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New Trends in Boundary Elements

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Introduction

Jean–C. Nédélec

The workshop was centred on new techniques to extend the capacities of the now largely used numerical methods based on integral equations.

Different issues can be noticed related to the actual practice in industrial applications;

- the solution of direct but large or very large (up to several millions of unknowns) problems of electrostatic, time-harmonic acoustic or electromagnetic origin;
- the solution of inverse problems, mostly in the case of time-harmonic acoustics (inverse scattering).

Several techniques are now widely studied and were presented in this workshop:

- The use of the fast multipole method under the supposition that good preconditioners are available.
- The use of wavelet approximations which can lead to a large compression of the linear system to be solved.
- The hierarchical matrix technique which is based on linear solver techniques and does not use the exact form of the discretization.

An informal plenary discussion took place on Wednesday evening. Some issues were raised on future tendencies and links with applications (see attached protocol). In particular, participants are convinced that the coupling of FEM and BEM will play an important role in multiphysics problems. Concerning problems in acoustics, it seems that currently no method is able to work robustly for all frequencies. Only few groups are working on transient, time-domain methods. Problems with unbounded surfaces are quite important in several applications. Many of our American colleagues are using collocation or Nystroem techniques while the Galerkin technique is more widely used in Europe. Only few realistic comparisons are available in the published literature.

The atmosphere of the Workshop was very pleasant but productive. The participants express their gratitude to the staff of the Oberwolfach institute for their great hospitality.

Abstracts

Wavelet approximation for mixed boundary value problems

Jens Breuer

(joint work with Olaf Steinbach and Wolfgang L. Wendland)

We consider a symmetric Galerkin boundary element method for the symmetric boundary integral formulation. In particular, a wavelet basis is used to obtain a data sparse representation of the discrete boundary integral operators. The Galerkin discretization of the hypersingular integral operator is done by applying the Maue trick, i.e. integration by parts and by using a piecewise constant wavelet basis for the resulting single layer potential. In the case of quadrangular or curved boundaries, the approach has to be modified by an additional projection onto the piecewise constant basis functions. For the discretization of the double layer potential we introduce a wavelet transformation of the piecewise linear hat functions, considered as discontinuous functions, onto piecewise linear wavelets. We then present a corresponding stability and error analysis for the wavelet compression in the energy norm. Numerical examples are in agreement with the theoretical results.

References

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Boundary elements for Maxwell equations on Lipschitz surfaces

Annalisa Buffa

We consider time-harmonic Maxwell equations in an heterogeneous, piecewise homogeneous material. First, we derive integral equations on the set of interfaces and establish the wellposedness for a suitable variational formulation of the problem. Second, we provide a general theory for finite elements approximation of a class of non-elliptic problems. This framework gives a suitable setting for the analysis of Raviart-Thomas approximations of the scattering problem. We prove then stability and quasi optimality for the related boundary element scheme.

Algebraic convergence is ensured by means of suitable anisotropic mesh refinement to compensate the field singularity.

High frequency boundary element methods for acoustic scattering problems

Simon N. Chandler-Wilde

As a case study, we consider the problem of a plane wave incident on straight impedance boundary with piecewise constant surface impedance. According to the geometrical theory of diffraction, the field consists of direct and reflected rays and rays diffracted from impedance discontinuities: these latter rays may be diffracted in turn at other impedance discontinuities. We propose a Galerkin finite element scheme which uses graded meshes with a conventional grading within one wavelength of discontinuities and exponential grading at larger distances. A key feature of the scheme is that the basis functions on each element are polynomials multiplied by the trace of the plane waves in the directions indicated by GTD. Numerical examples which illustrate the full error analysis are shown. The error analysis proves that, to maintain accuracy at a fixed level as the wavenumber k is increased, it is enough to increase the degrees of freedom proportional to some small power of $\log k$. We finish by discussing extending the method and analysis to general classes of scattering problems.

Boundary element methods for semilinear elliptic boundary value problems

Goong Chen

We first present a semilinear elliptic boundary value problem where nonlinearities occur in both the partial differential equation and the boundary condition. We show how the spatial and boundary nonlinearities interact and obtain error estimates between the exact solution and the boundary element solution based on the monotone iteration scheme, where there are multiple solutions and the boundary condition contains oblique derivatives.

We then discuss the Lane-Emden equation in astrophysics. This equation, under various homogeneous boundary conditions, are well known to have multiple solutions. Error estimates are not yet available, but we point out the advantage of BEM in the computation of the solution of such equations.

