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# Reelle Analysis

July 14th – July 20th, 2002

The organizers of this workshop were Detlef Müller (Kiel), Elias M. Stein (Princeton) and Hans Triebel (Jena).

43 other mathematicians participated and gave 28 talks. Their abstracts are listed in this report in the order the talks were given. Additionally, spontaneous meetings took place, where new developments were discussed. The work at this conference was mostly devoted to recent developments in several topics of harmonic analysis as well as in the theory of function spaces and their interplay.

We thank the "Mathematisches Forschungsinstitut Oberwolfach" for making this conference possible.

# Abstracts

### The Cauchy problem for fully non-linear Schrödinger equations

Carlos Kenig

In this talk I described recent work (joint with G. Ponce, C. Rolvung and L. Vega) on the well-posedness of the Cauchy problem, for short time, with data in Sobolev spaces in $\mathbb{R}^n$, intersected with weighted $L^2$ spaces, with power weights. Our result says that, under suitable ellipticity and asymptotic flatness assumptions, for data which generates (in a suitable way) metrics which are close to "non-trapping" ones, this well-posedness holds for any $n \geq 1$.

### Interpolation theory and compact operators

Fernando Cobos

In 1960, Krasnosel'skii proved that if $T$ is a linear operator which satisfies the hypothesis of the Riesz-Thorin theorem, that is, $T: L_{p_0} \to L_{q_0}$ and $T: L_{p_1} \to L_{q_1}$ are bounded, where $1 \leq p_0, q_0, p_1, q_1 \leq \infty$, and if, in addition, $q_0 < \infty$ and $T: L_{p_0} \to L_{q_0}$ is compact, then $T: L_p \to L_q$ is compact, where $1/p = (1-\theta)/p_0 + \theta/p_1$ , $1/q = (1-\theta)/q_0 + \theta/q_1$ and $\theta$ is any number such that $0 < \theta < 1$. At the beginning of the sixties with the foundation of abstract interpolation theory, this result led to the investigation of interpolation properties of compact operators between general Banach spaces. The first partial results were published in 1964 by Lions and Peetre and by Calderón, in their famous papers about the real interpolation method and the complex method, respectively. Many authors have worked on this subject since then, and still a lot of work is being done along different directions. As it was shown by Cwikel in 1992, compact operators can be interpolated by the real method. However, a similar result for the complex method is not yet known.

The aim of this talk is to survey old and new results on this subject, as well as some of the tools for their proofs which are intimately related to the structure of the interpolation method under study.

### Maximal operators related to the Ornstein-Uhlenbeck semigroup with complex time parameter

Giancarlo Mauceri and Peter Sjögren

In our two talks we have reported on two papers, one of which is joint work also involving J. Garcia-Cuerva, S. Meda & J.L. Torrea.

Let $\gamma$ be the Gaussian measure on $\mathbb{R}^d$ and $\{\mathcal{H}_t : t \geq 0\}$ the Ornstein-Uhlenbeck semigroup on $(\mathbb{R}^d, \gamma)$, whose generator is $-\frac{1}{2}\triangle + x \cdot \nabla$. For each $p$ in $[1, \infty)$ let $\mathbb{E}_p \subset \mathbb{C}$ be the closure of the region of holomorphy of $\{\mathcal{H}_t : t \geq 0\}$ on $L^p(\gamma)$. We examine the boundedness on $L^q(\gamma)$ of the maximal operator

$$\mathcal{H}_p^* f(x) = \sup_{z \in \mathbb{E}_p} |\mathcal{H}_z f(x)|.$$

We prove that, for $1 < p < 2$, $\mathcal{H}_p^*$ is of strong type $q$ for $p < q < p'$ and of weak type $p'$. However, $\mathcal{H}_p^*$ is not of weak type $p$ and not of strong type $p'$.

Here the strong type $p'$ estimate fails because of the behaviour of the semigroup $\mathcal{H}_z$ for $z$ near the subset $i\pi\mathbb{Z}$ of $\partial\mathbb{E}_p$. Indeed, if one modifies the definition of $\mathcal{H}_p^*$ by deleting from $\mathbb{E}_p$ an $\varepsilon$-neighbourhood of $i\pi\mathbb{Z}$, the resulting operator is of strong type $p'$.

A similar statement holds for the weak type $p$ estimate, with another discrete subset of $\partial\mathbb{E}_p$.

## Oscillatory integral operators with degenerate phases

Allan Greenleaf

(joint work with Andreas Seeger)

Consider an oscillatory integral operator

$$T_\lambda f(x) = \int_{\mathbb{R}^d} e^{i\lambda\Phi(x,y)} a(x,y) f(y) dy,$$

with phase $\Phi \in C^\infty_{\mathbb{R}}(\mathbb{R}^d \times \mathbb{R}^d)$ and amplitude $a \in C_0^\infty(\mathbb{R}^d \times \mathbb{R}^d)$, and in particular the decay properties of $\|T_\lambda\|_{L^2\to L^2}$ under various geometric assumptions on $\Phi$. Let $\pi_L(x,y) = (x, \nabla_x\Phi(x,y))$ and $\pi_R(x,y) = (y, -\nabla_y\Phi(x,y))$ be the projections to the left and right from the associated canonical relation, which we assume drop rank by at most one everywhere.

