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Numerical Integration and its Complexity

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This conference, which was organized by Harald Niederreiter, Knut Petras and Henryk Woźniakowski, brought together specialists in different areas of numerical integration. Main purpose was to examine limits of computability of integrals with respect to different types of difficulties (high dimensionality, unbounded regions etc.). Many of these difficulties come from finance or theoretical physics.

One major topic was small discrepancy, i.e., how to put finitely many points into a (possibly high dimensional) set with as little "gaps" as possible. Many new constructions have been presented. Further constructions of integration rules used polynomial exactness. Special types of integrands (absolute value of functions, convex functions) led to special rules. Particular difficulties arise from unbounded high dimensional domains.

Stochastic methods have been investigated in order to increase possible rates of convergence. New impulses in this direction come from quantum computing. There were also talks on applications in stochastic and finance and on integration in the frame of ill-posed problems.

On the real line, beside classical topics for integrals over bounded intervals, integration on unbounded or several intervals as well as integral transforms or integration of oscillatory functions have been discussed.

Complementary to research talks, surveys covered the different areas in numerical integration. There was also a session with surveys of open problems and research directions.

The conference profited from the exchange of methods that are typically used by groups of specialists.

Abstracts

Interval Gaussian Quadrature Formulae

Borislav Bojanov

In a joint work with Petar Petrov (Sofia University) we show that for every given finite interval $[a, b]$ and a system of numbers $h_1 \geq 0, \ldots, h_n \geq 0$ such that $h_1 + \cdots + h_n \leq b - a$, there exists a unique quadrature formula of the form

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{n} a_k \left(\frac{1}{h_k} \int_{x_k}^{x_k+h_k} f(x)\,dx \right), \tag{1}$$

which is exact for all algebraic polynomials of degree $2n - 1$. This is the highest algebraic degree of precision that can be attained by a formula of form (1).

Bounds for Peano Kernels – a survey

Helmut Brass

Various kinds of error estimations in quadrature theory are dependent on bounds for Peano kernels. Bounds of very general character can be obtained by using some duality theorems. These theorems shift the difficulty to approximation theory. I have compiled theorems from approximation theory useful for these problems.

Main points of my discussion: Norm bounds in C and L_1, behaviour of localized bounds at the boundary, Peano kernels of highest order, general Peano kernels versus Peano kernels of positive quadrature rules, application of the results to error estimation for functions with a special type of singularity.

Explicit Constructions of Point Sets with low L^2-Discrepancy

William W.L. Chen

We discuss first various techniques for obtaining Davenport's theorem on the classical L^2-discrepancy of aligned rectangles. Such techniques include the use of diophantine approximation, Fourier analysis, probability theory and symmetry. We then discuss the recent work of Chen and Skriganov on explicit construction of point sets in the k-dimensional unit cube which satisfy the best possible upper bound estimates for the L^2-discrepancy of aligned rectangular boxes. The new ideas include the use of vector spaces over finite fields, duality, Fourier-Walsh analysis, as well as the use of two metrics on digits.

An Encyclopaedia of Cubature Formulas

Ronald Cools

In this talk I describe a project that I started several years ago and that will go on for several more years. The main aim of the project is to collect all known cubature rules for standard regions and to make the points and weights electronically available to users. Obviously that requires validation and re-computation of all published results.

The current version is available at

`http://www.cs.kuleuven.ac.be/~nines/research/ecf/ecf.html`

Numerical Integration, Energy and Weighted Approximation

Steven Damelin

This talk will discuss recent work of the author, P. Grabner and G. Mullen. We will first discuss the relationship between Numerical integration and Energy functionals on the sphere and show that points on the sphere that minimize certain energy functionals are well distributed in the sense that their error of numerical integration is small. An example of a point system which admits t-designs for some t and good energy estimates is constructed.