Stable iterative method for solving the electric field integral equation at low frequencies

Snorre H. Christiansen

We consider the scattering of a time-harmonic electromagnetic wave by a bounded perfect conductor. This problem is attacked using the electric field integral equation. Standard discretizations of this equation are ill-suited for iterative methods both because of the mapping properties of the underlying operator at fixed frequencies and because we are interested in small frequencies. These two problems are overcome by the use of discrete Hodge decompositions – scaled by the frequency – of the Galerkin spaces and the construction of a preconditioner based on the Calderón relations linking integral operators. For the proposed method we prove a discrete LBB Inf-Sup condition uniform in both the frequency and the meshwidth (on non-trivial intervals beginning at 0). We furthermore prove that the spectral condition number of the preconditioned system is bounded independently of the frequency and the meshwidth.

Maxwell boundary integral equations and wavelets

Martin Costabel

The electric field integral equation can be transformed into a strongly elliptic system of pseudodifferential equations by using Hodge decompositions of $H_{div}^{-1/2}$ on the boundary manifold. To avoid the necessity of C^1 elements one can use a non conforming or equivalently a mixed formulation. This formulation can be discretized using any C^0 finite element method. The method discussed here, and used successfully for the numerical modelling of microwave antennas, uses a nodal wavelet basis of piecewise linear elements. A complete analysis of this method on open surfaces with piecewise smooth boundary is possible including matrix compression, simple block-diagonal preconditioning and quasioptimal error estimates.

Coupling of fast multipole method and microlocal discretization for the integral equations of electromagnetism

Eric Darrigrand

We are concerned with integral equations of scattering. In order to deal with the well-known high frequency problem, we suggest a coupling of two kind of methods that reduce the numerical complexity of iterative solution of these integral equations. The microlocal discretization method introduced by T. Abboud, J.-C. Nédélec and B. Zhou enables one to reduce efficiently the size of the system considering an approximation of the phase function of the unknown. However, the method needs an expensive precalculation. We suggest the use of the fast multipole method introduced by V. Rokhlin in order to speed up the precalculation. This work is an original application of the fast multipole method for acceleration of a microlocal discretization method within the new integral formulation written by B. Després. Numerical results obtained for Helmholtz's equation are very satisfying. For Maxwell's equations they are also quite interesting.

Boundary value problems on smooth hypersurfaces

Roland Duduchava

(joint work with Dorina Mitrea and Marius Mitrea)

Boundary value problems (BVPs) for tangent differential operators on a hypersurface $\mathcal{M}$ in $\mathbb{R}^n$ encounter in applications rather often. The purpose of the investigation is to give an alternative and simple formulation of classical problems which applies only a unit normal vector to the surface.

We find explicit representation of the Deformation tensor **Def** (which produces killing vector fields), the Laplace-Beltrami $\Delta_{\mathcal{M}}$, the Láme-Beltrami, operator of isotropic elasticity and the Stokes system on the hypersurface $\mathcal{M}$. It is proved that these operators perturbed by non-zero and non-negative function have fundamental solutions on $\mathcal{M}$. Based on the explicit Green's formulae the uniqueness and solvability of the classical BVPs on an open subsurface $\mathcal{C} \subset \mathcal{M}$ for the operators listed above and for the square of Laplace-Beltrami operator $\Delta^2_{\mathcal{M}}$ is proved.

On the numerical solution of periodic inverse diffraction problems

Johannes Elschner

The talk is devoted to inverse diffraction problems to recover a two-dimensional periodic structure from scattered waves measured above and below the structure. We consider grating profile reconstruction based on finite element/boundary element and optimization techniques. The dependence on regularization parameters is analyzed and convergence results for (non-smooth) grating structures are given. Numerical results demonstrate the practicability of the inversion algorithm.

Adaptive boundary element methods

Birgit Faermann

The refinement process in adaptive methods is generally controlled by local a posteriori error indicators (or briefly error indicators). Investigations of error indicators are already well established in finite element methods. Due to the non-locality of integral operators, such investigations are more involved for boundary element methods.

In this talk we introduce efficient and reliable error indicators for the discretization of boundary integral equations. For integral operators of negative order, we use local double-integral semi-norms of the residual as error indicator. For integral operators of positive order, we consider error indicators, introduced by Babuška and Rheinboldt for finite element methods. These latter indicators can also be regarded as local norms of the residual.

Finite elements on degenerate meshes: Inverse-type inequalities and applications

Ivan G. Graham

(joint work with Wolfgang Hackbusch and Stefan A. Sauter)

In this paper we obtain a range of inverse-type inequalities which are applicable to finite element functions on general classes of meshes, including degenerate meshes obtained by anisotropic refinement. These are obtained for Sobolev norms of positive, zero and negative order. In contrast to classical inverse estimates, negative powers of the minimum mesh diameter are avoided. We give two applications of these estimates in the context of boundary elements: (i) to the analysis of quadrature error in discrete Galerkin methods and (ii) to the analysis of the panel clustering algorithm. Our results show that degeneracy in the meshes yields no degradation in the approximation properties of these methods.