**Theorem**

(i). If both $\pi_L$ and $\pi_R$ have at most simple cusp ($S_{1,1,0}$) singularities, or more generally are of finite type $\le 2$, then

$$\|T_\lambda\|_{L^2\to L^2} \le c\lambda^{-\frac{d-1}{2}-\frac{1}{4}}.$$

(ii). If one of $\pi_L$ or $\pi_R$ has at most swallowtail ($S_{1,1,1,0}$) singularities, or more generally is of finite type $\le 3$, then

$$\|T_\lambda\|_{L^2\to L^2} \le c\lambda^{-\frac{d-1}{2}-\frac{1}{8}}.$$

Part (i) sharpens and extends a result of A. Comech and S. Cuccagna. As an application, consider averaging operators $A_j f(x) = \int_{\mathbb{R}} f(x-\gamma_j(t))\chi(t)dt$, $\chi \in C_0^\infty$, with $\gamma_1(t) = (t, t^2, t^4)$ and $\gamma_2(t) = (t, t^3, t^4)$ in $\mathbb{R}^3$ and $\gamma_3(t) = (t, t^2, t^3, t^4)$ in $\mathbb{R}^4$, so that $A_j\colon L^2 \to L^2_{\frac{1}{4}}$ by van der Corput. Then, averages over arbitrary smooth (non translation-invariant) perturbations of the translation-invariant families $\{x - \gamma_j(t)\}_{x\in\mathbb{R}^d}$ satisfy the same estimate.

## Maximal functions on the discrete Heisenberg group

Stephen Wainger

We discussed a recent Theorem of A. Magyar, E. M. Stein and myself concerning a maximal function on the discrete Heisenberg group, $\mathcal{H}^d$. As a set $\mathcal{H}^d = \{h | h = (m,k) = (m_1, m_2, k)\}$ with $m_j \in \mathbb{Z}^d$ and $k \in \mathbb{Z}$. We introduce a multiplication by setting $(m,k)\cdot(n,l) = (m+n, k+l+m_2\cdot n_1)$. For $f$ defined on $\mathcal{H}^d$ and $(m,k)$ in $\mathcal{H}^d$, put

$$M_N f(m,k) = \frac{1}{(2N+1)^{2d}} \sum f((n,0)\cdot(m,k)).$$

Then we have the following result.

**Theorem:**(A. Magyar, E.M. Stein, S. Wainger)

$$\|\sup_N |M_n f|\|_{\ell^2(\mathcal{H}^d)} \le A(d)\|f\|_{\ell^2(\mathcal{H}^d)}.$$

Applications to Ergodic theory are given.

## Envelopes in function spaces

DOROTHEE D. HAROSKE

We present our recently developed concept of envelopes in function spaces – a relatively simple tool for the study of spaces, say, of Sobolev type $H^s_p$, or Besov type $B^s_{p,q}$. Arising from the famous Sobolev embedding theorem it is, for instance, well-known that $B^{n/p}_{p,q} \hookrightarrow L_\infty$ if, and only if, $0 < p < \infty$, $0 < q \leq 1$; a natural question thus is in what sense the unboundedness of functions belonging to $H^s_p$ with $1 < p < \infty$, and $B^s_{p,q}$ with $1 < q \leq \infty$, respectively, can be qualified. Concentrating on this particular feature we introduce the concept of growth envelope functions $\mathcal{E}^X_{\mathsf{G}} = \sup\limits_{\|f|X\| \leq 1} f^*(t)$, $t > 0$, 'measuring' the unboundedness of functions belonging to some function space $X \subset L^1_{\mathrm{loc}}$ by means of the non-increasing rearrangement $f^*(t)$. Surprisingly enough one finds rather simple and final answers characterising spaces like $B^s_{p,q}$ and $H^s_p$; in fact, the results contain an even finer description of this feature than measured by $\mathcal{E}^X_{\mathsf{G}}$ merely. It turns out that in rearrangement-invariant spaces there is a connection between $\mathcal{E}^X_{\mathsf{G}}$ and the fundamental function $\varphi_X$; we derive further properties and give some examples : One verifies for the Lorentz spaces $\mathfrak{E}_{\mathsf{G}}(L_{pq}) = (t^{-1/p}, q)$, where $\mathfrak{E}_{\mathsf{G}}(X)$ is the so-called *growth envelope* of a space $X$. More interesting, however, are the results for $B^s_{p,q}$ or $H^s_p$.
Likewise we investigate limiting situations when questions of (un)boundedness of functions are replaced by enquiries about (almost) Lipschitz continuity; for $X \hookrightarrow C$ it makes sense to replace $f^*(t)$ by $\frac{\omega(f,t)}{t}$, where $\omega(f,t)$ is the modulus of continuity. Now the *continuity envelope function* $\mathcal{E}^X_{\mathsf{C}}$ and the continuity envelope $\mathfrak{E}_{\mathsf{C}}$ are introduced completely parallel to $\mathcal{E}^X_{\mathsf{G}}$ and $\mathfrak{E}_{\mathsf{G}}$, respectively, and similar questions are studied.
Apart from natural applications to inequalities, these sharp assertions imply not only new (and so far final) results on unboundedness and Lipschitz-continuity; besides there are also interesting connections between growth and continuity envelopes and lift operators, as well as with related problems of compactness and, say, approximation numbers.

