Mathematisches Forschungsinstitut Oberwolfach

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Branching Processes

July 6th – July 12th, 2003

This meeting was organized by Don Dawson (Ottawa) and Achim Klenke (Köln). Being the third Oberwolfach meeting on branching processes (after 1967 and 1995), it brought together 45 of the top specialists and promising young mathematicians from all over the world. There were six longer survey lectures and twenty talks of 45 minutes as well as one problem session.

The meeting was aimed at giving an overview about the state of the art in the theory of branching processes which has undergone quite a development within the last ten years. A lot of methods, which are available now, make it possible to shift attention more and more to increasingly realistic models, for example in biological applications. A special emphasis was laid on five topics:

- combinatorial aspects of discrete and continuous trees
- analytical aspects of genealogies of superprocesses and their approximating particle systems
- branching processes in random environments
- branching processes with interactions
- biological applications and biologically motivated models.

The lectures were received very well by the audience and stimulated lively discussions. The meeting was quite successful in highlighting some of the most important aspects of research in the field of branching processes and in giving the participants a perspective for their future work.

Abstracts

Some open problems involving applications of branching processes to computer science

Krishna B. Athreya, Ithaca

In many randomized algorithms on binary trees the running time T(n) at depth n is a random variable that exhibits a behavior that is like that of a heavy tailed one. Since for each n the random variable T(n) is a bounded random variable the traditional notion of heavy tailedness is not applicable. In this talk, we propose a definition of heavy tailed distributions with bounded support and that of asymptotically heavy tailed sequences of random variables. We then apply these ideas to trees that are regular in particular those generated by a Galton-Watson branching process. We also outline a few open problems.

Uniqueness for martingale problems arising from super-processes

Siva Athreya, Indian Statistical Institute, Delhi

(joint work with R. Bass and E. Perkins)

We introduce a new method called the semigroup method to show norm estimates for the resolvent of operators $\mathcal{L}$ that come up during the study of martingale problems related to measure valued diffusions. The method was originally applied to the infinite dimensional Laplacian by Cannarsa and Daprato. Norm estimates are a key step in the perturbation technique to show weak uniqueness.

Fragmentation, subordinators and branching random walks

Jean Bertoin, Laboratoire de Probabilités, Paris 6

Homogeneous fragmentations describe the evolution of a mass that breaks down into pieces as time passes. They can be thought of as continuous time analogs of branching random walks. Using Kingman's representation of exchangable partitions of $\mathbb{N}$, we adapt to fragmentations the method of probability tilting of Lyons, Pemantle and Peres. Some applications to the asymptotic behavior of the fragmentation are derived.

Branching random walk in space-time i.i.d. random environment and collision times of random walks

Matthias Birkner, Frankfurt

We consider discrete branching random walks in space-time i.i.d. random environment where all the individuals at a given space-time point use the same random offspring law and move independently according to a (deterministic) random walk kernel. We assume that the mean number of offspring per particle is equal to one when averaged over the distribution of the environment. Starting off e.g. from a shift-invariant Poisson field, the expected number of particles per site is constant, and we ask about the long-time behaviour of the system. We show that it dies out locally if the symmetrization of the individual random walk is recurrent and that in the transient case it converges to a non-trivial equilibrium provided that the variance of the mean offspring number is less than a threshold value which is expressed via exponential moments of the collision time of two random walk paths, conditional on one of them. We establish a regime in which equilibria have infinite variance of the number of particles per site.

Super Brownian Motion with Interactions

JEAN-FRANÇOIS DELMAS, ENPC-CERMICS, MARNE LA VALLÉE
(joint work with J.S. Dhersin)

Consider a super-Brownian motion $X = (X_t, t \geq 0)$ started at $X_0 = \delta_x$, the Dirac mass at $x \in \mathbb{R}^d$. This is a random process taking values in $\mathcal{M}_f$, the set of finite measures on $\mathbb{R}^d$. It is the unique solution of the martingale problem:

$$(X_t, \varphi) = (\delta_x, \varphi) + \int_0^t (X_s, \frac{\Delta}{2}\varphi)\, ds + M(\varphi)_t,$$

where $M(\varphi)$ is a continuous martingale (with respect to the filtration generated by X) with quadratic variation

$$\langle M(\varphi)\rangle_t = 4\int_0^t (X_s, \varphi^2)\, ds,$$

where φ is a sufficiently smooth function defined on $\mathbb{R}^d$.

