Mathematisches Forschungsinstitut Oberwolfach

Report No. 18/2003

Inverse Problems in Wave Scattering and Impedance Tomography

April 20th – April 26th, 2003

In organizing this conference the organizers have tried to adopt the general Oberwolfach style, which is meant to concentrate on fewer talks, leaving as much time as possible for joint collaborations. All participants, even those who could not present a paper this way, seemed to enjoy this experience, since this also left ample time for discussions at the end of each talk, allowing everybody – including the younger researchers – to actively participate.

The format itself consisted of three morning sessions with two one hour lectures each, which were delivered by invitation, and further sixteen half hour talks most of which were appointed on the eve of the conference at the institute. There were large breaks from noon to 4pm to allow discussions, and there have been obvious interactions between the participants until late in the evening. In addition there was one tutorial lecture on photonic crystals on Tuesday night, introducing the audience to an area not directly in the main theme of the conference but with high potential for applications. This lecture was as well attended as the other talks, and generally thought to be an excellent idea, one which should be continued at future meetings.

The invitation list had been made intentionally wide to achieve a good mixture of well established colleagues as well as very young people (there has even been one very interesting talk by a graduate student about her preliminary results towards a phD). The breakdown was approximately even with ten participants each from Germany, the other European countries, and the US, but there have also been participants from Tunisia, Israel, Korea, and Japan.

In selecting speakers the intention was to get a broad range of areas represented, rather than focussing on one particular subarea. There have been talks on theoretical results and on numerical analysis, as well as on new applications and their mathematical modelling, including the use of statistical models.

About one third of the talks was devoted to uniqueness results, namely the characterization of the minimal information required to recover the unknown quantities. For example, it is a long-standing conjecture in inverse scattering theory that one single incident wave with a full far-field measurement is sufficient to determine the shape of an obstacle. Over the years there have been incremental results reducing the restrictions on the obstacle that allow global and/or local uniqueness. This particular result also gained a significant amount of time in the discussions at the conference, since three of the talks were devoted to this topic and led to further substantial improvements. At the end, some participants even raised the idea to apply for a later miniworkshop at the Oberwolfach institute to join forces and work on a general proof of this conjecture.

A second theme was the question of stability. This is a critical issue because all the problems considered in this conference are ill-posed, that is, the object to be found does not depend continuously on the data. Therefore, it is a particularly difficult question to quantify the achievable error for a given amount of data, or vice versa, to specify the least precision in the measurement data to allow for a certain resolution in the reconstruction. As an alternative to determistic estimates, two talks discussed the potential of statistical methods in certain applications to model the uncertainty in the solution and in the reconstruction. This may well open a door to a lot more applications in the future.

Numerical algorithms were treated by quite a number of people. As compared to previous meetings on inverse problems, pointwise reconstruction methods (such as the sampling method or the factorization method) seem to have gained substantial interest because these methods do not require any a priori information about the number of scatterers or inclusions, and also are much faster than iterative methods and therefore suitable for real-time computations. Another alternative are level-set methods which share at least the first of the two aforementioned advantages. There was quite a lot of discussion and also some controversy about the relative merits of either of these methods, and of the competing iterative methods that are still in use.

There was a wide range of applications including a session on ultrasound imaging (and appropriate mathematical models for it) and diffraction gratings, which is of enormous importance in optical sciences. On the last day of the conference there was also one talk of the potentials in combining (static) MRI data with (dynamic) impedance tomography techniques. People were curious to learn that Korea has funded an interdisciplinary institute to work on this project for a total of nine years.

Last but not least, excellent talks were given by two of the pioneers in analytic methods for inverse scattering problems, David Colton and Rainer Kress. It has been publicly acknowledged at the meeting that the contributions of these two outstanding individuals have influenced directly and indirectly many of the participants.

The staff in Oberwolfach has maintained their usual high standards of support, even though Monday has been an Easter holiday in Germany. Everybody enjoyed the wonderful Müsli, the general healthful menu, and the nice landscape around the institute. The great library and the many possibilities to sit together in small groups to work on a problem contributed to create a wonderful and inspiring atmosphere. If there is any complaint to make, however, it should be adressed to the organizers who made a miscalculation in the inverse problem of determining the optimal time for the hike given the weather data from Monday and Tuesday. The numerical methods that seem to have been used failed to predict the heavy shower on Wednesday around three o'clock on the most exposed part of the ridge.

Abstracts

Lipschitz stability for the inverse conductivity problem

Giovanni Alessandrini

(joint work with S.Vessella)

We consider the basic inverse boundary problem of electrical impedance tomography. It is well known, [1], that if one assumes an a-priori regularity bound (of finite order) on the conductivity γ, then the dependence of the conductivity γ on the Dirichlet-to-Neumann map Λ_γ is controlled by a logarithmic modulus of continuity. In fact, after an example of Mandache, [2], it is also known that logarithmic stability is best possible when an a-priori regularity bound of any finite order is available.

This situation is evidently prejudicial to the effectiveness of any reconstruction procedure. In order to circumvent such difficulty two different strategies might be attempted:

(1) To introduce stronger, but still physically acceptable, a-priori assumptions on the conductivity which enable to prove better stability results.

(2) To identify few, maybe finitely many, parameters associated to the conductivity, which are physically significant, and at the same time depend in a stable fashion on the Dirichlet-to-Neumann map.

We shall show some positive results in the direction of the strategy (1) and shall propose some open problems related to (2).

References

[1] G. Alessandrini, *Stable determination of conductivity by boundary measurements*, Applicable Anal., **27** (1988), 153–172.