An application of the BIE method in petroleum engineering

Tuong Ha-Duong

We consider the problem of the simulation of complex wells in layered reservoirs. A simplified model consists in solving the heat equation in a reservoir with a non linear and non-local boundary condition along the well surface.

If this surface is represented by a cylinder of length L and of radius r_w then the pressure on the well is given by Dikken in 1990: the tangential gradient of the pressure is proportional to the square of the flow rate at a section of the well.

We prove that, with some restrictions (small friction factor), the model is mathematically correct (the problem is well-posed), and describe a BIE method to solve it, taking into account the geometric character ($r_w \ll L$) of the well.

$\mathcal{H}$-matrix structure of inverse FEM-matrices

Wolfgang Hackbusch

The inverse FEM matrix is related to the Green's function of the underlying elliptic problem. Assuming L^∞-coefficients of the elliptic operator and a general Lipschitz domain, the green function has little smoothness. Therefore, this situation differs extremely from the case of analytic fundamental solution studied in BEM. Nevertheless, the hierarchical matrix technique developed for BEM-matrices can also be applied to inverse FEM-matrices. For this purpose we prove that the matrix blocks have singular values decaying exponentially.

The p-version of the BEM revisited

Norbert Heuer
(joint work with Benqi Guo)

We study the p-version of the boundary element method for two-dimensional problems on polygons and open curves. Using Jacobi-weighted Besov and Sobolev spaces, we analyze lower and upper bounds for the approximation errors for hypersingular and weakly singular integral operators. We prove the optimal rate of convergence for the p-version in the energy norms of $\widetilde{H}^{1/2}$ and $\widetilde{H}^{-1/2}$, respectively.

$\mathcal{H}$-matrix approximation via weak admissibility

Boris N. Khoromskij
(joint work with Wolfgang Hackbusch and R. Kriemann)

A class of hierarchical matrices ($\mathcal{H}$-matrices) allows the data-sparse approximation to integral and more general nonlocal operators with almost linear complexity. In this talk, a method is described for the coarsening of the hierarchical $\mathcal{H}$-matrix formats based on the weakened admissibility condition. A coarsening process improves the structural constant of the hierarchical format and thus leads to lower complexity $\mathcal{H}$-matrix arithmetic. On the other hand, our approach preserves the approximation power provided by the standard admissibility criteria [1].

References

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Boundary element tearing and interconnecting methods

Ulrich Langer
(joint work with Olaf Steinbach)

In this paper we introduce the Boundary Element Tearing and Interconnecting (BETI) methods as boundary element counterparts of the well-established Finite Element Tearing and Interconnecting (FETI) methods. In some practical important applications such as far field computations, handling of singularities and moving parts etc., BETI methods have certainly some advantages over their finite element counterparts. This claim is especially true for the sparse versions of the BETI preconditioners resp. methods. Moreover, there is an unified framework for coupling, handling and analyzing both methods. In particular, the FETI methods can benefit from preconditioning components constructed by boundary element techniques. The first numerical results confirm the efficiency and the robustness predicted by our analysis.

This work has been supported by the Austrian Science Fund 'Fonds zur Förderung der wissenschaftlichen Forschung (FWF)' under grant SFB F013 'Numerical and Symbolic Scientific Computing' and the German Research Foundation 'Deutsche Forschungsgemeinschaft (DFG)' under the grant SFB 404 'Multifield Problems in Continuum Mechanics'.

Error indicators for Signorini interface problems

Matthias Maischak

We discuss an interface problem consisting of a linear partial differential equation in $\Omega \subset \mathbb{R}^n$ (bounded, Lipschitz, $n \geq 2$) and the Laplace equation in the unbounded exterior domain $\Omega_c := \mathbb{R}^n \backslash \bar{\Omega}$ fulfilling some radiation condition, which are coupled by transmission conditions and Signorini conditions imposed on the interface. The interior PDE is discretized by a mixed formulation, whereas the exterior part of the interface problem is rewritten using a Neumann to Dirichlet mapping (NtD) given in terms of boundary integral operators.

We treat the general numerical approximation of the resulting variational inequality and discuss the non-trivial discretization of the NtD mapping. Assuming some abstract approximation properties and a discrete inf-sup condition we prove existence and uniqueness and show an a-priori estimate. Choosing Raviart-Thomas elements and piecewise constants in Ω and hat functions on $\partial\Omega$ the discrete inf-sup condition is satisfied. We present a solver based on a modified Uzawa algorithm, reducing the solution procedure of the saddle point problem with an inequality constraint to the repeated solution of a standard saddle point problem and the solution of a variational inequality based on an elliptic operator. Finally, we present a residual based a-posteriori error estimator compatible with the Signorini condition and a corresponding adaptive scheme.