Let ϕ be a C^1 function defined on $\mathbb{R}^+$, s.t. $\phi(0) = 0$ and $\phi'(t) > 0$ for all $t \geq 0$. If we consider the changed time process $Y_t = X_{\phi(t)}$, for $t \geq 0$, then it is easy to check that $Y = (Y_t, t \geq 0)$ is a solution to the martingale problem:

$$(Y_t, \varphi) = (\delta_x, \varphi) + \int_0^t (Y_s, \phi'(s)\frac{\Delta}{2}\varphi)\, ds + M(\varphi)_t,$$

where $M(\varphi)$ is a continuous martingale with quadratic variation

$$\langle M(\varphi)\rangle_t = 4\int_0^t (Y_s, \phi'(s)\varphi^2)\, ds.$$

Now we want to use a random time change procedure to transform the martingale problem. It will be done using the Brownian snake representation of the super Brownian motion.

Recall that the Brownian snake is a Markov path valued process $W = (W_s, s \geq 0)$. Each path W_s is distributed according to a Brownian motion started at point x and stopped at its lifetime ζ_s. The lifetime process $\zeta = (\zeta_s, s \geq 0)$ is the law of reflected linear Brownian motion. Let L_s^t be its local time at time s and level t. Let τ the first time where L^0 reaches the value 1. It is well known, that the measure valued process $(X_t(W), t \geq 0)$ defined by

(1) $$X_t(W) = \int_0^\tau \delta_{W_s(\zeta_s)} dL_s^t,$$

is the super Brownian motion started at δ_x.

We intend to replace the local time of the lifetime process at level t, by the local time, $L^{\phi(t)}$, along a random curve $\phi(t) = (\phi_s(t), s \geq 0)$. This curve $\phi(t)$ needs to have particular properties to have a local time w.r.t. the lifetime process. And then we consider the random measure defined as X_t with L^t replaced by $L^{\phi(t)}$. We shall denote it by Y_t. We also perform a path transformation of the underlying process W to V, which allows one to replace the underlying Brownian motion by a diffusion.

The random time change will formally be given by the following equations:

- Stochastic differential equation and time change for the path W_s of the Brownian snake:

(2) $$d_t V_s(t) = \sigma(Y_t, V_s(t)) d_t W_s(\phi_s(t)) + b(Y_t, V_s(t)) d_t \phi_s(t).$$

- Differential equation for the time change at time t:

$$d_t\phi_s(t) = \theta(Y_t, V_s(t))dt \quad \text{for} \quad \phi_s(t) \le \zeta_s. \tag{3}$$

- Definition of the random measure Y_t:

$$Y_t = \int_0^\tau \delta_{V_s(t)}\, dL_s^{\phi(t)}, \tag{4}$$

where $L^{\phi(t)}$ is the "local time" of the lifetime process ζ on the random curve $\phi(t)$.

Notice that in general, the function $s \mapsto \phi_s(t)$ is not adapted to the filtration generated by the snake, because the measure Y_t take into account paths $W_{s'}$ for $s' \ge s$.

We give a discrete version of those equations and prove that $(Y^\varepsilon, \varepsilon > 0)$, the discrete versions of Y, is tight and that any limit is solution to the martingale problem:

$$(Y_t, \varphi) = (\delta_x, \varphi) + \int_0^t (Y_s, \theta(Y_s)A(Y_s)\varphi)\, ds + M(\varphi)_t,$$

where $A(\mu)$ is the infinitesimal generator of a diffusion, with diffusion coefficient $\sigma(\mu, \cdot)$ and drift $b(\mu, \cdot)$, and $M(\varphi)$ is a continuous martingale with quadratic variation

$$\langle M(\varphi)\rangle_t = 4\int_0^t (Y_s, \theta(Y_s)\varphi^2)\, ds.$$

The functions θ, b and σ are bounded continuous functions defined on $\mathcal{M}_f \times \mathbb{R}^d$, taking values respectively in $\mathbb{R}, \mathbb{R}^d$ and $\mathbb{R}^{d\times d}$. The functions θ and σ are also positive.

