Mathematisches Forschungsinstitut Oberwolfach

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Dynamics of Structured Systems

December 14th – December 20th, 2003

The meeting was organized by Wolf–Jürgen Beyn (Bielefeld), Bernold Fiedler (Berlin) and John Guckenheimer (Cornell).

The main purpose of the conference was to present and discuss current progress in the mathematical analysis as well as in numerical methods for dynamical systems that show special structures. Among the topics treated were:

- different time scales, in particular singularly perturbed systems,
- invariant manifolds and dimension reduction,
- symmetries or Hamiltonian structures of the underlying vector field,
- spatio–temporal phenomena in time–dependent partial differential equations, such as shock waves, fronts and spiral waves,
- synchronized and desynchronized behavior of coupled oscillator chains.
- statistical analysis of longtime behavior.

The various sessions were organized in such a way that each of the topics was discussed from an analytical and a numerical point of view. Several new approaches were presented and new areas of applications appeared, mainly to biological and physical systems.

Many of the presentations revealed that, in spite of substantial progress over the last years, our mathematical understanding of the additional structures in dynamical systems has to be deepened further in order to fully grasp their influence on asymptotic behavior and to exploit far–reaching applications to real–world problems. For such an achievement further joint efforts by mathematicians working in the areas of theory, numerics and applications of dynamical systems are desirable.

Special sessions

On one afternoon two parallel problem sessions were held:

(1) ”Multiscale Dynamics”, organized by Klaus Schneider, WIAS, Berlin)
Issues under considerations were: Singularly perturbed differential-delay equations (Hadeler, Mallet-Paret, Schneider), approaches in case of loss of hyperbolicity (Szmolyan, Schneider), biological applications (Doedel, Hadeler), open problem in continuum mechanics (Mielke).

(2) ”Rigorous numerics and numerical dynamics”, organized by Lars Grüne (Bayreuth) and Oliver Junge (Paderborn)
The issues were related to the question ’how false conclusions from numerical computations about the dynamics of the underlying dynamical system can be avoided’. In particular the discussion focused on high-dimensional systems and on methods that use the Conley index.

Abstracts

A novel preserved partial order for cooperative networks of units with overdamped second order dynamics, and application to Frenkel-Kontorova chains

Claude Baesens

We introduce a novel partial order on the state space for cooperative networks of units with overdamped second order dynamics. Among other things, we use it to prove that for chains of particles in a tilted periodic potential with overdamped inertial dynamics, for each class of spatially periodic states if there are no rotationally-ordered equilibria then there is a globally attracting periodically sliding solution.

Freezing solutions of equivariant evolution equations: Theory

Wolf-Jürgen Beyn

(joint work with Vera Thümmler)

We consider general evolution equations

$$\dot{u} = F(u),\ u(0) = u_0, \quad F : D(F) \overset{\text{dense}}{\subset} X \mapsto X \text{ (B-space)} \tag{1}$$

that are equivariant with respect to the action $a : G \mapsto GL(X)$ of a finite-dimensional (not necessarily compact) Lie group G.

We propose to solve (1) by introducing new variables $\gamma(t) \in G$, $v(t) \in D(F)$ such that

$$u(t) = a(\gamma(t))v(t), \quad t \geq 0.$$

The extra degrees of freedom are compensated for by $n = \dim(G)$ so called phase conditions. The triple $v \in D(F),\ \gamma \in G,\ \lambda \in T_\gamma G$ then satisfies a differential algebraic equation (PDAE).

The phase condition ψ is selected so as to minimize at each time instance the time variation $|\dot{v}|^2$ for some Hilbert norm $|\cdot|$ on X. Solving this PDAE numerically rather than the original problem (1) provides a method for separating the motion $\gamma(t)$ on the group from a 'frozen profile' $v(t)$ that is forced to vary as little as possible. This is particularly suitable near relative equilibria of the equivariant evolution equation (1).

