Mathematisches Forschungsinstitut Oberwolfach

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Groups and Geometries

September 1st – September 7th, 2002

The meeting Groups and Geometries centered around simple groups, building geometry and their interaction. One central topic of this meeting was the new classification proof of (simple) groups of characteristic p-type proposed by U. Meierfrankenfeld, B. Stellmacher and G. Stroth. The whole Monday morning (and some additional talks) were devoted to this subject, with an introductory lecture of 90 minutes given by U. Meierfrankenfeld.

One further central topic of the conference were applications of the classification of the finite simple groups to other branches of mathematics. Here the talk of B. Guralnick on *Genus Zero Groups in Positive Characteristic*, in which the classification was applied to algebraic geometry, should be mentioned.

Finally the theory of Moufang buildings has, thanks to the classification of Moufang polygons by J. Tits and R. Weiss, taken a new boom. For this reason Tuesday afternoon was completely devoted to Moufang buildings and the corresponding groups.

Abstracts

Small modules for groups of Lie type

BARBARA BAUMEISTER
(joint work with U. Meierfrankenfeld)

In the talk we discuss quadratic modules for the groups of Lie type in the defining characteristic. We prove

Theorem: Let M be a quotient of ${}^{\sigma}G_{\Phi}(K)$, $0 < p = \text{char } K$, V an irreducible, faithful GF$(p)M$-module, $D \leq M$, $|D| > 2$ a p-subgroup and $A \leq Z(D)$ such that $M = \langle A^M \rangle D$, $[V, A, D] = 0$, $|\Phi_D| \geq 2$. Then (M, V) are members of some well-known list.

There are the results of Thompson ($p \geq 5$), Premet & Suprunenko ($p \neq 2$) and Stroth & Englund ($p = 2$) on quadratic modules. We give a uniform proof for all primes which is based on the result of Curtis, that every irreducible module $V(\lambda)$, λ p-restricted, for a group of Lie type, is also an irreducible module for the related Lie algebra.

Black Box Groups and the Andrews-Curtis Conjecture

ALEXANDRE V. BOROVIK
(joint work with E.I. Khukhro, A. Lubotzky and A.G. Myasnikov)

Experimental evidence suggests that the problem of generation of random elements in a normal subgroup of a black box group can be efficiently solved by a version of the product replacement algorithm. From the theoretical point of view, this algorithm is a random walk over a certain graph associated with the group and its normal subgroup. The graph can be defined for infinite groups, and in the case of the free group viewed as a normal subgroup of itself, the well-known Andrews-Curtis Conjecture theoretical point of view, this algorithm is a random walk over a certain graph associated with the group and its normal subgroup. The graph can be defined for infinite groups, and in the case of the free group viewed as a normal subgroup of itself, the well-known Andrews-Curtis Conjecture says that the graph is connected. The talk discusses approaches to the Andrews-Curtis Conjecture via finite group theory and possible implications for the theory of black box finite groups.

Groups of rank 2 in local characteristic p

ANDREW CHERMAK
(joint work with U. Meierfrankenfeld)

Part of the Meierfrankenfeld program for understanding groups of local characteristic p concerns the case where G contains a pair $(P, \tilde{P})$ of minimal parabolic subgroups over a fixed Sylow p-subgroup S, such that $O_p(\langle P, \tilde{P} \rangle) = 1$. This talk consisted of a report on progress on this case.

Theorem: Set $Z = \Omega_1(Z(S))$ and assume that $[Z, P] \neq 1 \neq [Z, \tilde{P}]$. Assume also that $Y_P Y_{\tilde{P}}$ is normal in neither P nor $\tilde{P}$. Then one of the following holds:

1. $p = 2$ and $F^*(G) \simeq L_3(2^n)$, $Sp(4, 2^n)$, A_6 or M_{23},
2. $p = 3$ and $(P, \tilde{P})$ determines a weak BN-pair of type $G_2(3^n)$,

3. p is odd and $O^{p'}(P) \simeq O^{p'}(\tilde{P})$ is of the form$p^{2n} : \mathrm{SL}(2, p^n)$.
(The condition on $Y_P Y_{\tilde{P}}$ in the statement of the Theorem may be dropped, once the "Pushing Up Theorem" of Meierfrankenfeld is complete.) One may therefore assume, henceforth, that $[Z, P] \neq 1$ and $[Z, \tilde{P}] = 1$.