## The $\overline{\partial}_b$-complex on decoupled domains

ALEXANDER J. NAGEL

This is a report on joint work with E.M. Stein. Our objective is to study the relative fundamental solutions for the operator $\square_b = \overline{\partial}_b\overline{\partial}_b^* + \overline{\partial}_b^*\overline{\partial}_b$ on domains $\Omega \subset \mathbb{C}^{n+1}$ of the form $\{(z_1, \ldots, z_n, z_{n+1}) \in \mathbb{C}^{n+1} | \mathrm{Im}(z_{n+1}) > \sum_{j=1}^n P_j(z_j)\}$ where each $P_j$ is a subharmonic, non-harmonic polynomial. We show that the singularities and regularity properties of the solutions involve different phenomena that arise in the cases of strictly pseudoconvex domains, domains of finite type in $\mathbb{C}^2$, or domains in which the eigenvalues of the Levi form degenerate at comparable rates. Instead of being variants of Calderón-Zygmund singular integral operators or fractional integral operators, the relative fundamental solution in the decoupled case is best viewed as a "quotient" of product type operators. This helps to explain the failure of maximal subelliptic estimates.

The following is an example of the kind of result we obtain: if $\Omega \subset \mathbb{C}^3$ is decoupled and if $\{\bar{z}_1, \bar{z}_2\}$ is the standard basis for the tangential $(0,1)$ vector fields, consider the operator $\square_b = z_1\overline{z}_1 + z_2\overline{z}_2$. Let $S$ denote the orthogonal projections from $L^2(\partial\Omega)$ to the null space of $\square_b$.

**Theorem:** There is a relative fundamental solution $K$ so that $\square_b K = K\square_b = \mathrm{I} - S$. Also

(1) The operators $Z_1\bar{Z}_1K$ and $Z_2\bar{Z}_2K$ are bounded on $L^p(\partial\Omega)$, $1 < p < \infty$.
(2) The operators $\bar{Z}_1Z_1K$ and $\bar{Z}_2Z_2K$ need not be bounded on $L^2(\partial\Omega)$.
(3) If $B_1$ and $B_2$ are smooth functions on $\partial\Omega$ and if $|B_1(p)|\triangle P_1(p) \leq C\triangle P_2(p)$ and $|B_2(p)|\triangle P_2(p) \leq C\triangle P_1(p)$ then $[B_1\bar{Z}_1Z_1 + B_2\bar{Z}_2Z_2]K$ is bounded on $L^p(\partial\Omega)$ for $1 < p < \infty$.

## A restriction theorem for twisted sub-Laplacians

MICHAEL KEMPE

For $n \in \mathbb{N}$ consider the $2n+1$-dimensional Heisenberg Group $\mathbb{H}_n$. Its Lie algebra is spanned by vector fields $P_j, Q_j$ and $U$ ($j = 1, \dots, n$) fulfilling the canonical relations $[P_j, Q_j] = U$. By a Fourier transform in the central variable of $\mathbb{H}_n$, the convolution on $L^1(\mathbb{H}_n)$ induces a non-commutative convolution product on $L^1(\mathbb{R}^{2n})$, the so-called twisted convolution. To $P_j, Q_j$ there correspond vector fields $\widetilde{P_j}, \widetilde{Q_j}$ on $\mathbb{R}^{2n}$ (and these are given by twisted convolution with a certain kernel).

We consider differential operators given by $L_S := -\langle Z, SZ\rangle$ where $S$ denotes a real symmetric $2n \times 2n$-matrix and $Z = (\widetilde{Q}_1, \dots, \widetilde{Q}_n, \widetilde{P}_1, \dots, \widetilde{P}_n)$. $L_S$ is called a twisted sub-Laplacian, if $S$ is positive definite. Thangavelu proved a "restriction theorem" for the special case $L := L_{\mathrm{Id}}$, namely

$$\|\mathbf{1}_{[\lambda,\lambda+1]}(L)\|_{L^p\to L^2} \leq C\lambda^{\beta(p)},$$

where $\beta(p) := n\big(\frac{1}{p} - \frac{1}{2}\big) - \frac{1}{2}$ and $1 \leq p \leq 2\frac{2n}{2n+2}$. Although the exponent $\beta(p)$ is always optimal, the range of $p$ for which the above inequality holds can be improved to $1 \leq p < 2\frac{2n+1}{2n+3}$, and it also holds for all $L_S$ instead of $L$, if $S$ is positive definite. This result is analog to the well-known restriction theorem for the Fourier transform by Tomas and Stein.

As usual it can also be used to obtain better convergence of the corresponding Riesz means.

## Old and new results on BMO($\mathbb{R}^n$)

GERARD BOURDAUD

We consider the following subspaces of BMO($\mathbb{R}^n$): VMO $= \overline{\mathrm{UC} \cap \mathrm{BMO}}$, CMO $= \overline{C_0}$, $\mathrm{BMO}_0 = \overline{\mathrm{BMO}_c}$ ($\mathrm{BMO}_c$ is the set of compactly supported functions in BMO). We also consider the corresponding subspaces of bmo($\mathbb{R}^n$).

1. We give various characterizations of these spaces. For instance, we have the following properties:

(1) $\mathrm{BMO}_0 = R((L_0^\infty)^{n+1})$
(2) $\mathrm{CMO} = R((C_0)^{n+1})$

(Here $R = (\mathrm{Id}, R_1, \dots, R_n)$, where $R_j$ are the Riesz transforms; and $L_0^\infty$ is the closure of $L_c^\infty$ in $L^\infty$.)

Assertion (1) is likely new. Assertion (2) was claimed in the 70's, but the proof seems to have not been published. The two proofs rely upon a "kind of" $L^\infty \to$ BMO continuity of the commutator $[v, R]$ for $v \in$ BMO.

