Mathematisches Forschungsinstitut Oberwolfach

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Set Theory

January 20th – January 26th, 2002

The meeting was organized by Sy Friedman (Vienna), Ronald Jensen (Berlin), Menachem Magidor (Jerusalem), and Hugh Woodin (Berkeley). Talks were given on current research in various areas, including forcing techniques, applications of descriptive set theory, set theory of the continuum, core model theory, and determinacy. In addition there were many fruitful and stimulating interchanges outside the talks.

We wish to thank the staff of the Mathematisches Forschungsinstitute Oberwolfach for the care and dedication which, in no small measure, contributed to the success of the meeting.

Abstracts

Cardinalities of boldface point-classes

Alessandro Andretta, Torino, Italy
(joint work with Greg Hjorth)

We show that, assuming $\mathtt{AD}+\mathtt{V}=\mathtt{L}(\mathbb{R})$, there are more sets of the form $F \setminus G$ where $F, G \in F_\sigma$, than there are F_σ sets. More precisely, letting $\mathrm{Diff}(\alpha; \Sigma^0_2)$ be the collection of all α-differences of Σ^0_2 sets, we have:

Theorem. Assume $\mathtt{AD}+\mathtt{V}=\mathtt{L}(\mathbb{R})$ and $1 \leq \alpha < \beta < \omega_1$. Then

$$|\mathrm{Diff}(\alpha; \Sigma^0_2)| < |\mathrm{Diff}(\beta; \Sigma^0_2)|.$$

Indestructibility and Strong Compactness

Arthur W. Apter, New York, USA

Starting from a ground model satisfying "$\mathtt{ZFC}+\kappa$ is supercompact", we describe how to force and construct a model in which κ is both the least strongly compact and least strong cardinal and in which κ in addition satisfies certain indestructibility properties. If our ground model contains no Mahlo cardinals above κ, then in the generic extension, κ's strong compactness will be indestructible under κ-directed closed forcing, and κ's strongness will be indestructible under κ-strategically closed forcing. If we drop the restriction that our ground model contains no Mahlo cardinals above κ, then in the generic extension, κ's strong compactness will be indestructible under κ-directed closed forcing which is in addition κ-strategically closed and κ's strongness will be indestructible under κ-strategically closed forcing.

Weakly Dominating Families

Andreas Blass, Michigan, USA

We use the customary symbols for cardinal characterisitics of the continuum: $\mathtt{d}$ = dominating number, $\mathtt{b}$ = unbounding number, $\mathtt{s}$ = splitting number, $\mathtt{r}$ = unsplitting (=refining) number, $\mathtt{g}$ = groupwise density number, $\mathtt{u}$ = minimum character of nontrivial ultrafilters on ω, and $\mathtt{r}_\sigma$ = minimum number of infinite subsets of ω such that no countably many sets split them all. Let $\omega \nearrow \omega$ be the set of non-decreasing maps $\omega \to \omega$. Call $\mathcal{D} \subseteq \omega \nearrow \omega$ *k-dominating* if every $f \in \omega \nearrow \omega$ is eventually majorized by the (pointwise) maximum of k members of $\mathcal{D}$. The continuum hypothesis implies that the notions of k-dominating are different for all k; in contrast, $\mathtt{u} < \mathtt{g}$ implies that they are the same for all $k \geq 2$. If a dominating family is partitioned into $< \mathtt{g}$ pieces, some piece is 2-domination. (Various other results about k-dominating families were briefly mentioned in the talk.)

Most of the following is from the (forthcoming) PhD. thesis of my student, Jason Aubrey. Call $\mathcal{D} \subseteq \omega \nearrow \omega$ *pseudo-dominating (ψdom)* if

$$\begin{gathered}\forall f \in \omega \nearrow \omega \quad \exists \text{ a partition } \Pi \text{ of } \omega \text{ into finite intervals} \\ \forall\ 2- \text{ colorings of blocks of } \Pi \\ \exists g \in \mathcal{D} \quad \text{On one of the colors, } g \text{ eventually majorizes } f.\end{gathered}$$