The remainder of the report consisted in explaining how other parts of the Meierfrankenfeld program (the Theorems called E!, P!, and the Structure Theorem) provide the necessary background information from which to complete the classification (in characteristic 2) or the determination of p-local structure (in odd characteristic) of the rank-2 groups.

Artin groups, their representations and BMW algebras

Arjeh M. Cohen
(joint work with D. Gijsbers and D. Wales)

Let M be a spherical Coxeter diagram of type A, D, E and of rank n. The BMW algebra of type M (where B=Birman, M=Murahami, W=Wenzl) is the algebra $\mathcal{B}$ over $\mathbb{Z}[l^{\pm}, m^{\pm}]$ generated by $g_1, \ldots, g_n$ subject to the relations
(B1) $g_i g_j = g_j g_i$ if $i \not\sim j$,
(B2) $g_i g_j g_i = g_j g_i g_j$ if $i \sim j$,
(D1) $e_i := m^{-1} l(g_i^2 + m g_i - 1)$,
(R1) $g_i e_i = l^{-1} e_i$,
(R2) $e_i g_j e_i = l e_i$ when $i \sim j$.

For $M = A_{n-1}$ this gives the known BMW algebra, corresponding to the braid group on n braids. There is a homomorphism from the Artin group of type M on generators $s_1, \ldots, s_n$ to the group of invertible elements in $\mathcal{B}$, determined by $s_i \mapsto g_i$. Since Krammer's representation occurs in $\mathcal{B}$, this homomorphism is faithful. We prove that $\mathcal{B}$ is finite dimensional, and determine the group of type M on generators $s_1, \ldots, s_n$ to the group of invertible elements in $\mathcal{B}$, determined by $s_i \mapsto g_i$. Since Krammer's representation occurs in $\mathcal{B}$, this homomorphism is faithful. We prove that $\mathcal{B}$ is finite dimensional, and determine the structure of I_1/I_2, where I_j is the two sided ideal of $\mathcal{B}$ generated by all products $e_{i_1}, \ldots, e_{i_j}$ with $\{i_1, \ldots, i_j\}$ a coclique of M. By the way $\mathcal{B}/I_1$ is (close to) the Hecke algebra of type M.

Genus Zero Actions of Finite Groups

Daniel Frohardt
(joint work with R.M. Guralnick and K. Magaard)

G is a *genus* g group [in characteristic 0] if G is isomorphic to the monodromy group of (X, ϕ) where X is a compact Riemann surface of genus g and $\phi : X \to P^1C$ is a cover. The associated permutation action is a genus g action of G.

In a 1990 paper in the Journal of Algebra, Guralnick and Thompson conjectured that for every $g \geq 0$ there is a finite set $\mathcal{E}_g$ consisting of the nonabelian finite simple groups S such that S is not an alternating group and S is a composition factor of a genus g group. They also conjectured that $\mathcal{E}_0$ would be of ' manageable' size.

The finiteness of $\mathcal{E}_g$ for all g has now been established by the combined work of many authors, culminating in a 2001 paper by Frohardt and Magaard in Annals of Mathematics.

Current efforts are directed to finding not only the precise list of groups lying in $\mathcal{E}_0$ but also all of the actions and generating tuples that lead to primitive genus 0 groups with classical composition factors. In work that appears in the von Neumann Conference Proceedings (AMS, 2002), the authors did this for the groups of Lie rank 1. It is believed that there are precisely 16 groups of larger Lie rank lying in $\mathcal{E}_0$, and that, in particular, all of the actions have degree smaller than 300.