Simple Quadrature Rules to Evaluate the Hilbert Transform on the Real Line

Maria Carmela De Bonis

(joint work with Giuseppe Mastroianni)

The Hilbert transform $H(G)$, $G : \mathbb{R} \to \mathbb{R}$, is defined as follows

$$H(G,t) = \int_{\mathbb{R}} \frac{G(x)}{x-t}\,dx = \lim_{\varepsilon \to 0} \int_{|x-t|>\varepsilon} \frac{G(x)}{x-t}\,dx,$$

where $t \in \mathbb{R}$. The authors propose some simple algorithms to compute the Hilbert transform on the real line, using Markov-Sonin zeros. Error estimates are proved and some numerical tests are shown.

Harmonic Blending Approximation

Franz-Jürgen Delvos

The concept of harmonic Hilbert space was introduced by the author as an extension of periodic Hilbert spaces defined by Babuska. We studied approximation by exponential-type functions in these spaces and derived error bounds in the uniform norm for special functions of exponential type which are defined by Fourier partial integrals $S_b(f)$ and related interpolation operators T_b. In this talk, we will investigate more general approximation operators S_ψ and we will use Boolean methods to construct new operators.

Estimation of Quadrature Errors in Terms of Fractional Derivatives

Kai Diethelm

When dealing with the problem of numerical integration on a compact interval, one of the standard error estimates is of the form

$$|R[f]| \leq c_s(R)\|f^{(s)}\|$$

where R is the remainder functional of the quadrature formula in question, $s \in \mathbb{N}$, $c_s(R)$ is some constant that depends on s and R but not on f, and $\|\cdot\|$ is a norm on a suitable set of functions. In this talk we want to generalize these results in such a way that we allow s to be an arbitrary nonnegative real number. We will give answers to a number of questions arising in this context, such as:

- How do we have to interpret the expression $f^{(s)}$ if $s \notin \mathbb{N}$?
- Under what conditions can we expect to obtain such estimates?
- Assuming that we want to look at a sequence $(R_n)_{n=1}^{\infty}$ of quadrature errors (or, respectively, the corresponding quadrature formulas), what can we say about the asymptotic behaviour of $c_s(R_n)$ as $n \to \infty$?

In particular, we shall see that there is more than one answer to the first question, and that the answer to the second question depends on which answer for the first question we choose.

Error Bounds for the Integration of Singular Functions using Equidistributed Sequences

Elise de Doncker

We consider integral approximations using equidistributed point sequences, which converge for Riemann integrable functions. Under certain conditions, Sobol (1973) showed convergence for certain classes of singular functions. We focus on asymptotic error bounds and give a scheme for multivariate extension, thereby obtaining insight in the asymptotic error structure. Numerical examples are presented, validating the principle of "ignoring the singularity".

The Beauty of Discrepancy

Karl Entacher

Using several numerical experiments I will present different insights in the concept of discrepancy. Special graphical illustrations, derived from classical two-dimensional Monte Carlo- and quasi Monte Carlo point sets exhibit the beauty of discrepancy.

Irregular Oscillatory Integration using Generalised Quadrature Methods

Gwynne Evans

Generalised quadrature methods rely on generating quadrature rules for given irregular oscillatory weight functions $w(x)$ commonly belonging to the class $C^n[a,b]$, for some usually small n. If these weight functions are known to satisfy $Lw = 0$ for a differential operator L, then Lagrange's identity

$$wLu - uMw = Z'(u, w)$$

can be used to generate a quadrature rule by forcing exactness for a set of basis functions.

Theorems which give conditions under which the computed quadrature rules will yield machine precision results underpin the practical rule, and finite range integrals with weights such as $\sin(q(x))$ and $J_n(q(x))$ have been successfully integrated, for $q(x) \in C^2[a,b]$. Doubly oscillatory weights also become feasible with weights such as $J_n(q_1(x))J_m(q_2(x))$.

More recent work has considered multiple quadratures and the special problems which arise with the commonly occurring infinite range integrations. In the latter case, the direct approach results in violations of the conditions of the underlying theorem and requires some modification for success.