$k - \psi$dom is similar, using $\max\{g_1, \ldots, g_k\}$ in place of g. If a dominating family is partitioned into $< \mathtt{s}$ pieces, one of the pieces is $2 - \psi$dom. (There need not be a k-dominating piece for any k.) If $\mathcal{D}$ is $k - \psi$dom but not l-dom for any l, then there is a Borel map $(\omega \nearrow \omega) \to \mathcal{P}\omega$ sending $\mathcal{D}$ to a subbase for a nontrivial ultrafilter, in particular, $\mathtt{u} \leq |\mathcal{D}|$. (The last two results lead to a new proof of Mildenberger's theorem that $\mathtt{s} \leq \mathrm{cf}(\mathtt{d})$.) If $\mathcal{R} \subseteq [\omega]^\omega$ is unsplittable, then $\{$ next $(R, -)$: $R \in \mathcal{R}\}$ is $2 - \psi$dom, where next(R,n) means the smallest number $\geq n$ that is in R. Consequences include: If $\mathtt{r} < \mathtt{d}$ then $\mathtt{r} = \mathtt{u} = \mathtt{r}_\sigma$.

Recent Iteration Techniques

Jörg Brendle, Kobe, Japan

Most iterated forcing constructions adjoining real numbers are – like e.g. finite support iterations of ccc forcing or countable support iteration of proper forcing – of the form $\langle \mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha \mid \alpha < \kappa \rangle$ where $\mathbb{P}_\alpha$ is the *initial segment* of the iteration while $\dot{\mathbb{Q}}_\alpha$ is the iterand with which we force at stage α after having forced with $\mathbb{P}_\alpha$. Typical features of such an iteration include

(1) the initial segments are produced *recursively* (i.e. the underlying order structure is well founded)
(2) the initial segments are linearly ordered
(3) iterands are handed down simultaneously with the initial segments (along the same structure).

While (1) seems to be crucial in many situations (e.g. when it comes to adjoining dominating reals, by a theorem of Hjorth), there is no apriori necessity for (2) and (3), and indeed, techniques which do not satisfy these properties have been considered recently, in particular

(A) iterations along templates (Shelah)
(B) shattered iterations.

We focused on the second technique (which provides a way to adjoin many random reals together with other kinds of reals) and gave a brief outline of the consistency proof of "$\kappa =$ cov $(\mathcal{M})$ and $\lambda =$ cov $(\mathcal{N})$" where $\aleph_2 \leq \kappa < \lambda$ are regular cardinals and cov $(\mathcal{M})$ (cov $(\mathcal{N})$, resp.) denotes the size of the smallest covering of the real line by meager (null, resp.) sets. The latter consistency proof involves the *amalgamated limit*, a new limit construction for systems of complete Boolean algebras satisfying some additional properties which encompasses both the amalgamation (of two algebras over a common subalgebra) and the direct limit.

Classification problems in continua theory

Riccardo Camerlo, Torino, Italy

The study of the complexity of some classes of continua is undertaken using descriptive set theory. In particular:

(-) For a graph P, P-like continua form a G_δ set;
(-) For a dendrite D with finitely many branch points, the homeomorphism class $[D]$ is Π^0_3-complete;
(-) The homeomorphism class of the Warsaw circle is $D_2(\Sigma^0_3)$-hard;
(-) The class of graphs and the class of trees are both $D_2(\Sigma^0_3)$-complete.

Canonical functions, non-regular ultrafilers and Ulam's problem on ω_1

Oliver Deiser, Munich, Germany
(joint work with Dieter Donder)

We show the following theorems; using Jensen's core model $\mathtt{K}_{\mu_0}$ for measures of order zero:

$(D1) \geq_{\text{con}}\ ILM =$ "there is an inaccessible limit of measurables"
$(UP), (NR) \geq_{\text{con}}\ SLM =$ "there is an inaccessible stationary limit of measurables".