The key ingredients in the analysis are combining the Cauchy-Frobenius formula with the Riemann-Hurwitz formula, a careful analysis of the fixed point ratios of classical groups on 1-spaces of their natural modules, and Scott's Theorem on the size of the commutators of generators of matrix groups.

Groups acting on locally recognizable graphs

Ralf Gramlich
(joint work with A.M. Cohen and H. Cuypers)

A graph Γ is called *locally homogeneous*, if for any vertices $x, y \in \Gamma$ we have $\Gamma(x) \simeq \Gamma(y)$. An example of a locally homogeneous graph is the graph $H_n(\mathbb{F})$ on the nonincident point-hyperplane pairs of the projective space of dimension n over the division ring $\mathbb{F}$; the local structure of $H_n(\mathbb{F})$ is isomorphic to $H_{n-1}(\mathbb{F})$.

Conversely, for sufficiently large n, any connected graph that is locally $H_n(\mathbb{F})$, is isomorphic to $H_{n+1}(\mathbb{F})$. This allows for a characterization of the group $\mathrm{PSL}_{n+2}(\mathbb{F})$ using its natural action on the graph $H_{n+1}(\mathbb{F})$.

Generalizations to other graphs and groups are possible.

Genus Zero Groups in Positive Characteristic

Robert M. Guralnick

Let k be an algebraically closed field of characteristic $p > 0$. Let X, Y be curves defined over k and $\phi : X \to Y$, a separable rational map of degree n. We are interested in determining the possibilities for the monodromy group G of the cover if we fix g, the genus of X. In characteristic 0, much progress has been made on this problem over the past 12 years. Until 1999, there had not been a single example of a simple group that could be eliminated as being a composition factor of such a group (even for $g = 0$).

We discuss the various methods used in reducing this problem to the case of almost simple groups – including the Riemann-Hurwitz formula, the Tate module and fixed point ratios. The main result we discuss is that if L is a type of Chevalley group (in particular has fixed rank), then the minimal genus of any group containing $L(q)$ as a composition factor grows linearly with q as long as q is not a power of p. Abhyankar has shown that the classical groups in characteristic p are genus zero groups. This gives strong evidence for the conjecture that there are only finitely many cross characteristic groups of a given genus.

Amalgams determined by locally projective actions

Alexander A. Ivanov
(joint work with S.V. Shpectorov and V.I. Trofimov)

Let Γ be a regular tree of valency $2^n - 1$ for some $n \geq 3$ and G be a locally finite, vertex-transitive automorphism group of Γ, such that for every edge $\{x, y\}$ of Γ we have

$$G(x)/O_2(G(x)) \simeq L_n(2), \qquad G\{x,y\}/O_2(G\{x,y\} \cap G(x)) \simeq L_{n-1}(2) \times 2$$

(where $G(x)$ and $G\{x,y\}$ are stabilizers in G of the vertex x and the edge $\{x,y\}$, respectively). We show that the amalgam $\{G(x), G\{x,y\}\}$ either belongs to one of two infinite series associated, respectively with the action of $\mathrm{AGL}_n(2)$ on the set of vectors in an n-dimensional GF(2)-space and with the action of the orthogonal group $O^+_{2n}(2)$ on its dual polar space graph; or is one of twelve explicitly described exceptional examples. For each of the exceptional examples we have $n = 3, 4$ or 5 and most of them are related one way or another to flag-transitive Petersen geometries.