Some numerical experiments are shown which illustrate the convergence behaviour of the uniform h-version with triangles and rectangles and the adaptive scheme as well as the bounded iteration numbers of the modified Uzawa algorithm, underlining the theoretical results.

Boundary methods for the fast solution of discrete potential problems

Per-Gunnar Martinsson

An efficient method for solving a harmonic boundary value problem is to convert it into a Fredholm integral equation of the second kind first and then to discretize and solve the integral equation for some boundary potential. The success of this method hinges on two facts: First that the resulting system of discrete equations is very well conditioned so that an iterative solver converges fast. Second that there exist fast methods for the matrix-vector multiplication incurred at each iteration, since the relevant matrix is dense. One such method is the Fast Multipole Method by Rokhlin and Greengard which performs a dense $n \times n$ matrix-vector multiplication of this kind in $O(n)$ operations.

In this talk we will demonstrate how the well-established technique described in the previous paragraph can be extended to solve discrete lattice equations that arise in the study of atomic crystals and certain composite materials. This extension involves (a) the development of techniques for deriving discrete boundary equations and (b) the development of a Fast Multipole Method that can handle a lattice Green's functions. It is our belief that the insights gained from our study of lattice equations can be used to formulate a similar method for continuum potential problems involving periodic media.

Multilevel preconditioning of boundary element equations

WILLIAM MCLEAN
(joint work with Mark Ainsworth)

The talk describes multilevel diagonal scaling as a preconditioner for the Galerkin equations arising from a symmetric positive-definite operator equation of positive order. The theory applies in particular to hypersingular boundary integral equations. We consider piecewise linear finite element spaces with local mesh refinement starting from an initial quasi-uniform mesh. Hanging nodes may be created and the global mesh ratio can grow exponentially in the number of levels. We use an exact solver on the coarsest level and on each finer level a simple diagonal scaling only for the degrees of freedom local to the refinement zone on that level. In this way, the preconditioned system has a condition number bounded independently of the number of degrees of freedom, the global mesh ratio, the number of levels and the size of the elements in the initial quasi-uniform mesh.

Fast multipole method for linear elastostatics

GÜNTHER OF
(joint work with Olaf Steinbach and Wolfgang L. Wendland)

For mixed boundary value problems we use the symmetric formulation of boundary integral equations. The standard boundary element method leads to a system of linear equations with a positive definite and block skew–symmetric matrix which is fully populated. Because of the high effort needed to compute these matrices and the tremendous memory requirements, practical problems can only be solved efficiently by using fast boundary element methods, as for example the fast multipole method.

In linear elastostatics the realization of the boundary integral operators respectively their bilinear forms can be reduced to the boundary integral operators of the Laplacian and their derivatives. Therefore it is sufficient to have fast realizations of these operators. First of all, the fundamental solution of linear elastostatics can be written as a linear combination of the fundamental solution of potential theory and its derivatives. Following [2], the double–layer potential of linear elastostatics can be reduced to single–layer and double–layer potentials of the Laplace equation by using the surface curl. Therefore the bilinear form of the adjoint double–layer potential can be realized, too. The bilinear form of the hypersingular operator can be transformed into single–layer potentials of linear elastostatics and potential theory [1, 2].

Finally, we present some numerical examples showing the efficiency of the presented approach.

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Boundary elements for inverse scattering

Roland Potthast

Boundary elements are widely used for direct and inverse obstacle scattering problems. Often, the direct problem is solved by boundary integral equations on the boundaries of the unknown obstacles. The inverse problem involves boundary integral operators which live on subsets of the unit sphere or on the boundaries of test domains, respectively.

First, a novel method by Luke and Potthast for the detection of unknown scatterers is presented, called the "no response test". The method allows to reconstruct the shape of obstacles from the far field pattern of one time-harmonic wave, if the boundary condition of physical properties of the object are not known and iterative schemes cannot be applied.

Second, we will provide a survey about recent developments in inverse scattering and discuss the use of boundary elements for the solution of inverse problems as employed by recent methods by Colton, Kirsch, Potthast, Ikehata, Luke-Potthast and Potthast-Sylvester-Kusiak. For the "point source method", the "method of singular sources" and the "no response test" a backprojection technique is used to map the measured far field patterns onto the scattered field or special indicator functions, respectively. Ikehata's schemes rely on the application of Green's theorem with boundary integrals on some outer boundary. The range-test of Potthast-Sylvester-Kusiak solves ill-posed boundary integral equations from the boundary of some test domain into the unit sphere.

