Mathematisches Forschungsinstitut Oberwolfach

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# Darstellungen endlicher Gruppen

March 23rd – March 29th, 2003

The meeting was organized by A. Kleshchev (Eugene), G. Malle (Kassel), J. Rickard (Bristol) and G.R. Robinson (Birmingham). Unfortunately, due to the war against Iraq which begun just two days before the start of the meeting and the resulting disturbances in international air traffic one of the organizers and several of the invited participants were unable to attend. In fourteen lectures of 50 minutes length each and nineteen shorter contributions of 30 minutes each, recent progress in representation theory was presented and interesting new research directions were proposed. Besides the lectures, there was plenty of time for informal discussion between the participants, either continuing ongoing research cooperation or starting new projects.

A noteworthy achievement at this meeting was the announcement by U. Riese of the proof by himself and P. Schmid, (and independently by Gluck and Magaard) of the $k(GV)$-problem for the prime $p = 5$, which was the last outstanding prime. This means that Brauer's famous $k(B)$-problem for $p$-solvable groups is solved for all primes $p$ ( the problem in this case was reduced to the $k(GV)$-problem over 40 years ago by N. Ito, and the $k(GV)$-problem was solved for large primes by G.R. Robinson and J.G. Thompson in 1996, work presented at the corresponding meeting at Oberwolfach in 1996). In the same vein, at the present meeting, G.R. Robinson announced a new (in some ways more precise) variant of the $k(B)$-problem for $p$-constrained groups, proving some cases of the conjecture.

L. Puig presented a new Grothendieck group interpretation of conjectures of Alperin and Dade which offers the prospect of a structural explanation of some of the numerical predictions of these and related conjectures.

J. Alperin offered a new Hecke-algebra explanation for some of the numerical character theoretic results recently obtained by G. Navarro for $p$-solvable groups.

Much progress has been made in the last few years on the classification of endo-trivial (and endo-permutation) modules for $p$-group algebras, and several talks at the meeting were related to this. The endo-trivial modules are classified by the elements of an abelian group $T(P)$. Thévenaz reported on his work with Carlson determining that, with a few well-understood exceptions, the torsion subgroup of this group is trivial, and Carlson presented a new method that produces a set of generators modulo torsion, thus completing the classification. For endo-permutation modules, which are similarly classified by a group $D(P)$, Thévenaz described some consequences of his and Carlson's results on $T(P)$, and Bouc talked on some open conjectures on the structure of $D(P)$. One of these is that $D(P)$

is generated by relative syzygies of the trivial module, and Mazza described a proof of this for some particular groups.

Another major topic of the meeting was Broué's Abelian Defect Group Conjecture on equivalences of derived categories of block algebras. Koshitani and Kunugi described some work verifying this, and some more general equivalences of derived categories, for many blocks in characteristic 3, while Chuang presented his recent remarkable theorem with Rouquier that the derived category of any block of a symmetric group is determined by its defect group, which (using an earlier result of Chuang and Kessar) proves Broué's conjecture for all blocks of symmetric groups.

Benson talked on some deep new applications of commutative algebra to group cohomology, with an application to computational calculations of cohomology rings.

One active area of research is the study of Hecke algebras for Weyl groups and more generally complex reflection groups. S. Ariki described his determination of the representation types of Iwahori-Hecke algebras of classical type. This completes his earlier results about which he talked at the meeting in 2001. M. Geck reported on the construction of canonical basic sets for all types of Iwahori-Hecke algebras. This work of his PhD-student N. Jacon which allows now a natural parametrization of simple modules of arbitrary specializations uses the relation between crystal graphs and decomposition numbers. L. Iancu presented her joint work with C. Bonnafé in which they obtain the explicit descriptions of two-sided Hecke algebras of type $B_n$ for certain asymptotic choices of parameters. B. Leclerc described bis joint work with H. Miayachi which interprets constructible characters of Iwahori-Hecke algebras of classical types in terms of suitable canonical bases. R. Rouquier presented common work with V. Ginzburg, N. Guay and E. Opdam giving a construction of $q$-Schur algebras associated to complex reflection groups, using the rational Cherednik algebra.

C. Bonnafé gave an introduction to his work on the Lusztig restriction of Gelfand-Graev characters of connected reductive groups over a finite field. This should be seen as a major step towards the proof of Lusztig's conjecture to the important open case of $\mathrm{SL}_n(q)$. P.H. Tiep gave an overview on applications of the representation theory of quasi-simple groups to the determination of low dimensional representations and on applications of this to various problems like the non-coprime $k(GV)$-problem. One further contribution to the problem of determining small modules was given by F. Lübeck who combined theoretical results and algorithmic methods to list all small modules of groups of Lie-type in their defining characteristic.

Another group of talks centred on the representation theory of the symmetric groups. One important method here are Schur algebras. S. Doty gave an overview talk on recent progress by K. Erdmann, A. Henke and himself in this area, while A. Henke showed how structural properties of decomposition matrices can be derived from this approach. D. Nakano reported on joint work with D. Hemmer showing that for primes $p \geq 5$ multiplicities in Specht filtrations are well-defined.

# Abstracts

### On Navarro's McKay correspondence for a special class of $p$-solvable groups

Jonathan L. Alperin

Let $P$ be a Sylow $p$-subgroup of the finite group $G$. Assume that $G$ is $p$-solvable. In the case that $P$ is self-normalizing, Navarro has given a proof of the McKay conjecture in terms of a natural character correspondence. It is our purpose to give results about related Hecke algebras which give structural reasons behind the character result. The Hecke algebra used is the one for the $G$-set $G/P'$, where $P'$ is the derived group of $P$. Results are in terms of the representations of this algebra and its ring structure.

A final result give a a new natural one-to-one correspondence. Assume that $G$ is still p-solvable and that $G$ has no normal non-identity subgroup of order prime to $p$. There is a one-to-one correspondence between the simple modules in characteristic $p$ of $N(P)/P'$ and the simple modules in characteristic $p$ of the above Hecke algebra.

### Representation types of Hecke algebras

Susumu Ariki

Let $W$ be a finite Weyl group (not necessarily irreducible), $P_W(x)$ its Poincare polynomial. Let $F$ be an algebraically closed field, $q \in F^\times$. We denote by $\mathcal{H}_W(q)$ the associated Hecke algebra. We assume that $W$ is of classical type and $q$ is a primitive $e^{th}$ root of unity with $e \geq 2$.

**Theorem 1.**

(1) Suppose that $e \geq 3$. Then $\mathcal{H}_W(q)$ is
- finite if $(x-q)^2$ does not divide $P_W(x)$,
- wild otherwise.

(2) Suppose that $e = 2$. Then $\mathcal{H}_W(q)$ is
- finite if $(x+1)^2$ does not divide $P_W(x)$,
- tame if $(x+1)^2$ divides but $(x+1)^3$ does not divide $P_W(x)$,
- wild otherwise.

Hence, we have determined the representation type of the Hecke algebras of classical types completely.

The proof uses the Fock space theory, which was developed for proving the LLT conjecture, and the Specht module theory, which was developed by Dipper, James and Murphy in this case, and many results from the theory of finite dimensional algebras. Note that the finiteness result was already proved by the author, based on my previous work with Mathas.

The details and the references can be found in math.QA/0302136.

### Commutative algebra in the cohomology of groups.

Dave Benson

Let $G$ be a finite group and $k$ a field of characteristic $p$. Then $H^*(G,k)$ is a finitely generated graded commutative $k$-algebra. One can compute the generators and relations up to a given degree by explicitly building a resolution on a computer. Jon Carlson has implemented algorithms for doing this. I discussed in my talk the problem of how you know when you have all the generators and relations. Based on some ideas of Carlson, I gave a method which depends on an analysis of the Koszul complex for a homogeneous system of parameters. This method is related to a conjecture, which states that the Castelnuovo-Mumford regularity of $H^*(G,k)$ is always equal to zero.