The method applies to semilinear parabolic systems on unbounded domains, such as

$$u_t = \Delta u + f(u),\ x \in \mathbb{R}^2, t \geq 0, \quad G = SE(2),\ [a(\gamma)u](x) = u(\gamma x), x \in \mathbb{R}^2, \gamma \in G.$$

By the above approach it is possible to 'freeze' traveling, rotating and spiral waves in a fixed reference frame.

Elemental Periodic Orbits of the Circular Restricted Three-Body Problem

Eusebius Doedel

Periodic solutions of the CR3BP have been studied extensively in the literature, with many important contributions. These orbits are important in space-mission design, for example, the Genesis mission, which was designed using dynamical systems concepts, such as stable and unstable manifolds, is currently in a so called Halo orbit.

In this talk I will show how boundary value continuation methods can be effective tools for studying these orbits and their bifurcations. In particular, I will show a selection of recent computational results for the families of periodic orbits, that originate from the five libration points and for some secondary bifurcating families. Specifically, I will show how extended boundary value systems can be used to track loci of such bifurcations. This results in a rather complete classification of the solution structure for all values of the mass-ratio of the primaries.

Edge bifurcations for singularly perturbed reaction-diffusion equations: a case study

Arjen Doelman

We consider the stability of front solutions to a certain class of bi-stable equations. A leading order analysis indicates that these fronts are destabilized by the essential spectrum of the associated linear operator. However, a more careful analysis shows that there may, or may not, appear (discrete) eigenvalues from the essential spectrum that cause the destabilization. In this talk we use geometric singular perturbation theory and the Evans function approach to establish a relation between the (geometric) character of the front solution and the occurrence of edge bifurcations.

Kolmogorov systems and permanence

Barnabas M. Garay

(joint work with Josef Hofbauer)

We present a sufficient condition for robust permanence of ecological Kolmogorov differential equations of the form $x_i = x_i f_i(x_1, x_2, \ldots, x_n),\ i = 1, 2, \ldots, n,\ x_i \geq 0, \sum_{i=1}^{n} x_i = 1$. It is assumed that $\sum_{i=1}^{n} x_i f_i(x_1, x_2, \ldots, x_n) \equiv 0$. This makes both the unit simplex $X = \{x \in \mathbb{R}^n \mid x = (x_1, x_2, \ldots, x_n),\ x_i \geq 0,\ \sum_{i=1}^{n} x_i = 1\}$ and its boundary $Y = \partial X$ invariant. In biological applications, x_i means the proportion of the $i-$th species in an ecosystem and permanence - Y is a repellor - means the ultimate survival of all species. Robustness means that permanence is preserved under small perturbations respecting the Kolmogorov form of the equations. The concept of permanence is investigated by using various mathematical tools including

a) average Liapunov functions of the form $R(x) = \sum_{i=1}^{n} r_i \log x_i,\ r_i \geq 0$,
b) the set $M_\phi(Y)$ of all invariant Borel probability measures with support on Y,
c) the minmax theorem,
d) the Conley index.

Then we study the robustness of permanence criteria against discretizations.

Coupled Cell Systems

Martin Golubitsky

(joint work with Ian Stewart, Marcus Pivato, Matt Nicol, and Yunjiao Wang)

A cell is just a system of ODEs. Each coupled cell system has a network architecture that indicates which cells are equivalent and which couplings are equivalent. Synchrony subspaces are defined by setting coordinates in different cells equal. For a fixed network architecture, a robust synchrony subspace is a synchrony subspace that is flow invariant for all coupled cell systems with that architecture. We show that (1) patterns of synchrony correspond to balanced colorings on the set of cells, and (2) the restriction of a coupled cell system to a robust synchrony subspace is itself a coupled cell system corresponding to a quotient network.

We consider two examples: a square lattice of cells with nearest neighbor coupling and a three-cell feed-forward network. In the square lattice we classify all balanced two-colorings and show that all balanced two-colorings lead to equilibria in codimension one steady-state bifurcations from a uniform equilibrium. We also show that Hopf bifurcation in the feed-forward forward network leads to states that are in equilibrium in the first cell and periodic in the second and third but where the amplitude growth in the second cell is to the expected 1/2 power and the third cell is to a surprising 1/6 power.