Some mathematical problems from population genetics (survey lecture)

Alison M. Etheridge, University of Oxford
(joint work with N. Barton and (in parts) A. Sturm and S. Baird)

Kingman's coalescent provides an elegant description of the genealogy of a neutral panmictic population of constant size. There is now a huge literature that modifies and extends the coalescent to encapsulate more realistic biological situations including variable population size, selection, recombination and structure (spatial or genetic).

The standard approach to structure, the so-called structured coalescent, assumes that the population is subdivided into demes of constant size. In the context of a very simple problem, that of a neutral locus embedded in a fluctuating genetic background by virtue of being linked to a selected site, we examine the accuracy of this approximation. We see that in the case of balancing selection, fluctuations can considerably suppress the effects of selection.

We then turn to directional selection and in particular the problem of detecting partial selective sweeps. This problem has received considerable recent attention. We describe work in progress, the ultimate aim of which is to develop tests for selection.

Geometry of tree-space and tree valued Markov processes (survey lecture)

Steven N. Evans, Berkeley
(joint work with J. Pitman and A. Winter resp. S. Holmes)

Part I (joint with J. Pitman and A. Winter): Our point of departure are two results of David Aldous.

The first is the fact that a finite variance, critical Galton-Watson tree conditioned on total size n converges (with suitable rescaling) as $n \to \infty$ to a tree-like object, the Brownian continuum random tree (CRT). The CRT is the tree inside a standard Brownian excursion. In particular, a uniform random tree with n vertices converges to the CRT as $n \to \infty$.

The second result is that there is a simple Markov chain which has the uniform random tree on n vertices as its stationary distribution. This chain was independently discovered by Broder.

We construct a continuous-time Markov process on the space of compact R-trees that has the CRT as its long term stationary distribution.

Part II (joint with S. Holmes): We discuss the geometry of the space of phylogenetic trees on n taxa and show how it is possible to construct a "Brownian motion" on this space. This process suggests a family of methods for "averaging" phylogenetic trees.

The shape of large Galton-Watson trees with possibly infinite variance

Jochen Geiger, Universität Kaiserslautern
(joint work with L. Kauffmann)

Let T be a critical or subcritical Galton-Watson family tree with possibly infinite variance. We are interested in the shape of T conditioned to have a large total number of vertices. For this purpose we study random trees whose conditional distribution given their size is the same as the respective conditional distribution of T. These random family trees have a simple probabilistic structure if decomposed along the lines of descent of a number of distinguished vertices chosen uniformly at random. The shape of the subtrees spanned by the sampled vertices and the root depends essentially on the tail of the offspring distribution: While in the finite variance case the spanned subtrees are asymptotically binary, other shapes do persist in the limit if the variance is infinite. In fact, we show that these subtrees are Galton-Watson trees conditioned on their total number of leaves. Based on our spinal decomposition we also obtain a gamma limit law for the rescaled total population size.

Long range dependence processes arising from occupation times of branching systems

Luis Gorostiza, Centro de Investigacion y de Estudios Avanzados, Mexico

Consider a branching particle system on $\mathbb{R}^d$ starting from a uniform Poisson random measure, with critical binary branching and symmetric alpha-stable particle motion. For dimensions $\alpha < d < 2\alpha$, the rescaled long time occupation time fluctuations of the system converge towards a distribution-valued Gaussian process whose time structure resembles fractional Brownian motion, but its increments on non-overlapping intervals are correlated more weakly than those of fractional Brownian motion. Properties of this long range dependence process, called "sub-fractional Brownian motion", are discussed.