Groups of both even and p-type type

Inna Korchagina
(joint work with R. Lyons and R. Solomon)

and I.K.: The result is related to the part of classification of finite simple groups which deals with the characterization of groups of both even and p-types with $e(G) = 3$. For the purpose of the talk, we consider slightly modified situation:

Theorem: Let G be a finite simple K-proper group of simultaneously $\tilde{2}$- and $\tilde{p}$-type (where p is an odd prime). Suppose

that $e(G) = m_{2,p}(G) = 3$ and the following conditions hold:

1. There exists a 2-local subgroup H of G with $m_p(H) = 3$ and $F^*(H) = O_2(H)$; and
2. If $z \in G$ is an involution with $m_p(C_G(z)) = 3$, then $F_z := F^*(C_G(z)) = O_2(C_G(z))$.

Then the following hold:

1. $p = 3$;
2. There exists the unique class of 2-central involutions z^G with $m_3(C_G(z)) = 3$. Moreover, $F_z \cong Q_8^l$ where $l \in \{3, 4\}$;
3. $H = C_G(t)$ for some $t \in z^G$; and
4. If $B \cong E_{27}$ is a subgroup of $C_G(z)$ and $b \in B$ is a nontrivial element with $C_{F_z}(b) \cong Q_8^2$, then $L_b := E(C_G(b)) \neq 1$ and is isomorphic to one of the following groups: $PSp_4(3)$, $G_2(3)$, $3_2(U_4(3))$. Such b exist.

Moreover, if $l = 3$, then $C_G(z)/F_z \cong \Omega_6^-(2)$, $L_b \cong 3_2U_4(3)$ and $G \approx Suz$, while if $l = 4$, then $C_G(z)/F_z \cong A_9$, $L_b \cong G_2(3)$ and $G \approx Th$.

Outer control

Ross Lawther

Let G be a simple algebraic group over an algebraically closed field. The conjugacy class structure of G is well understood, thanks to Jordan decomposition and nice properties of centralizers of semisimple elements. Now assume G has a non-trivial graph automorphism τ, and form the disconnected group $\langle G, \tau\rangle$; in this talk we seek control over the class structure in the outer coset $G.\tau$. We find that there is a construction, beginning with root systems and proceeding through root data, which gives a group $\bar{G}$ and a ma p $\phi : T \to \bar{T}$

of maximal tori, such that ϕ induces a bijective correspondence between G-classes meeting $T.\tau$ and $\bar{G}$-classes meeting $\bar{T}$ (i.e. semisimple classes of $\bar{G}$); moreover, the correspondence behaves well with respect to taking fixed points of appropriate Frobenius maps. In the case where the order of τ is prime to the field characteristic, ordinary Jordan decomposition may be applied to complete the picture; using Steinberg's notion of quasi-semisimplicity, we conjecture a generalized Jordan

decomposition may be applied to complete the picture; using Steinberg's notion of quasi-semisimplicity, we conjecture a generalized Jordan decomposition which would cover all cases uniformly.

Bases for primitive permutation groups

Martin W. Liebeck

(joint work with Y. Shalev)

Let G be a transitive permutation group on a finite set Ω of size n. A subset B of Ω is a *base* for G if its pointwise stabilizer in G is trivial. The minimal size of a base is denoted by $b(G)$. It is very easy to see that

$$\log_2 |G| \leq b(G) \leq \log |G| / \log n.$$

Pyber conjectured that there is a constant c such that for any primitive group G we have $b(G) < c \log |G| / \log n$. Seress has shown that it suffices to prove this for G either almost simple or of affine type.

For G almost simple, Liebeck and Shalev have proved that either $b(G) < c$, or $F^*(G)$ is an alternating group acting on an orbit of subsets or partitions, or $F^*(G)$ is a classical group acting on an orbit of subspaces. In the latter two cases, Benbenishty has shown tha t $b(G) < 3 \log |G| / \log n$.

For the affine case, Liebeck and Shalev have proved the following: if $H < GL(V)$ is irreducible and primitive (as a linear group), then either $b(H) < c$, or $b(H) < 18 \log |H| / \log n + 27$ (where $b(H)$ is the minimal base size for the action of H on vectors). However, the imprimitive linear case of the conjecture remains open.

