities on infinite dimensional manifolds were presented. Most of the talks, however, were directed to new applications in other areas of mathematics, such as nonlinear PDE, and also other sciences, including Mathematical Finance, population genetics in Biology, and Mathematical Physics, in particular statistical mechanics and fluid dynamics. Unfortunately, the conference suffered from the consequences of the terror attacks of September 11, 2001, in New York City, so that no mathematicians from across the Atlantic took part. We hope that the next time the world will be peaceful enough to see all our colleagues friends again in Oberwolfach.

Giuseppe Da Prato Michael Röckner Jerzy Zabczyk

Abstracts

Semigroups generated by solutions of elliptic equations for measures

Vladimir Bogachev

Let L be an elliptic operator on $\mathbb{R}^d$ on a Riemannian manifold M such that in local coordinates it has the form

$$Lf = \sum_{i,j\le d} a^{ij}\partial_{x_i}\partial_{x_j} f + \sum_{j\le d} b^j \partial_{x_j} f,$$

where $A = (a^{ij})$ is a nonnegative matrix valued mapping and $b = (b^j)$ is a vector field. It is known that if a probability measure μ satisfies the elliptic equation $L^*\mu = 0$ in the weak sense, i.e.

$$\int Lf\, d\mu = 0$$

for every smooth compactly supported function f, then, under broad assumptions on a^{ij} and b^j, there exists a sub-Markovian semigroup $(T_t^\mu)_{t\ge 0}$ on $L^1(\mu)$ such that its generator coincides with L on smooth compactly supported functions (this semigroup is called Stannat's semigroup and in the symmetric case it coincides with the semigroup generated by the Friedrichs extension of L). In addition, μ is sub-invariant for this semigroup. The lecture gives an account of recent progress in the study of various properties of such semigroups, in particular, the strong Feller property and certain uniqueness properties are discussed. Connections with diffusion processes are considered and open problems in this area are formulated.

Stochastic Nonlinear Beam Equation

Zdzisław Brzeźniak

(joint work with Jan Seidler)

Suppose that H is a real separable Hilbert space and let A and B be self-adjoint operators in H. Suppose that $B > 0$ and that $A \ge \mu I$ for some $\mu > 0$. We assume that $D(A) \subset D(B)$ and that $B \in \mathcal{L}(D(A), H)$ with $D(A)$ endowed with the norm $\|x\|_{D(A)} := \sqrt{\langle Ax, Ax\rangle}$. Suppose that G is another real separable Hilbert and that $W(t)$, $t \ge 0$ is an G-cylindrical Wiener process on some probability space $(\Omega, \mathcal{F}, \mathcal{F}_{t\ge 0}, \mathbb{P})$. Suppose that $m : [0,\infty) \to \mathbb{R}$ is a nonnegative C^1-class function. Finally, $f : D(A) \times H \to H$ is locally Lipschitz (i.e. Lipschitz on balls) and satisfies the following one sided linear growth condition (for some $L > 0$)

$$\langle y, f(x,y)\rangle \ge -L(1 + |x|^2_{D(A)} + |y|^2), \quad (x,y) \in D(A) \times H. \tag{1}$$

We also consider a locally Lipschitz and of linear growth map $\sigma : D(A) \times H \to \mathcal{L}_2(G,H)$, where $\mathcal{L}_2(G,H)$ is the space of all Hilbert-Schmidt operators from G to H. Then, given $(x_0, x_1) \in D(A) \times H$ we consider the following second order stochastic differential equation

$$\begin{aligned} x_{tt} + A^2 x + f(x, x_t) \quad &+ \quad m(|B^{1/2}x|^2)Bx = \sigma(x, x_t)dW, \\ x(0) \quad &= \quad x_0, \;\; x_t(0) = x_1. \end{aligned} \tag{2}$$

Problem (2) can be rewritten as a stochastic evolution equation (of first order) in the Hilbert space $\mathcal{H} = D(A) \times H$ (endowed with a natural scalar product)

$$du = (\mathcal{A}u + F(u))\,dt + \Sigma(u)\,dW(t), \quad u(0) = u_0, \tag{3}$$

where $u_0 = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$ and with, informally $u = (x, x_t)$. Here, as in the deterministic theory of second order equations, the operator $\mathcal{A}$ in $\mathcal{H}$ is defined by $\mathcal{A}(x,y) = (y, -A^2 x)$, $F : \mathcal{H} \to \mathcal{H}$ is defined by $F(x,y) = \big(0, -m(|B^{1/2}x|^2)Bx - f(x,y)\big)$ and $\Sigma : \mathcal{H} \to \mathcal{L}_2(G, \mathcal{H})$ is defined by $\Sigma(x,y)(g) = (0, \sigma(x,y)g)$. With $D(\mathcal{A}) = D(A^2) \times D(A)$ the operators $\mathcal{U}$ and $-\mathcal{U}$ are m-dissipative. By a solution $x(t)$ of the problem (2) we mean the first component of the solution $u(t)$ to the problem (3). Our main results is the following

Theorem 1 Set $M(s) = \int_0^s m(r)\,dr$, $s \geq 0$. If

$$\mathbb{E}\left(|u(0)|^2 + M(|B^{1/2}u_0|^2)\right) < \infty \tag{4}$$

then there exists a unique global mild solution to the problem (3). The paths of this solutions are continuous ($\mathcal{H}$-valued) a.s.