This approach has enabled several diverse practical problems to be attempted including integrals from financial market predictions, from chemical reactor analysis, from coherent optical imaging and from wave analysis on sloping beaches.

The Complexity of the Computation of Multivariate Normal and Multivariate T Probabilities

Alan Genz

Methods for the numerical computation of multivariate normal and multivariate t probabilities will be reviewed. The focus of the talk is a description of several effective methods for problems that arise in important practical applications, including some analysis of the computational complexity of these methods.

Orthogonal Rational Functions and quadrature formulae on the real half-line

Pablo González-Vera

Qudrature formulas based on rational functions with prescribed poles have become a rapidly interesting topic in the last years as a result of their connection with other fields like continued fractions, multipoint Padé approximants, orthogonal rational functions and so on. In this talk we shall be mainly concerned with the following situation: Let $\alpha = \{\alpha_k\}_{k=1}^{\infty}$ and $\beta = \{\beta_k\}_{k=1}^{\infty}$ be two sequences of negative real numbers satisfying $0 \leq |\alpha_k| \leq M$ and $|\beta_k| \geq N$, M and N being positive constants. Let μ be a positive measure supported on (a, b) with $0 \leq a < b \leq \infty$. Set $I_n(f) = \sum_{j=1}^{n} A_j f(x_j)$ a quadrature formula with distinct nodes on (a, b) so that $I_n(R) = I_\mu(R) = \int_a^b R(x) d\mu(x)$ for any function R in the class

$$\left\{ \frac{P(x)}{(x-\alpha_1)\dots(x-\alpha_p)(1-\frac{x}{\beta_1})\dots(1-\frac{x}{\beta_q})} : \ P \text{ a polynomial of degree } p+q \right\}$$

with p and q nonnegative integers as large as possible. The construction of such these quadratures will be analyzed making use of properties of certain sequences of orthogonal rational functions generalizing well known results concerning orthogonal polynomials.

Quantum Integration in Sobolev Classes

Stefan Heinrich

We study high dimensional integration in the quantum model of computation. We develop quantum algorithms for integration of functions from Sobolev classes $W_p^r([0,1]^d)$ and analyze their convergence rates. We also prove lower bounds which show that the proposed algorithms are, in many cases, optimal within the setting of quantum computing. This extends recent results of E. Novak on integration of functions from Hölder classes $C^{k,\alpha}([0,1]^d)$.

Some Applications of the Spectral Test

Peter Hellekalek

In this talk, we will study the uniform distribution of digital (t, m, s)-nets in base b with respect to a particular measure of uniform distribution, the so-called spectral test. We will prove upper bounds for the general case of arbitrary (t, m, s)-nets and lower bounds for the case of digital nets where the base b is prime. All bounds are best possible. We will also discuss relations of our approach to concepts used in the context of assessing rank-1 lattice rules.

Existence of Extensible Rank-1 Lattice Rules

Fred Hickernell

(joint work with Harald Niederreiter)

Lattice rules are one popular type of quasi-Monte Carlo method for multidimensional quadrature. A weakness of rank-1 lattice rules has been that the s-dimensional generating vector, $\mathbf{h}$, for the node set depends on s and the number of points, n. This talk shows the existence of good rank-1 lattice rules that are extensible in both n and s. For a fixed integer $b \geq 2$, there exists an ∞-dimensional $\mathbf{h}$ of Mahler integers with good P_α for all s and all $n = b, b^2, b^3, \ldots$. The upper bounds on P_α for the extensible lattice rules are only slightly worse than the best known upper bounds on P_α for lattice rules with fixed s and n.