[2] N. Mandache, *Exponential instability in an inverse problem for the Schrödinger equation*, Inverse Problems, **17** (October 2001), 1435–1444.

The factorization method for scattering by diffraction gratings

Tilo Arens

(joint work with Dr. N. Grinberg, Prof. Dr. A. Kirsch)

The two dimensional problem of scattering a plane wave by a sound soft periodic surface differs considerably from the corresponding problem for a bounded obstacle: in the far field, only a finite number of plane waves propagating away from the surface are observable. Additionally, the scattered field consists of an infinite countable number of evanescent modes which decay exponentially with distance from the surface. The expansion of the scattered field in this series of plane and evanescent waves is known as the Rayleigh series.

It is proposed to reconstruct the scattering surface from the knowledge of the coefficients in the Rayleigh series for all incident plane waves which are quasi-periodic (i.e. periodic up to a phase-shift) with the same parameter with respect to the horizontal direction. This set of data can be thought of as forming a discrete linear operator on the space l^2 which we term *Near Field Operator*. It is the idea of the *Factorization Method* to derive a factorization of this operator which makes it possible to reconstruct the scatterer from the operator's spectral data alone. The main ingredients in this theory are an appropriately defined Herglotz operator and coerciveness of the single-layer operator for the periodic surface with a Dirichlet half-space Green's function as its kernel for small enough wave number. Numerical experiments indicate that for surface features of the order of the wave length a reconstruction is only possible if data from both incident and scattered evanescent modes are taken into account.

Inverse Scattering Methods in Optical Tomography

Simon R. Arridge

Optical Tomography seeks to recover spatially varying optical absorption and scattering functions in the interior of a domain Ω from measurements of transmitted light on the boundary $\partial\Omega$. In one commonly used model the propagation of light is modelled as a second order elliptic PDE, with Robin boundary conditions and measured Nemuann data:

$$\begin{aligned} -\nabla \cdot \kappa \nabla u + (\mu + i\omega)u &= 0 \\ u + \alpha \partial_\nu u = &= f \\ \partial_\nu u &= g \end{aligned}$$

With this model the measured data can be described as a linear Robin-to-Neumann mapping $g = \Lambda f$.

In this talk we apply a singular value decomposition to the change in the mapping $\Lambda_1 - \Lambda_0$ induced by a change in functions κ, μ in the interior. Analysis of this decomposition leads naturally to an iterative reconstruction algorithm that minimises the Frobenius norm of $\Lambda_1 - \Lambda_0$. Simulations in 2D for complex data reveal the ability to reconstruct both μ and κ from relatively few input patterns f.

A point-source method for inverse scattering by rough surfaces

Simon Chandler-Wilde

We consider the problem of 2D electromagnetic scattering in TE polarization by an unbounded perfectly conducting surface which is the graph of a $C^{1,1}$ function f. We consider specifically the case where the incident field is that due to a point source. The direct problem is to compute the scattered field given the $C^{1,1}$ function f. This is a Dirichlet problem for the Helmholtz equation. The inverse problem we consider is to obtain f from measurements of the total field on a finite horizontal line above the scattering surface. We describe and analyse a point source method for this inverse scattering problem, inspired by the point source method for inverse scattering by bounded obstacles of Potthast (IMA J. Appl. Math 61, 119-140 (1998)). The method reconstructs the field above the surface and then locates the surface as the zero level curve of the modulus of the total field. For monofrequency waves we prove an error estimate for the reconstructed total field. We present numerical results showing the reconstructed total field in the time domain and frequency domain and reconstructions of the position of the scattering surface.

The Inverse Electromagnetic Scattering Problem for Partially Coated Obstacles

David Colton

We consider the inverse scattering problem of determining the shape and surface impedance of a partially coated obstacle in $\mathbb{R}^3$ where on an (unknown) part of the obstacle the field satisfies a perfectly conducting boundary condition and on the other (also unknown) portion of the obstacle the field satisfies an impedance boundary condition. We asume that the incident field is an electromagnetic plane wave at fixed frequency in the resonance region and use the linear sampling method to determine the shape of the (possibly multiply connected) obstacle.We then use a variational method and the solution of the far field equation to determine the L infinity norm of the surface impedance. Numerical examples are given of reconstructions.

A level set method for shape reconstruction in medical and geophysical imaging

Oliver Dorn

Recently, the level set method for describing propagating fronts has become quite popular in the application of medical or geophysical tomography. The goal in these applications is to reconstruct unknown objects inside a given domain from a finite set of boundary data. Mathematically, these problems define nonlinear inverse problems, where usually iterative solution strategies are required. Starting out from some initial guess for the unknown obstacles, successive corrections to this initial shape are calculated such that the so evolving shapes eventually converge to a shape which satifies the collected data. Since the hidden objects can have a complicated topological structure which is not known a priori, the shapes usually undergo several topology changes during this evolution before converging to the final solution. Therefore, a powerful and flexible tool for the numerical description of these propagating shapes is essential for the success of the inversion method of choice. In the talk, we present a recently developed shape reconstruction method which uses a level set representation of the shapes for this purpose. Numerical results will be presented for three different practically relevant examples: cross-borehole electromagnetic tomography using a 2D Helmholtz model, surface to borehole 3D electromagnetic induction tomography (EMIT) using a model based on the full 3D system of Maxwell's equations, and diffuse optical tomography (DOT) for medical imaging using a model based on the linear transport equation in 2D.