The super-Brownian motion limit of spread-out oriented percolation above $4+1$ dimensions (survey lecture)

Remco van der Hofstad, TU Eindhoven
(joint work with G. Slade, F. den Hollander and A. Sakai)

We consider oriented bond percolation on $\mathbb{Z}^d \times \mathbb{N}$, at the critical occupation density p_c, for $d > 4$. The model is a "spread-out" model having long range parameterised by L. We consider configurations in which the cluster of the origin survives to time n, and scale space by $n^{1/2}$. We prove that for L sufficiently large all the moment measures converge, as $n \to \infty$, to those of the canonical measure of super-Brownian motion. This extends a previous result of Nguyen and Yang, who proved Gaussian behaviour for the critical two-point function, to all r-point functions ($r \geq 2$). We use lace expansion methods for the two-point function, and prove convergence of the expansion using a general inductive method that we developed in a previous paper.

I will also describe some extension to the contact process.

From the Malthusian Controversy to the Polymerase Chain Reaction (PCR) Or: Branching Processes in Random Near-Critical Environments

Peter Jagers, Göteborg
(joint work with F. C. Klebaner)

In 1798 Thomas Malthus made his famous claim that populations not dying out grow "in a geometrical ratio. Subsistence increases only in an arithmetical ratio." This sparked a conflict in the social sciences that has not yet been settled (even though ecological awareness may have turned the enthusiasm for exponential growth less fervent).

Far from the battlefield, branching processes have shown that there is a possibility of non-exponential population growth, in particular linear growth. We show that this is the case for Galton-Watson processes in a random environment, that is not i.i.d. or stationary, but rather such that reproduction approaches criticality, as the population grows. If this occurs at the rate $1/(populationsize)^c$, the population grows like a polynomial of degree c. A gamma-distributed randomness may remain.

Interestingly enough, this result is relevant for the understanding of the linear growth of the number of molecules in quantitative PCR. If the reaction efficiency is assumed governed by Michaelis-Menten kinetics, we obtain precisely the above case, with $c = 1$.

Local extinction versus local exponential growth

Andreas E. Kyprianou, Utrecht
(joint work with J. Engländer)

Consider a branching diffusion in which each individual moves as a Markov diffusion with corresponding operator L and branches at a (spatial) rate b into precisely two particles at each fission point. Suppose the process begins from an individual particle. Given any ball and starting position, does a criterion exist which will guarantee the ball is visited (or charged) infinitely often by the branching process with positive/zero probability. The answer is yes and the criterion concerns the sign $(+/-)$ of the minimum l such that there exists a positive harmonic function with respect to the operator $(L + b - l)$. There is local extinction if and only if l is less than or equal to zero. The number l is called the generalized principal eigenvalue. We also discuss stronger quantitative results for the case that local extinction fails. Namely that, with positive probability, growth on compacta is faster than exponential with any rate less than l.

Fractal properties of continuous random trees (survey lecture)

Jean-François Le Gall, ENS Paris
(joint work with T. Duquesne)

We discuss various properties of the so-called continuous Lévy trees, which describe the genealogical structure of continuous-state branching processes, or of superprocesses, with a general branching mechanism ψ. Lévy trees can be obtained as scaling limits of discrete Galton-Watson trees (cf Duquesne and Le Gall, Astérisque vol.281). We explain how a Lévy tree is coded by the so-called ψ-height process, which is a local time functional of the Lévy process with no negative jumps and Laplace exponent ψ. This coding yields a mathematically precise definition of the tree as a quotient metric space, which is a particular instance of the $\mathbf{R}$-trees recently studied by several authors in a deterministic setting. We then discuss geometric and fractal properties of Lévy trees. In particular, we compute the Hausdorff and packing dimensions of the tree and its level sets in terms of the lower and upper indices of the function ψ at infinity.

Flows, coalescence, branching and noise

Yves Le Jan, Orsay

Three examples of stochastic flows are presented. First sticky flows, which appear as scaling limits of product of independent transition matrices defined by Beta variables. Second, some diffusion flows on the circle for which a non-uniqueness phase exists. The multiple solutions should be classified by stickiness. The third example is given by branching flows. They define three different types of noise: Black, white and Poisson.