The proof is in some way standard. We first prove existence of a maximal local mild solution $u(t)$, $t \in [0, \tau)$, see also [2]. Next we prove that this solution is global (i.e. $\tau = \infty$) by using the Khasminski test of non-explosion with the Lyapunov function $V : \mathcal{H} \to \mathbb{R}$ defined by $V(u) = \frac{1}{2}\left(|u|^2 + M(|B^{1/2}x|^2\right)$, where $u = (x, y)$. On an informal level the last part is trivial. However, since firstly $u(t)$ is not a semimartingale, and secondly, because of some peculiarity of the nonlinear term, the full proof requires some delicate approach. We also study Feller property of the process $u(t)$ and we plan to investigate existence of an invariant measure.

Example 2 Let $\mathcal{O}$ be a bounded domain in $\mathbb{R}^n$ with (sufficiently) smooth boundary. Let $H = L^2(\mathcal{O})$, let B be the $-$Laplacian with Dirichlet boundary conditions: $D(B) = H^{2,2}(\mathcal{O}) \cap H_0^{1,2}(\mathcal{O})$.

Define a self-adjoint operator C by $D(C) = \left\{\psi \in H^{4,2}(\mathcal{O}) : \psi = \frac{\partial \psi}{\partial \nu} = 0 \,\text{on}\; \partial\mathcal{O}\right\}$, $Cu := \Delta^2 u$. Define finally $A = C^{1/2}$. Then our problem (2) becomes the following stochastic beam equation

$$u_{tt} - m\Big(\int_{\mathcal{O}} |\nabla u|^2\,dx\Big)\Delta u + \gamma\Delta^2 u + f(x, u, \nabla u, u_t) = \pi(x, u, \nabla u, u_t)\dot{W} \tag{5}$$

with the clamped boundary conditions

$$u = \frac{\partial u}{\partial \nu} = 0 \,\text{on}\; \partial\mathcal{O} \tag{6}$$

Example 3 Equation (5) can also be studied with so called hinged boundary conditions

$$u = \Delta u = 0 \,\text{on}\; \partial\mathcal{O} \tag{7}$$

The problem (5), (7) is also of the form (2) with A defined to be equal to B. The coefficient $f : D := \mathcal{O} \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is assumed to be locally Lipschitz in the last three variables, $f(\cdot, 0, 0, 0) \in L^2(\mathcal{O})$ and

$$f(x, r, s, z)z \geq -L(1 + |z|^2),$$

for some $L \geq 0$ and all $(x, r, s, z) \in D$. There are also some natural assumptions on the diffusion coefficient π and on the Hilbert space G.

This research has been motivated by a paper [3].

References

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Large deviations estimates for stochastic reaction-diffusion systems with multiplicative noise

Sandra Cerrai

(joint work with Michael Röckner)

We consider the following class of reaction-diffusion systems perturbed by a multiplicative noise

$$
(1) \quad \left\{\begin{aligned}
\frac{\partial u_i}{\partial t}(t,\xi) = & \ \mathcal{A}_i\, u_i(t,\xi) + f_i(t,\xi,u_1(t,\xi),\dots,u_r(t,\xi)) \\
& + \epsilon \sum_{j=1}^{r} g_{ij}(t,\xi,u_1(t,\xi),\dots,u_r(t,\xi)) B_j \frac{\partial^2 w_j}{\partial t \partial \xi}(t,\xi), \quad t \geq s, \quad \xi \in \overline{\mathcal{O}}, \\
u_i(s,\xi) = & \ x_i(\xi), \quad \xi \in \overline{\mathcal{O}}, \qquad \mathcal{B}_i\, u_i(t,\xi) = 0, \quad t \geq s, \quad \xi \in \partial\mathcal{O}.
\end{aligned}\right.
$$

Here $\mathcal{O} \subset \mathbb{R}^d$ is a bounded open set with regular boundary, $\mathcal{A}_i$ are second order uniformly elliptic operators with coefficients of class C^1 endowed with boundary conditions given by the operators $\mathcal{B}_i$, B_i are bounded linear operators on $L^2(\mathcal{O})$, $\partial^2 w/\partial t \partial \xi$ are independent space-time white noises.

Concerning the non-linearities, we assume that $f : \overline{\mathcal{O}} \times \mathbb{R}^r \to \mathbb{R}^r$ and $g : \overline{\mathcal{O}} \times \mathbb{R}^r \to \mathcal{L}(\mathbb{R}^r)$ are continuous and $g(\xi, \cdot) : \mathbb{R}^r \to \mathcal{L}(\mathbb{R}^r)$ is Lipschitz-continuous, uniformly with respect to $\xi \in \overline{\mathcal{O}}$. $f(\xi, \cdot) : \mathbb{R}^r \to \mathbb{R}^r$ has polynomial growth, is locally Lipschitz-continuous and satisfies suitable dissipativity conditions.