2. With the help of Jones, Iwaniec, Tchamitchian and Russ we point out some inexactitudes on CMO and VMO which appeared in the recent literature. The following assertions turn to be FALSE:

(3) VMO($\mathbb{R}^n$) is the closure of BUC($\mathbb{R}^n$)

(4) for any $f \in \mathrm{CMO}(\mathbb{R}^n)$, the following limit does exist:

$$\lim_{t\to 0} \frac{1}{|B(0,t)|} \int_{B(0,t)} f(x)dx.$$

3. In collaboration with Sickel and Lanza de Cristoforis, we study the functional calculus in the above subspaces $X$ of BMO. That is: what functions $f$ define a superposition operator $Tf(g) := f \circ g$ on $X$; for what $f$ is $Tf$ continuous? differentiable? in the $\mathrm{BMO}(\mathbb{R}^n)$. Our results complete the former by Fominykh and Chevalier. For instance, we have the following "degeneracy" result:

If $f$ is not an affine function, then:

(i) $Tf$ is not continuous on VMO or on bmo.

(ii) $Tf$ is not differentiable on $\mathcal{D}(\mathbb{R}^n)$ endowed with the $\mathrm{bmo}(\mathbb{R}^n)$.

Papers appeared or to appear:

- Functional calculus on BMO and related spaces (with Sickel and Lanza de C.) J.F.A. 2002
- Remarques sur certains sous-espaces de $\mathrm{BMO}(\mathbb{R}^n)$ et $\mathrm{bmo}(\mathbb{R}^n)$ Ann. Institut Fourier 2002.

## Singular integrals on exponential growth groups

Waldemar Hebisch

The talk extends our earlier joint work with Tim Steger. We propose a simple abstract version of Calderón-Zygmund theory, which is applicable to spaces with exponential volume growth. In particular, we have an analog of Calderón-Zygmund decomposition on all amenable Lie groups.

We apply theory to the Riesz transforms on amenable Lie groups: Let $G = U \times \mathbb{R} \ltimes N$, where $N$ is $\mathbb{R}^Q$ or $\mathbb{C}^Q$, $U$ is a subgroup of the orthogonal (unitary) group on $N$ and the multiplication is given by $(u_1, a_1, n_1)(u_2, a_2, n_2) = (u_1u_2, a_1 + a_2, u_2e^{sa_2}n_1 + n_2)$, $s$ being a scalar. Let $X_1, \ldots, X_n$ be right invariant vector fields on $G$. Put $L = -\sum_{j=1}^n X_j^2$.

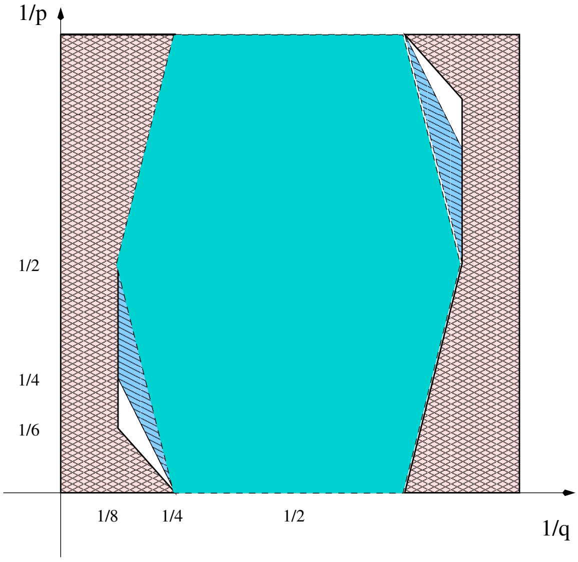
Then the operators $R_j = X_jL^{-\frac{1}{2}}$ are bounded on $L^p(G)$, $1 < p \leq 2$ and of weak type $(1,1)$. We have also boundedness for $p > 2$, but then we use stronger assumptions: $U$ trivial, $L$ of special form (coming from a symmetric space). Also under the strengthened assumptions, if $X_1$ corresponds to the derivative with respect to $a$, then $R_1^* = L^{-\frac{1}{2}}X_1$ is not of weak type $(1,1)$ (the other $R_j^*$ are of weak type $(1,1)$).

## Sharp estimates for the boundedness of Bergman projectors

Gustavo Garrigós

Let $\mathcal{D} = R^n + i\Omega$ be the tube domain over the light-cone $\Omega \subset R^n$. Let $P\colon L^2(\mathcal{D}) \to A^2(\mathcal{D})$ be the Bergman projector, and consider the mixed norm Lebesgue space $L^{p,q}(\mathcal{D}) = L^q(\Omega; L^p(R^n))$. We study the following problem: *Given* $p \in (1,\infty)$*, find the sharp range of* $q \in (1,\infty)$ *such that* $P$ *admits a bounded extension from* $L^{p,q}$ *into* $A^{p,q}$. When $n = 3$, the best known answer to this question is given in the following drawing:

It illustrates the regions of boundedness, unboundedness, and the open gap where for the moment no answer is known. Boundedness in the hexagonal region was shown in a paper by Békollé, Bonami, Peloso and Ricci from 1998, using the Plancherel theorem and a suitable discretization of the multiplier to obtain a sharp result for $p = 2$. In this talk we present sharp results for $2 \leq p \leq 4$, obtained by the author in collaboration with Békollé,




Bonami and Ricci. The techniques this time are based on a Littlewod-Paley decomposition adapted to the geometry of the cone, and the use of almost orthogonality results related to the Bochner-Riesz multiplier in 2 dimensions. We also present some necessary and sufficient conditions for the cases $4 < p \leq 6$, in terms of inequalities of Bochner-Riesz type. This research has been partially supported by the European Commission, *TMR Network "Harmonic Analysis: 1998-2002"*.