Robust Stability, Numerical Dynamics and Digital Control

Lars Grüne

In this talk we present a framework for the analysis of robust stability properties for dynamical systems subject to time varying perturbations. We use a dynamical systems extension of the "input–to–state stability" concept which was developed in nonlinear control theory. This way we derive a robust stability property which (i) allows for the explicit treatment of time–varying perturbations and (ii) allows a "tight" Lyapunov function characterization in the sense that the attraction rate and the robustness margin are exactly represented in the Lyapunov function.

Differential equations for granular matter

Karl-Peter Hadeler

Granular matter theory started early (Coulomb 1773: angle of repose and static friction; Monge 1781: transport problem; Felix Auerbach 1901: equilibrium configurations; R.A. Bagnold 1940 ff: blown sand) but only in the last few decades there is a rapid development in several research directions: Dynamic models for avalanches and for wind-blown sand, dune formation, vibrational instability, segregation and stratification.

Here a hyperbolic system for slow deposition of granular matter under the influence of gravity (without inertia) is presented. The characteristic directions, downhill for the rolling layer and uphill for the resting layer, explain a variety of phenomena such as the behavior at the wall of a silo or at the edge of a table (with C. Kuttler).

Boundary value problems and obstacle problems for the eikonal equation describe equilibrium configurations under gravity (with C. Kuttler and I. Gergert). A striking analogy between harmonic functions (Dirichlet problem) and a class of distinguished solutions of the eikonal equation can be shown. These functions can be characterized by a variational principle (maximizing the volume), by a Perron method of "subeikonal" functions and by transport paths (these are not the transport paths of the Monge-Kantorovich problem). These approaches work as well for the obstacle problems as for the boundary value problem and yield Lipschitz solutions under very weak regularity assumptions. For Lipschitz boundaries the different approaches are shown to be equivalent.

As limiting cases of the dynamical model one can design systems of ordinary differential equations for the instantaneous deposition from a finite number of sources (Aronsson, L.C. Evans). C. Kuttler has studied the one-dimensional case which develops strong singularities and she has found a way of smoothing the time course (rather than the profiles) to select physically correct solutions.

Joel Braun has studied the problem of segregation in polydisperse granular mixtures (Muesli effect, Brazil nut effect). He proposes a system of nonlinear diffusion convection equations where convection models the competition between grains of different types under gravity on a vertical scale while diffusion models the stochastic perturbation which keeps grains moving.

From FPU-type problems to nonlinear wave equations

Ernst Hairer

(joint work with Christian Lubich)

We aim in a better understanding of the long-time behaviour of numerical integrators when applied to differential equations with highly oscillatory solution components.

The so-called "modulated Fourier expansion" (see chapter XIII of our recent monograph on "Geometric Numerical Integration") gives much insight into the long-time near-conservation of the total and oscillatory energies, if the high oscillations stem from a linear part in the differential equation and if only one high frequency is present. We give an extension of these results to the situation where more than one high frequency give rise to the oscillations.

We then also consider space discretizations of nonlinear wave equations (partial differential equations) which lead to systems with a large range of frequencies. Numerical results will be presented on the near-conservation by symmetric numerical integrators of the total energy and of the oscillatory energies corresponding to high frequencies in the system. A theoretical explanation of the observations is still missing.

Global Analysis of the Forced van der Pol Equation

Kathleen Hoffman

(joint work with John Guckenheimer, Warren Weckesser etc.)

The forced van der Pol oscillator serves as a classic example of a multi-timescale system. I will discuss a hybrid system that approximates solutions of the forced van der Pol equation by decomposing the orbit into trajectories that follow the "slow flow" and "jumps" to approximate the fast subsystem. The global bifurcations of the fixed points and periodic points of this hybrid system lead to an understanding of the bifurcations in the periodic orbits (without canards) of the forced van der Pol system. In addition, I will describe a modified hybrid system that includes the canard solutions and briefly describe the additional bifurcations that occur in this extended map.