Inverse Problems for Diffraction Gratings: Uniqueness Results

Johannes Elschner
(joint work with G. Schmidt and M. Yamamoto)

We mainly consider the scattering of monochromatic plane waves by a 2D perfectly reflecting diffraction grating in an isotropic lossless medium, which is modeled by the Dirichlet problem (transverse electric polarization) or the Neumann problem (transverse magnetic polarization) for the periodic Helmholtz equation. The inverse problem consists in determining the profile function from the knowledge of *one* wave number, *one* incident direction and the total field measured on a straight line above the grating. Excluding the case of parallel half planes and the Rayleigh frequencies, we prove uniqueness for this inverse problem within the class of polygonal grating profiles. We also present a first uniqueness result for the inverse TE transmission problem.

Regularization of nonlinear statistical inverse problems

Thorsten Hohage
(joint work with N. Bissantz and A. Munk)

We consider nonlinear inverse problems described by operator equations $F(a) = g$ in Hilbert spaces in the following statistical framework: Suppose we have a finite number n of measurements, which are modelled by random variables $Y_i = g(X_i) + \epsilon_i$, $i = 1, \ldots, n$. Here the measurement errors are described by independent, identically distributed random variables ϵ_i with some unknown distribution satisfying $E\epsilon_i = 0$. g belongs to some L^2-Hilbert spaces, and the measurement points X_i are either random variables (random design) or fixed, depending on n (deterministic design). A statistic $\hat{a}_n$ for the unknown parameter a is constructed using a modified preconditioning technique of Mair and Ruymgaart and nonlinear Tikhonov regularization. We establish consistency in the sense that the mean integrated square error $E\|\hat{a}_n - a\|^2$ (MISE) tends to 0 as $n \to \infty$ under reasonable assumptions. Moreover, if a satisfies a source condition, we show a convergence rate result for the MISE. Our theoretical results are confirmed by numerical experiments with inverse scattering problems.

On nearfield acoustical holography

VICTOR ISAKOV
(joint work with Tom DeLillo, Nicolas Valdivia, Lianju Wang and Sean Wu)

We consider the problem of identifying the source of the acoustical noise and the normal velocity of the sound on the surface Γ of a three-dimensional domain Ω. The acoustical field u of frequency k in Ω satisfies the Helmholtz equation

$$\Delta u + k^2 u = 0 \quad \text{in} \quad \Omega \; (\quad \text{or in } \Omega_e = \mathbf{R}^3 \setminus \overline{\Omega}) \tag{1}$$

The application we are interested in deals with Ω which is a cabin of an aircraft or of a car. A solutions to (1) in Ω_e are assumed to be satisfying the Sommerfeld radiation condition. Such u are called radiating solutions. The acoustical sensors are located on a surface Γ_0 inside or outside the cabin. They can measure the field u and the problem is to recover from these measurements u inside Ω and in particular the so-called normal velocity $v = \partial_\nu u \quad \text{on} \quad \Gamma = \partial\Omega$. Here ν is the unit exterior normal to $\partial\Omega$. We will show that there is a unique representation of u in Ω by the single layer potential

$$u(x) = S_\Gamma \varphi(x) = \int_\Gamma K(x,y)\varphi(y) d\Gamma(y), \quad x \in \Omega \quad (\text{ or } x \in \Omega_e) \tag{2}$$

where $K(x, y)$ is the free space radiating fundamental solution to the Helmholtz equation. Now our problem is reduced to solving the linear integral equation

$$\int_\Gamma K(x,y)\varphi(y) d\Gamma(y) = u(x), \quad x \in \Gamma_0 \tag{3}$$

Given φ in the interior problem one can find the normal velocity from the standard formula which follows from (2) and the jump relations for the normal derivative of single layer potentials. This approach in principle allows to handle general domains Ω while most of existing methods are applicable to very special (rotationally symmetric Ω) when the Green's function of the Neumann problem for the Helmholtz equation can be found quite explicitly. In the paper [1] we considered a two-dimensional version of the interior problem. In the paper [2] we handle the complete three-dimensional case.

In a popular approach one uses the represenation by single and double layer potentials (the Helmholtz-Kirchhoff system). We show that this approach has no advantages over the single layer method.

The results were obtained together with Tom DeLillo, Nicolas Valdivia, Lianju Wang, and Sean Wu. The research is in part supported by the NSF grants DMS-9803816 and ITR/ACS 0081270.

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[3] V. Isakov, *Inverse Source Problems*, AMS, Providence, R.I., 1990.
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Electrostatic imaging via conformal mapping

RAINER KRESS

We present the solution of an inverse boundary value problem for harmonic functions arising in electrostatic imaging through conformal mapping techniques. The numerical method consists of two parts. In a first step, by successive approximations a nonlinear, nonlocal ordinary differential equation is solved to determine the boundary values of a holomorphic function on the outer boundary circle of an annulus. Then in a second step an ill-posed Cauchy problem is solved to determine the holomorphic function within the annulus. We discuss a convergence result for the iteration procedure and through numerical examples we illustrate the feasibility of the method.

Photonic crystals

PETER KUCHMENT

A photonic crystal (or a photonic band gap (PBG) material) is a composite medium which consists of a periodically modulated dielectric. The idea is that if the materials and geometry are chosen appropriately, PBG materials can exhibit with respect to light properties similar to the ones of semi-conductors with respect to electrons. In particular, band gaps, or prohibited ranges of frequencies of EM waves can arise. PBG materials promise a wide variety of applications ranging from lasers, efficient light sources, memories, optical integrated circuits, efficient optic cables, etc. This are a of research involves a wide variety of complex and interesting mathematical problems, resolving which requires usage of many techniques from PDEs, to spectral theory, to numerical analysis, to optimization, ... The talk provides a brief survey of the problems and techniques involved.