On polynomial collocation for second kind integral operators with fixed singularities of Mellin Type

Andreas Rathsfeld

(joint work with C. Frammartino and G. Mastroianni)

We consider a polynomial collocation for the numerical solution of a second kind integral equation with an integral kernel of Mellin convolution type. Using a stability result obtained together with P. Junghanns, we prove that the error of the approximate solution is less than a logarithmic factor times the best approximation and, using the asymptotics of the solution, we derive the rates of convergence. Finally, we describe an algorithm to compute the stiffness matrix based on simple Gauß quadratures and an alternative algorithm based on a recursion in the spirit of Monegato and Palamara Orsi. Altogether, an almost best approximation to the solution of the integral equation can be computed with $\mathcal{O}(n^2[\log n]^2)$ resp. $\mathcal{O}(n^2)$ operations, where n is the dimension of the polynomial trial space.

Fourier transform for multidimensional integral equations

Sergej Rjasanow

A new numerical method for the multifrequency analysis of the three–dimensional Helmholtz equation is introduced. A Collocation Boundary Element Method (BEM) is used for the discretization of the problem. The Fourier transform with respect to the wave number for the resulting system of linear equations is used. The matrix of the transformed system is shown to have a sparse structure leading to $O(m^2n)$ units of required memory, where m denotes the number of frequencies and n is the number of degrees of freedom by BEM discretization.

A new numerical method for the Boltzmann equation is developed. The gain part of the collision integral is written in a form which allows its numerical computation on a uniform grid to be carried out efficiently. The amount of numerical work is shown to be of order $O(n^6 \log(n))$ for the most general model of interaction and of order $O(n^6)$ for the Variable Hard Spheres (VHS) interaction model, while the formal accuracy is of order $O(n^{-2})$. Here n denotes the number of discretization points in one direction of the velocity space. Some numerical examples for Maxwell pseudo-molecules and for the hard spheres model illustrate the accuracy and the efficiency of the method in comparison with DSMC computations.

A numerical method for large-scale simulations of Stokesian emulsions

Gregor J. Rodin
(joint work with James R. Overfelt)

Stokesian emulsions are two phase-fluids in which inertial effects are insignificant in comparison to viscous and surface tension effects.

Mathematically, problems arising in micromechanical simulations of Stokesian emulsions can be advantageously formulated in terms of boundary integral equations because the numerical solution of such equations can be obtained using $O(N)$ operations and memory.

In this lecture, a numerical method for large-scale micromechanical simulations of Stokesian emulsions is presented. This method includes:

- $O(N)$ boundary element scheme for solving periodic problems,
- time-stepping scheme for economical handling of collisions,
- an accurate scheme for the normal vector and mean curvature computations ,
- parallel implementation.

In addition to presenting the numerical method, we discuss its applications to analysis of oil-in-water emulsions.

Alternative representation of boundary integral operators with applications to quadrature and panel clustering

Stefan A. Sauter
(joint work with Stefan Börm)

Alternative representations of boundary integral operators corresponding to elliptic boundary value problems are derived as a starting point for numerical approximations, as e.g. Galerkin BEM including quadrature and panel clustering. These representations have the advantage that the integrands of the integral operators have a reduced singular behaviour allowing to choose the order of the numerical approximations much lower than for the classical formulations. It turns out that the resulting fully discrete Galerkin method has linear complexity without any logarithmic terms.

Rapid solution of first kind boundary integral equations in $\mathbb{R}^3$

Gregor Schmidlin
(joint work with Christoph Schwab)

Weakly singular boundary integral equations (BIEs) of the first kind on polyhedral surfaces Γ in $\mathbb{R}^3$ are discretized by Galerkin BEM on shape-regular, but otherwise unstructured meshes of meshwidth h. Strong ellipticity of the integral operator is shown to give nonsingular stiffness matrices and, for piecewise constant approximations, up to $O(h^3)$ convergence of the farfield. The condition number of the stiffness matrix behaves like

$O(h^{-1})$ in the standard basis. An $O(N)$ agglomeration algorithm for the construction of a multilevel wavelet basis on Γ results in a preconditioner which reduces the condition number to $O(|\log h|)$. A class of kernel-independent clustering algorithms (containing the fast multipole method as special case) approximate the matrix-vector multiplication in $O(N(\log N)^3)$ memory and operations.

Iterative approximate solution of the linear system by CG or GMRES with wavelet preconditioning and clustering-acceleration of matrix-vector multiplication is shown to yield an approximate solution in log-linear complexity which preserves the $O(h^3)$ convergence of the potentials. Numerical experiments confirm the theory.