Moran models and coalescent processes

Vlada Limic, UBC Vancouver
(joint work with A. Greven and A. Winter)

We consider spatial Moran models and their dual, a coalescent marked by partition locations. It turns out that this coalescent can be constructed on d-dimensional tori $[-n, n]^d$ starting from an initial state with infinitely many atoms at each site, in such a way that at any positive time there are only finitely many equivalence classes remaining.

Moreover, on the time scale proportional to the volume, the marked coalescent behaves approximately as Kingman's coalescent. These properties are useful for studying fine clustering properties of interacting Fisher-Wright diffusions.

Regularity of the invariant density of branching diffusions with immigration

Eva Löcherbach, Creteil

We consider finite systems of interacting branching diffusions with immigrations in $\mathbb{R}^d$. Coexisting particles may interact in their spatial deplacement and also in their branching mechanism. The branching is supposed to be binary and uniformly strictly subcritical.

We are interested in the intensity measure of the invariant measure of the process. The goal of this work is to show that there exists a continuous, bounded, one times partially differentiable Lebesgue density of the intensity measure. This is shown by using Malliavin calculus. The specific difficulty in our context is the fact that the dimension of the underlying diffusion is random – therefore an exact control of the dependency of all constants arising in Malliavins's integration by parts formula on the dimension is necessary.

Existence of local time for a two-type superprocess

José-Alfredo López-Mimbela, Guanajuato

Dynkin defined the local time of a continuous superprocess as a stochastic integral with respect to a martingale measure, and gave a criterion for existence of local time. In this talk we consider a continuous superprocess $X \equiv \{X_t,\ t \geq 0\}$ whose values are finite measures on the space $S = \{1,2\} \times \mathbb{R}^d$, where $\mathbb{R}^d$ is d-dimensional Euclidean space, having Laplace functional $E[\exp\{-\int_S \phi(z)\, X_t(dz)\}] = \exp\{-\int_S u_t(z)\, \mu(dz)\}$, where μ is a given finite measure on S, $\phi : S \to [0,\infty)$ is bounded and measurable, and u_t is the unique positive solution to the semilinear initial value problem $\partial_t u_t(i,x) = Au_t(i,x) - C_i u_t^2(i,x), \quad u_0 = \phi$. Here $C_i > 0$, $i = 1,2$ and $A\phi(i,x) := \Delta_{\alpha_i}\phi(i,x) + V_i \sum_{j=1}^2 (m_{ij} - \delta_{ij})\, \phi(j,x), \quad (i,x) \in S, \ \phi(i,\cdot) \in \mathrm{Dom}(\Delta_{\alpha_i})$, where Δ_{α_i} denotes the generator of the symmetric α_i-stable process, $V_i > 0$ and $m_{ij} > 0$ with $\sum_{j=1}^2 m_{ij} = 1$, $i = 1,2$. We prove that the conditions in Dynkin's existence criterion are satisfied by the superprocess X if $d < 2\min\{\alpha_1, \alpha_2\}$ and μ has a bounded density with respect to a reference measure m on S. We also give a Tanaka formula-like representation of the local time which is used to show that the occupation measure of the multitype superprocess is absolutely continuous with respect to m, and that the corresponding density coincides a.s. with the local time.

Super-Brownian motion in a stable catalytic medium

Peter Mörters, Bath

We study the effect of a highly singular catalytic medium on the macroscopic behaviour of super-Brownian motion, both in low and high dimensions.

In the case of low dimension, $d = 1$, the clumping effect due to the branching dominates of the mixing effect due to the particle movement. Dawson and Fleischmann (1988) studied this in terms of a mass-time-space rescaling limit theorem. The clumping effect is enhanced when the branching is catalytic with a stable measure of index $0 < \gamma < 1$ acting as a catalyst. In a a joint paper with Dawson and Fleischmann (Ann. Probab. 30(4), 2002) we prove a functional limit theorem which shows this effect. There are three qualitative differences: a *stronger* scaling is needed to capture the macroscopic picture; the limiting field is again a Poissonian field of clumps, but the intensity is decaying faster and the mass of the individual clumps is *heavy-tailed*. The mass of individual clumps is no longer a Feller-diffusion but a *non-Markovian process*. A precise result can be given in terms of a Brownian snake picture.