Here $A \geq_{\text{con}} B$ means that Con $(\mathtt{ZFC} + A)$ implies Con $(\mathtt{ZFC} + B)$. $(D1)$, (UP), (NR) are the following combinatorial principles on ω_1:

$(D1)$ = "every function $f : \omega_1 \to \omega_1$ is dominated by a canonical function h_ν, $\nu < \omega_2$, on a club subset of ω_1"

(UP) = Ulam Property = "there is a sequence $\langle F_\alpha \mid \alpha < \omega_1 \rangle$ of ω_1-complete uniform filters on ω_1 s.t. each subset of ω_1 is measurable w.r.t. one F_α (i.e. $\mathcal{P}(\omega_1) = \bigcup_{\alpha<\omega_1} F_\alpha \cup I(F_\alpha)$, where $I(F)$ denotes the dual ideal of a filter F.)"

(NR) = "there is a non-regular ultrafilter on ω_1, i.e. there is a uniform ultrafilter U on ω_1 s.t. for every $\langle X_\alpha \mid \alpha < \omega_1 \rangle \subseteq U$ there is an infinite $A \subseteq \omega_2$ s.t.$\bigcap_{\alpha \in A} X_\alpha \neq \emptyset$."

Remark: Actually $(D1) =_{\text{con}}\ ILM$ holds; Larson and Shelah have shown recently that $ILM \geq_{\text{con}}\ (D1) + (CH)$.

Wild colourings of graphs

Mirna Dzamonja, Norwich, UK
(joint work with Peter Komjath, Charles Morgan)

I presented a theorem from a joint paper with Peter Komjath and Charles Morgan, where we consider the following situation for κ singular with $\mathrm{cf}(\kappa) = \aleph_0$:

Suppose X is a κ^+-chromatic graph on κ^+ and ask the question: Is there an edge colouring $f : E(X) \longrightarrow \kappa^+$ such that for every vertex colouring $g : X \longrightarrow \kappa$, there is a g-colour class on which f assumes every value?

We proved that it is consistent, modulo the consistency of a supercompact cardinal, that such a colouring f exists for every such graph X, for some strong limit singular κ with $\mathrm{cf}(\kappa) = \aleph_0$. In this model 2^κ can be as large as wished.

This work continues the similar results of Hajnal and Komjath for κ regular, and uses an iteration technique by the author and Shelah.

The Club Guessing Filter

Matt Foreman, Irvine, USA

I discuss results surrounding club guessing filters. Ishiu showed that these can be precipitous and in L the filter coincides with the closed unbounded filter. In joint work with Komjath, we show how to force a sequence where the club guessing filter is the closed unbounded filter and in an involved result the consistency that the club guessing filter is saturated.

Genericity and Woodin Cardinals

Sy Friedman, Vienna, Austria

I discussed the proof of (and difficulties surrounding) the following result: If V is "L-like" then there is a real R which is class-generic but not set-generic over V and preserves Woodin cardinals. The proof requires use of a special type of witness to Woodinness.

Actions of the Unitary Group

Su Gao, Denton, USA

In the first part of this talk we consider the classification problem for bounded linear operators up to unitary equivalence, where these operators are over a separable infinite-dimensional complex Hilbert space. We prove that this equivalence relation is Borel, using some (but not too much) descriptive set theory. One can interpret this result as saying that there is still hope for some kind of Spectral Theory to work for all bounded linear operators.

In the second part of the talk we mention in passing the result that no separable Banach space actions (where the space is viewed as an Abelian group under $+$) can generate equivalence relations not Borel reducible to any orbit equivalence relations generated by an action of the unitary group.

Wider gaps from weaker assumptions

Moti Gitik, Tel Aviv, Israel

We discussed the following:

Theorem Suppose that κ is a cardinal of countable cofinality in the core model and $\forall n < \omega \quad \{\alpha < \kappa \mid o(\alpha) = \alpha^{+n}\}$ is unbounded in κ. Then for every λ there is a cardinal preserving extension in which $2^\kappa \geq \lambda$ and κ is a strong limit.

By previous results of W. Mitchell and the author the above provides the equiconsistency.

Clones

Martin Goldstern, Vienna, Austria

For a finite nonempty set X let $\mathcal{O}_X = \bigcup_{n=1}^{\infty} {}^{X^n}X$. A subset $\mathcal{C} \subseteq \mathcal{O}_X$ is called a *clone* if $\mathcal{C}$ contains all projections $\pi_i^n : X^n \to X$ and is closed under composition.