**Lusztig restriction of Gel'fand-Graev characters**

Cédric Bonnafé

Let $G$ be a connected reductive group defined over a finite field with $q$ elements and let $F : G \to G$ be the corresponding Frobenius endomorphism. Lusztig constructed in 1985 an orthonormal basis $\mathrm{Ch}(G^F)$ of the space $\mathrm{Class}(G^F)$ of class functions on the finite group $G^F$ using geometric objects (perverse sheaves) called *character sheaves*. He also conjecture the explicit form of the transition matrix between $\mathrm{Irr}(G^F)$ and $\mathrm{Ch}(G^F)$, up to roots of unity. This conjecture has been solved (and the roots of unity determined) by Shoji (1990) whenever $Z(G)$ is connected and by Waldspurger (2002) if $G$ is symplectic or orthogonal. For the case of the special linear group, I have made the following progresses:

**1.** The Lusztig's induction is known (1996).

**2.** The conjecture holds for cuspidal character sheaves (1999).

**3.** Lusztig's restriction of Gel'fand-Graev characters have been determined explicitly (2001).

It seems reasonable to expect that these ingredients will be sufficient for solving Lusztig's conjecture for $SL(n, q)$ (plus a large amount of patience).

I will concentrate on the last point, namely the Gel'fand-Graev characters, particularly because I have now a better proof than my previous one in 2001. I must also say that this result on Gel'fand-Graev characters holds in any group, not only the special linear one. It is an improvement of a previous theorem of Digne, Lehrer and Michel : their theorem says that the Lusztig's restriction of a Gel'fand-Graev character is a Gel'fand-Graev character. However, whenever the centre of $G$ is not connected, there are several Gel'fand-Graev characters, and they were not able to determine which one was obtained after restriction. This explicit determination is the object of my new result.

First, using Alvis-Curtis duality, DLM's theorem is equivalent to the fact that the Lusztig restriction of the characteristic function of a regular unipotent class in $G^F$ is again a regular unipotent class. My improvement consists in finding this new regular unipotent class. First, I construct (independently of any huge theory like Deligne-Lusztig theory or character sheaves theory) an explicit map between the set of regular unipotent classes of $G^F$ and the corresponding set in an $F$-stable Levi subgroup $M$. This map has good properties (transitivity, compatibility with outer automorphisms of $G$ commuting with $F$...). Moreover, it coincides with the map induced by DLM's theorem whenever $M$ is the Levi subgroup of an $F$-stable parabolic subgroup.

Despite of these evidences, these two maps don't coincide in general (for instance, one can check on the character table of $SL(4, 3)$ given by `GAP` that this would lead to a contradiction). To determine the difference, one needs to introduce in DLM's proof a new ingredient, namely the explicit computation of the endomorphism algebra of an induced cuspidal character sheaves. A general treatment of this question was given in [1] for any cuspidal character sheaves, and we get an explicit result whenever the cuspidal character sheaf is supported by the regular unipotent class of $G$ (see [2]). Only these cuspidal character sheaves are relevant for our problem, and this explicit computation was exactly the missing point in DLM's proof for obtaining a more precise result.

[1] C. Bonnafé, *Actions of relative Weyl group I*, to appear in the J. of Group theory.

[2] C. Bonnafé, *Actions of relative Weyl group II*, preprint.

**Biset functors, rational representations, and the Dade group**

Serge Bouc

In this talk, I have described some results about biset functors, defined by the following framework : let $A$ be a commutative ring, and $p$ be a prime. Let $\mathcal{C}_{A,p}$ denote the category whose objects are the finite $p$-groups, with $\mathrm{Hom}_{\mathcal{C}_{A,p}}(P,Q) = A \otimes_{\mathbb{Z}} B(Q \times P^{op})$ (where $B$ is the Burnside group), composition being given by the usual product $({}_RV_Q, {}_QU_P) \mapsto V \times_Q U$. Let moreover $\mathcal{F}_{A,p}$ denote the category of $A$-linear functors from $\mathcal{C}_{A,p}$ to $A$-Mod (*biset functors* for short).

1. [*Ritter-Segal*] The natural transformation $B \to R_{\mathbb{Q}}$ is surjective in $\mathcal{F}_{\mathbb{Z},p}$.
One can improve this theorem, in an explicit form, leading to the notion of *basic subgroup* and *isometric section* of a $p$-group $P$. There is a natural equivalence relation on these isometric sections, whose classes are in one to one correspondence with the irreducible rational representations of $P$.

2. If $k$ is a field, the full *lattice of subfunctors of* $kR_{\mathbb{Q}}$ can be described. In particular if $p \neq 2$, then $kR_{\mathbb{Q}}$ is a uniserial object of $\mathcal{F}_{k,p}$ (with structure still depending on $char(k)$). In If $char(k) = p$ ($p = 2$ allowed), this leads to a determination of $\dim_k S_{L,k}(P)$ for the simple functors $S_{L,k}$, where $L$ is a $p$-group of normal $p$-rank 1, different from $C_p$. If $char(k) = p = 2$, there is a natural uniserial subfunctor of $kR_{\mathbb{Q}}$, attached to the quaternion group $Q_8$, denoted by $H_{Q_8}$.

3. For any $A$, any $F \in \mathcal{F}_{A,p}$, and any $P$, there are *explicit split injections* $\underset{(V,U)\in\mathcal{S}}{\oplus} \partial F(V/U) \to F(P)$, where $\mathcal{S}$ is a set of representatives of equivalence classes of isometric sections of $P$, and $\partial F(V/U)$ is the submodule of "faithful" elements in $F(V/U)$.

4. Let $D^{\Omega}(P)$ be the subgroup of the Dade group of $P$ generated by the relative syzygies $\Omega_X$ ($X$ finite $P$-set). Then $D^{\Omega} \in \mathcal{F}_{\mathbb{Z},p}$. Denote by $\omega_X$ the linear form on $B(P)$ defined by $\omega_X(P/Q) = 1$ if $X^Q \neq \emptyset$, and 0 otherwise. Then there is a *unique natural transformation* $\Theta : B^* \to D^{\Omega}$ such that $\Theta_P(\omega_X) = \Omega_X$ for any $P$ and $X$. This leads to an *exact sequence* $0 \to R^*_{\mathbb{Q}} \to B^* \to D^{\Omega}/D^{\Omega}_{tors} \to 0$. As a corollary, the structure of $D^{\Omega}_{tors}(P)$ is explicitly known.

5. Even if the whole Dade group is not an object of $\mathcal{F}_{\mathbb{Z},p}$, because of "Galois twists", the proof of 3 can be adapted to show that there are *split injective maps* $\underset{(V,U)\in\mathcal{S}}{\oplus} T_{tors}(V/U) \to D_{tors}(P)$ (same notation), where $T(V/U)$ is the group of endo-trivial modules.

6. *Conjecture A :* These maps are always isomorphisms.
This is known to hold in particular for $p \neq 2$, by a recent result of J. Carlson and J. Thévenaz. Equivalently $D_{tors}(P) \cong (\mathbb{Z}/4\mathbb{Z})^{a_P} \oplus (\mathbb{Z}/2\mathbb{Z})^{b_P}$, where $a_P$ and $b_P$ are integers determined by the irreducible rational representations of $P$, or also by combinatorial knowledge of the lattice of subgroups of $P$.

7. *Conjecture B :*
   - If $p \neq 2$, then $D = D^{\Omega}$.
   - If $p = 2$, there is an "exact sequence" $0 \to D^{\Omega} \to D \to H_{Q_8} \to 0$.

## Constructing endo-trivial modules

Jon F. Carlson

Suppose that $G$ is a p-group and $K$ is a field of characteristic $p$. A few years ago, Alperin found a set of generators for a subgroup of the torsion free part of the group of endotrivial modules for $G$, using relative syzygies. Bouc and Thevenaz, independently of Alperin, developed a set of generators using tensor induction. We show in this lecture that there is a third and very simple method for producing a set of generators using a small amount of group cohomology. In every case, it is known that the subgroup generated by the set of generators has the same rank as the torsion free part of the group of endotrivial modules.

During the course of the meeting it was discovered that a variation on the new method would actually produce a complete set of generators for the torsion free part of the group of endotrivial $kG$-modules. Moreover, it is possible to characterize the generators in terms of their restrictions to elementary abelian subgroups. It remains to be seen if the generators can be written as relative syzygies or tensor induced modules.