Construction of Lattice Rules achieving Strong Tractability Error Bounds when the number of points is a composite number

Stephen Joe

In some earlier work by Sloan, Kuo, and Joe, rank-1 lattice rules for weighted Korobov spaces of periodic functions and shifted rank-1 lattice rules for weighted Sobolev spaces of non-periodic functions were constructed. Under the assumption that n, the number of quadrature points, was a prime number, analyzes were given which showed that the rules so constructed achieved strong QMC tractability error bounds. Here we extend these earlier results by removing the requirement that n be prime. As in the prime case, the generating vectors and shifts characterizing the rules may be constructed 'component-by-component', that is, the $(d+1)$-th components of the generating vectors and shifts are obtained using 1-dimensional searches, with the previous d components kept unchanged.

Component-by-Component Constructions achieve the Optimal Rate of convergence in weighted Korobov and Sobolev spaces

Frances Kuo

It is known from the analysis by Sloan and Woźniakowski that the optimal rate of convergence for multivariate integration in weighted Korobov spaces is $O(n^{-\alpha/2+\delta})$ where $\alpha > 1$ is some parameter, and the optimal rate for weighted Sobolev spaces is $O(n^{-1+\delta})$. The existing theory behind the component-by-component constructions developed by Sloan, Kuo and Joe indicate that the rules constructed achieve $O(n^{-1/2})$ convergence. Here we present theorems which show that those lattice rules constructed by the component-by-component algorithms in fact achieve the optimal rate of convergence in the corresponding weighted function spaces.

Lattice Rules of moderate Trigonometric Degree

James Lyness

An elementary introduction to Lattices, Integration Lattices and Lattice rules was followed by a description of the role of the Dual Lattice in assessing the trigonometric degree of a lattice rule. The connection with the classical lattice packing problem was established; any s-dimensional cubature rule can be associated with an index

$$\rho(Q) = (\delta)^s / s!N$$

where δ is the enhanced degree of the rule and N its abscissa count. When Q is a lattice rule, this is the packing factor of the associated dual lattice with respect to the unit s-dimensional octahedron.

An individual cubature rule may be represented as a point on a plot of ρ against δ. Several of these plots were presented. They convey a clear impression of the relative cost-effectiveness of various individual rules and sequences of rules.

Numerical Integration on Unbounded Intervals

GIUSEPPE MASTROIANNI

The numerical approximation of integral transforms on unbounded intervals is of interest in different contexts, for instance in the numerical treatment of integral equations. This talk contains a short survey of results in the literature and some new ideas which allows to efficient procedures, whose convergence is optimal.

Numerical Integration using Markov Chains

PETER MATHÉ

Let $X_1, X_2, \dots$ be the successive samples from a Markov chain with initial distribution ν and transitions according to kernel k, having the invariant distribution, say π.

On finite state space, it is well known, that the mean square error of the sample mean $\vartheta_N(f) := \frac{1}{N}\sum_{j=1}^N f(X_j)$ against the true mean $\int f(x)\,\pi(dx)$ converges to 0 at a rate $N^{-1/2}$, independent of the initial distribution, if the chain was ergodic. On general state space, stronger ergodicity assumptions have to be made.

In this talk we discuss the concept of V-uniform ergodicity, as exhibited in Meyn/Tweedy "Markov Chains and Stochastic Stability". Under this assumption we provide analog asymptotic results as in the finite case. In particular, using interpolation type arguments, convergence at a rate $N^{-1/2}$ can be shown, uniformly for classes of functions, which are square integrable with respect to the measure $V\,d\pi$.

Some extensions and refinements are discussed.

Bit representation of band-dominant functions on the sphere

HRUSHIKESH MHASKAR

A band-dominant function on the Euclidean sphere embedded in $\mathbb{R}^{q+1}$ is the restriction to this sphere of an entire function of $q+1$ complex variables having a finite exponential type in each of its variables. We develop a method to represent such a function using finitely many bits, using the values of the function at scattered sites on the sphere. The number of bits required in our representation is asymptotically the same as the metric entropy of the class of such functions with respect to any of the L^p norms on the sphere.