Cl (X), the set of all clones (on X) is a complete algebraic lattice. Questions about the structure of Cl (X) are often of set-theoretic nature; if X is infinite.

Sample results:

(1) A clone $\mathcal{C} \subsetneqq \mathcal{O}_X$ is called precomplete if $\mathcal{C}$ is a coatom in Cl (X). [Rosenberg, 1976] There are $2^{2^{|X|}}$ many precomplete clones if $|X| \geq \aleph_0$.

(2) In fact, there is a clone $\mathcal{C}$ such that the interval $[\mathcal{C}, \mathcal{O}_X]$ is order isomorphic to the lattice of all filters on X; precomplete clones correspond to ultrafilters.

(3) Let $\mathcal{O}^{(1)} = \{f \circ \pi_i^n \mid f \quad \text{unary} \;, i \leq n < \omega\}$ be the clone of all essentially unary functions. The following are due to the author and Shelah, 1999:
   (a) If $|X|$ is weakly compact, then $[\mathcal{O}^{(1)}, \mathcal{O}]$ has exactly two precomplete clones
   (b) If $|X|$ satisfies a strong negative partition property, then there are $2^{2^{|X|}}$ many precomplete clones above $\mathcal{O}^{(1)}$.

Open Question: Is every clone $\mathcal{C} \subsetneqq \mathcal{O}_X$ below a precomplete clone?

The Automorphism Tower of a Group

Joel David Hamkins, New York, USA

The automorphism tower of a group is obtained by computing its automorphism group, the automorphism group of that group, and so on, iterating transfinitely, by taking the natural direct limit at limit stages. The question, known as the automorphism tower problem, is whether the tower ever terminates, whether there is eventually a fixed point, a group that is isomorphic to its automorphism group by the natural map. Wielandt (1939) proved the classical result that the automorphism tower of any finite centerless group terminates in finitely many steps. This was generalized to successively larger collections of groups until Thomas (1985) proved that every centerless group has a terminating automorphism tower. Here, it it proved that *every* group has a terminating automorphism tower. After this, an overview is given of the author's (1997) result with Thomas revealing the set-theoretic essence of the automorphism tower of a group: The very same group can have wildly different towers in different models of set theory.

Another Proof of the Strong Partition Relation on ω_1

Stephen Jackson, University of North Texas, USA

We assume AD throughout. We give here a new proof of the strong partition relation on ω_1. This proof is in some sense a derivative of the proof in [1], which is currently the only proof known to generalize to the higher odd projective ordinals (and a ways beyond). The idea is to find a proof that does not depend so much on the complete inductive analysis, and thus might be applicable to cardinals past the point to which the current inductive theory applies. This proof is perhaps a step in that direction. It develops a coding of the functions on ω_1 using not the full analysis of measures on ω_1 (as the proof in [1]), but rather only the first step in that analysis, which is a general step not involving the combinatorics at κ. Thus, this proof uses only the first trivial step in the analysis of the measure, together with a "cheap" coding for the rest of the analysis. While this seems promising, it is not clear at the moment if these ideas can be used to provide a new proof of the strong partition on δ^1_3 (say starting from a Δ^1_3 coding of the subsets of ω_ω; it may be unreasonable to expect a general proof starting from less). Finding a way to "decouple" the proofs of the strong partition relation from the detailed analysis below would have many interesting consequences.

References

[1] S. Jackson, "A new proof of the strong partition relation on ω_1," Trans. Amer. Math. Soc. 320 (1990) 737-745

[2] S. Jackson, "Structural consequences of AD," to appear in the *Handbook of Set Theory,*M. Foreman, A. Kanamori, M. Magidor eds.

Homogeneously Souslin Sets and Mitchell Models

Peter Köpke, Bonn, Germany

I showed that in "small" models of set theory every homogeneously Souslin set is Π^1_1:

Theorem 1 If V is a Mitchell model $L(\mathcal{U})$ for some coherent sequence of measures and $\mathcal{U} \neq \emptyset$, then Π^1_1 is exactly the class of homogeneously Souslin sets.