We show that for any initial datum $x \in C(\overline{\mathcal{O}}; \mathbb{R}^r)$ and $\epsilon > 0$ the system (1) has a unique mild solution $u^x_\epsilon \in L^p(\Omega; C((s,T]; C(\overline{\mathcal{O}}; \mathbb{R}^r)) \cap L^\infty(s,T; C(\overline{\mathcal{O}}; \mathbb{R}^r)))$. Moreover, we give some estimates for such solution.

We apply these results to the study of large deviations for the laws of the paths of the solution. Namely, we show that the family of probability measures $\mu^x_\epsilon = \mathcal{L}(u^x_\epsilon)$ on $C((s,T]; C(\overline{\mathcal{O}}; \mathbb{R}^r)) \cap L^\infty(s,T; C(\overline{\mathcal{O}}; \mathbb{R}^r))$ fulfils a large deviations principle with respect to a suitable action functional.

Symmetric Ornstein-Uhlenbeck generators: characterization and domains

ANNA CHOJNOWSKA-MICHALIK
(joint work with Beniamin Goldys)

We consider the transition semigroup (R_t) for Ornstein-Uhlenbeck (O-U) process Z on a Hilbert space H, satisfying the equation:

$$dZ = AZdt + Q^{1/2}dW; \quad Z(0) = x \in H. \tag{1}$$

In (1), A is the generator of strongly continuous semigroup (S_t) on H, W is an H-valued cylindrical Wiener process, $Q = Q^* \geq 0$.

First, we provide necessary and sufficient conditions (in terms of A and Q) for the existence of symmetrizing (probability) measure for the semigroup (R_t).

For a symmetric O-U semigroup (R_t) we show that the operators $S_Q(t) := Q^{-1/2}S_tQ^{1/2}$ define a C_0-semigroup of symmetric contractions on H. This fact allows us to provide explicit criteria in terms of A and Q for various properties in $L^p(H,\mu)$-spaces of the O-U semigroup (R_t) and its generator L. We show that the spectral gap of L, the compactness and the Hilbert-Schmidt property of R_t are determined by the corresponding properties of the semigroup $(S_Q(t))$. We provide necessary and sufficient conditions for the strong Feller property of (R_t). Finally, we give a complete characterization of the domain $\mathrm{dom}_p(L)$, $p \in (1,\infty)$, of the symmetric generator L acting in $L^p(H,\mu)$, in terms of appropriately defined Gauss-Sobolew spaces.

We also discuss some examples.

Deterministic invariant regions and Wong–Zakaï type approximation theorem for a class of stochastic parabolic PDE systems

IGOR CHUESHOV
(joint work with P. Vuillermot)

We consider the following system of semilinear Itô parabolic SPDE

$$\left\{ \begin{array}{rcl} du^l(x,t) & = & \left(-A^l(x,t,D)u(x,t) + f^l(x,t,u(x,t),Du(x,t))\right)\, dt \\ & & \\ & + & \sum_j g^l_j(x,t,u(x,t))\, dW_j(t), \quad l = 1,\dots m,\ x \in \mathcal{O},\ t > 0, \end{array} \right. \tag{1}$$

in bounded smooth domain $\mathcal{O} \subset \mathbf{R}^n$ with the boundary and initial conditions:

$$B^l(x,D)u^l(t,x) = 0,\ x \in \partial\mathcal{O},\ t > 0, \quad u^l(0,x) = u^l_0(x),\ x \in \overline{\mathcal{O}}, \quad l = 1,\dots m. \tag{2}$$

Here $u = (u^1,\dots,u^m) : \overline{\mathcal{O}} \times \mathbf{R}_+ \mapsto \mathbf{R}^m$ is a vector function, $\left(A^l, B^l\right)$ is an elliptic second order regular pair of differential operators for each $l = 1,\dots m$ and $\{W_j(t)\}$ is a family of independent standard scalar Wiener processes on the canonical Wiener space $(\Omega,\mathcal{F},\mathbf{P})$, $dW_j(t)$ is the corresponding Itô differential.

We discuss the problem of existence of deterministic regions $\mathbf{D}$ in $\mathbf{R}^m$ such that the property $u_0(x) \in \mathbf{D}$ for all $x \in \mathcal{O}$ implies that $u(x,t) \in \mathbf{D}$ for all $x \in \mathcal{O}$ and $t > 0$ almost surely. Here $u_0(x) = (u^1_0(x),\dots,u^m_0(x))$ is initial data and $u_(x,t) = (u^1(x,t),\dots,u^m(x,t))$ is the corresponding solution to (1) and (2). The main tool is a Wong–Zakaï type approximation theorem for stochastic problem (1) and (2). This theorem claims the possibility to construct smooth convergent approximations and allows us to invoke results from the theory of deterministic parabolic equations.