Complicated dynamics in spatially extended time-discrete dynamical systems

Oliver Junge

(joint work with Sarah Day, Konstantin Mischaikow)

We present a computational technique that allows to draw rigorous conclusions about the global dyamical behaviour of infinite dimensional maps. Based on a finite dimensional Galerkin approximation of the system and corresponding bounds on the truncation error, the technique employs set oriented numerical methods for the computation of coverings of invariant sets of the multivalued Galerkin system. Using the Conley index theory, rigorous statements abourt the dynamical behaviour within these coverings are obtained. By verifying certain conditions on the truncated modes it is possible to lift the index information to the full system.

Phase Synchronization of Coupled Chaotic Systems and the Influence of Noise

Jürgen Kurths

First recognized in 1665 by C. Huygens, synchronization phenomena are abundant in science, nature and engineering. I will report on recent developments on phase synchronization in coupled complex oscillators and distributed systems. Then numerical results on constructive effects of noise on inducing resp. enhancing synchronization as well as experimental results will be presented. Some explanations for these phenomena will be given, but also several open questions.

Bifurcations without parameters along manifolds of equilibria

Stefan Liebscher

(joint work with A. Afendikov, B. Fiedler)

Classical bifurcation theory studies dynamical systems $x' = f(x,p), x \in X$, that depend on parameters $p \in Y$. Frequently, trivial fixed points $x = 0$ are assumed to exist. In an extended phase space $X \times Y$, they form manifolds with a trivial transverse foliation.

In contrast, we consider vector fields with equilibrium manifolds that are not induced by any parameters. We address the failure of normal hyperbolicity in absence of any transverse flow-invariant foliation. We call our emerging theory "bifurcation without parameters".

Applications include coupled oscillators, travelling wave profiles in systems of hyperbolic balance laws, population dynamics, fluid mechanics, and many more.

Motivated by the Kolmogorov problem on an incompressible fluid flow in a plane channel, we discuss a reversible Bogdanov-Takens bifurcation without parameters. The analysis includes Kirchgässner reduction (spatial dynamics), normal form calculation, blow up, reduction to Hamiltonian form, averaging, Melnikov calculations, and the discussion of higher order terms.

The Role of the Pressure in Potential Blow-Up of Solutions of the Navier-Stokes Equations

Jens Lorenz

I consider the Cauchy problem for the incompressible Navier-Stokes equations with smooth, rapidly decaying initial data. It is not known if singularities can develop in finite time. To analyze the equations, it is common to first eliminate the pressure by applying the Helmholtz projector. However, since pressure differences drive the flow, the pressure term is essential for understanding blow-up conditions. In this lecture, I will describe properties of the pressure, assuming that blow-up does occur.

Symplectic multistep methods over long times

Christian Lubich

(joint work with Ernst Hairer)

For computations of planetary motions with symmetric multistep methods an excellent long-time behaviour is observed. Neither the total energy nor the angular momentum exhibit secular error terms. In this talk this behaviour is explained by studying the modified equations of these methods and by analyzing the remarkably stable propagation of parasitic solution components.

Continuation in large networks: direct and tensor products

Robert MacKay

Continuability (smooth and locally unique) of solutions of a dynamical system is a useful property. For systems consisting of a network of units, one can ask whether continuability is uniform in the network size and what implications any spatial decay structure of the coupling between units has on the continuations. If the network dynamics is on a direct product, there are uniform hypothesis which imply uniform continuability, and spatial decay of coupling implies spatial decorrelation of continuation (with some weaker functional form). An application is to prove existence and properties of "discrete breathers": time - periodic spatially localized solutions. For network dynamics on a tensor product, however, uniform continuability fails in standard norms. For example, the unique stationary probability for a network of Markov processes and the continuation of a spectral projection for a quantum lattice system move non-uniformly with typical coupling parameters. Some new norms are proposed to deal with this case.