Theorem 2 If there is a measurable cardinal and $\neg 0^{\text{long}}$ (= there is no mouse $J_\alpha[\mathcal{U}]$ with $\text{otp}(\mathcal{U}) \geq \min \text{dom}\mathcal{U}$), then the same conclusion holds.

I employed the following techniques:

- A set X is homogeneously Souslin iff it has a $2^{\aleph_0}$-closed Embedding Normal Form (ENF) $(M_s)_{s\in\omega^{<\omega}}, (\pi_{s,t})_{s\subseteq t\in\omega^{<\omega}}$: $x \in X$ iff the branch $(M_x \restriction m)_{m<\omega}$, $(\pi_{x\restriction m,x\restriction n})_{m\leq n<\omega}$ has a well founded limit (K. + Windßus).
- Under the hypothesis of the Theorems, the class of possible inner models M_s and elementary embeddings of an ENF can be described by finite pieces of information. Wellfoundedness along a branch of the ENF can be decided by a Π^1_1-formula in the finite pieces of information.
- Theorem 2 also uses the covering theorem for short sequences of measures.

Variations of Compactness and Large Cardinals

Menachem Kojman, Be'er Sheva, Israel

A topological space X is compact iff $\forall A \subseteq X$ there is a complete accumulation point. If one requires the same only for $|A| = \kappa > \aleph_0$, κ regular, then X is linearly Lindelöf. It is shown that if one requires that $\forall |A| = \kappa = \text{cf}(\kappa) > \aleph_0 \quad \kappa < w(X)$, A has a converging subset of the same cardinality, then such X exists unless there are Woodin cardinals in inner models. The proof uses good PCF scales.

Canonical Equivalence Relations in FIN_k

Jordi López-Abad, Paris, France

It is well known that the Banach space cannot be norm-distorted (James). Indeed, W.T. Gowers showed that every Lipschitz map from the unit sphere of c_0 to $\mathbb{R}$ must be almost constant in the unit sphere of some infinite dimensional subspace of c_0. The proof uses a discretization, $FIN_k^{\pm}$ of the unit sphere of c_0, which is a natural generalization of FIN, the set of finite subsets of $\mathbb{N}$. $FIN_k^{\pm}$ has Ramsey-like properties from which one can deduce the result about Lipschitz maps.

Our project consists in trying to understand the distortion of norms of a (infinite dimensional) Banach space in terms of some discrete structure of the unit sphere of X, in particular for the Tsirelson space. The first step is the case of c_0. We give a generalization of the results of A. Taylor about canonical equivalence relations on FIN, and we obtain the canonical list for FIN_k.

Borel Orders

Alain Louveau, Paris, France

I presented various old and new results about Borel orders, and especially the structure of the Borel reducibility ordering on Borel orders. The two main new results were:

(-) a dichotomy result for when a Borel order admits a Borel linearization

(-) a dichotomy result for when a Borel order with no perfect antichains is essentially closed.

Denser free subsets

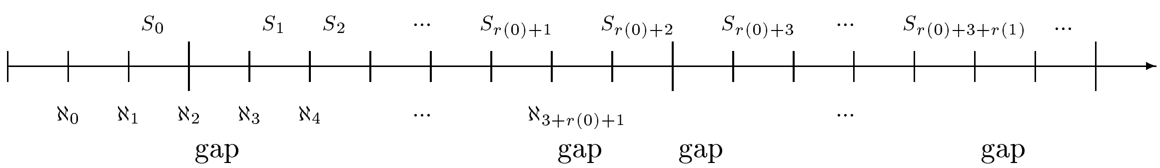
Heike Mildenberger, Vienna, Austria

We write Fr $(\aleph_\omega, \omega)$ if every structure $\mathcal{A}$ on $\aleph_\omega$ with countable signature has an infinite free subset S. S is free in $\mathcal{A}$ iff $\forall s \in S \quad s \notin [S \setminus \{s\}]^{\mathcal{A}}$. We discuss the possible locations of the members of free subsets on the line $\aleph_\omega$. We show that sets of indiscernibles for suitable functions give rise to free subsets.