$2D$-Navier-Stokes equations driven by a space-time white noise

Arnaud Debussche

(joint work with G. Da Prato)

We study the two-dimensional Navier-Stokes equation with periodic boundary condition perturbed by a space-time white noise. It is shown that, although the solution is not expected to be smooth, the nonlinear term can be defined without changing the equation. We first construct a stationary martingale solution. Then, we prove that, for almost every initial data with respect to a measure supported by a negative spaces, there exists a unique global solution in the strong probabilistic sense.

Local spectral gaps on loop spaces

Andreas Eberle

We prove Poincaré inequalities w.r.t. the distributions of Brownian bridges on sets of loops with jumps of limited size over compact Riemannian manifolds. Moreover, we study the asymptotic behaviour of the second Dirichlet eigenvalues as the time parameter T of the underlying Brownian bridge tends to 0. This behaviour depends crucially on the geodesics contained in the set of loops considered. In particular, for different choices of a Riemannian metric on the base manifold, qualitatively different asymptotic behaviours can occur. The proof of the basic Poincaré inequality is based on the construction of the Brownian bridge by consecutive bisection of the parametrization interval.

Stochastic Invariance with Applications to Interest Rate Models

Damir Filipovic

(joint work with Josef Teichmann)

A Heath–Jarrow–Morton type model for the forward curve $r_t : \mathbb{R}_+ \to \mathbb{R}$ can be considered as a stochastic equation

$$dr_t = (Ar_t + \alpha_{HJM}(r_t))\, dt + \sigma(r_t)\, dW_t \tag{1}$$

on a separable Hilbert space H which is characterized axiomatically. Here W is a d-dimensional Wiener process, A the generator of the strongly continuous semigroup of shift operators $S(t)r = r(t + \cdot)$, and $\alpha_{HJM}(r) = \sum_{j=1}^{d} \sigma^j(r) \int \sigma^j(r)$.

Let $m \in \mathbb{N}$ and $\mathcal{M}$ be a locally invariant m-dimensional submanifold of H; that is, for every $r_0 \in \mathcal{M}$ there exists a continuous local weak solution (r_t) to (1) and a stopping time $\tau > 0$ such that $r_{t\wedge\tau} \in \mathcal{M}$, for all $t \geq 0$. Under the appropriate regularity assumptions we can show that $\mathcal{M}$ is locally invariant if and only if $\mathcal{M} \subset D(A)$, $\mu(r) := Ar + \alpha_{HJM}(r) - 1/2 \sum_j D\sigma^j(r)\sigma^j(r) \in T_r\mathcal{M}$ and $\sigma^j(r) \in T_r\mathcal{M}$, for all $r \in \mathcal{M}$. Expressed in local coordinates this yields a diffusion process Z in $\mathbb{R}^m$ and a smooth embedding $\phi : V \subset \mathbb{R}^m \to U \cap \mathcal{M}$ such that $r_{t\wedge\tau} = \phi(Z_{t\wedge\tau})$. Hence $\mathcal{M}$ can be interpreted as a finite-dimensional realization (FDR) for the model (1). FDRs are very important from the point of view of applications. We show that their choice is very restricted. In fact, most of the commonly used parametrized families of forward curves,

$\mathcal{M} = \{G(\cdot, z) \mid z \in \mathbb{R}^m\}$, are inconsistent with (1). This calls for an existence result for invariant manifolds. We provide a Frobenius theorem which has to be formulated on the Fréchet space $D(A^\infty) := \bigcap_{n\in\mathbb{N}} D(A^n)$, since there the vector field μ is smooth. We obtain a global picture of FDRs: all generic FDRs are affine.

Forward-Backward stochastic differential equations in Hilbert spaces and applications

Marco Fuhrman and Gianmario Tessitore

We consider a forward-backward system of stochastic differential equations on infinite dimensional Hilbert spaces:

$$\begin{cases} dX_\tau = (AX_\tau + F(X_\tau))\,dt + G(X_\tau)dW_\tau \qquad \tau \in [t, T] \\ dY_\tau = \psi(X_\tau, Y_\tau, Z_\tau)dt + Z_\tau dW_\tau \qquad \tau \in [t, T] \\ X_t = x \\ Y(T) = \phi(X_T) \end{cases}$$

where W is a cylindrical Wiener process, Y, Z are Hilbert space valued adapted processes and Y is a real valued adapted process.

We prove existence and uniqueness of the solution and show that if ϕ and ψ are Gateaux differentiable then letting $Y_t = u(t, x)$ the function u is a mild solution of the nonlinear Kolmogorov equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} u(t, x) = \mathcal{L}u(t, x) = \psi(x, u(t, x), \nabla_x u(t, x)G(X)) \qquad t \in [0, T]\ x \in H \\ u(T, x) = \phi(x) \end{cases}$$

where H is an Hilbert space and (formally)

$$\mathcal{L}u(x) = \frac{1}{2}\mathrm{Tr}\left(G(x)G^*(x)\nabla^2 u(x)\right) + \langle Ax + F(x)\,,\, \nabla_x u(x)\rangle$$

The results are then applied to optimal control problems for stochastic partial differential equations with no nondegeneracy assumptions.