**The Bergman projection on Siegel domains over polyhedral cones**

FULVIO RICCI

(joint work with P. Ciatti)

For a convex proper open cone $\Gamma$ in $\mathbb{R}^n$, let $D_\Gamma = \mathbb{R}^n + i\Gamma$ be the associated tube domain in $\mathbb{C}^n$.

Given a Hermitian form $\Phi\colon \mathbb{C}^m \times \mathbb{C}^m \to \mathbb{C}^n$ that is $\Gamma$-positive, let also

$$D_{\Gamma,\Phi} = \{(z,w)\colon \mathrm{Im} w - \Phi(z,z) \in \Gamma\} \subset \mathbb{C}^{m+n}$$

be the associated type II Siegel domain.

For each of these domains, the Bergman space $A^p(D)$ is the space of $L^p$-integrable holomorphic functions on $D$. The orthogonal projection of $L^2(D)$ onto $A^2(D)$ is called the Bergman projection.

The general question is if the Bergman projection extends to a bounded operator from $L^p(D)$ to $A^p(D)$ for $p \neq 2$. If $n = 1$, i.e. if $\Gamma = \mathbb{R}^+$, the answer is positive if and only if $1 < p < \infty$.

Recent results (see e.g. G. Garrigós' abstract) prove that for tube domains over circular cones the answer is positive only for a more restricted range of values of $p$.

We prove that if $\Gamma$ is a polyhedral cone (i.e. the convex hull of a finite number of half-lines) then the Bergman projection is bounded if and only if $1 < p < \infty$, both for tube domains and type II domains.

The proof is based on a careful analysis of the Fourier multipliers $(e^{-y\cdot\xi}1_{\Gamma^*}(\xi))/\chi_{\Gamma^*}(\xi)$, on $\mathbb{R}^n$, where $\Gamma^*$ is the dual cone of $\Gamma$ and

$$\chi_{\Gamma^*}(\xi) = \int_\Gamma e^{-y\cdot\xi} dy$$

is its characteristic function.

### Stability and Energy functionals on Kähler geometry

Duong H. Phong

In the early 1980's, it was conjectured by S.T. Yau that the existence of Kähler-Einstein metrics should be equivalent to stability in the sense of geometric invariant theory. This notion of stability has proven difficult to exploit on geometric PDE's, since it is global and depends on the complex structure. In recent joint work with J. Sturm, we discuss its relation to lower bounds for energy functionals in Kähler geometry, as well as to notions of stability in real analysis. Central to our considerations is a new semi norm $\|\cdot\|_\sharp$ on $H^0(\mathrm{Gr})$

$$\log\|f\|_\sharp^2 = \frac{m+1}{(m+2)(d-1)}\frac{1}{D}\int_Z \log\frac{(\omega_{\mathrm{Gr}}^m\wedge\partial\bar\partial\frac{|f(z)|^2}{|\mathrm{Pl}(z)|^{2d}})}{\omega_{\mathrm{Gr}}^{m+1}}\omega_{\mathrm{GR}}^m + \frac{d-m-2}{(m+2)(d-1)}\frac{1}{D}\int_{\mathrm{Gr}}\log\frac{|f(z)|^2}{|\mathrm{Pl}(z)|^{2d}}\omega_{\mathrm{Gr}}^m$$

(Here $Z$ is the Chow variety), and a non-linear Radon transform, mapping the Mabuchi energy of a variety $X$ to the Mabuchi energy of the regular part $Z$, of its Chow variety.

### Symbolic calculus for pseudodifferential operators with periodic symbols

Karlheinz Gröchenig

We prove non-commutative versions of Wiener's Lemma on absolutely convergent Fourier series (a) for the case of twisted convolution and (b) for rotation algebras. Equivalently, these results can be seen as a symbolic calculus for pseudodifferential operators with periodic symbols. Such operators occur frequently in time-frequency analysis and the theory of Gabor frames. As an application we provide the solution of some open problems about Gabor frames, among them the problem of Feichtinger and Janssen that is known in the literature as the "irrational case".

### A theory of Gabor multipliers

Hans Georg Feichtinger

Let $\Lambda \lhd \mathbb{R}^d \times \widehat{\mathbb{R}^d}$ be a lattice in phase space, e.g. $\Lambda = a\mathbb{Z}^d \times b\mathbb{Z}^d$. For $l = (t,\omega)$ we write $\pi(l)g$ for $M_\omega T_t g(z) := e^{2\pi i\omega z}g(t-z)$ (=TF-shift). The pair $(g,\Lambda)$ generates a Gabor frame if $(\pi(l)g)_{l\in\Lambda}$ is a frame for $L^2(\mathbb{R}^d)$. It is called a tight Gabor frame if

$$f = \sum_{l\in\Lambda}\langle f,\pi(l)g\rangle\pi(l)g \quad \forall f\in L^2.$$

A Gabor multiplier is an operator of the form

$$G_m f := \sum_{l\in\Lambda} m_l\langle f,\pi(l)g\rangle\pi(l)g$$

for some sequence $(m_l)_{l\in\Lambda}$ on $\Lambda$.

Atoms should be taken from $S_0(\mathbb{R}^d) = \{f\in L^2, V_{g_0}f\in L^1(\mathbb{R}^{2d})\}$, where $V_{g_0}f(t,\omega) = \langle f, M_\omega T_t g_0\rangle$ is the short time Fourier transform with Gaussian window.