Dynamics of Max-Plus Operators

John Mallet-Paret

(joint work with Roger Nussbaum)

We study some dynamical issues associated with max-plus operators $T : C[0,1] \to C[0,1]$, namely operators given by

$$T\varphi(t) = \max_{s\in J(t)} (a(t,s) + \varphi(s)).$$

Here $a(t,s)$ is a given kernel function and $J(t) \subseteq [0,1]$ is a given t-dependent interval. Of particular interest are solutions of the additive eigenfunction problem $T\varphi = \varphi + r$. Such solutions arise in the description of patterns seen in periodic solutions of state-dependent delay-differential equations.

Atomic-scale localization of lattice waves

Karsten Matthies

(joint work with Gero Friesecke)

One-dimensional monatomic lattices with Hamiltonian $H = \sum_{n\in\mathbf{Z}} (\frac{1}{2}p_n^2 + V(q_{n+1} - q_n))$ of Fermi-Pasta Ulam-type are known to carry localized travelling wave solutions, for generic nonlinear potentials V . The profile of these waves in the high-energy limit $H \to \infty$ is derived for Lennard-Jones type interactions. The limit profile is proved to be a universal, highly discrete, piecewise linear wave concentrated on a single atomic spacing.

This shows that dispersionless energy transport in these systems is not confined to the long-wave regime on which the theoretical literature has been focused, but also occurs at atomic-scale localization.

Furthermore we describe the existence of longitudinal travelling waves in a two-dimensional square lattice with nearest and next nearest neighbour interaction via harmonic springs, where the needed overall unharmonicity is created by the geometry of the lattice.

Macroscopic Limits for Oscillator Chains

Alexander Mielke

Starting from the Newtonian dynamical system of infinitely many identical atoms coupled by identical (nonlinear) spring we derive several macroscopic limit equations depending on the particular set up of the initial condition

a) (nonlinear) elastic-wave equation
b) Wigner transport equation for the energy measure
c) the nonlinear Schrödinger equation
d) Whitham's modulatin equation for (strain, velocity, wave number, frequency).

We give theorems justifying these limits .

Rates of convergence of discretized attractors for parabolic equations

Sergei Yu. Pilyugin

We present a general estimate for the rate of convergence of approximate attractors to the global attractor A of an evolutionary system in the case where the system is structurally stable on A. The result is applied to various discretizations of a semilinear parabolic equation. A particular case of an attractor containing a non-hyperbolic fixed point (for the Chafee-Infante problem) is also considered.

Order Preserving Semiflows

Carlos Rocha
(joint work with Giorgio Fusco)

Dynamical systems generated by one-dimensional reaction-diffusion equations enjoy special properties that lead to a very simple structure for the semiflow. Among these properties, the monoton behaviour of the number of zeros of the solutions plays an essential role. This discrete Liapunov functional, the zero number, contains important information on the spectral behaviour of the linearizations and leads to the simple description of the dynamical system. Other systems possess this kind of discrete Lyapunov functional and we discuss classes of linear equations that generate semiflows with this property. Moreover, we ask if this property is characteristic of such problems.

On resonances and energy surfaces

Vered Rom-Kedar
(joint work with A. Litvak-Hinnenzon)

A framework for understanding the global structure of near integrable n d.o.f systems is proposed. The goal is to reach a similar situation to the near integrable 1.5 d.o.f. systems, where one is able in a glance of the integrable phase portrait, to understand where instabilities are expected to arise. It is suggested that the main tool for understanding the system structure is an energy-momentum bifurcation diagram and generalized Fomenko graphs. Choosing the appropriate co-ordinates in this diagram (these are determined by the form of the perturbation) leads to establishing a relation between bifurcations in such plots and resonances. Thus, a novel connection between changes in the topology of the energy surfaces and resonances is found. It is demonstrated that for some systems this procedure is sufficient for achieving a qualitative understanding of the near integrable dynamics. In particular, the persisten appearance of instabilites associated with resonant lower dimensional tori will be described.