## Derived equivalences for blocks of symmetric groups

Joseph Chuang

Rouquier and I have proved that two blocks of symmetric groups over an algebraically closed field are derived equivalent if and only if they have the same number of simple modules.

The tilting complexes used to realize the equivalences were constructed by Rickard in 1990. To prove that these complexes indeed work, we use the Lascoux-Leclerc-Thibon approach to modular representation theory of symmetric groups, especially the ideas of Ariki and Grojnowski. An analogy with the work of Cabanes-Rickard on Alvis-Curtis duality is another important guide.

Together with earlier results of Rickard, Marcus, and Kessar, this theorem implies that Broue's abelian defect group conjecture is true for all blocks of symmetric groups.

## The rational Schur algebra

Richard Dipper
(joint work with S. Doty)

Let $K$ be an infinite field, $V = K^n$ for some natural number $n$ and let r, s be non negative integers. Then the general linear group $G = GL_n(K)$ acts on the mixed tensor space $T = T_{r,s} = V^{\otimes r} \otimes V^{*\otimes s}$, where $V^*$ denotes the dual space $V^* = Hom_K(V, K)$. If $K$ is the complex field, it is known that this action is in Schur-Weyl duality with an action of the walled Brauer algebra $B^{(n)}_{r,s}$, a certain subalgebra of the Brauer algebra $B^{(n)}_{r+s}$.

The image of $KG$ in $End_K(T_{r,s})$ is the rational Schur algebra $S_K(n; r, s)$. It is shown that $S_K(n; r, s)$ satisfies $S$. Donkin's definition of generalized Schur algebras determined by a saturated set of weights in the poset of dominant weights for $G$. As a consequence, $S_K(n; r, s)$ possesses an integral form $S_{\mathbb{Z}}(n; r, s)$ such that $S_K(n; r, s) = K \otimes_{\mathbb{Z}} S(n; r, s)$ and in particular its $K$-dimension is independent of the choice of the field $K$.

The representations of rational Schur algebras determine all rational representations of $G$.

The hyper algebra $\mathcal{U}_K$ associated with the Lie algebra $\mathfrak{gl}_n(K)$ acts on $T_{r,s}$ such that the image of $\mathcal{U}_K$ in $End_K(Tr, s)$ is the rational Schur algebra $S_K(n; r, s)$. We give two presentations of the complex rational Schur algebra $S_{\mathbb{C}}(n; r, s)$ as epimorphic image of $\mathcal{U}_{\mathbb{C}}$.

All these constructions have $q$ deformations. As a consequence of a result of F. Stoll, generalizing Tit's deformation argument, it is shown that the action of the rational $q$-Schur algebra on mixed tensor space satisfies Schur-Weyl duality in the generic case.

## Generic Hecke algebras

Stephen R. Doty

(joint work with K. Erdmann and A. Henke)

We initiate the systematic study of endomorphism algebras of transitive permutation modules for symmetric groups. These finite-dimensional Hecke algebras include the group algebras of symmetric groups as a special case. They lead very naturally to the study of certain infinite-dimensional quotients of the hyperalgebra associated with the enveloping algebra $U(\mathfrak{gl}_n)$ of the general linear Lie algebra. These quotient algebras are closely related to (perhaps equal?) to the endomorphism algebra of an induced module $\mathrm{ind}_{T_n}^{GL_n} K_\lambda$; which is an injective $GL_n$-module. (Here $T_n$ is the maximal torus of diagonal elements in $GL_n$.) The finite-dimensional algebras mentioned earlier are quotients of the infinite-dimensional generic algebras. Moreover, the generic algebras come from Lusztig's modified form of a (quantized) enveloping algebra. All of these algebras — generic and their finite-dimensional quotients — are cellular algebras in the sense of Graham and Lehrer. We expect the study of these algebras to ultimately lead to new information about Schur algebras and symmetric groups. There is a $q$-version of the theory.

## Some consequences of the classification of blocks with trivial intersection defect groups.

Charles Eaton

Jianbei An and myself gave a complete list (up to Morita equivalence) of the blocks of finite groups with non-normal defect groups which intersect pairwise trivially.

Here we describe some applications, including to the following problems:
(i) Donovan's conjecture;
(ii) conditions for a block to be nilpotent;
(iii) Dade's conjecture and related conjectures;
(iv) bounds on the number of irreducible characters of at most a given height.

## Homomorphisms between Specht modules

Matt Fayers

In the early 1980s, James proved the Principle of Row Removal, a result relating decomposition numbers of symmetric groups to those of smaller symmetric groups. Here we prove an analogue (of Donkin's generalisation) of this result for the space $\mathrm{Hom}_{k\mathfrak{S}_n}(S^\lambda, S^\mu)$, where $\lambda$ and $\mu$ are partitions of $n$ and $k$ is a field of characteristic not two. Specifically, we prove the following.

Let $\lambda$ and $\mu$ be partitions of $n$, and suppose that for some $s > 0$ we have $\lambda_1 + \cdots + \lambda_s = \mu_1 + \cdots + \mu_s = m$, say. Define

$$^{\mathrm{T}} = (\lambda_1, \dots, \lambda_s), \qquad \mu^{\mathrm{T}} = (\mu_1, \dots, \mu_s),$$
$$\lambda^{\mathrm{B}} = (\lambda_s, \lambda_{s+1}, \dots), \qquad \mu^{\mathrm{B}} = (\mu_s, \mu_{s+1}, \dots).$$

Then, if $k$ is any field of characteristic not two, we have

$$\mathrm{Hom}_{k\mathfrak{S}_n}(S^\lambda, S^\mu) \cong \mathrm{Hom}_{k\mathfrak{S}_m}(S^{\lambda^{\mathrm{T}}}, S^{\mu^{\mathrm{T}}}) \otimes \mathrm{Hom}_{k\mathfrak{S}_{n-m}}(S^{\lambda^{\mathrm{B}}}, S^{\mu^{\mathrm{b}}})$$

as $k$-vector spaces.

A 'column removal' analogue is also true.

## Modular representations of Iwahori–Hecke algebras

Meinolf Geck

Let $W$ be a finite Weyl group and $H$ be the generic (one-parameter) Iwahori–Hecke algebra associated with $W$ over the ring of Laurent polynomials $A = \mathbf{Z}[u^{1/2}, u^{-1/2}]$. Let $K$ be the field of fractions of $A$ and $\mathcal{O} \supseteq A$ be a discrete valuation ring in $K$, with residue field $k$. We assume that the characteristic of $k$ is either 0 or a prime which is "good" for $W$. Then, by results of Rouquier and myself, we have a canonical subset $\mathcal{B} \subseteq \mathrm{Irr}(H_K)$ which is in bijection with $\mathrm{Irr}(H_k)$ and such that the corresponding decomposition matrix has a lower unitriangular shape with 1 on the diagonal. The set $\mathcal{B}$ might be called the *canonical basic set* (with respect to $\mathcal{O}$); it leads to a "natural" parametrization of the simple modules of $H_k$.

The purpose of the talk is to report on recent results which complete the explicit determination of these canonical basic sets for all types of $W$ and all possible choices of $\mathcal{O}$ as above. These recent results are due to Nicolas Jacon (a Ph. D. student of mine at the Université Lyon 1) who showed that, in types $B_n$ and $D_n$, the set $\mathcal{B}$ is parametrized by pairs of partitions arising from a certain crystal graph studied by Foda et al. The proof makes use of Ariki's theory relating decomposition numbers with canonical bases of Fock spaces. Type $A_n$ is covered by results of Dipper and James, while the canonical basic sets for the exceptional types can be deduced from the explicit tables of decomposition numbers obtained by Lux, Müller and myself.

## Schuralgebras: From numerical properties to structural properties.

Anne E. Henke

(joint work with Doty and Erdmann)

Let $char k = p > 0$. The Sierpinski gasket is the combinatorial structure underlying the representation theory of $S(2, r)$, or a certain quotient of $KS_r$ associated to two-part partitions. This combinatorics is described by binomial coefficients modulo p, and as fractal structure provides a rich source for repeating numerical patterns. A number of representation theoretical interpretations of such patterns is known; for example Schur subalgebras, Ringel-selfdual algebras or the fact, that the family $S(2, r)$, $r \in \mathbb{N}$ has only one generic block for a given number of simples.