We notice then that if G is invertible then a Bismut-Elworthy formula holds for the forward-backward system. This allows to prove existence and uniqueness of the solution of the nonlinear Kolmogorov equation relaxing regularity assumptions on ϕ and ψ.

Super replications of multiasset derivative in the degenerate case: an approach using viscosity solutions

Fausto Gozzi

The seminar is concerned with the problem of pricing European contingent claims in the multiasset case using the super hedging approach.

1 - Brief introduction to the problem where we introduce notation and recall main ideas of Black and Scholes pricing method. 2 - pricing in the case of stochastic volatility (incomplete market) using the super - hedging approach: setting of the problem in general and then in a special case when the volatility varies in a compact set Σ.

3 - Recall known results on super hedging approach in the so-called nondegenerate case (i.e. when show that the square of the volatility matrix is always invertible in Σ): the super

hedging price can be found solving a fully non linear PDE: the so-called Black-Scholes-Barenblatt equation. This equation is in fact a Bellman equation associated to a stochastic control problem. When this equation is non degenerate (i.e. the square of volatility matrix is always invertible) it admits a smooth solution and the solution is the super - hedging price. Moreover the space derivative of it represent a Markov superstrategy. This is known by papers of El-Karoui et al, Lyons and others.

4 - We consider the degenerate case (since it arises in applied problem: we give an example) and show how to deal with this case using the notion of viscosity solution. Main results: existence and uniqueness of a viscosity solution to the BSB equation, and characterize it as the super hedging price; moreover show that, in the convex case, the space derivative of it again give a Markov superstrategy.

5 - Few ideas on how to extend results of point 4 to the infinite dimensional case when we consider derivatives associated to interest rates following the Musiela model (or related infinite dimensional models)

Unitarizing measure for Virasoro algebras

Paul Malliavin

In the theory of automorphic functions in one complex variable it appears the automorphy factor. The point of this talk is to interpret the modulus of the automorphy factor as a Radon-Nykodym derivative of a reference measure through its transform under the group action. This problematic can be carried in complex dimension n using the Siegel automorphic forms and the Berezin measures.

The purpose of the talk is to develop the same methodology in infinite dimension for the gauge group of string theory that is a central extension of the group of diffeomorphisms of the circle.

The first part of this program appeared (Airault-PM Journal de Mathématiques Pures et Appliquées July 2001), there are explicitly determined the infinitesimal automorphy factors which can be remarkably related to two other theories the Sato Universal Grassmannian,

The theory of univalent functions in the unit disk.

The reference measure (Called the unitarizing measure) must satisfy a formula of integration by part prescribed by the automorphy factor. This formula characterizes the unitarizing measure which is realized on the quotient of the group of homeomorphisms of the circle by the group of homographic transformation : the extended universal Teichmuller space.

On strongly Petrovskiĭ's parabolic SPDEs in arbitrary dimension

Annie Millet

These following results are in a joint work with Caroline Cardon-Weber. We prove existence and uniqueness of a function-valued process $u(t,x)$ which is the weak solution to the following SPDE on a "smooth" bounded domain $Q \subset \mathbb{R}^d$

$$\frac{\partial}{\partial t} u(t,x) = A(t,x,D_x)\, u(t,x) + \sigma(t,x,u(t,x))\, \dot{F}(t,x) + \sum_{i=1}^{N} D_x^{k_i}\big(b_i(t,x,u(t,x))\big)\,,$$

with the initial condition $u(0,.) = u_0$, normal complementary homogeneous (Neumann's or Dirichlet's) boundary conditions and "well-chosen" k_i and b_i. We suppose that $\frac{\partial}{\partial t} - A$ is uniformly parabolic in the sense of Petrovskiĭ and that F is either the space-time white noise (for small d) or some space-correlated Gaussian process (for large d); in the last case we give necessary and sufficient conditions on the space correlation function. We also establish Hölder regularity of the trajectories of the solution.

The particular case of the Cahn-Hilliard equation ($Q=]0,\pi[^d$, $A=-\Delta^2$, $D_x^k=\Delta$ and b is a polynomial of degree 3 with positive dominant coefficient) is studied with more details in dimension 1 up to 5. We prove that for $t>0$ and pairwise distinct points $x_1,\cdots,x_l\in Q$, the law of $(u(t,x_1),\cdots,u(t,x_l))$ has a density. When $|\sigma|\geq C>0$, this density is "almost surely" positive.