Towards a classification of defects in oscillatory media

Björn Sandstede

Coherent structures, or defects, are interfaces between wave trains with possibly different wavenumbers: they are time-periodic in an appropriate coordinate frame and connect two, possibly different, spatially-periodic travelling waves. We propose a classification of defects into four different classes which have all been observed experimentally. The characteristic distinguishing these classes is the sign of the group velocities of the wave trains to either side of the defect, measured relative to the speed of the defect. Using a spatial-dynamics description in which defects correspond to homoclinic and heteroclinic connections of an ill-posed pseudo-elliptic equation, we relate robustness properties of defects to their spectral stability properties and determine the multiplicity with which they occur in generic reaction-diffusion systems.

Absolute instabilities of standing pulses

Scheel, Arnd

We analyze instabilities of standing pulses in reaction-diffusion systems that are caused by an absolute instability of the background state. Specifically, we investigate Turing and oscillatory bifurcations of the homogeneous background state and their impact on the standing pulse. At a Turing instability, symmetric pulses emerge that are spatially asymptotic to the bifurcating spatially-periodic Turing patterns. Oscillatory instabilities of the background state typically lead to modulated pulses that emit small wave trains. We analyse these three bifurcations by studying the standing-wave equation: the standing pulses correspond then to homoclinic orbits to equilibria that undergo reversible bifurcations. We use blow-up techniques to show that the relevant stable and unstable manifolds can be continued across the bifurcation point. To analyze stability, we construct, and extend across the essential spectrum, appropriate Evans functions for operators with algebraically decaying coefficients.

Time discretization of structured systems: selected examples and applications

Schropp, Johannes

Modelling real world phenomena very often leads to ordinary, delay or differential algebraic equations with a special structure reflecting the properties of the underlying problem. Typical examples are gradient, symplectic or monotonicity structure or one is interested in the phase portrait near spcial invariant sets. We present discretization results for a variety of these situations which are of relevance in applications. Moreover, computational examples are shown.

Kinetic models for chemotaxis and their drift-diffusion limits

Angela Stevens

A widespread phenomenon in moving microorganisms and cells is their ability to reorient themselves in dependence of chemical signals. This behavior is crucial for the cells to adapt to a changing extracellular environment.

During the talk kinetic models for chemosensitive movement and their macroscopic limits are discussed. Examples are given how the evaluation of the external signal by the cells influences the macroscopic process of structure formation.

Spectral stability of small amplitude shock waves

Peter Szmolyan
(joint work with Heinrich Freistühler)

In this talk resultson the stability of viscous shock waves in one and several space dimensions are presented. In the context of Evans function theory a geometric framework is established which allows to show that small amplitude viscous profiles are spectrally stable, i.e. the corresponding linearization along the wave has no eigenvalues λ with $Re(\lambda) \geq 0, \lambda \neq 0$. A suitable scaling brings out the slow-fast structure of the problem, which is used to construct the stable and unstable spaces of the eigenvalue problem by methods from invariant manifold theory. In one space dimesion this allows to reduce the stability question of small amplitude genuine nonlinear K-shocks to the corresponding problem for a fixed shock of Burgers equation, which are known to be stable. The stability question for shocks associated with conconvex or muliple modes is reduced to the stability of shocks at low-dimensional model problems. The stability of multi-d shock waves is reduced to the condition that a certain "residual" Lopatinski-Kreiss-Majda determinant does not vanish, which is shown to be true under natural assumptions.

Freezing Solutions of Equivariant Evolution Equations: Numerics

Vera Thümmler
(joint work with Wolf-Jürgen Beyn)

We first describe the numerical method used to solve the equations for the frozen system which have been derived in the corresponding talk by W.-J. Beyn. Then we present numerical computations for examples in one space dimension such as traveling pulses in the FitzHugh Nagumo system and rotating waves in the complex Ginzburg-Landau equation. In the plane we show examples of freezing rotating waves in $\lambda - \omega$-systems and rigidly rotating spiral solutions in the Barkley system.

Numerical difficulties arising from convective terms introduced by the freezing procedure and the influence of the choice of phase condition are discussed.