In the case of high dimensions the mixing effect of the motions dominates over the clumping effect. Holley and Stroock (1978) were the first to show this behaviour exhibiting convergence to a heat flow under the hydrodynamic scaling and Gaussian fluctuation behaviour. In an ongoing research project with Klaus Fleischmann we study super-Brownian motion with a (smeared-out) stable catalyst of index $0 < \gamma < 1$ in supercritical dimensions $d > 2/\gamma$. We show that there is still a law of large numbers under the hydrodynamic scaling, but the fluctuations are *stronger* and no longer Gaussian.

Regularity and irregularity of $(1+\beta)$-stable super-Brownian motion

Leonid Mytnik, Haifa
(joint work with E. Perkins and J.-F. Le Gall)

We study the regularity properties of the fixed time density and exit measure density for the $(1+\beta)$-stable super-Brownian motion. We prove the continuity of the fixed time density in dimension $d = 2$ and unboundedness of the density in all higher dimensions where it exists. For the exit measure density we establish its continuity in $d = 3$ and unboundedness in all higher dimensions where it exists. An alternative description of the exit measure and its density is also given via a stochastic integral representation.

High density Lotka-Volterra models converge to super-Brownian motion with drift - (survey lecture)

Edwin Perkins, UBC Vancouver
(joint work with T. Cox)

We show that for $d \geq 3$ a space-time rescaling of a Lotka-Volterra model introduced by Neuhauser and Pacala converges weakly to super-Brownian motion with drift. The Neuhauser-Pacala model $LV(\alpha_0, \alpha_1)$ is a two-parameter model which models the spatial distribution of two types, say 0's and 1's. Here α_0 measures the competitive effect of nearby 1's on a 0 and α_1 measures the competitive effect of nearby 0's on a 1. The self-competition parameters are both set to 1. $LV(1,1)$ is then the voter model. If $\alpha_i \to 1$ at an appropriate rate we get convergence of the rescaled LV to super-Brownian motion with drift. The theorem is then used to get information on the critical survival curve and coexistence region near $(1,1)$. It is a special case of a more general convergence result for perturbations of the voter model to super-Brownian motion with drift. In addition long range limit theorems are established for $\alpha_0 \in [0,1]$ and α_1 approaching 1. These lead to natural conjectures and partial results on the precise rate of convergence of the critical survival curve to that of the mean-field case.

The Finite System Scheme for state-dependent multitype branching processes

Peter Pfaffelhuber, FAU Erlangen-Nürnberg

We study branching processes on the lattice $\mathbb{Z}^d$. We introduce a multitype branching model in which the particles have some type $u \in [0,1]$, migrate on the lattice due to some random walk and branch. The branching rate for a particle at $\xi \in \mathbb{Z}^d$ is a function of the total mass at ξ. We develop the finite system scheme for this model, i.e. the comparison between infinite systems on $\mathbb{Z}^d$ and large finite systems on $(-N;N] \cap \mathbb{Z}^d$. It turns out that although the long-time behavior of finite and infinite systems is drastically different these two systems are still comparable up to a time that is of the size of the finite system. After this time the density of the finite system fluctuates, but locally it is in the equilibrium measure of the infinite system.

Asymptotic genealogy of critical branching processes

LEA POPOVIC, BERKELEY

Consider a continuous-time binary branching process conditioned to have population size n at some time t, and with a chance p for recording each extinct individual in the process. Within the family tree of this process, we consider the smallest subtree containing the genealogy of the extant individuals together with the genealogy of the recorded extinct individuals. We introduce a novel representation of such subtrees in terms of a point-process, and provide asymptotic results on the distribution of this point-process as the number of extant individuals increases. We motivate the study within the scope of a coherent analysis for an a-priori model for macroevolution.

Conditioned Super-Brownian Motion

THOMAS S. SALISBURY, TORONTO
(joint work with S. Athreya)

For a Euclidean domain, Salisbury and Verzani (1999, 2000) derived certain representations for super-Brownian motion with quadratic branching, conditioned in terms of its exit measure. These included both singular and non-singular conditionings, and in each case provided an explicit martingale change of measure, and a construction via a tree-like backbone, along which mass is immigrated.

