Mathematisches Forschungsinstitut Oberwolfach

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Miniworkshop: Stochastische Prozesse in zufälligen Medien

May 19th – May 25th, 2002

The mini-workshop was organized by Nina Gantert (Karlsruhe), Achim Klenke (Köln), Matthias Löwe (Nijmegen), and Peter Mörters (Bath).

The workshop focused on stochastic processes in random media. In thirteen lectures the fifteen participants learned about new developments in this area. Moreover a series of problems were introduced. Some of them were studied more intensively in four problem sessions.

The twelve talks presented considered problems on stochastic processes in random media, both, in a continuous and a discrete setting. Since one of the main goals was to initiate new collaboration among the participants of the workshop, the speakers were asked to devote a considerable amount of their time (90 to 120 minutes in total) to the presentation of promising open problems. This opportunity was taken by most of the speakers. Four of the problems posed were discussed in greater detail in four problem sessions (of 6-8 participants) and it is hoped that this activity – as well as the large number of informal discussions – will result in new, interesting projects.

The abstracts of the talks are listed below in alphabetical order. The inspiring atmosphere and the excellent working conditions (including the rich library) at Oberwolfach made this meeting very fruitful.

Abstracts

On branching particle systems in random environments – diffusion limits and long-time behaviour

Matthias Birkner (Frankfurt) and Anja Sturm (Berlin)

Consider a particle system in discrete time. At each time step the particles undergo a branch event, subsequently they migrate in their state space E. The offspring distribution of the particles is determined by a random field on E which is i.i.d. at different time steps but may be spatially correlated.

For a particular class of branching mechanisms we consider convergence of the empirical measure of the process to a diffusion limit in the annealed case (averaging over the environment). Depending on the choice of E the limit may be described by an infinite system of coupled SDEs or by SPDEs. In particular, starting with Brownian particles one obtains the stochastic heat equation with multiplicative coloured noise terms in the limit.

In the second part of this talk we ask questions about the long-time behaviour of the particle system itself. For simplicity we focus on a countable state space with offspring distribution i.i.d. in time and space points. For E finite it is well known that – unlike in a non-random environment – it is not the mean number of particles (averaged over the probability space and environment) which decides long-time survival.

For $E = \mathbb{Z}^d$ we look at a rather restricted class of random offspring laws which is inspired by the so-called "coupled branching process" of A. Greven: Each individual independently has K *potential* offspring, where K has a fixed distribution, who take a step from the position of their ancestor according to some random walk kernel p. Each potential child of an individual at site $x \in \mathbb{Z}^d$ survives with probability $U(n,x)$, where $U(n,x), n \in \mathbb{N}, x \in \mathbb{Z}^d$ are i.i.d. $[0,1]$-valued. Only the surviving children form the next generation. This model has the advantage that one can first consider the forest of potential family trees which is then subject to random pruning.

Calculations with a caricature where the trees are replaced by independent random walk paths suggest the following conjecture: Start initially with one individual per site and let the branching be critical in the classical sense that $\mathbb{E}U \, \mathbb{E}K = 1$ (in particular the particle intensity is constant for all times). Then the population becomes locally extinct iff

$$\mathbb{E}_{(0,0)} \left[\left((\mathbb{E}K)^2 \mathbb{E}(U^2) \right)^{\#\{i \,:\, X_i = \bar{X}_i\}} | \bar{X} \right] < \infty \quad \text{a.s.}$$

where $X, \bar{X}$ are two independent p-random walks. Otherwise it converges to a non-trivial equilibrium. This equilibrium will have finite variance iff the left hand side in the above display remains finite when averaged over $\bar{X}$.

Solutions of $\Delta u = 4u^2$ with Neumann's conditions using the Brownian snake.

