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Entanglement and Decoherence: Mathematics and Physics of Quantum Information and Computation

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Abstract. This is the report for the Oberwolfach workshop on Entanglement and Decoherence: Mathematics and Physics, held January 23 - 29, 2005.

Mathematics Subject Classification (2000): 94Axx.

Introduction by the Organisers

The meeting was attended by over 50 participants from more than 10 countries and three continents.

It brought together mathematicians, physicists and computer scientists working on quantum information and computation. Experts were present as well as young participants, both at predoctoral level as well as postdoctoral level. The focus was on the problems of entanglement and decoherence, aiming at the formulation and discussion of precise concepts, developments of models and their interrelations, and the discussion of experiments in relation with the theory. Special attention was given to recent developments, and to furthering interaction and co-operations between the different groups. The scientific programm focused mainly on the following subjects: 1) Quantum Entanglement and Nonlocality: including Bell-type inequalities (theoretical and experimental studies), equivalence of quantum states under local unitary transformations. 2) Quantum Separability: separability criteria for multiple quantum mixed states in arbitrary dimension. 3) Decoherence: study of models; quantum error corrections, fault tolerant computation, multiparticle problems. 4) Sources of Quantum Entanglement. 5) Quantum Information: quantum cloning, teleportation, key distribution, algorithms. 6) Quantum Measurement and Quantum Optics; Holonomic quantum gates; Quantum semigroups;

Continuum Observation. 7) Mathematical Quantum and non commutative structures in connection with quantum information theory.

20 lectures were presented, including 15 surveys and 5 concentrating on specific recent results. There were 10 informal evening lectures on 3 topics: entanglement, control and decoherence. Besides 7 informal talks were presented in 2 round table discussions.

All presentations and discussions amply demonstrated the vitality and actuality of this area of research and the fascinating interrelations between the different specialities it involves. The organizers and participants would like to take this opportunity to thank the Mathematisches Forschungsinstitut Oberwolfach for having provided a comfortable and inspiring environment for the meeting and the scientific work.

Workshop: Entanglement and Decoherence: Mathematics and Physics of Quantum Information and Computation

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Abstracts

Controllability of the Schrödinger Equation via Intersection of Eigenvalues

Riccardo Adami and Ugo Boscain

The issue of designing an efficient transfer of population between different atomic or molecular levels is crucial in atomic and molecular physics. In the experiments, excitation and ionization are often induced by means of a sequence of laser pulses. Mathematically, the description of such processes translates into the problem of controlling the Schrödinger equation.

We introduce two toy models and propose a method to prove approximate controllability of the Schrödinger equation. More specifically, given two arbitrary eigenstates of the uncontrolled system, we construct a path in the space of controls that steers the system from the first to the second; the target is reached only approximately, but the accuracy of the approximation can be arbitarily improved slowing the process down and correspondingly raising its duration. Our main technical tool is the adiabatic theorem ([5, 6] and refereces therein), which requires slowly varying controls and gives explicit estimates of the error.

It is worth pointing out that in order to apply our method we need a Hamiltonian with purely point spectrum that degenerates for some values of the controls. This seems to be in contradiction with the claimed use of the the adiabatic theory, which requires that during the whole time evolution the eigenvalues remain separated by a non vanishing gap ("gap condition").

The main idea is that such a difficulty can be overcome by a decoupling between the levels other than the adiabatic one. This observation is crucial in our analysis and we shall specialize it to the two toy models we deal with.

Before doing that, we stress that our strategy can be applied in many situations in which classical control theory would be too difficult or cumbersome, provides explicit expressions of controls (motion planning), and most of all is very robust, in the sense that similar controls produce similar population transfers. This last feature is very important in experiments.

Let us describe our two toy models. The former is the simplest generalization to infinite dimension of three-level models that describe STIRAP processes (see for instance [7]). As in that cases, it is given in the representation of the eigenfunction of the uncontrolled Hamiltonian, namely as an infinite dimensional matrix.

The full Hamiltonian reads $H(u, v) = H_0 + uB_1 + vB_2$, where the spectrum of the drift (or "free") Hamiltonian H_0 is discrete and non degenerate. The coupling B_1 couples level E_i with E_{i+1} for i even, while B_2 couples E_i with E_{i+1} for i odd.

For every value of u and v the spectrum of $H(u, v)$ is still discrete, but degeneracies occur at isolated points in the space of the controls. This phenomenon holds generically for Hamiltonians depending on two parameters, and one refers to it as to the "conical crossing" of eigenvalues (see e.g. [4]).