Finite groups of local characteristic p

Ulrich Meierfrankenfeld

(joint work with B. Baumeister, A. Chermak, A. Hirn, M. Mainardis, C.W. Parker, G. Parmeggiani, P. Rowley, B. Stellmacher and G. Stroth)

Let p be a prime and G a finite group. G is of *characteristic* p if $C_G(O_p(G)) \leq O_p(G)$. And G is of *local characteristic* p if every p-local subgroup of G is of characteristic p. In my talk I described the current status of the project to understand and classify the finite groups of local characteristic p. This project is joint work with Barbara Baumeister, Andy Chermak, Andreas Hirn, Mario Mainardis, Chris Parker, Gemma Parmeggiani, Peter Rowley, Bernd Stellmacher and Gernot Stroth.

Moufang buildings

Bernhard Mühlherr

In this talk the classification of 2-spherical Moufang buildings with no small residues was described.

By a generalization of the Curtis-Tits theorem any such building is determined by its local structure, which is a Moufang foundation. Hence the classification reduces to the classification of the Moufang foundations and a criterion for deciding whether a given Moufang foundation is integrable (i.e. the local part of a Moufang building).

The first problem reduces to the isomorphism problem of Moufang sets which are residues in Moufang polygons. This is dealt by using Jordan algebras.

The second question is answered first in the rank 3 case and then we use a rank 3 criterion.

A 5-local identification of the Lyons sporadic group

Christopher W. Parker

In this talk I briefly considered how K-groups of local characteristic p will be identified when p is an odd prime. It is expected that most of these groups will be identified via the geometry of their p-local subgroups. However, when the group doesn't have a simply connected geometry (for example when the rank is 2), different methods need to be exploited.

Using the Lyons sporadic simple group as an example, I demonstrated how the local characteristic p property and the existence of an elementary abelian subgroup of order p^2 in the centralizer of some involution can be used to determine the centralizer of an involution. Using this information the group is identified.

Permutation groups and normal subgroups

Cheryl E. Praeger

Primitive and quasiprimitive groups were proposed as alternative choices for 'basic' finite permutation groups. Different choices are required in different applications. The structure of the two classes, their overgroups, and applications to edge-transitive graphs were discussed, together with several new results about finite simple groups.

This lecture was a repeat of an invited lecture presented last week at the ICM2002 in Beijing.

Normalizers of primitive groups

Laszlo Pyber
(joint work with M. Abért and R.M. Guralnick)

We discuss the following

Theorem: Let G be a primitive subgroup of S_n and N its normalizer in S_n. Then $|N/G| \leq n-1$ if $n \geq n_0$. In fact we have $|Out(G)| \leq n-1$ for large n.

The statement of the theorem does not hold for $n \in \{81,\ 6561,\ 43046721\}$.

Exceptionality

JAN SAXL

Let A be a primitive permutation group on the finite set X, let G be a normal subgroup of A. At present we assume that A/G is cyclic. We say that the triple (A, G, X) is *exceptional* if no non-diagonal G-orbit on $X \times X$ is A-invariant. In a joint work with Guralnick and Muller, we obtained a classification of exceptional triples.

Exceptional triples came up in the work with Fried and Guralnick on exceptional polynomials. A slightly more general situation arose in our paper with Guralnick and Muller, where we considered a variation on a problem of Schur and investigated rational functions which give rise to a bijection on infinitely many residue fields.

Other applications are concerned with homogeneous partitions of complete graphs, and with line-transitive linear spaces.

Maximal subgroups of algebraic groups of exceptional type

GARY SEITZ
(joint work with M.W. Liebeck)

Let G be a simple algebraic group of exceptional type over an algebraically closed field. Martin Liebeck and I have determined the maximal subgroups of G having positive dimension. We make no assumption on the characteristic of the underlying field, thus completing the classification which began with Dynkin (char 0) and followed by Seitz (char $p > 7$). We also show how tilting modules and certa in variants appear naturally in the restrictions of the adjoint module of G to maximal subgroups.