Dynamics of a passive tracer

Szymon Peszat

Consider the ordinary differential equation

$$dx(t)=V(t,x(t))dt, \qquad x(0)=0. \tag{1}$$

Assume that V is a stationary solution to the following Ornstein–Uhlenbeck equation

$$dV=-AVdt+BdW, \tag{2}$$

where A and B are pseudo-differential operators $\widehat{A\psi}(k)=|k|^{2\gamma}\widehat{\psi}(k)$ and $\widehat{B\psi}(k)=\sqrt{2|k|^{2\gamma+1-d}\mathcal{E}(k)}\widehat{\psi}(k)$, $\gamma\geq 0$, and $\operatorname{Trace}\mathcal{E}(k)\sim|k|^{1-2\alpha}$ for a certain $\alpha>1$. These assumptions guarantee that there is a unique stationary solution V to (2). Moreover, V is Hölder continuous in t and x with the exponent $H<\alpha-1$. We are concerned with the existence and uniqueness of a solution to (1). The main results are:

1. Uniqueness of the law $\mathcal{L}_T(x)$ of the solution to (1) on the space of trajectories $C([0,T];R^d)$, under the condition $\gamma>1/2$.
2. The construction of a measurable and adapted solution, under the conditions $\alpha+\gamma>2$ and $\alpha+3\gamma>3$.

The proofs are based on the observation that the Lagrangian environment process $\eta(t,x)=V(t,x+x(t))$, $t\geq 0$, $x\in R^d$ satisfies the SPDE,

$$d\eta(t,x)=(-A\eta(t,x)+\eta(t,0)\cdot\nabla\eta(t,x))dt+Bd\tilde{W}(t),$$

with a certain Wiener process $\tilde{W}$. The talk presents some of the results obtained in the following two joint papers:

A. Fannjiang, T. Komorowski, and S. Peszat, *Lagrangian dynamics for a passive tracer in a class of Gaussian Markovian flows*, Stochastic Processes Appl., to appear.

T. Komorowski and S. Peszat, *Transport of a passive tracer in an Ornstein–Uhlenbeck velocity field*, in preparation.

Liouville theorems for Ornstein-Uhlenbeck processes

Enrico Priola
(joint work with J. Zabczyk)

We consider an infinite dimensional Ornstein-Uhlenbeck process X_t on a separable Hilbert space H, satisfying the equation: $dX_t = AX_t dt + \sqrt{Q} dW_t$, $t \geq 0$, where A generates a $\mathcal{C}_0$-semigroup in H, $Q = Q^* \geq 0$ and W_t is a cylindrical Wiener process in H. Under some conditions which include strong Feller and irreducibility properties for X_t, we prove the following result:

Theorem There exists a nonconstant bounded harmonic function for the Ornstein-Uhlenbeck process if and only if the spectrum of the operator A is not contained in the set $\{\lambda \in \mathbf{C} : \mathrm{Re}(\lambda) \leq 0\}$.

Our proof uses also control theoretic arguments. We give also a probabilistic interpretation of the main result in terms of the paths properties of the process X_t.

On the singularities of solutions to the Stochastic Navier-Stokes equations

Marco Romito
(joint work with F. Flandoli)

Among the most important open problems for the Navier-Stokes equations there is the existence of possible singularities. Let $u = u(t, x)$ be the velocity field of an incompressible homogeneous Newtonian fluid in a bounded domain D subject, with the pressure $P = P(t, x)$, to the Navier-Stokes equations. Denote by $S(u)$ the set of singular points of u. Even if the data are regular, it is not known whether $S(u)$ is empty (only local in time regular solutions are guaranteed). The best known result, by Caffarelli, Kohn and Nirenberg, states that the one-dimensional Hausdorff measure of $S(u)$ is zero, for the so called suitable weak solutions, that are weak solutions satisfying a local energy inequality.

We approach this problem with a probabilistic technique. We consider both the classical deterministic Navier-Stokes equations and a stochastic version, where we add a white in time noise. Our first result is that the above statement holds for martingale solutions. Moreover we consider stationary statistical solutions: probability measures on the space of suitable weak solutions that are invariant for the time-shift. Let μ be such a measure and consider the projection ν at time 0 of the statistical solution μ (ν should play the role of an invariant measure). We prove that for every fixed $t > 0$, the set $S_t(u)$ of singular points at time t is empty for ν-almost all u_0, where u is a solution starting at u_0. In addition, under suitable non-degeneracy assumptions on the white noise, the support of the measure ν is the whole space of initial conditions with finite energy.

Finally our approach to the problem shows that the above results depend heavily on the regularity assumptions on the data. We can present a (deterministic) suitable weak solution to the Navier-Stokes equation with a singular set of positive one-dimensional Hausdorff measure.