Theorem Suppose there are ω compact cardinals. Let $r : \omega \to \omega$. Then the following is consistent:

$$\begin{gathered} \forall f : [\aleph_\omega]^{<\omega} \to \aleph_\omega \text{ regressive} \quad \exists \langle S_n \mid n \in \omega \rangle \text{ s.t.} \\ \forall k \in \omega \; \forall i_0 < \cdots < i_k \forall \alpha_0, \alpha_0' \in S_{i_0} \forall \alpha_1, \alpha_1' \in S_{i_1} \ldots \forall \alpha_k, \alpha_k' \in S_{i_k} \\ f(\alpha_0, \ldots, \alpha_k) = f(\alpha_0', \ldots, \alpha_k'), \end{gathered}$$

and such that the S_n's have fairly large cardinality and lie in $\aleph_\omega$ as follows:



A Hypothesis Related to the Mahlo Property OR "That's not what I wanted to know"

William Mitchell, Gainesville, USA

I prove the following theorem, answering a question of Zapletal.

For sequences $\vec{A} = \langle A_\xi \mid \xi < \omega_2 \rangle$ with each $|A_\xi| = \omega_1$, write

$$B_{\vec{A}} = \{\nu < \omega_2 \mid \exists D \subseteq \nu \quad (\text{otp}(D) = \omega_1 \wedge \bigcup D = \nu \wedge \forall \beta < \nu \quad D \cap \beta \in A_\beta)\}.$$

Set

$$(*) \quad \forall \vec{A} \quad B_{\vec{A}} \text{ is non stationary.}$$

Note that (CH) implies $\exists \vec{A} \quad B_{\vec{A}}$ contains a club.

Theorem

$$\mathrm{Con}(*) \iff \mathrm{Con}(\exists\kappa \quad \kappa \text{ is } \kappa^+ - \text{ Mahlo }).$$

The proof of the forward direction involves a slight modification of the forcing for no special Aronszajn trees, followed by forcing to add κ^+ many club sets. The first stage makes $B_{\vec{A}}$ costationary for all $\vec{A}$, and the second makes it non stationary for all $\vec{A}$.

The subtitle is the reaction of Zapletal and of Cummings when told of this result. What Cummings wants to know is the following modification on $(*)$:

For $\vec{a} = \langle a_\xi \mid \xi < \omega_2 \rangle$ let $B'_{\vec{a}} = \{\nu < \omega_2 \mid \exists D \subseteq \nu \quad (\mathrm{otp}(D) = \omega_1 \wedge \bigcup D = \nu \wedge \forall \beta < \nu \quad D \cap \beta \in \{a_\xi \mid \xi < \nu\}\}$

Question: Is it consistent that $B'_{\vec{a}}$ is non stationary for all $\vec{a}$?

Remark: The statement of the problem in this abstract is a correction of that in the talk. The error was pointed out to the author by Magidor.

α-semiproper Iterations

Tadatoshi Miyamoto, Nagoya, Japan

1. Proper, semiproper, α-proper and α-semiproper preorders are characterized by preservation of analogs of stationary and semistationary sets.
2. We have preservation theorems on these notions of forcing, e.g. α-semiproper.

All of these are originated from Shelah. We give an account.

Lemma For $\mathbb{P}$ a preorder and $\alpha < \omega_1$, the following are equivalent:

1. $\mathbb{P}$ is α-proper (α-semiproper)
2. $\mathbb{P}$ preserves every (A, α)-stationary ((A, α)-semistationary) set, for every infinite set A.

Lemma Let $\langle \mathbb{P}_\alpha \mid \alpha \leq \nu \rangle$ be a *simple* iteration of ζ-semiproper preorders, i.e. $\forall \gamma < \nu \quad \Vdash_{\mathbb{P}_\gamma}$ "$\mathbb{P}_{\gamma,\gamma+1}$ is ζ-semiproper" . Then $\forall \gamma \leq \nu \quad \Vdash_{\mathbb{P}_\gamma}$ "$\mathbb{P}_{\gamma,\nu}$ is ζ-semiproper" . In particular, $\mathbb{P}_\nu$ is ζ-semiproper.

Here, $I = \langle \mathbb{P}_\gamma \mid \gamma \leq \nu \rangle$ is a simple iteration, if I gets constructed recursively as usual by taking the *simple* limit at every limit stage.