Primitive groups of squarefree degree

ÁKOS SERESS
(joint work with Cai Heng Li)

We classify all primitive groups of squarefree degree, and all primitive groups of squarefree degree that contain a regular subgroup. Applications of these results are the determination of all vertex-primitive non-Cayley graphs of squarefree order, all vertex-primitive Cayley graphs of squarefree order, and all Burnside groups of squarefree order. (A group is called a Burnside group if all primitive groups containing it as a regular subgroup are 2-transitive.)

On Curtis-Tits amalgams

SERGEY V. SHPECTOROV
(joint work with J. Dunlap)

While reproving the first Phan's theorem, C. Bennett and the speaker had to deal with the problem of the uniqueness of the related group amalgam. In dealing with the problem we introduced a method that may have much more general applications. In particular, at present speaker's Ph.D. student J. Dunlap is applying this method to the amalgams arising in the Curtis-Tits theorem. Although the uniqueness of those amalgams can be obtained indirectly using work of F. Timmesfeld and J. Tits' classification of spherical buildings, the results of Dunlap will be the first direct proof of the uniqueness.

In the talk we discussed our proof of Phan's theorem and how it works in the Curtis-Tits situation.

Characterizations of Lie incidence geometries by a class of maximal singular subspaces

Ernest Shult

Throughout Γ is a parapolar space and $\mathcal{M}$ is a class of maximal singleton subspaces such that every line is a member of $\mathcal{M}$.

Theorem 1: Suppose the following:

1. For every point $x \in \mathcal{P} - M, M \in \mathcal{M}$: (i) $x^{\perp} \cap M$ has finite projective dimension and (ii) if $y \in x^{\perp} - M$, $y^{\perp} \cap M$ properly contained in $x^{\perp} \cap M$ implies $y^{\perp} \cap M = \emptyset$.

2. There is a pair $(p, M) \in \mathcal{P} \times \mathcal{M}$, p not in M such that $p^{\perp} \cap M$ contains a plane, and M has finite projective rank.

Then Γ is a polar space or a homomorphic image of a half-spin geometry.

Theorem 2: Suppose some line lies in two members of $\mathcal{M}$, and some member of $\mathcal{M}$ has finite rank. For $(p, M) \in \mathcal{P} \times \mathcal{M}$ with $p \in \mathcal{P} - M$, $p^{\perp} \cap M$ is empty or is a line.

Then Γ is a polar space or a Grassmannian of d-spaces of a vector space V, $d > 1$ ($\dim V$ need not be finite), or $\Gamma \simeq A_{2n-1}/\langle\sigma\rangle$, where σ is a polarity of Witt index at most $n - 5$.

A similar theorem for which $x^{\perp} \cap M = \emptyset$, a point or a PG$(d)$, $d \geq 3$ when $(x, M) \in \mathcal{P} \times \mathcal{M}$ and $x \in \mathcal{P} - M$, implies $d = 3$ and Γ is a homomorphic image of a building with diagram $Y_{2,1,m}$, $m \geq 1$, or else Γ is a polar space.

Connections of finite group geometries with algebraic topology

Stephen D. Smith

Algebraic topologists, especially those working in cohomology of finite groups, are increasingly using (in effect) group geometries and related techniques – for similar as well as different reasons from the ones we are familiar with.

The talk gives an informal survey of progress in several active areas:

1. Explicit computation of cohomology for sporadic groups (Adem, Milgram, Tezuka, Yagita and others).

2. "Homology approximations" (Webb, Dwyer, Grodal and others); applications (Ryba, Smith, Yoshiara, Sawabe).