Triviality and non-triviality effects by SPDEs

FRANCESCO RUSSO

We consider a stochastic heat equation of parabolic type of the following nature

$$\left\{ \begin{array}{lcl} \partial_t U & = & LU + F(U)\chi + \dot{W} \\ U(0,\cdot) & = & U_0 \end{array} \right. \tag{1}$$

where $U_0 \in \mathcal{S}'(\mathbb{R}^d), L$ is a uniformly elliptic operator with smooth coefficients, F is a Lipschitz function, χ a truncating real function with compact support and $\dot{W}$ is a Gaussian space-time white noise.
If $d \geq 2, F = 0$, the solution of (1) is a random Gaussian Schwartz distribution. Replacing $\dot{W}$ with a regularized noise $\dot{W}^\epsilon$ in (1), we denote by U^ϵ the corresponding solution.
We say that (1) produces a **triviality effect** if there is a Gaussian random field $X \in C(\mathbb{R}_+; \mathcal{S}'(\mathbb{R}^d))$ such that

$$\lim_{\epsilon \to 0} \int U^\epsilon(t,x)\varphi(x)dx \overset{\epsilon \to 0}{\longrightarrow} X(t,\varphi) \tag{2}$$

in probability.

We give a large class of functions F for which (1) produces a triviality effect. A sufficient condition is that there are $a, b \in \mathbb{R}, a$ real bounded function, G such that the Fourier transform of G is **massless** at zero in some sense to be precise with $F(x) = G(x) + ax + b$. On the other hand, there are functions F not fulfilling this property for which (2) does not hold. Similar results are obtained in the case of a stochastic wave equation and elliptic equations.

In fact, triviality effects can arise in other asymptotical situations.
For instance we consider in dimension $d = 1$, the equation

$$\left\{ \begin{array}{lcl} \partial_t U & = & \frac{\partial^2 U}{\partial x} + F(U) + \eta \dot{W} \\ U(0,x) & = & U_0(x), \qquad x \in [0,1] \end{array} \right. \tag{3}$$

with Neumann boundary condition, where F is a bounded continuous (not necessarily Lipschitz) real function, $\eta \in \mathbb{R}, U_0$ is a continuous function.

One is interested to compare the solution $U(\eta)$ with the solution $X(\eta)$ of (3) in the free case ($F \equiv 0$) when $\eta \to \infty$ (large noise). In fact, we can exhibit natural sufficient conditions on F such that

$$U(\eta) - X(\eta) \longrightarrow 0$$

when $\eta \to \infty$, give examples of F such that $U(\eta) - X(\eta)$ converges to some non-Gaussian field and illustrate cases when there is a non converging real sequence $C(\eta)$ when

$$\frac{U(\eta) - X(\eta)}{C(\eta)}$$

goes to a non-Gaussian field.

Similar large noise effects can be described in the case that (3) is replaced by an ordinary SDE driven by large noise. In that case, in dimension 1, we can precise better the triviality effect, by proving a central limit theorem and large deviations estimates.

This presentation includes joint works with S. Albeverio, Z. Haba, M. Oberguggenberger and S. Peszat.

Logarithmic estimates for the density of hypoelliptic two-parameter diffusions

Marta Sanz-Solé

(joint work with A. Kohatsu-Higa and D. Márquez-Carreras)

We consider the stochastic partial differential equation on R^m

$$\frac{\partial^2 X_{s,t}(x)}{\partial s \partial t} = A(X_{s,t}(x))\dot{W}_{s,t} + B(X_{s,t}(x)),$$

whit initial condition $X_{s,t}(x) = x$ if $st = 0$, where W is a d-dimensional Brownian sheet.

Our purpose is to analyse the behaviour of the density of $X_{s,t}(x)$ for $st \neq 0$ as $st \to 0$, that is, the density at "small time", under Hörmander type assumptions on the vector field A. More precisely, we prove that

$$\lim_{\epsilon \to 0} 2\epsilon^2 \log p_\epsilon(x, y) = -d^2(x, y),$$

where $\epsilon = \sqrt{st}$ and the quantity $d^2(x, y)$ is described in terms of the control equation associated with the spde.

This question has been widely studied for diffusion processes. However, in spite of the formal analogy, the problems arising in this two-parameter situation cannot be solved by a straightforward generalization of the arguments used in the one-dimensional case. Under a global coercivity condition, our result has been proved by Léandre and Russo in 1990.

The main difficulties we have encountered in the extension of Léandre and Russo's result are related to the control of the inverse of the Malliavin matrix and the proof of the finiteness of $d^2(x, y)$. The first one has been solved by establishing sharp estimates for the inverse of the Malliavin matrix in "small time". The second one has been approached using Taylor's expansion in time for the control skeleton.

Long-time behaviour and regularity of measure-valued processes

Wilhelm Stannat

We present several results on regularity of transition semigroups of Fleming-Viot (FV) and Dawson-Watanabe (DW) processes with immigration. These processes can be interpreted as diffusion approximations for the random evolution of a given population. Given a Feller operator A on a compact type space S modelling the mutation we will specify conditions on A implying that the semigroup (T_t) generated by the corresponding FV-process with no selection and no recombination (i) converges to equilibrium with exponential rate (moreover, we determine explicit bounds on the rate of convergence in terms of A), (ii) is hyper- and ultra contractive, and (iii) is compact. We give applications of the last result to the existence of invariant measures for FV-operators with interactive selection. We also present similar results on transition semigroups of DW-processes with immigration and general, possibly type-dependent, branching mechanism Ψ, but mutation $A = 0$. Using an explicit construction of corresponding invariant probability measures, we study again the long-time behaviour and hypercontractivity of the corresponding transition semigroups. In particular, we obtain estimates on the exponential rate of convergence towards equilibrium in terms of Ψ which are independent of the immigration.