JEAN-FRANÇOIS DELMAS (MARNE LA VALLÉE)
(joint work with Romain Abraham (Paris))

We consider a Brownian snake $(W_s, s \geq 0)$ with underlying process a reflected Brownian motion in a bounded domain D. We construct a continuous additive functional $(L_s, s \geq 0)$ of the Brownian snake which counts the time spent by the end points $\hat{W}_s$ of the Brownian snake paths on ∂D. The random measure $Z = \int \delta_{\hat{W}_s} dL_s$ is supported by ∂D. Then we represent the solution v of $\Delta u = 4u^2$ in D with weak Neumann boundary condition $\varphi \geq 0$ by using exponential moment of (Z, φ) under the excursion measure of the Brownian snake. We also show Z is absolutely continuous with respect to the surface measure on ∂D for dimension 2 and 3. In particular Z is more regular than the so-called exit measure of the Brownian snake of D.

Improving performance of third generation mobile phone systems

REMCO VAN DER HOFSTADT (EINDHOVEN)
(joint work with Marten Klok)

In the coming years, the wireless communication system UMTS will be introduced in Europe and Japan. This is the third generation wireless system which uses a technique called Code Division Multiple Access (CDMA). Instead of users transmitting in different time slots, in CDMA the signals are coded and decoded to reduce the interference from other users.

In this talk I will explain the basics of the mathematical model behind CDMA, and explain why CDMA has greater potential than the currently used GSM system. Then I will go into ways to improve performance for these systems using an iterative estimation scheme.

I will also introduce several open problems related to this work. These among other things involve different systems and extensions of our results to more realistic systems.

This is joint work with Marten Klok.

Random Walk, and Directed Polymers in Random Environment

YUEYUN HU (PARIS)
(joint work with Ph. Carmona and Z. Shi)

This talk is based on some joint works with Ph. Carmona and with Z. Shi. In the first part, we discuss the directed polymers model introduced by Imbrie and Spencer (1988) in the Gaussian random environment case: Let (S_n) be a simple random walk on $\mathbb{Z}^d$ and let $(g(i,x), i \geq 1, x \in \mathbb{Z}^d)$ be a sequence of i.i.d. standard Gaussian variables. Fix $\beta > 0$ and consider the polymer measure $\langle \cdot \rangle^{(n)}$ defined by

$$\langle F(S) \rangle^{(n)} = \frac{1}{Z_n(\beta)} E_S \Big(e^{\beta \sum_1^n g(i, S_i)} F(S) \Big),$$

where E_S denotes the expectation with respect to the random walk S. We are interested in the asymptotic behaviours of $\langle \cdot \rangle^{(n)}$ as $n \to \infty$. We obtain

Theorem $(d = 1, 2)$ For any $\beta > 0$, there exists some constant $c(d, \beta) > 0$ such that

$$\limsup_{n\to\infty} \max_x \langle \mathbf{1}_{(S_n=x)} \rangle^{(n)} \geq c(d,\beta), \qquad \text{a.s..}$$

In the second part, we discuss the one-dimensional random walk in random environment, and in particular the recurrent case whose rate of renormalisation was obtained by Sinai (1982) using the method of valley. We present some asymptotic results as well as some questions about Sinai's walk. We have shown that

Theorem Let (X_n) be Sinai's walk. We have

$$\limsup_{n\to\infty} \frac{X_n}{(\log n)^2 \log\log\log n} = \frac{8}{\pi^2\sigma^2}, \qquad \text{a.s..}$$

$$\liminf_{n\to\infty} \frac{\log\log\log n}{(\log n)^2} \max_{0\le k\le n} |X_k| = \frac{1}{\sigma^2}, \qquad \text{a.s..}$$

Rescaled Interacting Diffusions converge to Super Brownian Motion

Achim Klenke (Köln)
(joint work with Ted Cox)

Super Brownian motion is known to occur as the limit of properly rescaled interacting particle systems such as branching random walk, the contact process and the voter model.

Here we show that certain linearly interacting diffusions converge to super Brownian motion if suitably rescaled in time and space. The results comprise nearest neighbour interaction as well as long range interaction.

The parabolic Anderson model and intersection of random paths

Wolfgang König (Berlin)

We consider the heat equation with random potential on $(0, = \infty) \times \mathbb{Z}^d$, i.e., $\partial_t u(t,x) = \Delta u(t,x) + u(t,x)\xi(x)$, $u(0,\cdot) = \delta_0(\cdot)$, which is usually referred to as the parabolic Anderson model. Here $\xi = (\xi(x))_{x\in\mathbb{Z}^d}$ is a random i.i.d. potential having all positive exponential moments finite. The model appears in the description of population dynamics or chemical reactions, e.g.