Then $m\in\ell^1/\ell^2/\ell^\infty$ implies $G_m$ trace class/$\mathcal{HS}$/ bounded.

Moreover the corresponding operators depend continuously in the respective norms on their ingredients (window in $S_0$, but even the lattice $\Lambda$!)

### Singular maximal functions and Radon transforms near $L^1$

Andreas Seeger

(joint work with Terence Tao, James Wright)

We consider a class of maximal functions which are known to give $L^p$-bounded operators and for which the weak-type (1-1) inequality is unknown. This class includes maximal functions associated to parabola in the plane and the lacunary spherical maximal function in $\mathbb{R}^d$. We prove a weak-type $L \log\log L$ inequality, i.e.

$$\operatorname{meas}\Big(\{x\colon |Tf(x)| > \alpha\}\Big) \lesssim \int \Phi\Big(\frac{|f(x)|}{\alpha}\Big)dx$$

with $\Phi(s) = s\log\log(10+s)$.

The proofs are based on stopping time arguments involving quantities of "length" and "thickness". We also obtain a related result on singular Radon transforms.

### On parametrices of semi-linear elliptic boundary problems

Jon Johnsen

The talk concerns a parametrix formula for semi-linear elliptic boundary problems, established by the speaker in recent years. The formula shows how a given solution depends on the data, whence one can read off its regularity direncly. In a broader context, with derivatives in the $L^p$-sense, this question may be technically rather demanding to answer by bootstrap methods (e.g. in cases with a large integrability gap between the "initial" and "final" spaces for the solution). It is explained how to deduce the formula, which gives a purely analytical way to obtain such regularity properties (in fact with weaker assuptions on the data). The construction has been completed for non-linerarities of product type, but there remain fundamental questions for those of composition type, like $f(u)$, where one is lead to pseudo-differential operators in a Hörmander class $S^0_{1,\delta}$ with $\delta > 1$.

### Restriction and decay for flat curves and hypersurfaces

Sarah N. Ziesler

In this talk I describe recent work with A. Carbery on restriction theorems for hypersurfaces $\Gamma(t) = (t, \gamma(t))$ in $\mathbb{R}^n$ ($t \in \mathbb{R}^{n-1}$, $\gamma\colon \mathbb{R}^{n-1} \to \mathbb{R}$) with the affine curvature $K_\Gamma(t)^{\frac{1}{n+1}} = (\det \mathrm{Hess}\gamma(t))^{\frac{1}{n+1}}$, introduced as a mitigating factor. Our work shows that, for $n \geq 3$, there is no universal restriction theorem for hypersurfaces with non-negative curvature, in contrast with the case $n = 2$, where Sjölin proved a universal restiction theorem for all convex curves. We also discuss decay estimates for the Fourier transform of the density $K_\Gamma^{\frac{1}{2}}$ supported on the surface and give results on the relationship between restriction and decay.

## A non-linear Fourier transform

CRISTOPH THIELE
(joint work with C. Mascalu, T. Tao)

Define the partial Fourier integrals of a function $F$ on the real line by

$$h(\alpha, k) = \int_{-\infty}^{\alpha} F(x)e^{ikx}dx.$$

The Fourier transform of $F$ is the limit of these partial sums as $\alpha \to \infty$. If we write $g = \exp h$, then we have the following ODE for $g$:

$$\partial_\alpha g(\alpha, k) = F(\alpha)e^{ik\alpha}g(\alpha, k), \quad g(-\infty, k) = 1.$$

This exponentiated Fourier transform can be generalized to the matrix case, e.g.

$$\partial_\alpha G(x, k) = \begin{pmatrix} 0 & F(\alpha)e^{ik\alpha} \\ \overline{F(\alpha)e^{ik\alpha}} & 0 \end{pmatrix} G(\alpha, k), \quad G(-\infty, k) = \text{id}.$$

The coefficient matrix is in the Lie algebra of $\mathrm{SU}(1,1)$ and thus the solution is of the form $\begin{pmatrix} a(\alpha,k) & b(\alpha,k) \\ \overline{b(\alpha,k)} & \overline{a(\alpha,k)} \end{pmatrix}$. It is known that $\int \log |a(\infty, k)| dk = C \int |F|^2$ for some universal $C$. This is a version of Plancherel's identity. We conjecture

$$\int \sup_x \log |a(\alpha, k)| dk \leq C \int |F|^2.$$

We can prove a variant of this where $e^{ik\alpha}$ is replaced by characters of the Cantor group: $\gamma^n = 1$, $(\omega(\alpha, k) = \sum_{n \in \mathbb{Z}} \gamma^{x_n k_{-n}}$ where $x = \sum x_n d^n$, $k = \sum k_n d^n$.

## On the absence of positive eigenvalues of Schrödinger operators with rough potentials

ALEXANDRU D. IONESCU
(joint work with D. Jerison)

We consider the problem of proving the absence of positive eigenvalues of Schrödinger operators for a certain class of rough potentials in $\mathbb{R}^n$. Let $H = -\triangle + V$ denote a Schrödinger operator. Assume that $V \in L^{\frac{n}{2}}_{\text{loc}}(\mathbb{R}^n)$ if $n \geq 3$ and $V \in L^k_{\text{loc}}(\mathbb{R}^n)$, $k > 1$, if $n = 2$. Assume also that for some exponent $q \in [\frac{n}{2}, \infty]$ (or $q \in (1, \infty]$ if $n = 2$) we have

$$\lim_{R \to \infty} \|V\|_{L^q(|x| \in [R, 2R])} \cdot R^{1 - \frac{n}{2q}} = 0.$$

Then the operator $H$ has no positive eigenvalues. The case $q = \infty$ is a well-known theorem of Kato. Our proof is based on establishing a Carleman inequality of the form

$$\|W_m u\|_{\ell^1(L^{p'(q)})} \leq C \|W_m |x|^{1 - \frac{n}{2q}} (\triangle + 1) u\|_{\ell^1(L^{p(q)})}$$

for a certain sequence of weights $W_m$, $m \to \infty$. This inequality holds uniformly as $m \to \infty$ and $p(q)$ and $p'(q)$ are dual exponents with the property that $\frac{1}{p(q)} - \frac{1}{p'(q)} = \frac{1}{q}$.