Semi-smooth Newton methods for BV-regularized inverse problems

KARL KUNISCH

(joint work with Dr. M. Hintermüller)

We show that, while the Fenchel dual of BV-regularized problems has an extremely difficult structur, its pre-dual can be elegantly characterized in a Hilbert space setting as a bilaterally constrained optimization problem. For the latter we propose and analyse semi-smooth Newton methods for their efficient numerical realisation.

Inverse scattering problem for Schroedinger operator with random potentials

MATTI LASSAS

(joint work with L.Päivärinta, E.Saksman)

In these talk we consider high-frequency scattering from random media and the inverse problem for it. More precisely, we study the problem how the stochastical properties of a random scatterer can be determined from scattered fields on high frequencies. Motivation for this kind of problems is evident: As a practical example, consider scattering from a surface, where the macro-scale structure of surface is smooth but in very small scale, the surface is rough. If we forget the micro-scale roughness, and approximate the surface with a smooth surface, on the high frequencies waves scatter according to Kirchoff law, that is, like from a mirror. This contradicts with many everyday experiences. So, for a valid model one has to include the roughness of the micro-structure in the goal of inverse problem is to find the functions describing the properties of the micro-structure.

In this talk we consider high-frequency scattering for Schrödinger equation

$$(\Delta + q + k^2)u(x, y, k) = \delta_y$$

with a random potential $q(x)$. The potential $q(x)$ is assumed to be a Gaussian random function that defines a Markov random field. We show how realizations of the scattered field $u_s(x, y, k)$ can be used to determine stochastical properties of q, for instance the properties of covariance function $E(q(x)q(y))$. In physical terms, this corresponds to e.g. determination of typical sizes of particles in random medium.

In contrast to applied literature, we approach the problem with methods that do not require approximations, for instance linearization the problem. In technical point of view, we analyze the scattering from the random potential by combining methods of harmonic and microlocal analysis to stochastic, in particular to theory of ergodic processes and Wiener chaos.

Using interior elastic wave displacement data to create images of stiffness variation in tissue

Joyce R. McLaughlin

(joint work with Lin Ji, Jeong Rock Yoon and Dan Renzi)

Using extensions of Doppler ultrasound techniques and time reversal methods (Diffraction field of a low frequency vibration in soft tissues using transient elastography, Catheline, Thomas, Wu, Fink, IEEE Trans. Ultrasonics, Ferroelectrics and Control, 1999) the time and space dependent interior displacement of a propagating shear wave can be measured. This shear wave propagates at 1 to 3 meters/second in normal tissue and can travel more than twice that speed in abnormal tissue. This substantial change in the shear wave speed motivates our considerations of using shear wave speed images to detect abnormalities.

The data is time and space dependent displacement and the experiment begins with the tissue at rest and a boundary or locally confined interior excitation. So we consider several problems of which we list two here:

(1) Find μ/ρ or μ and ρ separately in the wave equation

$$\begin{aligned} \nabla \cdot (\mu \nabla u) &= \rho\, u_{tt}, && \underset{\sim}{x} \in \Omega,\ \ 0 < t < T, \\ u_t(\underset{\sim}{x}, 0) = \underset{\sim}{u}(\underset{\sim}{x}, 0) &= 0, && \underset{\sim}{x} \in \Omega, \\ \mu \frac{du}{dn} &= f(x, t), && \underset{\sim}{x} \in \partial\Omega, 0 < t < T. \end{aligned}$$

(2) Find μ/ρ given λ or find μ and ρ separately given λ in

$$\begin{aligned} \rho\, \underset{\sim tt}{u} &= \nabla(\lambda \nabla \cdot \underset{\sim}{u}) + \frac{1}{2} \nabla \cdot (\mu(\nabla \underset{\sim}{u} + (\nabla \underset{\sim}{u})^T)), && \underset{\sim}{x} \in \Omega, \quad 0 < t < T, \\ \underset{\sim}{u_t}(\underset{\sim}{x}, 0) &= \underset{\sim}{u}(\underset{\sim}{x}, 0) = 0, && \underset{\sim}{x} \in \Omega, \\ f &= [\lambda \nabla \cdot \underset{\sim}{u} \underline{I} + \mu(\nabla \underset{\sim}{u} + (\nabla \underset{\sim}{u})^T)] \cdot \underset{\sim}{n} && \underset{\sim}{x} \in \partial\, \Omega,\ 0 < t < T. \end{aligned}$$

For each of these models we present new uniqueness theorems for two coefficients utilizing this rich data set. A major simplifying feature of the proof results from using the fact that the solution of each equation set has a propagating front. We note here that the data is nearfield data.

In addition we show the results of two new algorithms. One utilizes a Hankel function expansion for the case where the data is confined to displacements in a plane and the boundary source is nearly a point source. Prior to this, when the boundary source can be approximated by a plane wave source, a geometric optics expansion was utilized to develop a differential algebraic equation set that forms the basis of a reconstruction algorithm. A heuristic reason for why this succeeds in this nearfield inverse problem is that the geometric optics expansion is exact for some special cases. For the point source problem in two dimensions the geometric optics expansion is a good approximation only in the far field. Hence we utilize a Hankel function expansion that is exact in the nearfield for some special cases.

The result is a (different) differential algebraic equation set that forms the basis of this new reconstruction algorithm. Numerical calculations will be presented; robustness with respect to noise will be exhibited.