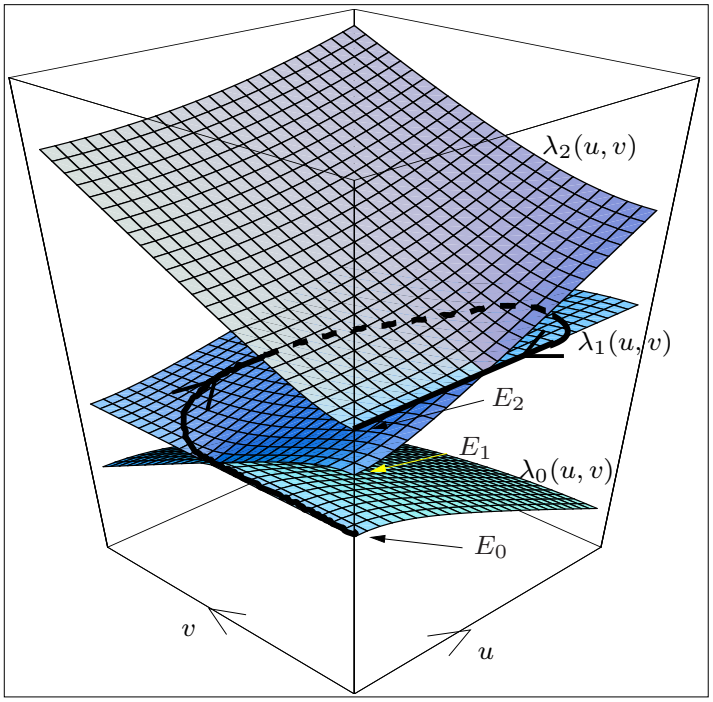

FIGURE 1.

Assume that at time zero $u = v = 0$ and the system lies in the ground state of the drift H_0. The adiabatic theorem asserts that, employing slow varying controls $u(\varepsilon t)$ and $v(\varepsilon t)$ such that $H(u(\varepsilon t), v(\varepsilon t))$ has no degeneracies for any t, then at time t the system lies close to the ground state of $H(u(\varepsilon t), v(\varepsilon t))$.

As widely known, the situation becomes more complicated when the system is driven near eigenvalues intersections.

Nevertheless, we exhibit paths in the control space that pass exactly through an eigenvalue intersection and force the system to perform a transition from the old to a new level. In Fig. 1 an example of a path steering the system from E_0 to E_2 is portrayed.

Notice that, in order to employ such a strategy, we need to move controls along a surface, so we must have at our disposal at least two controls: the crucial point is to cross the eigenvalues intersections in one direction only, and this would not be possible with one control.

The strategy presented here is inspired to a numerical method developed for finite dimensional systems in [7].

Although this toy model is easily solvable using classical control theory, in our opinion it clearly illustrates how our method works.

The second model we present consists of the Schrödinger picture of a quantum particle in a one-dimensional infinite potential well with some additional controlled external fields.

The main difficulty to overcome is due to the fact that in a one dimensional quantum system the presence of degeneracies in the discrete spectrum is a highly

nonstandard feature. In particular the non degeneracy of the ground state holds in any dimension for systems subject to a locally integrable potential field. Therefore our strategy consists in producing degeneracies by means of potentials with non integrable singularities. To this purpose we use point interaction potentials (Dirac δ and δ') with a coupling constant to be sent to infinity. Let us recall that interactions like Dirac δ and δ' are widely used in modeling of quantum system, since Fermi's paper [3] up to contemporary applications [1, 2].

We consider a particle confined to the interval $(-\pi/2, \pi/2)$, whose Hamiltonian reads $H(u, v, w) := -\partial_x^2 + u\delta(x - \pi/2) + v\delta'(x - \pi/2) + w\theta(x - \pi/2)$ and the drift coincides with $H(0, 0, 0) = -\partial_x^2$.

Assume that at time zero the system lies in the ground state of the drift Hamiltonian. Then we switch a Dirac's delta interaction on, located at the center of the well, with a coupling constant $u(\varepsilon t)$. Then the energy of the ground state is slowly increasing with time, while the energy of the first excited level remains unchanged. In the limit $u(\varepsilon t) \to \infty$ the two energy levels coincide, but the associate eigenfunctions do not. We then use a Heaviside function in order to perform a rotation in the two dimensional eigenspace of the degenerate eigenvalue, and reach the eigenfunction of the first excited level of the drift Hamiltonian. In this way we obtain a transition from the ground level to the first excited.

In contrast with the previous model, here we exploit an intersection obtained letting the control u diverge; however the above Hamiltonian is well defined also for an infinite value of u and v.