Refining the idea of looking at binomial coefficients modulo p to looking at the actual p-adic expansions seems to provide even further information. An example is given by analysing the p-Kostka numbers: the corresponding binomial coefficients reflect a set of primitive orthogonal idempotents of the algebras $End_{KS_r}(M^\lambda)$, the Hecke algebras associated to two-part partitions. As consequence, a description of the blocks and PIMs of these algebras has been obtained.

## Projective summands in tensor products of simple modules of finite dimensional Hopf algebras

Gerhard Hiss

This is a talk on joint work with Hui-Xiang Chen.

Let $H$ be a finite dimensional Hopf algebra over an algebraically closed field $k$ of characteristic $p$. I shall discuss a couple of results on projective direct summands of tensor products of simple $H$-modules.

(1) Suppose that there is a number $n$ such that every projective indecomposable $H$-module is contained in the tensor product of $n$ suitable simple modules. (If $H = kG$ is the group algebra of the finite group $G$ this is the case if and only if $O_p(G) = 1$.) What can be said about the smallest such number $l(H)$? How is $l(H)$ related to $l(L)$ for certain Hopf subalgebras $L$? Some examples of $l(H)$ for group algebras $H$ are given.

(2) Suppose that $H$ is co-commutative. Then the following reciprocity law holds for any three simple $H$-modules $V_1$, $V_2$ and $V_3$: The number of direct summands of $V_1 \otimes_k V_2$ isomorphic to the projective cover of $V_3$ is equal to the number of direct summands of $V_1 \otimes_k V_3^*$ isomorphic to the projective cover of $V_2^*$.

## Kazhdan-Lusztig cells in type $B_n$ and Robinson-Schensted correspondence

Lacri Iancu

(joint work with Cédric Bonnafé)

We study left cells in Iwahori-Hecke algebras of type $B_n$ with unequal parameters, for a certain class of choices of the parameters, which we call "the asymptotic situation". We give a combinatorial description of the left cells in terms of a Robinson-Schensted correspondence for the Coxeter group $W_n = W(B_n)$. Namely, one can associate to each element $w \in W_n$ a pair $(\mathbf{A}(\mathbf{w}), \mathbf{B}(\mathbf{w}))$ of standard bitableaux of total size $n$ and of the same shape (shape given by a bipartition $\lambda = (\lambda_\mathbf{1}, \lambda_\mathbf{2})$ of $n$).

Our main result states that two elements $x, y \in W_n$ are in the same left cell iff $\mathbf{B}(\mathbf{x}) = \mathbf{B}(\mathbf{y})$. We also give a description of the representation carried by a left cell in terms of its shape $\lambda$.

## Symmetric group blocks of small defect

Gordon James

It is usually the case that the $p$-modular decomposition numbers for a block are easiest to find when the block has small defect.

For the symmetric groups, the defect of a block is the same as the weight $w$ of the block provided that $p > w$, so let us assume that $p > w$. For blocks of weight $0, 1$ or 2 the combinatorics of partitions can be exploited to determine the decomposition numbers (see the work of Scopes and Richards). The situation is much harder for weight 3. It is claimed in [S. Martin and L. Russell, Defect 3 blocks of symmetric group algebras, J. Algebra 213 (1999) 304-339] that the decomposition numbers can be evaluated recursively, and that they are all 0 or 1, but the proof contains gaps and mistakes which mean that the case $w = 3$ is still open. We showed ways of making progress, and pointed out where the difficulties lie.

## Polynomial functors in prime degree

Steffen König

(joint work with Alexander Zimmermann)

Polynomial functors frequently occur in algebraic topology, K-theory and cohomology of finite groups, for example in the following subject areas: classification problems in homotopy theory, unstable modules over the Steenrod algebra, stable K-theory and functor cohomology, calculus of functors.

Drozd computed quadratic functors from free abelian groups to modules over 2-adic integers and cubic functors from free abelian groups to modules over 3-adic integers. In each case, the functors are modules over a classical order, which he was able to describe explicitly. He conjectured that for any prime p, the functors of degree up to p from free abelian groups to modules over p-adic integers are modules over an explicitly given classical order. A consequence of this conjecture is that in each case there are only finitely many torsionfree such functors and the torsion functors can be classified (tame situation).

We prove this conjecture by combining information about symmetric groups and Schur algebras with Friedlander and Suslin's result on functor cohomology and with an equivalence of categories.

## Broué's and Rickard's conjecture on $p$-blocks with abelian defect groups of finite groups

Shigeo Koshitani

In representation theory of finite groups there is an important and well-known conjecture due to Michel Broué, which is called **Broué's abelian defect group conjecture** (Broué's conjecture, for short). He conjectures that, for any prime number $p$, if $A$ is a $p$-block of a finite group $G$ with abelian defect group $P$, and if $B$ is a $p$-block of the normalizer $N_G(P)$ of $P$ in $G$ which is the Brauer corresponding block of $A$, then the (bounded) derived category $D^b(\text{mod}{-}A)$ of the category mod$-A$ of finitely generated $A$-modules and that of mod$-B$ are equivalent as triangulated categories. There are several cases where Broué's conjecture is checked. In this talk we present that Broué's conjecture is true when $A$ is a non-principal 3-block with elementary abelian defect group $P$ of order 9, and $G$ is one of the following (sporadic) simple groups such as; $G = HS$ (Higman-Sims), $O'N$ (O'Nan), $Suz$ (sporadic) and $He$ (Held).

## Examples of Splendid Equivalent Blocks with Non-Abelian Defect Groups

Naoko Kunugi

Let $k$ be an algebraically closed field of characteristic $p > 0$. We consider equivalences between principal blocks of two finite groups having the same $p$-local structure. If such groups have abelian Sylow $p$-subgroups then Broué and Rickard conjectured that these blocks would be splendid equivalent. We consider here cases of blocks with non-abelian defect groups.

In the following we assume that the field $k$ is of characteristic 3.

**Theorem 1** (with Y. Usami). *The principal blocks of $kPGL(3, 2^2)$ and $k[(Z_3 \times Z_3) : SL(2, 3)]$ are splendid equivalent.*

**Corollary 2.** *Let $q$ be a prime power such that $q + 1$ is divisible by 3 and is not divisible by 9. Then the principal blocks of $kPGL(3, q^2)$ and $kPGU(3, q^2)$ are splendid equivalent.*

**Theorem 3** (with T. Okuyama). *Let $q$ be a prime power such that $q + 1$ is divisible by 3. Then we have the following.*

(1) *The principal blocks of $kPSL(3, q^2)$ and $kPSU(3, q^2)$ are splendid equivalent.*

(2) *The principal blocks of $kPGL(3, q^2)$ and $kPGU(3, q^2)$ are splendid equivalent.*

We also consider central extensions of these groups. ¿From the constructions of the equivalences above, we have the following.

**Theorem 4** (with T. Okuyama). (1) *The principal 3-blocks of $kSL(3, q^2)$ and $kSU(3, q^2)$ are splendid equivalent.*

(2) *The principal 3-blocks of $kGL(3, q^2)$ and $kGU(3, q^2)$ are splendid equivalent.*

## Constructible characters and canonical bases

Bernard Leclerc

In a joint work with Hyohe Miyachi, we have given closed formulas for all vectors of the canonical basis of a level 2 irreducible integrable representation of the quantum algebra $U_v(sl_\infty)$. These formulas coincide at $v = 1$ with some formulas of Lusztig for the constructible characters of the Iwahori-Hecke algebras of type B and D. This suggests that the canonical basis vectors for level higher than 2 could be some natural candidates for constructible characters of Ariki-Koike algebras.

## Small degree representations of Chevalley groups in defining characteristic

Frank Lübeck

In this talk I discussed the following problem. Given a simple simply connected reductive group $G$ over an algebraically closed field $k$, and given a bound $M \in \mathbb{N}$. What are the (characters/degrees of) irreducible representations of G over $k$?