The simple limit, denoted by Smp (I) for $I = \langle \mathbb{P}_\gamma \rangle \mid \gamma < \nu$ with ν limit is defined by

$$\begin{aligned}
\mathrm{Smp}\,(I) \;=\; \{p \in I^* := \text{ the inverse limit } \mid \exists \langle \dot{\alpha}_n \mid n < \omega \rangle \text{ inverse limit names s.t.} \\
(1) \quad \Vdash_{I^*} \text{“ } \dot{\alpha}_n \leq \dot{\alpha}_{n+1} \leq \nu \text{ ”} \\
(2) \quad \text{If } x \Vdash_{I^*} \text{“ } \dot{\alpha}_n = \check{\eta} \text{ ”, then } x|\eta^\frown \mathbb{1} \Vdash_{I^*} \text{“ } \dot{\alpha}_n = \check{\eta} \text{ ”} \\
(3) \quad p \Vdash_{I^*} \text{“ } \dot{\alpha}_n < \check{\nu} \text{ ”} \\
(4) \quad \Vdash_{I^*} \text{“ } p \restriction \sup \langle \dot{\alpha}_n \mid n \in \omega \rangle \in \dot{G}_{I^*} \restriction \sup \langle \dot{\alpha}_n \mid n \in \omega \rangle \\
\longrightarrow p \in \dot{G}_{I^*} \text{ ” } \}
\end{aligned}$$

Hence, the simple iterations enjoy “the kind richness” in conditions. They are beautiful!

Games of length ω_1

Itay Neeman, Los Angeles, USA

We discuss determinacy at the level of games of length ω_1. For starters we have:

Theorem Suppose there is an iterable inner model with indiscernible Woodin cardinals. Then definable *open* length ω_1 games are determined.

Let $W^{\#}$ denote the assumption that there is an iterable inner model with indiscernible Woodin cardinals. Let M be the minimal class iterable model with a cub class of indiscernible Woodin cardinals. Open games of length ω_1 suffice to recover the Σ_1 theory of M, much as length ω games with Π^1_1 payoff suffice to recover the Σ_1 theory of $\mathtt{L}$. Drawing on the analogy with $\mathtt{L}$ we are lead to search for games which recover the full theory of indiscernibles for M, in much the same way that length ω games with $< \omega^2 - \Pi^1_1$ payoff recover $0^{\#}$. We describe such games in the talk. These games have length ω_1. Their determinacy follows from $W^{\#}$. The games are stronger than open games of length ω_1. They are strong enough that the associated game quantifiers recover the full theory of indiscernibles in M.

Mutual Stationarity in $\mathtt{L}[E]$

Ralf Schindler, Vienna, Austria

Let $A \neq \emptyset$ be a set of uncountable regular cardinals. Let $\vec{S} = \langle S_\kappa \mid \kappa \in A\rangle$ be such that $S_\kappa \subseteq \kappa$ for all $\kappa \in A$. The sequence $\vec{S}$ is called mutually stationary if for all regular $\theta \geq \sup(A)$ and for all models $\mathcal{M} = \langle H_\theta; \in, \ldots\rangle$ of finite type there is some $\mathcal{N} = \langle X; \in\restriction X, \ldots\rangle \prec \mathcal{M}$ such that $\kappa \in X \cap A \longrightarrow \sup(X \cap \kappa) \in S_\kappa$. We generalize a theorem of Foreman and Magidor's by showing the following

Theorem Suppose that $0^{\P}$ ("zero hand grenade") doesn't exist, and suppose that $\mathtt{V} = \mathtt{K}$ where $\mathtt{K}$ denotes the core model. There is then a sequence $\langle S^n_\kappa \mid n < \omega,\ \kappa$ an uncountable successor cardinal$\rangle$ such that the following holds:

- $S^n_\kappa \subseteq \mathrm{cf}(\omega_1)$ for all n, κ
- S^n_κ is a stationary subset of κ for all n, κ, and
- for all limit cardinals λ and for all $f : \lambda \longrightarrow \omega$, $\langle S^{f(\kappa)}_\kappa \mid \kappa$ is an uncountable successor cardinal $< \lambda\rangle$ is mutually stationary if and only if there is a partition $A_1, \ldots, A_m$ of the uncountable successor cardinals $< \lambda$ such that $\sup(A_i) \leq \min(A_{i+1})$ and $|f``A_i| = 1$ for all i.