We survey the derivation of the large-t asymptotics of the expectation of the total mass $\sum_x u(t,x)$ for two important special cases: the double-exponential distribution investigated by Gärtner and Molchanov (1998) and the case of fields bounded from above considered by Biskup and the speaker. An interesting open project is the investigation of the large-t asymptotics for the class of fields "in between" these two classes (including a proper formulation what this should mean).

If the random field ξ is replaced by the product $\xi\ell$, where $\ell(x) = \int_0^t ds\, 1\{Y_s = x\}$ denote the local times of an independent simple random walk Y on $\mathbb{Z}^d$ for $d > 2$, then the study of the model naturally leads to the question about large-deviation principles for the shape of the intersection local times of several independent simple random walks. We survey (almost finished) existing work in the Brownian case of this question (joint with Mörters, Bath) and formulate and conjecture such principles for the discrete case.

Title of the talk: Brownian motion in scaled Gibbsian potentials

Franz Merkl (Bielefeld)

The talk describes the almost sure infinite volume asymptotics of the ground state energy of random Schroedinger operators with scaled Gibbsian potentials. The random potential is obtained by distributing soft obstacles according to an infinite volume grand canonical tempered Gibbs measure with a superstable pair interaction. There is no restriction on the strength of the pair interaction: it may be taken, e.g., at a critical point. The potential is scaled with the box size in a critical way, i.e. the scale is determined by the typical size of large deviations in the Gibbsian cloud. The almost sure infinite volume asymptotics of the ground state energy is described in terms of two equivalent deterministic variational principles involving only thermodynamic quantities. The qualitative behaviour of the ground state energy asymptotics is analyzed: Depending on the dimension and on the critical exponents of the free energy density, it is identified which cases lead to a phase transition of the asymptotic behaviour of the ground state energy.

Thin rays in Galton-Watson trees and a problem on intersections of Brownian paths

Peter Mörters (Bath)
(joint work with N.R. Shieh (Taipeh))

In a supercritical Galton-Watson tree the size of the nth generation behaves asymptotically like Mm^n, where $m > 1$ is the mean offspring number and M a random variable characterizing the thickness of the tree. A ray in a Galton-Watson tree is thin if infinitely many subtrees along the ray have an exceptionally small value of M. In the talk I show that the extent of thinness one can expect is qualitatively different for two types of offspring distributions and discuss the Hausdorff dimension of the set of thin rays in both cases. This result is a simplified model for the problem of thin points of Brownian paths and intersections of Brownian paths, and I explain the possible implications on the latter problem, which is open. The first part of the talk is based on joint work with N.R. Shieh (Taipeh).

Reinforced random walks: Results and open problems

Silke Rolles (Bielefeld)

Reinforced random walk was introduced by Coppersmith and Diaconis in 1987 as follows: Let G be a locally finite graph. All edges are given weights; initially all weights are equal to one. Reinforced random walk is a nearest-neighbour random walk on G, where in each step an edge is traversed with probability proportional to its weight. The weights change in time; each time an edge is traversed, its weight is increased by 1.

Diaconis asked whether reinforced random walk on $mathbbZ^d$ is recurrent or transient. This fundamental question, posed 15 years ago, seems still open for $d \geq 2$. In the case of $\mathbb{Z}$ it is known that reinforced random walk is positive recurrent.

A seemingly simpler model is once-reinforced random walk, where the weight of an edge is increased by δ the first time the edge is traversed, but the weight does not change from the second traversal on. Also for this model, the question of recurrence and transience seems open for $\mathbb{Z}^d$, $d \geq 2$.

Another model is directed-edge-reinforced random walk. In this model, the underlying graph is directed and the weights of the directed edges increase linearly. Directed-edge-reinforced random walk has the same law as a random walk in random environment. This equality in law was used to prove recurrence on $\mathbb{Z} \times G'$ for any finite directed graph G'. For $\mathbb{Z}^d$, $d \geq 2$, the recurrence question seems also open.