These representations are parametrized by highest weights. If the characteristic is zero or big enough (depending on the highest weight) then the characters and degrees are known by efficient formulae from the classical theory of finite dimensional complex Lie algebras.

When the characteristic is big enough compared to the rank of $G$ then there is a conjectured formula for the characters of the representations by Lusztig. In small characteristic nothing like that is known.

Nevertheless, there is an algorithm that in principle allows to compute for a given root system of $G$ and given highest weight the character of the corresponding irreducible representations for all characteristics of $k$. The idea is to construct a $\mathbb{Z}$-lattice in the Weyl module corresponding to the highest weight over the complex numbers. There is a computable integral bilinear form on that lattice and the reduction of the Gram matrix modulo a prime $p$ describes the desired character if $k$ has characteristic $p$.

I have explained how this idea could be made practical to obtain systematic results for the question above for reasonable bounds $M$ (e.g., $M = 300$ for $G = Spin_7(k)$, $M = 100000$ for $G = E_8(k)$ or $M \sim l^3$ for $G$ of rank $l \geq 12$).

I remarked that in the case $k = \bar{\mathbb{F}}_p$ and a Frobenius morphism $F : G \to G$ the irreducible representations of the finite group of Lie type $G^F$ over $k$ are obtained by restrictions of representations of $G$, thus justifying the title of the talk.

## Alternating Hecke algebras

Andrew Mathas

(joint work with Leah Ratliff)

Let $W$ be a (finite) Coxeter group and let $A = Alt(W)$ be its alternating subgroup; that is, the kernel of the sign representation. Just as with Coxeter groups, it is possible to completely characterize "alternating groups" in terms of their presentations.

Let $H_W$ be the Iwahori-Hecke algebra of $W$. Then $H_w$ has a natural subalgebra $H_A$ which is a deformation algebra of the the alternating subgroup $A$. Using a generalization of of Reidemeister-Schreier, we give a presentation for $H_A$. The proof is case free.

## The Dade group of (some) extra-special $p$-groups.

Nadia Mazza

Let $p$ be a prime number, $k$ be an algebraically closed field with characteristic $p$ and $P$ be a finite $p$-group. We denote by $D(P)$ the Dade group of $P$ and by $D^{\Omega}(P)$ the subgroup of $D(P)$ generated by all relative syzygies of $P$. At present, it is conjectured that in "many" cases, that we can explicitly describe, we should have the equality $D^{\Omega}(P) = D(P)$.

Using previous results of S. Bouc, J. Carlson and J. Thévenaz, we could prove that this conjecture holds in the following two cases:

**Theorem 1.** Assume that $p = 2$ and $P$ is an extra-special 2-group of the shape $\underbrace{D_8 * \cdots * D_8}_{n \geq 2}$, where $D_8$ denotes the dihedral group of order 8. Let $\mathcal{Q}$ be a set of representatives of the conjugacy classes of subgroups of $P$ of index at least 4, and $Q_0 \in \mathcal{Q}$ be a maximal element such that $Q_0$ is not normal in $P$. Then, the subset $\{\Omega_{P/Q} \mid Q \in \mathcal{Q} \, : \, Q \neq Q_0\}$ of $D^{\Omega}(P)$ is a $\mathbb{Z}$-basis of $D(P)$.

In particular, we have $D(P) = D^{\Omega}(P) \cong \mathbb{Z}^{|\mathcal{Q}|-1}$.

To prove this statement, we first use a result of J. Carlson and J. Thévenaz, which tells us that the torsion subgroup of $D(P)$ is trivial. Then, we notice that $D(P)$ decomposes as a direct sum $D(P) = \ker(\mathrm{Def}^P_{P/Z}) \oplus D(P/Z)$, where $Z$ denotes the centre of $P$ and $\mathrm{Def}^P_{P/Z}$ the deflation map. Finally, we proceed by induction on the order of $P$ (starting with $|P| = 32$) and using known facts on the group of endo-trivial modules of $P$ (which is isomorphic to $\mathbb{Z}$, generated by $\Omega_P$) and on the Dade group of the dihedral group of order 8.

**Theorem 2.** Let $p$ be an odd prime number and $P$ an extra-special $p$-group of order $p^3$. Then, $D(P) = D^{\Omega}(P)$. That is,

- If $P$ has exponent $p^2$, then $D(P) \cong \mathbb{Z}^2 \oplus (\mathbb{Z}/2\mathbb{Z})^{p+2}$.
- If $P$ has exponent $p$, then $D(P) \cong \mathbb{Z}^{p+2} \oplus (\mathbb{Z}/2\mathbb{Z})^{p+2}$.

The first case follows a previous author's result, concerned with the Dade group of metacyclic $p$-groups for an odd prime number $p$ (see J. of Alg., to appear). For the second case we apply similar techniques to the ones used in the extra-special 2-group considered above. However, in this case an additional cohomological argument due to J. Carlson was necessary to determine the group of endo-trivial modules.

## Some degenerate unipotent blocks

Hyohe Miyachi

**Definition 0.1.** *Let $G$ be a finite group of Lie type. Let $A$ be a unipotent block ideal of $G$ with $\Phi_e$-defect torus $T$ and canonical character $\lambda$ in $M = \mathcal{Z}_G(T)$. Let $W(M, \lambda)$ be the inertial group of $A$. We say that $A$ is a Rouquier block if there exists a Levi subgroup $L$ of $G$ such that*

1. *there exists parabolic subgroup $P$ of $G$ with Levi decomposition $P = LU_P$,*
2. *$L$ contains $M$,*
3. *$H = L \cdot W(M, \lambda)$ is a proper subgroup of $G$,*
4. *There exists a block $B$ of $H$ with canonical character $\lambda$ such that $A$ is Morita equivalent to $B$.*

One aim of this note is to report that there are Rouquier blocks which are not of type $A$ and are not included in Puig's theorem.

In through this note, we assume that a prime number $\ell$ and a prime power $q$ satisfy the following condition:

$$\text{(1)} \qquad \text{(i) } \ell \neq 2, 3, \quad \text{(ii) } \ell \text{ divides } q^2 + 1, \text{ and } \quad \text{(iii) } q \text{ is odd.}$$

**Proposition 0.2** (Leclerc-Miyachi)**.** *Let $A$ be the unipotent $\ell$-block ideal of $D_8(q)$ in which $\phi_{11.6}$ lies. Put $H$ to be $N_{D_8(q)}(D_6 \times A_1(q))$. Let $B$ be the "unipotent" $\ell$-block ideal of $H$ such that $B \cdot (\phi_{1.5} \uparrow^H_{D_6 \times A_1(q)}) \neq 0$. Then, $A$ and $B$ are Morita equivalent.*

**Remark 0.3.** *It is interesting to think about the following problems:*

1. *To generalize the above Proposition to the case for any even multiplicative order $e$ of $q$ and any $e$-weight.*
2. *To construct Lascoux-Leclerc-Thibon's decomposition number version by Jimbo-Misra-Miwa-Okado's labelling of crystals.*

**Proposition 0.4.** *The principal $\ell$-block ideal of $E_6(q)$ is Morita equivalent to the principal $\ell$-block ideal of $N_{E_6(q)}(D_4(q))$.*

**Remark 0.5.** *Using the above Proposition, we can determine the decomposition numbers for the principal $\ell$-block of $D_4(q)$ except $d_{\mathrm{St},\overline{\phi}}$ where* St *is the Steinberg character and $\overline{\phi}$ is an $\ell$-modular reduction of the unique cuspidal unipotent character $\phi$ of $D_4(q)$. Moreover, once we know $d_{\mathrm{St},\overline{\phi}}$ we can determine the decomposition numbers for the principal $\ell$-block of $E_6(q)$.*

**Proposition 0.6.** *There are two Scopes' equivalences $\tau_{\pm}$ between the principal $\ell$-block ideal of $E_6(q)$ and $A$. Here, $A$ is the unipotent $\ell$-block ideal of $E_8(q)$ in which $\phi_{28,8}$ lies.*

## Brauer trees for the Schur cover of the symmetric group

Jürgen Müller

For $n \geq 4$ the Schur covering group $\tilde{\mathcal{S}}_n$ of the symmetric group $\mathcal{S}_n$ is a non-split central extension of $\mathcal{S}_n$ by a cyclic group of order 2, uniquely defined up to isoclinism; for $n \geq 8$ the Schur covering group $\tilde{\mathcal{A}}_n$ of the alternating group $\mathcal{A}_n$ is a non-split central extension of $\mathcal{A}_n$ by a cyclic group of order 2, uniquely defined up to isomorphism. While the ordinary character theory of $\mathcal{S}_n$ and its relatives is well-understood, the $p$-modular Brauer characters of $\tilde{\mathcal{S}}_n$ are not known. The aim of this talk is to describe the easiest of the non-trivial faithful $p$-blocks of the groups $\tilde{\mathcal{S}}_n$ and $\tilde{\mathcal{A}}_n$, namely their blocks of cyclic defect, which are of defect 1, in odd characteristics $p$.