Unlike in the case of quadratic branching, stable branching coupled with the singular conditioning causes mass to blow up severely. In joint work with Siva Athreya, we derive a representation for the non-singular conditioning in the case of a general branching mechanism. This is used to make precise the nature of the blowup in the singular case.

On superprocesses with Neveu's branching mechanism

ANJA STURM, TU BERLIN
(joint work with K. Fleischmann)

In this talk, a class of finite measure-valued càdlàg superprocesses X with Neveu's (1992) continuous-state branching mechanism is constructed. To this end, we start from certain supercritical (α, d, β)-superprocesses $X^{(\beta)}$ with symmetric α-stable motion and $(1+\beta)$-branching and prove convergence on path space as $\beta \downarrow 0$. The log-Laplace equation related to X has the locally non-Lipschitz function $u \log u$ as non-linear term (instead of $u^{1+\beta}$ in the case of $X^{(\beta)}$). The process X has infinite expectation, is immortal in all finite times, propagates mass instantaneously everywhere in space, and has locally countably infinite biodiversity.

Branching processes in random environment

VLADIMIR A. VATUTIN, STEKLOV MATHEMATICAL ISTITUTE, MOSCOW

Let Z_n be the number of particles at time n in a branching process in random environment and $Z_{m,n}$ be the number of particles in this process at time $m \le n$ which have non-empty offspring at time n. Assuming that Z_n is "critical" in a certain sense we prove limit theorems for the process $Z_{nt,n},\ t \in (0,1]$ conditioned on the event $\{Z_n > 0\}$. In particular, we describe the "bottleneck" phenomena for such processes occurring both under the quenched and annealed approaches.

Hierarchical branching equilibria and super Feller cascades (survey lecture)

Anton Wakolbinger, Frankfurt

Consider a population distributed over (infinitely) many islands with a hierarchically structured geography (in the sense of Sawyer and Felsenstein ('83)). The individuals perform independent critical branching and migrate according to a symmetric random walk which is transient but may be almost recurrent.

Then the diffusion limit (order of N individuals per island, N islands per archipelago, etc, $N \to \infty$) of the population densities in a sequence of nested blocks (island, archipelago,...) leads to a cascade of Feller branching diffusions in (quasi-)equilibrium in the asymptotically separated time scales $N^\ell, \ell = 1, 2, \ldots$, cf. Dawson and Greven ('96).

In recent work with Dawson and Gorostiza, we consider a strongly transient individual migration on the borderline to weak recurrence (and in that sense analogous to a $4+0$ dimensional random walk) which counterbalances strong fluctuations of the population densities caused by two-level (individual plus family) branching. The diffusion limit of the family size process in a sequence of nested blocks leads to a cascade of superprocesses built over subcritical Feller branching diffusions. Now the asymptotic time scales are $N^{\ell/2}$, and an entrance law of iterated subordinators encodes the genealogical relationship of individuals within an island as $N \to \infty$.

A stochastic log-Laplace equation with applications

Jie Xiong, Knoxville

We study a nonlinear stochastic partial differential equation whose solution is the conditional log-Laplace functional of a superprocess in a random environment. We establish its existence and uniqueness by smoothing out the nonlinear term and making use of the particle system representation developed by Kurtz and Xiong. We also derive the Wong-Zakai type approximation for this equation. As an application, we give a direct proof of the moment formulas of Skoulakis and Adler. As further applications, we derive the longtime limit of the process as well as that of a related immigration process.

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Branching in and out of Oberwolfach:
Sneaking in the archives, digging in the memories, and looking ahead

Anton Wakolbinger, Frankfurt

The present conference was preceded by two ancestors in 1967 and 1995. The 1967 conference was organised by D. Kendall (Cambridge) and H. Dinges (Frankfurt), among the participants were A. Kolmogorov, T. Harris, A. Renyi and P. Jagers. The vigorous development and the fruitful extensions which the field of branching processes has experienced since then were the subject of a reflection, accompanied by some personal reminiscences of Peter Jagers, at the end of the present conference.

Edited by Roland Alkemper

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