Vertex-reinforced random walks

Pierre Tarres (Lausanne)

Vertex-Reinforced Random Walks, defined by Pemantle (1988), is a random process in a continuously changing environment, which is more likely to visit states it has visited before.

Pemantle and Volkov (1997) have proved that, when the underlying graph is $\mathbb{Z}$, the random walk eventually gets stuck at a finite set of vertices and, with positive probability, at a set of five vertices. They conjectured that this second event holds with probability one. We have recently proved this conjecture.

The purpose of this talk is to introduce to some tools and models in the area of random walks with reinforcement (urn models, dynamical systems approach), and to give an outline of our proof of the conjecture of Pemantle and Volkov.

OK Corral and the battle of Trafalgar: a probabilistic model

Stas Volkov

(joint work with J.F.C. Kingman)

I will talk about the OK Corral gunfight model formulated by Williams and McIlroy (1998) and later studied by J.F.C. Kingman (1999). Recently Kingman (2002) discovered that a deterministic version of this model was applied by a British engineer and mathematician F.W. Lanchester at least half a century ago to the battle of Trafalgar.

For example, Admiral Lord Nelson , had fewer ships at Trafalgar than the combined French and Spanish fleets. If he did battle in the conventional manner, he would be defeated.

I will describe the probabilistic model and demonstrate recent sharp results obtained by Kingman and myself. In this is a model of the famous gunfight, two lines of gunmen face each other, there being initially N on each side. At every shot, a person is killed. The chances are higher that a killed person is on the side which has less gunmen. The shooting continues till there are no gunmen on one of the sides. For S, the number of survivals, we compute the exact p.m.f., as well as (very accurate!) asymptotical one as N goes to infinity. Note that the right scaling for S is $N^{3/4}$ (computed in the 1998 paper)

The connection between this model and Friedman's urn, Rubin's construction, (incidentally quite useful for studies of reinforced random walks) will be shown.

Finite system scheme for historical interacting Fisher-Wright diffusions and questions on spatially marked coalescents

Anita Winter (Erlangen)
(joint work with Andreas Greven and Vlada Limic)

For any fixed time a particle representation for the historical interacting Fisher-Wright diffusions is provided on the same probability space by means of a look-down process.

The latter allows for defining genealogies. Based on these we give a representation of the equilibrium historical measure, and describe the behaviour of large finite systems in comparison with the infinite system on the level of the historical processes.

In the second part we state questions on the asymptotic behaviour of spatially marked coalescents which are dual to the interacting Fisher-Wright diffusions in a strong sense.

The talk is based on joint work with Andreas Greven and Vlada Limic.

A functional central limit theorem for the stationary branching random walk

Iljana Zähle (Erlangen)
(joint work with Matthias Birkner and Anton Wakolbinger (Frankfurt))

We study fluctuations of the occupation time of the stationary branching random walk $(\xi_t)_{t\geq 0}$ on $\mathbb{Z}^d$, $d \geq 3$. A functional central limit theorem for $\int_0^{Nt} \xi_s(0)\, ds$ is proven. For $d = 3$ the norming function $N^{-3/4}$ leads to convergence to fractional Brownian motion, while for $d = 4$ and $d \geq 5$ the norming functions $(N \log N)^{-1}$ resp. $N^{-1/2}$ yield convergence to Brownian motion. The methods are based on recent work by Jankowski, Quastel and Sheriff. In the case $d \geq 4$ the main idea is to decompose the centered occupation time into a martingale and a remainder term. For the martingale part we apply a central limit theorem for martingales by Rebolledo, which ensures convergence to Brownian motion. The remainder term is shown to be asymptotic negligible. In the case $d = 3$ we decompose the centered occupation time into two terms, where one is negligible and the other converge to a Gaussian process. Calculating the asymptotic covariance determines the limiting process uniquely.

This is joint work with Matthias Birkner and Anton Wakolbinger (Frankfurt), which is still in progress.

Edited by Matthias Löwe

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