On the Dynamics of Certain Subgrid Turbulence Models

Edriss S. Titi

In recent years many new turbulence models have been introduced based on semi-rigorous mathematical analysis. Such as the Navier-Stokes-alpha model, the Leray-model, the Clark model as well as the Smagorinsky turbulence model. In this talk I will present results about the global well-posedness and regularity of these models as well as provide estimates for their global attractors and relate these estimates to heuristic physical arguments.

Ordered Upwind Methods for Approximating Invariant Manifolds

Alexander Vladimirsky

(joint work with John Guckenheimer)

We show how the problem of constructing an invariant manifold (of co-dimension k) can be locally reduced to solving a (system of k) quasi-linear PDE(s), which can be efficiently solved using an Ordered Upwind Method (OUM). Such methods, originally introduced in a joint work with J.A. Sethian for static Hamilton-Jacobi PDEs, rely on careful use of the direction of information propagation to systematically advance the computed "boundary" and to de-couple the discretized system. We illustrate our approach by constructing invariant manifolds of hyperbolic saddle points for several "geometrically stiff" systems.

Singularly Perturbed Systems in $\mathbb{R}^3$ Existence and Bifurcation of Canards

Martin Wechselberger

We give a geometric analysis of canard solutions in singularly perturbed systems with $2-d$ folded critical manifold. Under the violation of a cerain transversality condition singular canards are detected in the reduced system near isolated points of the fold-curve. These canard points are classified in correspondence with the phase portrait of the reduced flow as folded saddles, folded nodes or folded saddle-nodes. Based on the blow-up technique the existence of canards in the case of folded saddles and folded saddle-nodes is proved. Furthermore it is shown that bifurcations of canards occur in the case of folded nodes. We give a detailed geometric explanation for this phenomenon. All these types of canards are found in e.g. the famous forced van der Pol oscillator, in chemical oscillators and in models of coupled neurons.

Canards and Horseshoes in the Forced van der Pol Equation

Warren Weckesser

(joint work with John Guckenheimer, Kathleen Hoffman)

Cartwight and Littlewood discovered "chaotic" solutions in the periodically forced van der Pol equation in the 1940's. Subsequent work by Levinson, Levi, and others has made this singularly perturbed system one of the archetypical dissipative systems with chaotic dynamics. Despite the extensive history of this system, many questions concerning its bifurcations and chaotic dynamics remain. We use a combination of analysis of the singular limit and numerical simulation to investigate the bifurcations of this system. In particular, we give a clear picture of a horseshoe map that arises in a cross-section of the three-dimensional phase space. The canards that form at a "folded saddle" play a crucial role in this analysis.

Attractors of semilinear parabolic equations on the circle

Matthias Wolfrum

Under suitable conditions on the nonlinearity, the scalar semilinear parabolic reaction-advection-diffusion equation

$$u_t = u_{xx} + f(u, u_x),\ x \in S^1,\ t > 0$$

possesses a global attractor, consisting of equilibria, rotating waves, and heteroclinic connections between them. Using the nodal properties of the solutions, we study the geometry of the attractors. We introduce a criterion, deciding which equilibria and waves are connected by a heteroclinic orbit, and classify different types of possible attractors.

Approximate Momentum Conservation for Spatial Semidiscretizations of Semilinear Wave Equations

Claudia Wulff
(joint work with Marcel Oliver, Matt West)

We prove that a standard second order finite difference uniform space discretization of the semilinear wave equation with periodic boundary conditions, analytic nonlinearity, and analytic initial data conserves momentum up to an error which is exponentially small in the stepsize. Our estimates are valid for as long as the trajectories of the full semilinear wave equation remain real analytic. The method of proof is that of backward error analysis, whereby we construct a modified equation which is itself Lagrangian and translation invariant, and therefore also conserves momentum. The modified equation interpolates the semidiscrete system for all time, and we prove that it remains exponentially close to the trigonometric interpolation of the semidiscrete system. These properties directly imply approximate momentum conservation for the semidiscrete system.

Edited by Vera Thümmler

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