As these $p$-blocks are Brauer tree algebras, their structure is given in terms of their Brauer trees. The general pattern is easily described: All the Brauer trees are 4-fold stars, possibly with two different branch lengths occurring. The precise shape of the Brauer trees as well as the labelling of its vertices is given in terms of combinatorial data related to the ordinary character theory of $\tilde{\mathcal{S}}_n$.

As a consequence, keeping $p$ fixed but letting $n$ vary, there are $\lfloor \frac{p+3}{4} \rfloor$ Morita types of faithful $p$-blocks of $\tilde{\mathcal{S}}_n$ of cyclic defect; this contrasts the situation for $\mathcal{S}_n$, where there is only one Morita type of $p$-blocks of cyclic defect. Furthermore, by a result of Feit, all Brauer trees occurring for finite groups already occur as unfoldings of Brauer trees for finite quasi-simple groups; using the classification of finite simple groups, much is known for the latter class of groups. One of the last remaining gaps, the case of $\tilde{\mathcal{A}}_n$, is settled now as well.

## Specht filtrations for symmetric groups (and Hecke algebras of type A)

Daniel K. Nakano

(joint work with David J. Hemmer)

Let $S_d$ be the symmetric group on d letters and $k$ be a field of characteristic $p > 0$. Using the results from the cohomology of the general linear group and recent results about the Schur and adjoint Schur functor we show that (contrary to expectations) that for $p$ greater than or equal to 5, the multiplicities in a Specht or dual Specht module are well-defined. Necessary and sufficient conditions are also provided for such filtrations to exist. Applications will be later given pertaining to this result as well as a number of interesting open questions.

## The local Grothendieck group: a refinement of the counting characters conjectures

Lluis Puig

We introduce two new invariants of a block — the *local Grothendieck groups* — as the inverse limits of the usual Grothendieck groups in both characteristics, zero and $p$, of the $k^*$-localizers of chains of selfcentralizing pointed groups of the block; as a matter of fact, in the general case this definition demands the existence of a $k^*$-*lifting* of the *localizing functor* introduced in 1990 in a series of lectures at the MSRI, and the proof of this existence has been completed in the Institut. On the one hand, we prove that local Grothendieck groups fulfill the obvious translation of Alperin's conjecture in Robinson's form, and we hope to get a similar result for Dade's conjecture; more precisely, we prove the exactness of the standard differential sequences with coefficients in $\mathbb{Z}(p)$. Actually, the usual relationship between the Grothendieck group in characteristic zero and those of the centralizers of the $p$-elements in characteristic $p$ still holds for the local Grothendieck groups, and the proof can be carry out in the more general framework of the Frobenius systems.

On the other hand, we prove comparison results, both for global and for local Grothendieck groups, between a group and suitable normal subgroups, in order to reduce the final comparison between *global and local* to a suitable precise statement for any simple group. Naturally, we cannot help to conjecture that, at least with coefficients in $\mathbb{Z}(p)$, *global and local Grothendieck groups are isomorphic*; moreover, since they are mathematical structures — not just numbers — it seems reasonable to conjecture the existence of an isomorphism compatible with the suitable outer automorphisms (the so-called *equivariant conjecture*), with the suitable Galois automorphisms (Navarro's conjecture), with a conjectural *height filtration* (Dade's conjecture), with a *reduced degree* in the graded height sections (Isaac-Navarro's conjecture in the first step and Uno's conjecture in general) and with the *determinant homomorphisms*.

The conjecture is trivially true in the $p$-solvable case and there is some hope in finding a general pattern in the infinite series of simple groups which would lead to the verification of the conjecture in them. Presently, all the elements seem ready to guarantee that the coincidence of global and local Grothendieck groups *with coefficients in* $\mathbb{Q}$ can be reduced to a suitable precise statement for any simple group.

## On the $k(GV)$-Conjecture

Udo Riese

Richard Brauer's famous $k(B)$-problem states that

$$k(B) \leq |D|$$

whenever $B$ is a $p$-block of the finite group $G$ with defect group $D$.

For $p$-solvable groups this reduces to the so-called $k(GV)$-conjecture. Very recent work, joint with Peter Schmid, solves the last open case in characteristic $p = 5$. David Gluck and Kay Magaard solved this case at the same time with the same methods (basically). It is intended to join these two solutions in some form.

In the talk we give an overview on the now completed proof of the $k(B)$-problem for $p$-solvable groups. The basic steps where first the reduction of Nagao to the $k(GV)$-conjecture, then the concept of real vectors (Knörr, Gow, Robinson-Thompson), the classification of all nonreal pairs $(G, V)$ (Goodwin, Gluck-Magaard, Köhler-Pahlings, R.). At this stage of the proof one was left with the possible characteristics $p = 3, 5, 7, 11, 13, 19$ and 31.

The final part of the proof began with the reduction to induced modules (Gluck-Magaard for $p = 31$, R.-Schmid) and the solution for the resulting groups in characteristic different from $p = 5$ (R.-Schmid). As mentioned above this last case ($p = 5$) was solved by two groups at the same time.

### Bounding the number and heights of irreducible characters of $p$-constrained groups

Geoffrey R. Robinson

We consider a finite group $G$ which has an Abelian normal subgroup $V$ such that $C_G(V)$ is a $p$-group. We let $S$ be a Sylow $p$-subgroup of $G$, $H = G/V$, $P = S/V$. For each $v \in V$, we set $H(v) = C_G(v)/V$. We present a new conjecture in both a weak and a strong form. For $d$ a non-negative integer, we let $k_d(G)$ denote the number of irreducible characters $\chi$ of $G$ which satisfy $p^d\chi(1)_p = |G|_p$. We let $G_{p'}$ denote the set of $p$-regular elements of $G$.

**CONJECTURE:** i) ( Strong form ) We have

$$\sum_{d=0}^{\infty} \frac{k_d(G)}{p^{2d}} \leq \frac{1}{|V|} max\{\frac{|H(v)|_{p'}}{|H(v)|} : v \in C_V(S)\}.$$

i) ( Weak form ) We have

$$\sum_{d=0}^{\infty} \frac{k_d(G)}{p^{2d}} \leq \frac{1}{|V|},$$

with equality only possible when $V = S$.

We remark that it is a consequence of the famous paper of Brauer and Feit that we have

$$\sum_{d=0}^{\infty} \frac{k_d(G)}{p^{2d}} \leq 1,$$

and implicitly that

$$\sum_{d=0}^{\infty} \frac{k_d(G)}{p^{2d}} \leq \frac{|G|_{p'}}{|G|}.$$

In the case that $G$ is $p$-solvable, the bound of the new conjecture is genuinely stronger than the bound of Brauer's $k(B)$-problem. In general, the predicted bound means that the smaller the value of $d$, the more restrictive the bound becomes. Fore example, there should be at most $|V|$ irreducible characters of the minimal possible defect $log_p(|P|)$.

We prove the conjecture in the case that there is an element $v \in C_V(S)$ such that $Res^G_{C_G(v)}(V)$ has a submodule $U$ on which no non-trivial $p$-regular element acts trivially and which affords a real-valued Brauer character.