Our second new set of results for a completely different algorithm utilizes the fact that the solution has a propagating front. This front, called the arrival time surface, is extracted from the displacement data using level set methods. To utilize this front, we establish, using unique continuation, that when the arrival time surface is C^2 it satisfies the eikonal equation. The eikonal equation is then used to compute the solution of the inverse problem which is the spacial variation in the shear wave speed.

Phase space analysis in tomography

Victor Palamodov

The phase space analysis is a method of microlocal operating with functions. Gabor's 'elementary signal' is the same as *coherent state* in physics. A coherent state is a function (halfdensity) which is maximally localized and the x- and ξ-representation simultaneously. An arbitrary function in free space having finite energy admits an integral representation by means of the family of coherent states when the support point (x, ξ) runs through the phase space. This provides a tool for featuring its microlocal properties.

An inverse problem is typically non-linear and ill-posed and only partial information can be stably recovered from real data. This is the case for the acoustic equation in inhomogeneous medium. The boundary measurements done for a fixed time frequency are used for estimating of Gabor coefficients of the refraction coefficient in the frame of rigorous theory. An explicitly shaped domain in the phase space is shown to be the stability domain for reconstruction of the refraction coefficient. For a small perturbation of the velocity this domain is the union of local "Ewald balls" of variable radius depending on the background velocity.

In the method of electric impedance imaging the admittivity coefficient γ in a domain D is reconstructed from series of measurements of voltage and current on ∂D. A modified Gabor system is fabricated by means of the action of the linear conformal group to blow up a neighborhood of ∂D. The elements of this system shows to be close to the eigenfunctions of the sensitivity operator which is the linearization of the operator: from γ to the Dirichlet-to-Neumann operator on ∂D. The estimates for action of the sensitivity operator on the Gabor functions show the zone in the phase space, where a perturbation of γ can not be stably reconstructed. This zone is defined by an upper bound for the product of the distance from x to ∂D and of the scalar frequency $|\xi|$. The linear envelope of Gabor functions that belong to the zone can be tested as a reasonable subspace of admittivities where the inverse problem has a stable solution. No standard regularization machinery is needed.

Solving time-dependent inverse acoustic scattering problems
Roland Potthast

Over the last seven years several non-iterative methods for the reconstruction of obstacles or the shape of media have been proposed. We outline the basic ideas of these methods and describe their difference. In particular, we introduce the no-response test by Luke and the author, which uses test domains and special incident waves which are small on the test domain to find the unknown scatterer and its shape. Then, the point-source method (PSM) for the reconstruction of time-dependent waves from remote measurements for one incident wave is described. The PSM reconstructs the scattered and total field by backprojection and, then, uses the boundary condition for reconstructions. It is shown how a singular sources scheme can be developed with the same tools as used for the PSM. This singular sources scheme does not need to know the boundary condition, but it needs measurements for many incident waves. We relate these methods to the schemes of Colton-Kirsch, Kirsch and Ikehata and to the range test of Potthast-Sylvester-Kusiak.

Finally, we numerically demonstrate the power of the point-source method by reconstructing a full time-dependent wave from remote measurements (produced by simulation).

Uniqueness for the determination of non-smooth scatterers
Luca Rondi

We present a uniqueness result for the determination of finitely many sound-soft scatterers by a finite number of acoustic far-field measurements.

The class of admissible scatterers is very large and includes non-smooth obstacles (that is a scatterer which is the closure of a domain) and, more notably, non-smooth open surfaces.

The possible presence of scatterers whose interiors are empty requires an argument which is different from the classical one by Schiffer, and its extension due to Colton and Sleeman, which dealt with the case of obstacles. However, a careful study of the nodal set of the total field, using an optimal estimate of decay of the solution to the direct problem and a characterization of the level sets of harmonic functions due to Alessandrini and Di Benedetto, allows us to recover a situation in which Schiffer's argument is applicable. Therefore we obtain unique determination of non-smooth (and possibly with empty interiors) scatterers by a finite number of far-field measurements. Even if we consider a quite larger class of admissible scatterers, the result is completely analogous to the one by Colton and Sleeman for obstacles. The only price we have to pay for allowing scatterers represented by open surfaces is that we might need more measurements than those needed to identify obstacles.

Electrical Impedance Tomography in the half space

Birgit Schappel

Let $\Omega = \mathbb{R}^3_+$ be the upper half space in $\mathbb{R}^3$ and $\Gamma = \mathbb{R}^2$ its boundary. For a nonnegative scalar conductivity σ the electrostatic potential u inside Ω fulfills the PDE $\nabla\cdot(\sigma\nabla u) = 0$. If a current density $f \neq 0$ is induced on the boundary Γ the potential u fulfills the Neumann boundary condition $\sigma\frac{\partial u}{\partial \nu} = f$ on Γ. Assuming that Ω is homogeneous except for some inclusions D with conductivities smaller than the background conductivity our aim ist the reconstruction of D from the knowledge of the Neumann-to-Dirichlet operator Λ_σ. Using suitable weighted Sobolev spaces we show that this operator is compact and self-adjoint. Denoting bei $\Lambda_\mathbb{1}$ the Neumann-to Dirichlet operator for the domain without inclusions we obtain that $\Lambda_\sigma - \Lambda_\mathbb{1}$ is positiv and give a characterization of the range of the operator $(\Lambda_\sigma - \Lambda_\mathbb{1})^{1/2}$. Using this we show that the trace of a function g_z which is essentially the restriction of the fundamental solution of the Laplace equation onto the boundary Γ belongs to $R((\Lambda_\sigma - \Lambda_\mathbb{1})^{1/2})$ iff z is a point lying in the inclusion. Using the Picard criterion as well as the eigenvalues and eigenfunctions of $\Lambda_\sigma - \Lambda_\mathbb{1}$ this can be tested numerically.