Gauss-type quadrature formulae for Hermite weight function and associated inequalities

GENO NIKOKLOV

We present two results related to Gauss-type quadrature formulae for the Hermite weight function $w(x) = \exp(-x^2)$. First, we show that the associated Christoffel function is bell-shaped. This result fully describes how the weights in a Gauss-type quadrature formula are arranged in magnitude. Our second result is an inequality of Duffin and Schaeffer type in $L_2(w;\mathbb{R})$ norm.

New constructions of digital nets

HARALD NIEDERREITER

Digital nets form a well-known class of low-discrepancy point sets. It is an important problem for many applications, e.g. in computational finance, to construct digital (t,m,s)-nets in very high dimensions s. In this joint work with C.P. Xing (Singapore) we study the asymptotics of digital nets as $s \to \infty$ and provide new constructions of digital nets with good asymptotic behavior. One of the constructions is based on coding theory and yields, for any integer $d \geq 2$, a sequence of binary digital $(t_n, t_n + d, s_n)$-nets with $\lim_{n\to\infty} s_n = \infty$ and

$$\lim_{n\to\infty} \frac{t_n}{\log_2 s_n} = \left\lfloor \frac{d}{2} \right\rfloor .$$

This result is best possible since for any sequence of binary digital $(t_n, t_n + d, s_n)$-nets with $\lim_{n\to\infty} s_n = \infty$ we have

$$\liminf_{n\to\infty} \frac{t_n}{\log_2 s_n} \geq \left\lfloor \frac{d}{2} \right\rfloor .$$

New interpolatory quadrature formulae with Gegenbauer abscissae

SOTIRIOS NOTARIS

We study interpolatory quadrature formulae, relative to the Legendre weight function on $[-1,1]$, having as nodes the zeros of the Gegenbauer polynomial of degree n, n even, plus one of the points 1 or -1. In particular, we establish the convergence or nonconvergence for continuous and Riemann integrable functions on $[-1,1]$, we determine the precise degree of exactness, we obtain asymptotically optimal error bounds, and we examine the definiteness or indefiniteness for these quadrature formulae. In addition, we investigate numerically the question of positivity of the quadrature weights.

How many random bits do we need for Monte Carlo integration?

ERICH NOVAK

(joint work with Stefan Heinrich and Harald Pfeiffer)

To compute an integral

$$I_d(f) = \int_{[0,1]^d} f(x)\,dx$$

up to some error $\varepsilon > 0$, the classical Monte Carlo method needs about $d\varepsilon^{-2}$ random numbers from $[0,1]$. We want to construct Monte Carlo methods that

a) use a small number of function values for a given class F of integrands and

b) use a small number of random bits instead of random numbers. (This kind of randomness can be easily simulated on a quantum computer.)

Result: For many classes F we only need about $d \log \varepsilon^{-1}$ random bits to achieve the optimal rate of convergence. Lower bounds show that this is optimal.

Optimal recovery and best quadratures for Hardy-Sobolev classes

Konstantin Yu. Osipenko

Denote by H^r_∞ the class of analytic in the unit disk D functions f for which $|f^{(r)}(z)| \leq 1$, $z \in D$. Using a general approach for the construction of optimal recovery methods of linear functionals, we obtain optimal recovery methods and best quadrature formulas for Hardy-Sobolev classes H^r_∞. We find a linear space of analytic functions which play the same role as polynomial splines in the similar problem for Sobolev classes.

Sufficient conditions for fast quasi-Monte Carlo convergence

Anargyros Papageorgiou

We study the approximation of d-dimensional integrals. We present conditions for fast quasi-Monte Carlo convergence. Our approach applies to isotropic and non-isotropic problems and, in particular, to a number of problems in computational finance. We show that the convergence rate of quasi-Monte Carlo is of order $n^{-1+p\{\log n\}^{-1/2}}$, $p \geq 0$. Since this is a worst case result, compared to the expected rate $n^{-1/2}$ of Monte Carlo it shows the superiority of quasi-Monte Carlo for this type of integrals.