This work was supported in part under the TMR network "Multiscale Methods in Numerical Analysis" of the EC by the Swiss government under grant number BBW 97.0404

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Adaptive wavelet methods for the fast solution of boundary integral equations

Reinhold Schneider
(joint work with Helmut Harbrecht)

Wavelet bases are offering an efficient way for the numerical approximation of integral operators. These methods have been developed theoretically and a successful implementation has been improved through the past years. We like to review these results. An important recent breakthrough has been established by the adaptive wavelet approximation in terms of best N–term approximation by Cohen, Dahmen, deVore. We realized their method with some modifications in our wavelet boundary element method. Since the computation of the matrix entries is the most expensive part in our algorithm, the present refinement invokes only one subsequent scale.

On the numerical approximation of the derivatives of potentials on a smooth 3D simply or multiply connected boundary

Antoine Sellier

We present a new recursion scheme to successively obtain higher and higher order Cartesian derivatives of potentials on a simply or multiply connected surface, either for interior or exterior 3D problems. The advocated procedure solely appeals to boundary integral equations and rests on a new boundary integral equation that relates the potential gradient to the normal flux on the surface. For derivatives of any order, the only needed metric informations on the geometry are the normal vector and the mean curvature. Using a collocation method, the approach is tested against a few analytical solutions in three cases: the case of a unit sphere embedded in a uniform electric field with a Neumann boundary condition, the case of an ellipsoidal particle with a source located inside and finally the case of a 2-sphere cluster with unit sources at each centre. Excellent agreement between the computed and analytical derivatives is observed. Finally, it is possible to extend the method to deal with the Helmholtz equation and linear elasticity in 3D problems.

On the effective computation of the partial indices of a regular matrix function

BERND SILBERMANN

Let $W_{N\times N}$ denote the algebra of all functions with values in $\mathbb{C}^{N\times N}$ such that the entries are in the Wiener class W. It is well-known that $a \in W_{N\times N}$ admits a Wiener-Hopf factorization if and only if $\det a$ does not vanish. The topic of the talk is the computation of the partial indices. The proposed method is based on the so-called k-splitting property of the singular values of modified finite sections of related Toeplitz operators.

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Inverse acoustic scattering in three dimensions

IAN H. SLOAN

(joint work with X. Chen, M. Ganesh, Ivan G. Graham and R. Womersley)

This talk reviews recent work on the problem of scattering of acoustic waves by a three-dimensional object (called the 'direct' problem), and the 'inverse' problem of detecting the shape of an object by measurement of the scattered wave. The inverse problem for three-dimensional scattering remains a challenging task. In the approach described here the solution to the inverse problem rests on a very efficient spectral solution to the direct problem, together with an optimisation approach to refining the description of the surface. Current progress will be described.

A preconditioned fast multipole boundary element method for Neumann boundary value problems

OLAF STEINBACH

(joint work with Günther Of)

For a Neumann boundary value problem we consider a modified hypersingular boundary integral equation leading to a $H^{1/2}(\Gamma)$–elliptic variational problem. The involved rank one approximation can be determined explicitly yielding almost optimal spectral equivalence inequalities of the modified hypersingular bilinear form and the form induced by the single layer potential. The latter is used as a preconditioner. All boundary integral operators are realized by applying a fast multipole algorithm. Numerical examples for complicated three–dimensional structures show the efficiency of the proposed approach.

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An overlapping additive Schwarz method for boundary integral equations

Thanh Tran
(joint work with Ernst P. Stephan)

We study a two-level overlapping additive Schwarz preconditioner for the Galerkin solution of hypersingular integral equations. These equations arise from the boundary integral reformulation of the Laplace or Lamé equation in the exterior of an open surface in $\mathbb{R}^3$ with Neumann boundary condition. We prove that the condition number of the preconditioned system is bounded by $O(1+\log^2 \frac{H}{\delta})$, where H is the size of the coarse mesh and δ the overlap size. A salient feature of the method is that the overlaps do not need to be quadrilateral domains.

Plenary discussion on "New Trends in Boundary Elements"

Christoph Schwab

The plenary discussion was held on the evening of Wednesday, Dec 4th 2003, from 2000h-2130h and was chaired by Prof. W.L. Wendland. All participants in the meeting were present. Some points which were raised in the meeting are summed up here.

Q: Why are Nystroem methods, which are often used in the US for large scale computations, not presented/discussed at the meeting?

A: These methods have many problems for polyhedral domains with corners. The methods used in the US are also not the classical Nystroem schemes, but rather modified versions of it (e.g. collocation plus low order quadrature).

A: Prof. J.-C. Nédélec remarks that codes in the US based on Nystroem/Collocation often need a very large number of unknowns to reach a certain accuracy which is produced with carefully implemented Galerkin BEM codes with much less work.