Inversion with transducer of laser generated waves

Otmar Scherzer

(joint work with R. Kowar)

In material testing with ultrasound for non-destructive evaluation a wave is generated at the surface of the material to be tested. From measurement data of the wave field on the surface of the material defects are recognized and reconstructed.

We are simulating waves propagating through inhomogeneous media. The waves are generated by common transducers or lasers. Transducers such as commonly used in medical imaging generate a focused wave, which gives good spatial resolution near the focal point. In industrial processes laser generated ultrasound is used for contactless evaluation of materials, such as investigation of hot metal sheets.

The governing model is a wave equation for the pressure field. In medical applications the wave is initiated by a transducer. It consisting of piezo crystals which strike a body and initiate a wave. Mathematically this can be modeled as harmonic forcing terms with small support, which are initiated at exactly specified locations and time. Lasers generate waves are smooth initial functions (typically Gaussians).

Under suitable assumptions the wave equation can be transformed into a nonlinear integral equation of the first kind. The data (right hand side of the integral equation) is the measured pressure field on the investigated material. The kernel function very much depends on the initiated wave, i.e., it depends on the construction and control of the transducer, and on the density of the material, which eventually should be recovered.

The Born-Neumann approximation linearizes the nonlinear integral equation. The resulting integral equation is linear with respect to the density. The kernel function depends on the design and control of the transducer. For modern complex transducers the simulation of the kernel function (of the linear integral equation) is extremely computationally expensive. Once the Born-Neumann approximation of the kernel function is calculated it can be used for the recovery of the density with the same transducer. The Born-Neumann approximation just models single reflections in the material; multiple scattering is neglected. We show that the Born-Neumann approximation can be iterated, which eventually approximates the nonlinear integral equation. Laser generated waves result in smoother kernel functions and consequently the recovery problem for the density is considerably more unstable.

Several numerical simulations are presented showing the calculation of the kernel function. Moreover, we compare simulated and measured waves for nondestructive evaluation of aluminum sheets.

Magnetic Resonance Electrical Impedance Tomography (MREIT)

Jin Keun Seo

Numerous experimental findings have shown that different biological tissues in the human body have different electrical properties at the frequency range of tens of Hz to several MHz. The electrical conductivity and permittivity of a biological tissue change with cell concentration, cellular structure, molecular composition, membrane capacitance, and so on. Therefore, these properties manifest structural, functional, metabolic, and pathological conditions of the tissue providing valuable diagnostic information. Cross-sectional imaging of electrical conductivity distributions within a human body has been a research goal in Electrical Impedance Tomography (EIT). However, EIT suffers from the ill-posed characteristics of the corresponding inverse problem due primarily to nonlinearity and low sensitivity. In order to overcome this technical difficulty, we started looking at internal information so that we can transform the ill-posed problem into a well-posed one. This initiated the new research area called Magnetic Resonance Electrical Impedance Tomography (MREIT). In MREIT, we use a Magnetic Resonance Imaging (MRI) scanner to measure the induced magnetic flux density due to an injection current. We address the image reconstruction problem in MREIT as a well-posed inverse problem taking advantage of this additional internal information. We first describe a three-dimensional forward solver as a basic tool in experimental design and verification as well as implementation of image reconstruction algorithms. For image reconstruction algorithms, we summarize the J-substitution algorithm where we use the internal current density obtained from the induced magnetic flux density. The main drawback of this method is the requirement of subject rotations to measure all three components of the induced magnetic flux density. We introduce two other algorithms that utilize only one component of the induced magnetic flux density thereby removing the subject rotation procedure. Once we have reconstructed images of conductivity distributions, we can produce images of internal current density distributions for any given injection currents. Showing numerical and experimental results in MREIT, we suggest MREIT for static imaging of cross-sectional conductivity distributions leaving EIT for dynamic imaging and lesion detections.

Some inverse problems associated with acoustic propagation in dispersions

Brian Sleeman

(joint work with O.G. Harlen, M.J.Holmes, M.J.W.Povey, Y.Qiu. Supported by EPSRC.)

Ultrasound techniques for the characterization of colloidal systems are gaining wide acceptance and many new ultasound instruments have recently appeared on the market. The majority of these instruments claim to accurately measure colloidal particle size distribution. They all determine particle size from a measurement of ulrasound attenuation as a function of frequency, together with thermophysical data for the continuous and dispersed phases such as thermal diffusivity. The claims depend on an accurate model of acoustic scattering in such systems. Recently we have developed a low frequency theory (i.e for small acoustic wave number K and small thermal wave number L) based on a potential theory idea of Kleinman. A theory has also been developed for small K and large L based on a novel combination of the potential theory and the geometrical theory of diffraction.

Characterisation of soft solid systems depends on a number of features, i.e.

(i) The structure of the gel, which is described by the particle distribution. The structure may range from a nearly homogeneous distribution, to a sparse, fractal network of interlinked clumps of particles.

(ii) Weak, long-range interactions between particles. These interactions are involved in the flocculation/aggregation processes and determine the network structure.

(iii) The elastic properties of the soft solid. These properties depend on the structure and interactions of the system.

The fundamental inverse problems are to determine the effect on ultrasound of each of the above features and hence to determine what features of the underlying microstructure can be ascertained from ultrasound measurements.

These problems form a large area of research. We describe progress on understanding the direct problem as a precursor to attacking the inverse problems.