3. Topological constructions modelling finite group aspect - "p-local finite groups" (Oliver, Grodal and others).

4. Group actions – developments continuing the spirit of P.A. Smith theory – actions on spaces of prescribed homology, with restrictions on fixed points, e.g. results of Oliver, Segev.

Generalized hexagons regularly embedded in a projective space

Anja Steinbach
(joint work with H. Van Maldeghem)

For a generalized hexagon Γ, we defined a regular embedding in a projective space (over a skew field).

It turns out that Γ admits a regular embedding in $\mathrm{P}(V)$ if and only if Γ is Moufang with its little projective group induced by $\mathrm{GL}(V)$ such that $[V, A]$ is 2-dimensional and $[[V, A], A] = 0$, for any long root subgroup A.

Using the classification of Moufang hexagons due to Tits and Weiss, regular embeddings have been classified completely:

For the hexagons of type G_2 and ${}^3D_4$, ${}^6D_4$, there is a unique embedding (in orthogonal space). But for the hexagons of mixed type G_2 in characteristic 3, we found several new embeddings (in unbounded dimension), which are quotients of some universal embedding.

Split BN-pairs of rank 2

Katrin Tent

If G is a group with a split BN-pair of rank 2 (i.e. there is a nilpotent $U \lhd B$ with $B = U(B \cap N)$) then G is a group of Lie type.

The proof uses the geometric interpretation of such a BN -pair as a generalized polygon. This generalizes the corresponding result for finite groups due to Fong and Seitz, and easily extends to split BN-pairs of rank ≥ 2.

Parts of this are joint work with H. Van Maldeghem and B. Mühlherr, respectively.

Low dimensional representations of finite quasisimple groups and applications

Pham Huu Tiep

Recently there has been considerable interest in finding the smallest degree $d_l(G)$ of faithful irreducible representations of finite quasisimple groups G in characteristic l, and in classifying representations of G of degree less than $(d_l(G))^{2-\varepsilon}$. We report on recent results concerning this problem.

In the case of $G = \widehat{A}_n$ (and $\widehat{S}_n$), the results are joint with Kleshchev, and these results have allowed us to make substantial progress on (i) describing modular spin representations of G that are irreducible over a proper subgroup, and (ii) proving that in general the tensor product of modular representations of G are reducible if $l \neq 2, 3$. The results on $\mathrm{SL}_n(q)$ and $\mathrm{Sp}_{2n}(q)$, q even, are joint with Guralnick; and the results on $\mathrm{SU}_n(q)$ and $\mathrm{Sp}_{2n}(q)$, q odd, are joint with Guralnick, Magaard and Saxl.

We outline the main ideas behind the proofs, including results using Deligne-Lusztig theory, study of local properties, gluing method. We also discuss some recent applications, including (i) a new approach to $k(GV)$-problem and (ii) Larsen's conjecture (joint work with Guralnick).

The Curtis Tits presentation

Franz Georg Timmesfeld

Let $\mathcal{B}$ be an irreducible spherical Moufang building of rank $l \geq 2$ with root system Φ and fundamental system Π. For each $r \in \Phi$ let A_r be the root-group corresponding to r (in the sense of Tits), $X_r := \langle A_r, A_{-r} \rangle$ and let $G := \langle A_r \mid r \in \Phi \rangle \leq \mathrm{Aut}(\mathcal{B})$. For $r \neq s \in \Pi$ let $X_{rs} := \langle X_r, X_s \rangle$. Then the following is known as Curtis-Tits presentation of G:

Let $\widehat{G}$ be the amalgamated product of the X_{rs}; $r, s \in \Pi$, amalgamated over the X_r, $r \in \Pi$. Then $\widehat{G}$ is a perfect central extension of G.

Now in this generality there is no proof for this theorem in the literature. In my talk I presented the following

Theorem: Let Φ and Π be as above and let G be a group generated by rank one groups X_r, $r \in \Pi$, satisfying:

(1) $[X_r, X_s] = 1$ if r and s are not connected in the Dynkindiagram Δ of Π.