Along the way, some results of general interest are proved.

### $q$-Schur algebras for complex reflection groups

Raphaël Rouquier

(joint work with V. Ginzburg, N. Guay and E. Opdam)

We explain how to construct a "quasi-hereditary cover" of Hecke algebras associated to complex reflection groups, over a field of characteristic 0. As a consequence, the decomposition matrices of these Hecke algebras are triangular.

In type $A$, such a cover is given by the double endomorphism ring of a $q$-tensor space. A direct construction has be given for type $B$. These $q$-Schur algebras play an important role is the modular representation theory of finite reductive groups in non-describing characteristic.

Our approach makes use of the rational Cherednik algebra. Using the triangular decomposition of that algebra, we define a category $\mathcal{O}$ and show it is a highest weight category. This category is equivalent to the category of modules over a finite dimensional quasi-hereditary algebra : this is our $q$-Schur algebra. We relate representations of the rational Cherednik algebra to Hecke algebra representations via a "Knizhnik-Zamolodchikov functor".

We conjecture that, in type $A$, our algebras are isomorphic to the usual $q$-Schur algebras. We can at least prove that the images of the standard modules by the KZ functor are Specht modules (in general, modules coming from Kazhdan-Lusztig theory).

## On the classification of endo-permutation modules

Jacques Thévenaz
(joint work with Jon Carlson)

Let $k$ be a field of characteristic $p$ and let $P$ be a finite $p$-group. Let $D(P)$ be the set of isomorphism classes of endo-permutation modules which are indecomposable with vertex $P$. Tensor product induces a structure of abelian group on $D(P)$, called the *Dade group* of $P$). Let $T(P)$ be the subgroup of $D(P)$ consisting of endo-trivial modules.

Recall that the torsion-free ranks of $T(P)$ and $D(P)$ have been determined recently (Alperin, Bouc-Thévenaz). We have now decisive new results about the torsion subgroup of $T(P)$ and $D(P)$.

**Theorem 1.** If $P$ is not cyclic, generalized quaternion, or semi-dihedral, then the torsion subgroup of $T(P)$ is trivial.

This is a consequence of the following detection theorem.

**Theorem 2.** If $P$ is not cyclic, generalized quaternion, or semi-dihedral, then the restriction homomorphism $T(P) \to \prod_E T(E)$ is injective, where $E$ runs over the set of elementary abelian subgroups of order $p^2$ (up to conjugation).

The assumption on $P$ is harmless since the complete structure of $T(P)$ and $D(P)$ is known if $P$ is cyclic, generalized quaternion, or semi-dihedral. The proof of Theorem 2 is rather involved and requires a large amount of group cohomology, recent bounds for the number of Bocksteins in Serre's theorem, Carlson's recent theorem expressing Serre's theorem in terms of modules, and finally the theory of varieties attached to modules.

The following detection theorem for $D(P)$ is deduced from Theorem 2.

**Theorem 3.** The deflation-restriction homomorphism $D(P) \to \prod_{H/K} D(H/K)$ is injective, where $H/K$ runs over the set of all sections of $P$ (up to conjugation) which are elementary abelian of order $p^2$, cyclic of order $p$, and in addition when $p = 2$, cyclic of order 4 and quaternion of order 8.

¿From this, the structure of the torsion subgroup of $D(P)$ is easily obtained using earlier work of Bouc-Thévenaz:

**Theorem 4.** When $p$ is odd, the torsion subgroup of $D(P)$ is isomorphic to $(Z/2Z)^s$, where $s$ is the number of conjugacy classes of non-trivial cyclic subgroups of $P$.

When $p = 2$, partial results on the torsion subgroup of $D(P)$ are known and an explicit conjecture, due to S. Bouc, is open.

Finally, an open question asks if $D(P)$ is generated by the relative syzygies of the trivial module when $p$ is odd (and another version of the question can be stated when $p = 2$). This has been proved by N. Mazza for a few families of $p$-groups where a complete description of $D(P)$ is obtained.

**Low dimensional representations of finite quasisimple groups and applications**

Pham Huu Tiep
(joint work with R. M. Guralnick)

Let $\mathbb{F}$ be an algebraically closed field of characteristic $\ell$. For any finite group $G$, let $d_\ell(G)$ denote the smallest degree of faithful $\mathbb{F}G$-representations. Fix a (small) positive $\epsilon$ ($\epsilon = 1/2$ would be good enough). In the first part of the talk we report on recent results concerning the following problem.

*Given a finite quasisimple group $G$, classify all irreducible $\mathbb{F}G$-representations of degree less than $d_\ell(G)^{2-\epsilon}$.*

Next we outline some applications that use results on this problem.

The first application is concerned with a conjecture of Larsen that originates from work of Deligne and Katz (on the Zariski closure of the monodromy group of some local systems associated to a given smooth projective complex variety). The conjecture says

*Let $V = \mathbb{C}^d$ with $d \geq 5$ and let $\mathcal{G} \in \{GL(V), Sp(V), O(V)\}$. Assume $X$ is a closed subgroup of $\mathcal{G}$ such that $X^\circ$ is reductive and $\dim(End_X(V^{\otimes 4})) = \dim(End_{\mathcal{G}}(V^{\otimes 4}))$. Then $X$ contains $\mathcal{G}^\circ$.*

We prove that Larsen's conjecture is true, with a single exception that $d = 6$, $\mathcal{G} = Sp(V)$, and $X = 2J_2$. This is a consequence of the following more general result. Let $\mathbb{F}$ be an algebraically closed field of characteristic $\ell$, $V = \mathbb{F}^d$ with $d \geq 5$, and $\mathcal{G} \in \{GL(V), Sp(V), O(V)\}$. Then we classify all closed subgroups $X$ of $\mathcal{G}$ such that $X$ is irreducible on every $\mathcal{G}$-composition factors of $V \otimes V^*$. This result is related to previous work of Liebeck-Seitz, Magaard, Malle, and Tiep. Among other things, this result has applications on generation results for finite groups of Lie type.

The second application is concerned with the following non-coprime version of the $k(GV)$-problem:

*Let $H$ be a finite group with a normal elementary abelian $p$-subgroup $V$ such that $C_H(V) = V$ and $O_p(H/V) = 1$. What can we say about $H$ if $k(H) > |V|$, where $k(H)$ is the number of conjugacy classes of $H$ ?*

This problem is closely related to Brauer's $k(B)$-problem, and to a recent conjecture of G. Robinson. We show that the $k(GV)$-problem can be reduced to the case where $H$ is a semidirect product of $V$ by $G$ and $G$ acts semisimply on $V$. Assume $G$ is almost quasisimple with $S$ the unique non-abelian composition factor of $G$. Then the problem can be further reduced to the case where $G$ is irreducible on $V$. Our first result shows that either $k(GV) \leq |V|$, or $S \in Lie(p)$, or $S$ belongs to a finite explicit list (currently containing 87) of finite simple groups. Next assume $S = PSL_n(q)$ and $n \geq 13$. Then we show that either $k(GV) \leq |V|$, or $G \rhd SL_n(q)$ and $V$ comes from the natural module $\mathbb{F}_q^n$ for $SL_n(q)$.

**A Hecke theoretic shadow of tensoring the crystal of the basic representation with a level 1 perfect crystal**

Monica Vazirani

The irreducible representations of the symmetric group $S_n$ are parametrized by partitions of $n$. Furthermore, one can use the data from the partition to construct the module algebraically.

Over a field of characteristic $p$, the irreducible representations of $S_n$ are parametrized by the "$p$-regular" partitions. They can be constructed as quotients of the characteristic 0 irreducibles taken $\mod p$.

We give an alternate construction of the modules, motivated by viewing Kleshchev's "socle of restriction" graph of $p$-regular partitions, which is also the crystal graph of the basic representation of $\widehat{\mathfrak{sl}}_p$, as a limit of tensor products of level 1 perfect crystals.