Necessary and sufficient conditions for the existence of weighted L^p-solutions for evolution equations with spatially homogeneous noise

Jan van Neerven

(joint work with Zdzisław Brzeźniak)

In this talk we study space-time regularity of solutions of the following linear stochastic evolution equation in $\mathcal{S}'(\mathbb{R}^d)$, the space of tempered distributions on $\mathbb{R}^d$:

$$\begin{aligned} du(t) &= Au(t)\,dt + dW(t), \quad t \geq 0, \\ u(0) &= 0. \end{aligned} \tag{1}$$

Here A is a pseudodifferential operator on $\mathcal{S}'(\mathbb{R}^d)$ whose symbol $q : \mathbb{R}^d \to \mathbb{C}$ is symmetric and bounded above, and $\{W(t)\}_{t\geq 0}$ is a spatially homogeneous Wiener process with spectral measure μ. We prove that for any $p \in [1,\infty)$ and any nonnegative weight function $\varrho \in L^1_{\mathrm{loc}}(\mathbb{R}^d)$, the following assertions are equivalent:

1. The problem (1) admits a unique $L^p(\varrho)$-valued solution;
2. The weight ϱ is integrable and
$$\int_{\mathbb{R}^d} \frac{1}{C - \operatorname{Re} q(\xi)}\, d\mu(\xi) < \infty$$
for sufficiently large C.

Under stronger integrability assumptions we prove that the $L^p(\varrho)$-valued solution has a continuous, resp. Hölder continuous version.

A Stochastic Parabolic Obstacle Problem

Lorenzo Zambotti

We study a white-noise driven semilinear stochastic partial differential equation with reflection, introduced by Nualart and Pardoux. The main result is a completely rigorous interpretation of such equation as an infinite-dimensional Skorokhod problem. The relationship between the Nualart-Pardoux equation and the excursion measure, i.e. the 3-d Bessel Bridge, is explored: in particular, a fundamental tool is an integration by parts formula on the excursion measure, where explicit infinite-dimensional boundary terms appear. The connection between the N-P equation and the 3-dimensional stochastic string is investigated.

Rotations and Tangent Processes on Wiener Space

M. Zakai

(joint work with Y. Hu and A.S. Üstunel)

Measure preserving transformations on Wiener space are called 'Rotations'.

If $\{e_i, i = 1, 2, \cdots\}$, is a CONB on H, $R(w)$ is a unitary transform on the Cameron Martin space H satisfying that for all $h \in H$, $Rh \in \mathbb{D}_{2,1}(H)$ and ∇Rh is quasi nil potent, then $\sum_1^\infty \delta(R(w)e_i)e_i$ is a rotation. Conversely if η_i are i.i.d., $N(0,1)$, then the rotation $\Sigma \eta_i e_i$ has a representation as $\Sigma\delta(R(w)e_i)e_i$ where $R(w)$ in unitary and ∇Rh is quasi-nil-potent.

Let A be an operator on H, and for all $F \in \mathbb{D}$, $A\nabla F \in \mathbb{D}_{2,1}$ then $\mathcal{L}_A F$ is defined as

$$\mathcal{L}_A F =: \delta(A\nabla F)\,.$$

$\mathcal{L}_A F$ is said to be a tangent operator and if $A + A^T = 0$ it is a derivation.

Let $A(w)$ be as above, if $\Sigma\delta(Ae_i)e_i = \Sigma\mathcal{L}_A(\delta e_i)e_i$ converges weakly into, say, Y, then Y is said to be the *tangent process* generated by A and denoted $Y = \mathcal{L}_A w$. For example: If W is the d-dimensional classical Wiener process and

$$(A(w)h)_i = \sum_j \int_o^{\cdot} \sigma_{ij}(s, w)h'(s)ds$$

where a is an $n \times n$ antisymmetric matrix adapted to $\mathcal{F}_t$ (for all v), then

$$(\mathcal{L}_A w)_i = \int_0^{\cdot} \sum_j \sigma_{ij}(s, w)dw_s\,.$$

and

$$\mathcal{L}_{e^{tA}} w = \sum_j \int_0^{\cdot} \exp t\sigma_{ij} dw_s\,.$$

The interpretation of $\mathcal{L}_A w$, under $A + A^T = 0$ as a generator for rotations is reflected in the following result:
Let $A_t(w)$ be an antisymmetric operator on H, and satisfy some technical conditions then the equation

$$\frac{dT_t w}{dt} = (\mathcal{L}_{A_t(w)} w) \circ T_t w, \quad T_\circ w = w$$

defines a flow of rotations.

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