### Function spaces in presence of symmetries: compactness of embedings, decay and smoothness of functions

Leszek Skrzypczak

We are interested Sobolev embeddings of function spaces of Besov and Triebel-Lizorkin type consisted of distributions invariant with respect to the action of a compact group of isometries of an underlying space. The underlying space means in this context the Euclidean space or a Riemannian manifolds with bounded geometry. The following problems are regarded:

- characterization of Besov and Triebel-Lizorkin spaces on manifolds with bounded geometry via heat semi-group,
- compactness of Sobolev and Trudinger-Strichartz embeddings,
- improved smoothness properties of Sobolev embeddings on compact manifolds,
- entropy numbers of embeddings of radial functions on $\mathbb{R}^n$,
- local smoothness and decay of functions,
- smoothing properties and compactness of Riesz-Bessel potentials on symmetric spaces of noncompact type.

### A complex analytic view point on the $2d$ Euler equations

Nets Katz

For the $2d$ Euler equation of fluid motion, two basic problems remain open.

(1) Do the Sobolev norms of any solution with smooth initial data grow as fast as double exponential in time?
(2) Does a solution with initial vorticity in $H^1 \cap L^\infty$ remain in $H^1$?

These properties are on the case of what can be proved trivially by Littlewood Paley theory. We present an explicit model in which both might be studied.

### Product BMO and second order commutators

Michael Lacey

Given a function $b$ on the plane $M_b f = b \cdot f$ is the operator of multiplication by $b$. $H_1$ and $H_2$ are 1-dimensional Hilbert transforms performed in the 2 canonical directions of the plane. A theorem of Sarah Ferguson and Cora Sadosky concerns the commutator

$$\|[[M_b, H_1], H_2]\|_{L^2 \to L^2} \simeq \|b\|_{\mathrm{BMO}(\mathbb{C}_+ \times \mathbb{C}_+)}$$

What is most important is that the BMO norm is that of the dual of product $H^1(\mathbb{C}_+ \times \mathbb{C}_+)$, as indicated by S.-Y. Chang and R. Fefferman.

This theorem, as in the one dimensional case, admits equivaltent formulations in terms of Hankel operators and weak factorization of product $H^1$.

## A multilinear generalization of the Cauchy-Schwarz inequality

Anthony Carbery

For a nonnegative measurable function $K$ defined on a product $X_1 \times \cdots \times X_n$ of measurable spaces, let

$$Q(K) = \Big( \int K(s_1, x_2^0, \ldots, x_n^0) K(s_1, s_2, \ldots, x_n^1) \cdots K(x_1^n, \ldots, s_n) d(s,x) \Big)^{\frac{1}{n+1}}$$

Then $| \int K(x_1, \ldots, x_n) \prod_{i=1}^n f_i(x_i) dx_i | \leq Q(K) \prod_{i=1}^n \|f_i\|_{n+1}$.

We give a proof of this result, which generalizes a lemma of Katz and Tao.

## Tailored function spaces on fractals

Michele Bricchi

We have considered a generalization of the idea of $d$-sets and $(d, \psi)$-sets as follows.

Let $h: (0,1) \to \mathbb{R}$ be a positive and continuous monotone function. Then a non-empty compact set $\Gamma \subset \mathbb{R}^n$ is called $h$-set if there exists a finite Radon measure $\mu$ with supp$\mu = \Gamma$ and $\mu(B(\gamma, r)) \sim h(r)$, $\forall \gamma \in \Gamma$ and all $r \in (0,1)$. It turns out that, given $h$ as above, there exists an example of $h$-set in $\mathbb{R}^n$ if, and only if, $h(2^{-k-l})/h(2^{-l}) \gtrsim 2^{-kn}$ $\forall k, l \in \mathbb{N}_0$. Here "$\gtrsim$" means "up to an equivalent function $\widetilde{h}$" the estimate holds with the usual $\geq$ symbol.

The main theorem we have proved reads as follows.

Let $\Gamma$ be an $h$-set fulfilling a certain geometrical condition (ball condition). Then, for $0 < p < \infty$ and $0 < q \leq \min(1, p)$

$$\mathrm{Tr} B_{pq}^{h_p}(\mathbb{R}^n) = L_p(\Gamma).$$

Here $h_p$ is the sequence $h_p = \{h(2^{-j})^{\frac{1}{p}} 2^{\frac{n}{p}j}\}_{j \in \mathbb{N}_0}$ and the related generalized Besov spaces can be defined in analogy to the classical ones.