As to the question on trends, the following was mentioned:

- Problems with 10^7 degrees of freedom are today possible, mainly due to the development of fast algorithms.
- Boundary elements will play a significant role in wave propagation problems, i.e. acoustics, electromagnetics, in particular with high wave numbers.
- There appear to be problems with stable formulations and fast multipole methods for low frequency electromagnetics.
- To date, there seem to be no methods which work robustly for all frequencies.
- In high frequency scattering, there is the problem of coupling different methods in different parts of the domain.
- The coupling of FEM with BEM is on solid grounds and will play a role in the future, in particular for multiphysics problems.
- There is little work to date for transient, time-domain methods. The only work is that of the French school.

- Adaptivity is relevant and should be pushed to industrial applications. All industrial BEM codes to date are nonadaptive.
- Unbounded surfaces appear to be very little analyzed, but arise in several applications/models.
- Free boundary problems with linear field equations appear to be a 'natural' problem class where BEM promise to be superior.
- Problems in optics (laser).
- Robustness of BEM with respect to critical material parameters, such as thin plates appear to be promising.
- Ill posed problems often are formulated with integral operators; iterative solution methods require fast and stable evaluation of the discretized integral operators.

Edited by Wolfgang L. Wendland

Participants

Dipl.-Math. Jens Breuer
breuer@mathematik.uni-stuttgart.de
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57
D–70569 Stuttgart

Dr. Annalisa Buffa
annalisa@dimat.unipv.it
annalisa@ian.pv.cnr.it
Istituto di Analisi Numerica
Via Ferrata, 1
I–27100 Pavia

Prof. Dr. Carsten Carstensen
Carsten.Carstensen@tuwien.ac.at
Institut für Angewandte und
Numerische Mathematik
Technische Universität Wien
Wiedner Hauptstraße 8 - 10
A–1040 Wien

Dr. Simon N. Chandler-Wilde
Simon.Chandler-Wilde@brunel.ac.uk
mastsnc@brunel.ac.uk
Dept. of Mathematical Sciences
Brunel University
Kingston Lane
GB–Uxbridge, Middlesex, UB8 3PH

Prof. Dr. Goong Chen
gchen@math.tamu.edu
Department of Mathematics
Texas A & M University
College Station, TX 77843-3368 - USA

Dr. Snorre Harald Christiansen
snorrec@math.uio.no
Matematisk Institutt
Universitetet i Oslo
P.B. 1053 - Blindern
N–0316 Oslo

Prof. Dr. Martin Costabel
costabel@univ-rennes1.fr
Département de Mathématiques
Université de Rennes I
Campus de Beaulieu
F-35042 Rennes Cedex

Prof. Dr. Wolfgang Dahmen
dahmen@igpm.rwth-aachen.de
Institut für Geometrie und
Praktische Mathematik
RWTH Aachen
Templergraben 55
D–52056 Aachen

Dr. Eric G. Darrigrand
eric.darrigrand@math.u-bordeaux.fr
darrigra@math.udel.edu
Department of Mathematical Sciences
University of Delaware
409 Ewing Hall
Newark DE 19716 - USA

Prof. Dr. Roland Duduchava
duduch@rmi.acnet.ge
duduch@mathematik.uni-stuttgart.de
Ramzadze Institute of Mathematics
Georgian Academy of Sciences
Z. Rukhadze, 1
380093 Tbilisi - Georgia

Dr. Johannes Elschner
elschner@wias-berlin.de
Weierstraß-Institut für
Angewandte Analysis und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39
D–10117 Berlin

Birgit Faermann
bf@numerik.uni-kiel.de
Mathematisches Seminar II
Universität Kiel
Hermann-Rodewald-Str. 3
D–24098 Kiel

Dr. Stefan A. Funken
stefan.funken@femlab.de
Femlab GmbH
Berliner Str. 4
D–37075 Göttingen

Prof. Dr. Ivan G. Graham
i.g.graham@bath.ac.uk
Department of Mathematical Sciences
University of Bath
Claverton Down
GB-Bath, BA2 7AY

Prof. Dr. Tuong Ha-Duong
tuong.ha-duong@utc.fr
GI/EMA
Université de Technologie
de Compiegne
BP 20529
F-60205 Compiegne Cedex

Prof. Dr. Wolfgang Hackbusch
wh@mis.mpg.de
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
D–04103 Leipzig

Dr. Norbert Heuer
norbert@ing-mat.udec.cl
Departamento de Ingenieria
Matematica
Universidad de Concepcion
Casilla 160-C
Concepcion - Chile