Haar null sets in product groups

Slawomir Solecki, Urbana, USA

The talk discussed properties of Haar null sets in groups which are infinite products of countable groups. The notion of Haar nullness is a generalization of the notion of being of Haar measure zero to all Polish groups. Answering some questions of Mycielski, I showed that there exists a solvable Polish group G with an invariant metric for which there exist closed Haar positive sets $A, B \subseteq G$ with $1 \notin \mathrm{int}\,(AA^{-1})$ and with B having a left transversal (= a Borel probability measure μ such that $\forall g \in G \quad \mu(gB) = 0$). This indicates that properties of Haar null sets on non-Abelian groups are rather different from the properties of such sets in Abelian groups. Furthermore, I discussed how prevalent the above phenomenon is among product groups.

Long Ehrenfeucht-Fraïssé games and forcing

Boban Velickovic, Paris, France

Given two structures $\mathcal{A}, \mathcal{B}$ in the same 1^{st} order language L, we study the E-F game of length ω_1 between $\mathcal{A}$ and $\mathcal{B}$. Two players, $\forall$ and $\exists$, pick elements of $\mathcal{A} \cup \mathcal{B}$ s.t. if $\forall$ picks an element of $\mathcal{A}$, then $\exists$ picks an element of $\mathcal{B}$ and vice versa. $\exists$ wins if she produces a partial isomorphism.

$\exists$ has a winning strategy if there is a σ-closed forcing notion $\mathbb{P}$ which makes $\mathcal{A}$ and $\mathcal{B}$ isomorphic. We ask: When is this equivalent to the existence of a σ-closed set of partial isomorphisms which has the back and forth property? This is simply a positional winning strategy for $\exists$.

We show that these two notions are not equivalent and raise some open questions.

Illfounded Iterations

Jindrich Zapletal, Gainesville, USA

Theorem (ZFC+LC) Every suitably definable strongly proper forcing can be iterated along an arbitrary linear ordering.

Here $\langle \mathbb{P}, \leq, \dot{r}_{gen} \rangle$ is suitably definable if $\mathbb{P}$ is a set of reals and all sets mentioned are projective, $\dot{r}_{gen}$ is a $\mathbb{P}$-name for a real and there is a projective formula $\varphi(\cdot, \cdot)$ such that

$$\Vdash_{\mathbb{P}} \dot{G} = \{p \in \check{\mathbb{P}} \mid \varphi(p, \dot{r}_{gen})\}.$$

$\langle \mathbb{P}, \leq \rangle$ is strongly proper if $\forall M \prec H_\theta$ countable $\forall p \in M \cap \mathbb{P}\ \forall \{D_n \mid n \in \omega\} \quad D_n \subseteq M \cap P$ open dense $\exists q \leq p \quad \forall n \quad D_n$ is predense below q.

An iteration of a poset $\mathbb{P}$ along a linear order L is a poset $\mathbb{Q}$ adding reals $\langle r_i \mid i \in L \rangle$ such that r_i is $\mathbb{P}^{\mathtt{V}[r_j \mid j<i]}$-generic over the model $\mathtt{V}[r_j \mid j < i]$.

The iteration I produce coincides with Kanovei's notion of iteration in the case of Sacks forcing.

Stationary Reflection in $\mathtt{L}[E]$

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We show that in $\mathtt{L}[E]$, every stationary subset of κ^+ has a stationary subset which can reflect only at points from $\mathcal{O}(\kappa) = \{\nu < \kappa^+ \mid E_\nu \neq \emptyset\}$. Moreover, $\mathcal{O}(\kappa)$ can reflect only at points from $\mathcal{O}(\kappa)$. This has the consequence that stationary reflection at κ^+ implies the existence of many subcompact cardinals $< \kappa$ that are superstrong. This shows that the consistency strength of stationary reflection at κ^+ is larger than that of the failure of square and indicates that it should be possible to force $\neg\square_\mu$ starting from a subcompact cardinal.

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