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The Scattering Support for Inverse Source and Inverse Scattering Problems

John Sylvester
(joint work with R.Potthast, S.Kusiak)

I will give an introduction to scattering and inverse scattering for the Helmholtz equation, emphasizing the connection between the Paley-Wiener theorems, the integral equations of scatttering theory (Lipmann-Schwinger equation), and the classical expansions of the far field (asympotic representation of the solution) in terms of Bessel and Hankel functions.

Inserting these functions into the right hand side of Green's identity yields a series of inequalities which we can use to find what we call the convex scattering support of a far field. This set must be a subset of the support af any source that could possibly produce that far field.

I will describe methods for computing the scattering support (joint work with Steve Kusiak) based on special functions and other methods based on characterizing the range of certain integral operators (joint work with Roland Potthast and Steve Kusiak).

Uniqueness in inverse scattering problems with a single incident wave

Masahiro Yamamoto
(joint work with J.Cheng)

Let $D \subset \mathbb{R}^2$ be a bounded domain and $k \in \mathbb{R}$. For $x \in \mathbb{R}^2$, we set $r = |x|$. We consider a scattering problem with sound-soft obstacle:

$$\Delta u + k^2 u = 0 \qquad in \quad \mathbb{R}^2 \setminus \overline{D} \tag{1}$$

$$u = 0 \qquad on \quad \partial D \tag{2}$$

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial}{\partial r}u^S(x) - iku^S(x)\right) = 0. \tag{3}$$

Henceforth we consider $d \in S^1 \equiv \{x \in \mathbb{R}^2\,;\,|x| = 1\}$ and let

$$u^S(x) = u(x) - e^{ikx\cdot d},$$

which is called the scattered field, while u is called the total field. We consider $d \in S^1$ and $k \in \mathbb{R}$ respectively as the direction of the incoming plane wave (i.e., $e^{ikx\cdot d}$) and the wave number given by the medium in $\mathbb{R}^2 \setminus \overline{D}$.

Condition (3) is the Sommerfeld radiation condition. Under suitable conditions on D, for $k \in \mathbb{R}$ and $d \in S^1$, there exists a unique H^1-solution $u(x) = u(D)(x)$ to (1) - (3), and we can define the far field pattern $u_\infty(D)\left(\frac{x}{r}\right)$:

$$u^S(D)(x) = \frac{e^{ikr}}{\sqrt{r}}\left\{u_\infty(D)\left(\frac{x}{r}\right) + O\left(\frac{1}{r}\right)\right\} \quad \text{as } r \longrightarrow \infty. \tag{4}$$

Inverse scattering problem: Determine D from the far field pattern $u_\infty(D)$ for given k and d (possibly by changing them).

This inverse problem is also physically significant and has been studied by many authors. We refer for example to Colton and Kress [1].

The first basic topic for this inverse problem is the uniqueness: Does

$$u_\infty(D_1)(x) = u_\infty(D_2)(x), \qquad |x| = 1 \tag{5}$$

(for possible several d and k) imply $D_1 = D_2$?

There is a classical uniqueness result within smooth D_1, D_2 if (5) holds for an infinite number of $d \in S^1$, which is proved based on Schiffer's idea (see Theorem 5.1 in [1]).

For the uniqueness by means of a finite number of $d \in S^1$, see Colton and Sleeman [2], Theorem 5.2 in [1]. Moreover the uniqueness is known with *a single* d, provided that D_1, D_2 are contained in a ball of radius ρ such that $k\rho < \pi$. See Corollary 5.3 in [1], [2].

An important open problem is the uniqueness in the inverse scattering problem with *a single* (d, k). This problem is interesting from the theoretical point of view, because the far field patterns with many d are overdetermining data for determination of D and we can expect the uniqueness with a single far field pattern. Moreover the formulation with a single (d, k) is helpful for justification of numerical reconstruction of D, because one can usually use far field patterns observed by taking a single or a finite number of d.

The purpose of this presentation is to give a positive answer to the uniqueness within polygonal (but not necessarily convex) obstacles.

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Edited by Bastian Gebauer and Birgit Schappel

Participants

Prof. Dr. Giovanni Alessandrini
alessang@univ.trieste.it
Dipartimento di Scienze Matematiche
Universita di Trieste
Via Alfonso Valerio 12/1
I-34127 Trieste

Dr. Tilo Arens
arens@numathics.com
Mathematisches Institut II
Universität Karlsruhe
Englerstr. 2
D–76131 Karlsruhe

Prof. Dr. Simon R. Arridge
arridge@cs.ucl.ac.uk
Department of Computer Science
University College London
Gower Street
GB-London WC1E 6BT

Prof. Dr. Amel Ben Abda
amel.benabda@enit.rnu.tn
ENIT-LAMSIN
Campus Universitaire le Belvedere
BP 37
1002 Tunis - Tunisie

Prof. Dr. Lester F. Caudill
lcaudill@richmond.edu
Dept. of Math. and Computer Science
University of Richmond
Richmond, VA 23173 – USA

Dr. Simon N. Chandler-Wilde
Simon.Chandler-Wilde@brunel.ac.uk
mastsnc@brunel.ac.uk
Dept. of Mathematical Sciences
Brunel University
Kingston Lane
GB-Uxbridge, Middlesex, UB8 3PH

Prof. Dr. David L. Colton
colton@math.udel.edu
Department of Mathematical Sciences
University of Delaware
501 Ewing Hall
Newark, DE 19716-2553 – USA