Prof. Dr. Ralf Hiptmair
hiptmair@na.mathematik.uni-tuebingen.de
ralf@hiptmair.de
Departement Mathematik
ETH-Zentrum
Rämistr. 101
CH–8092 Zürich

Prof. Dr. George C. Hsiao
hsiao@math.udel.edu
Department of Mathematical Sciences
University of Delaware
501 Ewing Hall
Newark, DE 19716-2553 - USA

Dr. Boris Khoromskij
bokh@mis.mpg.de
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
D–04103 Leipzig

Prof. Dr. Ulrich Langer
ulanger@numa.uni-linz.ac.at
Institut für Numerische Mathematik
Johannes Kepler Universität Linz
Altenbergstr. 69
A–4040 Linz

PD Dr. Matthias Maischak
maischak@ifam.uni-hannover.de
Institut für Angewandte Mathematik
Universität Hannover
Welfengarten 1
D–30167 Hannover

Priv. Doz. Dr. Per-Gunnar Martinsson
per-gunnar.martinsson@yale.edu
Department of Mathematics
Yale University
10 Hillhouse Avenue
New Haven CT 06520-8283 - USA

Prof. Dr. William McLean
w.mclean@unsw.edu.au
Department of Applied Mathematics
University of New South Wales
Sydney NSW 2052 - Australia

Prof. Dr. Giovanni Monegato
monegato@polito.it
Dipartimento di Matematica
Politecnico di Torino
Corso Duca degli Abruzzi, 24
I-10129 Torino

Prof. Dr. Peter Monk
monk@math.udel.edu
Department of Mathematical Sciences
University of Delaware
501 Ewing Hall
Newark, DE 19716-2553 - USA

Prof. Dr. Jean-Claude Nedelec
nedelec@cmapx.polytechnique.fr
Centre de Mathématiques Appliquées
Ecole Polytechnique
U. R. A. - C. N. R. S. 169
F-91128 Palaiseau Cedex

Dipl.-Math. Guenther Of
ofgr@mathematik.uni-stuttgart.de
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57
D–70569 Stuttgart

Dr. Roland Potthast
potthast@math.uni-goettingen.de
Institut für Numerische
und Angewandte Mathematik
Universität Göttingen
Lotzestr. 16-18
D–37083 Göttingen

Dr. Andreas Rathsfeld
rathsfeld@wias-berlin.de
Weierstraß-Institut für
Angewandte Analysis und Stochastik
Mohrenstr. 39
D–10117 Berlin

Prof. Dr. Sergej Rjasanow
rjasanow@num.uni-sb.de
Fachrichtung 6.1 - Mathematik
Universität des Saarlandes
Postfach 151150
D–66041 Saarbrücken

Prof. Dr. Gregory J. Rodin
gjr@ticam.utexas.edu
rodin@lms.polytechnique.fr
Laboratoire Mécanique des Solides
Ecole Polytechnique
F-91128 Palaiseau Cedex

Prof. Dr. Stefan A. Sauter
stas@amath.unizh.ch
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH–8057 Zürich

Dipl.-Math. Gregor Schmidlin
gschmidl@sam.math.ethz.ch
Seminar für Angewandte Mathematik
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Reinhold Schneider
reinhold@mathemtik.tu-chemnitz.de
Fakultät für Mathematik
T. U. Chemnitz
D–09107 Chemnitz

Prof. Dr. Christoph Schwab
schwab@sam.math.ethz.ch
Seminar für Angewandte Mathematik
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Antoine Sellier
sellier@ladhyx.polytechnique.fr
Ladhyx
Ecole Polytechnique
F-91128 Palaiseau Cedex

Prof. Dr. Bernd Silbermann
silbermann@mathematik.tu-chemnitz.de
Fakultät für Mathematik
T. U. Chemnitz
Reichenhainer Str. 41
D–09126 Chemnitz

Prof. Dr. Ian H. Sloan
sloan@maths.unsw.edu.au
School of Mathematics
The University of New South Wales
Sydney NSW 2052 - Australia

Dr. Olaf Steinbach
steinbach@mathematik.uni-stuttgart.de
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57
D–70569 Stuttgart

Prof. Dr. Ernst P. Stephan
stephan@ifa.uni-hannover.de
Institut für Angewandte Mathematik
Universität Hannover
Postfach 6009
D–30060 Hannover

Prof. Dr. Thanh Tran
thanh@maths.unsw.edu.au
School of Mathematics
The University of New South Wales
Sydney NSW 2052 - Australia

Prof. Dr.-Ing. Wolfgang L. Wendland
wendland@mathematik.uni-stuttgart.de
Institut für Angewandte Analysis
und Numerische Simulation
Universität Stuttgart
Pfaffenwaldring 57
D–70569 Stuttgart