(2) If $r, s \in \Pi$ are connected in Δ, then there exists a surjective homomorphism $\varphi : \langle X_r, X_s \rangle \to R_{rs}$, where R_{rs} is a group of Lie type of rank two in the above sense, with $\ker \varphi \leq Z(\langle X_r, X_s \rangle)$, mapping $A_{\pm r}$ and $A_{\pm s}$ onto corresponding fundamental root groups of R_{rs}.

Then G is a perfect central extension of a group of Lie type $\mathcal{B}$ in the above sense.

This theorem contains the Curtis Tits presentation as a special case.

Affine Moufang buildings

Hendrik Van Maldeghem

(joint work with B. Mühlherr)

In this talk we comment on some difficulties arising in the proof of the following

Theorem: All Moufang buildings of type $\widetilde{C}_2$ are known.

We point out the equivalence of a Moufang building of type $\widetilde{C}_2$ with valuations on root groups of Moufang quadrangles satisfying certain conditions. We then address the question of how to identify the local

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The general curve covers $\mathbf{P}^1$ with monodromy group A_n

Helmut Völklein

(joint work with Gerhard Frey and Kay Magaard)

Let C be a general curve of genus $g \geq 2$. Then C has a cover to $\mathbf{P}^1$ of degree n if and only if $2(n-1) \geq g$. This is a classical fact of algebraic geometry. If C has a cover to $\mathbf{P}^1$ of degree n then there is such a cover that is simple, i.e., has monodromy group S_n and all inertia groups are generated by transpositions. The question arises whether C admits other types of covers to $\mathbf{P}^1$.

If there is a cover $C \to \mathbf{P}^1$ branched at r points of $\mathbf{P}^1$ then $r \geq 3g$. Zariski used this to show that if $g > 6$ then there is no such cover with solvable monodromy group.

The condition $r \geq 3g$ was further used by Guralnick and various co-authors to restrict the possibilities for the monodromy group. Let $C \to \mathbf{P}^1$ be a cover of degree n. Its monodromy group G is a transitive subgroup of S_n. If G is a primitive subgroup and $g \geq 4$ then $G = S_n$ or $G = A_n$.

It was not known whether the case $G = A_n$ actually occurs. This is answered to the positive in this talk.

The classification of thick irreducible spherical buildings of rank at least three

Richard Weiss

(joint work with J. Tits)

The classification of Moufang polygons can be used to give a new proof of the classification of thick irreducible spherical buildings of rank at least three. Let Δ be such a building. It is a consequence of 4.1.2 of Tits' Lecture Notes that Δ is Moufang. Hence each residue of rank two is a Moufang polygon and therefore determined by a "root group sequence," a certain sequence of root groups of Δ. It follows that the building Δ is uniquely determined by a "root group labeling" of the Coxeter diagram Π of Δ which consists of labellings which assign

(i) to each vertex u of Π a group $\nu(u)$,

(ii) to each directed edge (u, v) of Π the root group sequence Θ_{uv} associated with the residue of type $\{u, v\}$ of Δ containing a fixed chamber c so that Θ_{vu} is the sequence Θ_{uv} in reverse order and

(iii) to each directed edge (u, v) of Π an isomorphism θ_{uv} from $\nu(u)$ to the first term of Θ_{uv}.

We describe a proof of the classification of thick irreducible spherical buildings of rank at least three based on the classification of Moufang polygons and the concept of a root group labelled Coxet er diagram. For details see J.Tits & R.Weiss, "Moufang Polygons," Springer, 2002.

Minimal polynomials of elements of prime order in complex representations of quasi-simple groups

Alexandre E. Zalesskii

Let G be a quasi-simple group and $g \in G$ be an element of prime order p. We list all complex irreducible representations φ of G such that the number of distinct eigenvalues of $\varphi(g)$ is strictly less than p.

Edited by Sergei Haller

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