Numerical integration on nonconnected sets

Franz Peherstorfer

Let $a_1 < a_2 < \ldots < a_{2l}$, $E_j = [a_{2j-1}, a_{2j}]$ and put $E = \bigcup_{j=1}^{l} E_j$. First we study the location and behaviour of the nodes of the Gaussian integration formulas (abbreviated in the following by G-QF) on the set of several intervals E or in other words the zeros of polynomials (p_n) orthogonal on E. Surprisingly even such elementary questions as: how many nodes has the G-QF in the interval E_j, $j \in \{1, \ldots, l\}$, when does there appear a node in a gap $(a_{2j} a_{2j+1})$, are the accumulation points of the nodes dense in the gaps,...; remained open so far.

In this talk we answer the following questions.

First we give the number of nodes in the interval E_j, $j = 1, \ldots, l$. Then criteria are given such that the G-QF has nodes in arbitrarily given gaps and the behaviour of the associated quadrature weights is studied. Since it turns out that the nodes in gaps cause troubles, an alternative numerical integration procedure is suggested.

Finally, the denseness of the nodes of the G-QFs in the gaps $(a_{2j} a_{2j+1})$ is discussed. As a consequence of the above results we obtain that every point from $[a_1, a_{2l}] \setminus E$ is an accumulation point of nodes if the harmonic measures of the intervals are independent over the rationals.

Numerical integration of functions that are the solutions of ill-posed problems

Sergei Pereverzev

(joint work with Peter Mathe and Alex Goldenshluger)

We study the efficiency of the linear-functional strategy, as introduced by Anderssen (1986), for inverse problems with observations blurred by Gaussian white noise with known intensity. The optimal accuracy is presented and it is shown how this can be achieved by linear-functional strategy based on the noisy observations. This optimal linear-functional strategy is obtained from Tikhonov regularization of some dual problem. Moreover, we develop an adaptive estimator that is rate optimal within a logarithmic factor simultaneously over a wide collection of balls in the Hilbert scale.

Investigation of Niederreiter-Xing-nets

Gottlieb Pirsic

In quasi-Monte Carlo numerical integration, digital (t, m, s)-nets are among the best possible node sets, especially for high-dimensional problems. Among them, Niederreiter-Xing nets achieve the best theoretic results, i.e. their quality parameter t is of optimal order. The talk presents (mainly numerical) results on their distribution properties and performance in numerical integration experiments with the Genz test function package.

Complexity of weighted integration of stochastic processes over unbounded domains

Leszek Plaskota
(joint work with K. Ritter and G.W. Wasilkowski)

Let X be a zero mean Gaussian stochastic process defined over $\mathbb{R}^d$ with kernel $K(s,t) = E(X(s)X(t))$, $s,t \in \mathbb{R}^d$. For a given weight $\rho : \mathbb{R}^d \to [0,\infty)$, we study the complexity of approximating the integral

$$\mathrm{Int}_\rho X \;=\; \int_{\mathbb{R}^d} X(t)\rho(t)\,dt$$

by quadratures $Q_n X = \sum_{j=1}^n a_j X(t_j)$. By ε-complexity, comp(ε), we mean the minimal n for with there exists a quadrature Q_n with the expected error

$$E|\mathrm{Int}_\rho X - Q_n X|^2 \;\le\; \varepsilon^2.$$

For Int_ρ to be well defined a.e. it is necessary and sufficient to assume that

$$\int_{\mathbb{R}^d} \rho(t) K^{1/2}(t,t)\,dt < \infty.$$

If this condition is met then $\mathrm{comp}(\varepsilon) = o(\varepsilon^{-2})$.