Dr. Oliver Dorn
odorn@math.uc3m.es
Departamento de Matematicas
Campus de Leganes
Universidad CarlosIII de Madrid
Avda. de la Universidad 30
E-28911 Leganes Madrid

Dr. Johannes Elschner
elschner@wias-berlin.de
Weierstraß-Institut für
Angewandte Analysis und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39
D–10117 Berlin

Bastian Gebauer
Gebauer@math.uni-mainz.de
Fachbereich Mathematik
Johannes-Gutenberg-Universität Mainz
Staudingerweg 9
D–55099 Mainz

Prof. Dr. Tuong Ha-Duong
tuong.ha-duong@utc.fr
GI/EMA
Université de Technologie
de Compiegne
BP 20529
F-60205 Compiegne Cedex

Prof. Dr. Martin Hanke-Bourgeois
hanke@math.uni-mainz.de
Fachbereich Mathematik
Johannes-Gutenberg-Universität Mainz
Staudingerweg 9
D–55099 Mainz

Dr. Frank Hettlich
hettlich@math.uni-karlsruhe.de
Mathematisches Institut II
Universität Karlsruhe
Englerstr. 2
D–76131 Karlsruhe

Dr. Thorsten Hohage
hohage@math.uni-goettingen.de
Institut für Numerische
und Angewandte Mathematik
Universität Göttingen
Lotzestr. 16-18
D–37083 Göttingen

Prof. Dr. Victor Isakov
victor.isakov@wichita.edu
Department of Mathematics
and Statistics
Wichita State University
Wichita, KS 67260-0033 – USA

Prof. Dr. Andreas Kirsch
Andreas.Kirsch@math.uni-karlsruhe.de
Mathematisches Institut II
Universität Karlsruhe
Englerstr. 2
D–76131 Karlsruhe

Prof. Dr. Rainer Kreß
kress@math.uni-goettingen.de
Institut für Numerische
und Angewandte Mathematik
Universität Göttingen
Lotzestr. 16-18
D–37083 Göttingen

Prof. Dr. Peter Kuchment
kuchment@math.tamu.edu
Department of Mathematics
Texas A & M University
College Station, TX 77843-3368 – USA

Prof. Dr. Karl Kunisch
karl.kunisch@uni-graz.at
Institut für Mathematik
Karl-Franzens-Universität
Heinrichstr. 36
A-8010 Graz

Prof. Dr. Jaroslav Kurylev
Y.V.Kurylev@lboro.ac.uk
Department of Mathematical Sciences
Loughborough University
Loughborough
GB-Leicestershire LE11 3TU

Dr. Matti Lassas
lassas@cc.helsinki.fi
Rolf Nevanlinna Institute
P.O. Box 4
Yliopistonkatu 5
FIN-00014 Helsinki

Prof. Dr. Joyce R. McLaughlin
mclauj@rpi.edu
Dept. of Mathematics
Rensselaer Polytechnic Institute
110 8th Street
Troy, NY 12180-3590 – USA

Prof. Dr. Peter Monk
monk@math.udel.edu
Department of Mathematical Sciences
University of Delaware
501 Ewing Hall
Newark, DE 19716-2553 – USA

Prof. Dr.Dr.h.c. Frank Natterer
natterer@math.uni-muenster.de
Institut für Numerische und
Instrumentelle Mathematik
Universität Münster
Einsteinstr. 62
D–48149 Münster

Prof. Dr. Lassi Päivärinta
lassi@tols16.oulu.fi
Department of Mathematics
University of Oulu
FIN-90570 Oulu

Prof. Dr. Viktor P. Palamodov
palamodo@tau.ac.il
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv
Tel Aviv 69978 – Israel

Prof. Dr. Michael K. Pidcock
mkpidcock@brookes.ac.uk
The School of Computing and
Mathematical Sciences
Oxford Brookes University
Wheatley Campus
GB-Oxford OX33 1HX

Dr. Roland Potthast
potthast@math.uni-goettingen.de
Institut für Numerische
und Angewandte Mathematik
Universität Göttingen
Lotzestr. 16-18
D–37083 Göttingen

Dr. Luca Rondi
rondi@mathsun1.univ.trieste.it
Dipartimento di Scienze Matematiche
Universita di Trieste
Via Alfonso Valerio 12/1
I-34127 Trieste

Prof. Dr. William Rundell
william.rundell@math.tamu.edu
Department of Mathematics
Texas A & M University
College Station, TX 77843-3368 – USA

Birgit Schappel
schappel@math.uni-mainz.de
Fachbereich Mathematik
Johannes-Gutenberg-Universität Mainz
Staudingerweg 9
D–55099 Mainz

Prof. Dr. Otmar Scherzer
otmar.scherzer@uibk.ac.at
Institut für Informatik
Viktor-Franz-Hess Haus
Technikerstr. 25/7
A-6020 Innsbruck

Prof. Dr. Jin Keun Seo
seoj@yonsei.ac.kr
Department of Mathematics
Yonsei University
Seoul 120-749 – Korea

Dr. Valery Serov
vserov@sun3.oulu.fi
Department of Mathematical Sciences
University of Oulu
P.O.Box 3000
FIN-90014 University of Oulu

Prof. Dr. Brian D. Sleeman
bds@amsta.leeds.ac.uk
School of Mathematics
University of Leeds
GB-Leeds, LS2 9JT

Prof. Dr. John Sylvester
sylvest@math.washington.edu
Dept. of Mathematics
Box 354350
University of Washington
Seattle, WA 98195-4350 – USA

Prof. Dr. Masahiro Yamamoto
myama@ms.u-tokyo.ac.jp
Department of Mathematical Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153 – Japan