Afterwards, omitting details, one defines $B_{pq}^s(\Gamma)$ or even $B_{pq}^\sigma(\Gamma)$ (for a given "positive" sequence $\{\sigma_j\}$) as

$$B_{pq}^s(\Gamma) = \mathrm{Tr}_\Gamma B_{pq}^{2^{js} h_p}(\mathbb{R}^n) \quad \text{for } 0 < p, q \leq \infty.$$

Here $2^{js} h_p$ means the sequence $\{2^{js} h(2^{-j})^{\frac{1}{p}} 2^{\frac{n}{p}j}\}$. Once Besov-type spaces are defined, one can provide some more direct characterizations and exploit their definition for applications to PDE's.

## Riesz transform, Littlewood-Paley-Stein functions and heat kernels on non-compact Riemannian manifolds

Thierry Coulhon

(joint work with Xuan Thinh Duong)

Robert Strichartz has asked in 1983 for which complete non-compact Riemannian manifolds $M$ and which $p \in ]1, +\infty[$ one has

$$C_p^{-1} \|\Delta^{1/2} f\|_p \leq \| |\nabla f| \|_p \leq C_p \|\Delta^{1/2} f\|_p, \ \forall f \in \mathcal{C}_0^\infty(M).$$

The second inequality above means that the Riesz transform is bounded on $L^p(M)$. If true, it implies the first inequality for the conjugate exponent of $p$. We proved in [2] that the Riesz transform is bounded on $L^p(M)$, $1 < p \leq 2$, and has weak type $(1,1)$ if:

1. $V(x, 2r) \leq C\, V(x, r)$, $\forall x \in M$, $r > 0$

2. $p_t(x,x) \leq \frac{C}{V(x,\sqrt{t})}$, $\forall x \in M$, $t > 0$.

Here $V(x,r)$ is the Riemannian volume of the geodesic ball of center $x \in M$ and radius $r > 0$, and $p_t(x,y)$, $t > 0$, $x, y \in M$ is the heat kernel on $M$. The example of two Euclidean planes glued by a cylinder shows that the above statement is false for $p > 2$. On Vicsek manifolds (see [1]), the even weaker inequality $\|\Delta^{1/2} f\|_p \leq C_p \||\nabla f|\|_p$ is false for $p < 2 - \varepsilon$. In [5], we prove that the multiplicative inequality $\||\nabla f|\|_p^2 \leq C_p \|\Delta f\|_p \|f\|_p$ is valid on any complete Riemannian manifold for $1 < p \leq 2$, and for $p > 2$ on every complete Riemannian manifold satisfying

3. $|\nabla e^{-t\Delta} f| \leq C e^{-t\Delta}(|\nabla f|)$, $\forall f \in \mathcal{C}_0^\infty(M)$, $\forall t > 0$.

This relies heavily on the use of Littlewood-Paley-Stein functions and on the work of P-A. Meyer. The Riesz transform itself is bounded on $L^p(M)$ for $p > 2$ if 1 and 2 above are satisfied, and in addition the heat kernel on function suitably dominates the heat kernel on 1-forms (which generalizes the results of Bakry). Finally, we were recently able to reach the same conclusion under the weaker set of assumptions 1, 2, 3.

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## Rigidity of nilpotent Lie groups


Hans Martin Reimann


Carnot groups $N$ are nilpotent Lie groups, which are equipped with a generalized contact structure, a non integrable subbundle $HN$ of the tangent bundle $TN$. A (generalized) contact mapping on a Carnot group is a diffeomorphism, which preserves $HN$. The group $N$ is rigid, if the Lie algebra of infinitesimal generators for contact mappings is finite dimensional.

The rigid nilpotent Lie groups which appear in the Iwasawa decomposition of parabolic subgroups of simple Lie groups have been classified by Yamaguchi.

$H$-type groups are shown to be rigid if dim(center) $\geq 3$, global results for contact mappings on rigid nilpotent groups are discussed.

## On 1-quasiconformal maps of Carnot groups


Michael Cowling

(joint work with Luca Capogna)


A Carnot group $G$ is a connected, simply connected nilpotent Lie group, whose Lie algebra $\mathfrak{g}$ is stratified, that is, $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \cdots \oplus \mathfrak{g}_r$, where $[\mathfrak{g}_1, \mathfrak{g}_j] = \mathfrak{g}_{j+1}$; further, $\mathfrak{g}$ carries an inner product such that the various $\mathfrak{g}_j$ are mutually orthogonal.

The identification of $\mathfrak{g}$ with the set $\tilde{\mathfrak{g}}$ of left-invariant vector fields on $G$ leads to the definition of the horizontal tangent space $HT(G)$: $HT_p(G) = \{\tilde{X}_p : X \in \mathfrak{g}_1\}$; this carries an invariant subriemannian metric. Suppose that $U$ is an open subset of $G$. A differentiable map $f : U \to G$ is said to be *conformal* if $df$ maps $HT(U)$ to $HT(G)$, and the restriction of $df$ to each $HT_p(U)$ is a multiple of an orthogonal map.

The inner product on $\mathfrak{g}$ induces a Carnot–Caratheodory metric on $G$: we define $d(x, y)$ to be the infimum of the lengths of all curves from $x$ to $y$ whose tangent vectors are horizontal. A homeomorphism $f : U \to G$ is said to be $\lambda$-*quasiconformal* if it is $\lambda$-quasiconformal relative to this metric (i.e., as the radius of balls become smaller, the ratio of the outer radius to the inner radius of their images becomes at most $\lambda$). Capogna showed that if $f$ is 1-quasiconformal, then the first component of $f$ is smooth. Our result extends this to all $f$. In particular, conformal and 1-quasiconformal maps coincide, and both are smooth.

*Edited by Michael Kempe*

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