We also give more specific complexity formulas in the case $d = 1$. For instance, suppose that X is the r-fold Wiener process on $[0,\infty)$. Let the weight ρ be asymptotically non-increasing. If, in addition, $\|\rho^{1/\Gamma}\|_{L_1} := \int_0^\infty \rho^{1/\Gamma}\,dt < \infty$ with $\Gamma = r + 3/2$, then

$$\mathrm{comp}(\varepsilon) \;=\; \Theta\left(\left(\frac{\|\rho^{1/\Gamma}\|_{L_1}^{\Gamma}}{\varepsilon}\right)^{1/(r+1)}\right),$$

For this result, the asymptotic monotonicity of ρ is crucial. On the other hand, if $\|\rho^{1/\gamma}\|_{L_1} = \infty$ then the exponent at $(1/\varepsilon)$ can be arbitrarily close to 2. (Almost) optimal quadratures use a regular sampling.

Convexity results and sharp error bounds for multivariate analogues of the midpoint and the trapezoidal rule

Gerhard Schmeisser

(joint work with Allal Guessab)

Let f be a convex function on an interval $[a, b]$. The inequalities

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{1}{2}(f(a)+f(b)), \qquad (*)$$

also known as the Hermite-Hadamard inequalities, have been a starting point for one-sided approximation of integrals by the midpoint and the trapezoidal rule. Moreover, these inequalities have inspired various investigations, including error estimates under relatively weak regularity conditions on the function f.

We present multivariate analogues of the inequalities $(*)$ and interpret the resulting lower and upper bounds as multivariate analogues of the midpoint and the trapezoidal rule, respectively. We also study convex combinations of these formulae and establish sharp error bounds for Lipschitzian functions and for twice continuously differentiable functions. These bounds are in terms of the Lipschitz constant and the Hessian matrix, respectively.

Minimal Cubature Formulae of an even degree for integrals over the surface of the Torus

Hans Jaochim Schmid

(joint work with M.V. Noskov)

In this talk we derive a characterization of minimal even degree formulae for the 2-torus in the trigonometric case. All such formulae are obtained by solving several matrix equations. As far as we know this is the first approach to determine all formulae of this type. Computational results by using a Computer Algebra System are given. They verify that up to degree 30 there is only one minimal formula of even degree (and its dual), if one node is fixed. In all cases computed it turned out that the known lattice rules of rank 1 are the only minimal formulae.

On the step by step construction of randomly shifted lattice rules in weighted Sobolev spaces

Ian Sloan

In this talk , the second stage of a joint project with F. Kuo and S. Joe (Waikato), describes the step by step construction of randomly shifted lattice rules for an arbitrary number of dimensions, that achieve strong tractability error bounds in weighted Sobolev spaces. The new feature is that the shifts are now chosen randomly, instead of being determined as part of the algorithm. The cost of the algorithm, for obtaining an n-point rule (with n prime) in all dimensions up to d, is thereby reduced from $O(n^3d^2)$ to $O(n^2d^2)$. Moreover, a probabilistic error estimate is available by repeating the calculation of a desired integral with several different randomly selected shifts.

Multivariate integration and related questions

Vladimir N. Temlyakov

A survey on optimal cubature formulas for classes of functions with bounded mixed derivative will be given. A connection between optimal rates of errors of cubature formulas and the discrepancy problem will be discussed. Some results on the r-discrepancy will be presented.

Probabilistic bounds for the discrepancy

Robert Tichy
(joint work with Walter Philipp)

We survey on classical metric discrepancy results and present some recent bounds on the pair correlation of pseudo random number sequences. We focus on probabilistic results and extend recent theorems of Rudnick and Zaharescu. The proofs depend on martingale inequalities, exponential sums and limit theorems for weakly depending random variables.