*Edited by Michael Cuntz*

# Participants

**Prof. Dr. Jonathan L. Alperin**
alperin@math.uchicago.edu
Department of Mathematics
The University of Chicago
5734 South University Avenue
Chicago, IL 60637-1514 - USA

**Prof. Dr. Susumu Ariki**
ariki@kurims.kyoto-u.ac.jp
Research Institute for
Mathematical Sciences
Kyoto University
Kitashirakawa, Sakyo-ku
Kyoto 606-8502 - Japan

**Prof. Dr. David J. Benson**
djb@byrd.math.uga.edu
Department of Mathematics
University of Georgia
Athens, GA 30602-7403 - USA

**Prof. Dr. Christine Bessenrodt**
bessen@math.uni-hannover.de
Institut für Mathematik
Universität Hannover
Welfengarten 1
D–30167 Hannover

**Dr. Robert Boltje**
boltje@math.ucsc.edu
Dept. of Mathematics
University of California
Santa Cruz, CA 95064 - USA

**Prof. Dr. Cedric Bonnafe**
bonnafe@vega.univ-fcomte.fr
Département de Mathématiques
UFR de Sciences et Techniques
Universite de Franche-Comté
Route de Gray
F-25000 Besancon

**Dr. Serge Bouc**
bouc@math.jussieu.fr
U.F.R. de Mathématiques
Case 7012
Université de Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

**Prof. Dr. Michel Broué**
broue@ihp.jussieu.fr
Institut Henri Poincaré
11, rue Pierre et Marie Curie
F-75231 Paris Cedex

**Prof. Dr. Jon F. Carlson**
jfc@sloth.math.uga.edu
Department of Mathematics
University of Georgia
Athens, GA 30602-7403 - USA

**Dr. Joseph Chuang**
Joseph.Chuang@bristol.ac.uk
Department of Mathematics
University of Bristol
University Walk
GB-Bristol, BS8 1TW

**Michael Cuntz**
cuntz@mathematik.uni-kassel.de
FB 17 - Mathematik/Informatik -
Universität Kassel
Heinrich-Plett-Str. 40
D–4132 Kassel

**Dr. Maud De Visscher**
m.devisscher@qmul.ac.uk
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road
GB-London, E1 4NS

**Prof. Dr. Richard Dipper**
rdipper@mathematik.uni-stuttgart.de
Institut für Algebra und
Zahlentheorie
Universität Stuttgart
Pfaffenwaldring 57
D–70569 Stuttgart

**Prof. Dr. Stephen Doty**
doty@math.luc.edu
Dept. of Mathematics and Statistics
Loyola University of Chicago
Chicago, IL 60626-5385 – USA

**Dr. Charles W. Eaton**
eatonc@for.mat.bham.ac.uk
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham, B15 2TT

**Dr. Matt Fayers**
m.fayers@dpmms.cam.ac.uk
Magdalene College
GB-Cambridge CB3 OAG

**Prof. Dr. Meinolf Geck**
geck@desargues.univ-lyon1.fr
Institut Girard Desargues
Université Lyon 1
Bâtiment Braconnier (ex-101)
21 Avenue Claude Bernard
F-69622 Villeurbanne Cedex

**Dr. Anne Henke**
A.Henke@mcs.le.ac.uk
Dept. of Mathematics
University of Leicester
University Road
GB-Leicester, LE1 7RH

**Dr. Gerhard Hiß**
Gerhard.Hiss@math.rwth-aachen.de
Lehrstuhl D für Mathematik
RWTH Aachen
Templergraben 64
D–52062 Aachen

**Prof. Dr. Lacri Iancu**
iancu@desargues.univ-lyon1.fr
Institut Girard Desargues
Université Claude Bernard
43, Bd. du 11 Novembre 1918
F-69622 Villeurbanne Cedex

**Prof. Dr. Gordon D. James**
g.james@ic.ac.uk
Dept. of Mathematics
Imperial College of Science
and Technology
180 Queen's Gate, Huxley Bldg
GB-London, SW7 2BZ

**Dr. Radha Kessar**
kessar@maths.ox.ac.uk
Department of Mathematics
Ohio State University
231 West 18th Avenue
Columbus, OH 43210-1174 – USA

**Dr. Steffen König**
sck5@mcs.le.ac.uk
Dept. of Math. and Computer Science
University of Leicester
University Road
GB-Leicester LE1 7RH

**Prof. Dr. Shigeo Koshitani**
koshitan@math.s.chiba-u.ac.jp
Department of Mathematics and Informatics
Faculty of Science
Chiba University
Yayoi-cho, 1-33
Chiba-Shi 263-8522 – Japan

**Prof. Dr. Burkhard Külshammer**
kuelshammer@uni-jena.de
Mathematisches Institut
Universität Jena
Ernst-Abbe-Platz 1-4
D–07743 Jena

**Dr. Naoko Kunugi**
kunugi@cc.ocha.ac.jp
School of Mathematics
Ochanomizu University
2-1-1, Otsuka
Bunkyo-ku
Tokyo 112-8610 – Japan

**Prof. Dr. Bernard Leclerc**
leclerc@math.unicaen.fr
Dept. de Mathematiques
Universite de Caen
F-14032 Caen Cedex

**Prof. Dr. Markus Linckelmann**
linckelm@math.ohio-state.edu
Department of Mathematics
Ohio State University
231 West 18th Avenue
Columbus, OH 43210-1174 – USA

**Dr. Frank Lübeck**
frank.luebeck@math.rwth-aachen.de
Lehrstuhl D für Mathematik
RWTH Aachen
Templergraben 64
D–52062 Aachen

**Dr. Gunter Martin Malle**
malle@mathematik.uni-kassel.de
FB 17 - Mathematik/Informatik -
Universität Kassel
Heinrich-Plett-Str. 40
D–34132 Kassel

**Prof. Dr. Andrew Mathas**
mathas@maths.usyd.edu.au
School of Mathematics & Statistics
University of Sydney
Sydney NSW 2006 - Australia

**Prof. Dr. Nadia Mazza**
Nadia.Mazza@ima.unil.ch
Institut de Mathématiques
BCH Faculté des Sciences
Université de Lausanne
CH-1015 Dorigny-Lausanne

**Prof. Dr. Hyohe Miyachi**
miyachi@ma.noda.sut.ac.jp
miyachi@ihes.fr
Institut des Hautes Etudes Scientifiques
Le Bois Marie
35, route de Chartres
F-91440 Bures-sur-Yvette

**Dr. Jürgen Müller**
mueller@math.rwth-aachen.de
Lehrstuhl D für Mathematik
RWTH Aachen
Templergraben 64
D–52062 Aachen

**Prof. Dr. Daniel K. Nakano**
nakano@math.uga.edu
Department of Mathematics
University of Georgia
Athens, GA 30602-7403 – USA

**Prof. Dr. Jörn Börling Olsson**
olsson@math.ku.dk
Matematisk Afdeling
Kobenhavns Universitet
Universitetsparken 5
DK-2100 Kobenhavn

**Prof. Dr. Lluis Puig**
puig@math.jussieu.fr
UFR de Mathématiques
Université de Paris VII
2, place Jussieu
F-75251 Paris Cedex 05

**Prof. Dr. Jeremy Rickard**
j.rickard@bris.ac.uk
School of Mathematics
University of Bristol
University Walk
GB-Bristol BS8 1TW

**Dr. Udo Riese**
udo.riese@uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
D–72076 Tübingen

**Prof. Dr. Geoffrey R. Robinson**
grr@for.mat.bham.ac.uk
g.r.robinson@bham.ac.uk
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

**Prof. Dr. Raphael Rouquier**
rouquier@math.jussieu.fr
Inst. de Mathématiques de Jussieu
Université Paris VI
175 rue du Chevaleret
F-75013 Paris

**Prof. Dr. Jacques Thevenaz**
jacques.thevenaz@ima.unil.ch
Institut de Mathématiques
Université de Lausanne
CH-1015 Lausanne Dorigny

**Prof. Dr. Pham Huu Tiep**
tiep@math.ufl.edu
Dept. of Mathematics
University of Florida
358 Little Hall
P.O.Box 118105
Gainesville, FL 32611-8105
USA

**Dr. Monica Vazirani**
vazirani@math.berkeley.edu
Mathematics 253 - 37
California Institute of Technology
Pasadena, CA 91125
USA