Tractability of weighted integration and approximation over $\mathbb{R}^d$

Grzegorz Wasilkowski

We study tractability and strong tractability of multivariate approximation and integration in the worst case deterministic setting. Tractability means that the number of functional evaluations needed to compute an ε-approximation of the multivariate problem with d variables is polynomially bounded in ε^{-1} and d. Strong tractability means that this minimal number is bounded independently of d by a polynomial in ε^{-1}. Both problems are considered for certain Sobolev spaces of functions defined over the whole space $\mathbb{R}^d$. These spaces are characterized by a number of parameters: r is the smoothness of functions, $\gamma_{d,k}$ is a space weight which measures the relative importance of the kth variable for d-variate functions, and a weight function ψ that monitors the behavior of the functions at infinity. The approximation and integration problems are defined in a weighted sense with respect to a probability density ω and variances $\sigma_{d,k}$. We find conditions on the weights ω and ψ such that the approximation and integration are well defined. For the approximation problem, we consider two classes of functional evaluations: Λ^{all} consisting of all linear continuous functionals and Λ^{std} consisting of function evaluations. Of course, for integration we only consider Λ^{std}. Under natural assumptions on the weight functions ω and ψ, we prove that strong tractability holds iff $\sup_{d\geq 1}\sum_{k=1}^{d}(\gamma_{d,k}\,\sigma_{d,k}^{2r-1})^b < \infty$, and tractability holds iff $\sup_{d\geq 1}\sum_{k=1}^{d}(\gamma_{d,k}\,\sigma_{d,k}^{2r-1})^b/\ln(d+1) < \infty$. Here b can be any positive number for approximation in Λ^{all}, and $b=1$ for approximation and integration in Λ^{std}.

Tractability of Absolute Value Integration

Henryk Woźniakowski

Motivated by finance computation examples, we consider the absolute value integration problem which is defined as

$$\int_{\mathbb{R}^d} \rho_d(t)|f(t)|\,dt, \qquad \text{where } \rho_d \geq 0,\ \int_{\mathbb{R}^d} \rho_d(t)\,dt = 1,$$

and f belongs to a given class F_d. Hence, we integrate the absolute value of f instead of integrating f as it is done in the classical integration problem. For many examples of F_d, we have $|f| \notin F_d$, and the absolute value integration problem is different from classical integration.

Tractability means that the minimal number n of function values needed to reduce the initial error by a factor ε is bounded by a polynomial in $1/\varepsilon$ and d. It can be studied in the worst case, average case and randomized settings. In the worst case setting, the absolute value integration problem is equivalent to approximation in the L_1-norm, and it is therefore not easier than integration and no harder than approximation in the L_2-norm. Since tractability conditions are the same for integration and approximation in the L_2-norm for many classes F_d, the same conditions are also needed for the absolute value integration problem.

We also study the worst case error of QMC algorithms. By the use of maximal inequalities for stochastic processes we show the error of many QMC is of order $c/\sqrt{n}$, with c independent of d, if the metric ε-entropy of F_d in the sup-norm is bounded by $c_1(1/\varepsilon)^p$ with c_1 independent of d and $p < 2$. The last condition holds, for example, for some weighted Korobov spaces.

Cubature Formula and Common Zeros of Orthogonal Polynomials

Yuan Xu

A cubature formula $Q(f) = \sum_{i=1}^{N} \lambda_i f(x_i)$ has degree n if $\int f\, d\mu = Q(f)$ for $f \in \Pi_n^d$ – polynomials of degree at most n in d variables. We say a cubature formula is generated by a polynomial ideal $I \subset \mathbb{R}[x_1, \ldots, x_d]$ if its set of nodes $\{x_1, \ldots, x_N\}$ is the variety of I. A polynomial p is called an m-orthogonal polynomial if $\int pq\, d\mu = 0$ for all q such that $qp \in \Pi_m^d$. The following characterization of cubature formulae holds: Let I be an ideal generated by a set of $(2n-1)$-orthogonal polynomials. Assume that the variety of I is real and has zero-dimension. If $\operatorname{codim} I = |V|$, then I generates a cubature formula of degree $2n-1$. This gives a characterization of cubature formulae and examples include formulae ranging from product type formulae to those that satisfy Möller's lower bounds.

Edited by Knut Petras

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