Moreover, the gap condition is fulfilled because of the parity selection rule: the δ potential and the ground state of the drift are even, therefore even and odd levels are decoupled during the entire evolution and the effective gap is the one between the ground state and the second excited.

Replacing the delta potential with a "delta prime" interaction, one can repeat this procedure and induce a transition from the first excited state to the second; more generally, alternating delta and delta prime one can reach any energy level. This control strategy can be generalized to any symmetric (coercive) potential.

It is worth mentioning that, unlike the first toy model, in this case it seems extremely difficult to prove that it is possible to steer the system from two eigenstates using classical control theory (for instance using finite dimensional techniques on a Galerkin approximation of the system, and then passing to the limit).

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Reflection symmetries for multiqubit density operators

Claudio Altafini
(joint work with Timothy F. Havel)

The Wigner Theorem affirms that unitary and antiunitary operations exhaust all possible symmetric transformations (intended as operations that preserve the trace, the Hermitian structure and the inner product of density operators) applicable to the wavefunction of a quantum mechanical system. For a density operator, an antiunitary operation corresponds to transposition, and in a two-dimensional Hilbert space this means a reflection, i.e. an orientation changing rotation in $O^-(3) = O(3) \setminus SO(3)$ of the corresponding Bloch vector. Up to unitary equivalence, this operation corresponds to spin flip [4], or (unconditional) NOT operation. For multiqubit systems, local reflections correspond to partially antiunitary transformations such as partial transposition, and they can be used to detect bipartite entanglement, as is well-known [8, 6].
When multiqubit density operators are represented as Stokes tensors (obtained by taking the "envelope" of the juxtaposition of the affine Bloch vectors, see [1, 7]), then other similar discrete reflection symmetries, i.e., orientation changing rotations, arise:

(i): local reflections applied simultaneously to two or more qubits;
(ii): nonlocal reflections, i.e. reflections applied at the *joint density* of two or more qubits.

The two cases are *qualitatively different*: while (i) is equivalent, up to local unitary operations, to multiqubit partial transposition and to multiple spin-flip, (ii) is a genuinely new operation and does not correspond to any local operation on two or more qubits.
All local and nonlocal reflections originate from the presence of two disconnected components (one orientation-preserving, the other orientation-changing) in the group of rotations acting on the Stokes tensor. The parametrization used makes this observation rather natural and allows to classify as reflection symmetries a number of nonunitary operations used in the detection and synthesis of entanglement. Operations reconducible to reflections appear in the Peres-Horodecki test and in the various measures of entanglement relying upon "spin-flip" operations (like concurrence, negativity and tangle) for what concerns (multiple) 1-qubit reflections. Also nonlocal reflections are used in the literature: for example a total reflection corresponds to what is normally referred to as "taking the complement of a density", used for example in the construction of bound entangled states from

an Unextendible Product Basis of orthogonal states [2]. In between local and total reflections lies a class of "nonlocal yet partial" reflections which also belong to the type (ii) above. These maps resemble very closely those used in the so-called reduction criterion [3, 5], although the reduction criterion is not trace-preserving. More in detail, a total reflection acts by changing sign to all components of the Stokes tensor other than the expectation value of the identity. Since the convex sum of a density and of its reflection is the random state, a total reflection can be intended as a nonlocal multiparty NOT operation. This operation does not necessarily yield a valid (positive semidefinite) density operator. However, it does so on sufficiently mixed states.
For the purposes of further understanding the structure of composite quantum systems, we find it useful to have a unifying perspective on these nonunitary yet symmetric (in the sense of Wigner Theorem) transformations.

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Quantum Convex Structures and their Physical Interrelations

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(joint work with *QUIT Group*, Università di Pavia, Italy, *http://www.qubit.it*)

This talk reviews some results contained in Refs.[1]-[7], and the present extended abstract mainly recalls the open problems posed during the talk.

The quantum convex structures that will be considered are those of Quantum States, Quantum Operations (in particular trace-preserving, i. e. channels) and POVM's (Positive Operator Valued Measures). More than focusing only on the convex structures themselves, I will analyze some physically meaningful interrelations that link them each other: 1) one-to-one maps between States and Quantum Operations, and between States and POVM's, corresponding to Quantum Calibration; 2) dilation maps from the convex set of States to those of Quantum Operations and of POVM's, corresponding to Quantum Programmability; 3) mapping POVM's to POVM's via channels, corresponding to pre-processing of POVM's.

Quantum Calibration. The convex Quantum Operations and that of bipartite states are connected each-other (apart from a normalization) by the Choi-Jamiolkowski isomorphism between CP-maps and positive bipartite operators. Such correspondence can be extended to the following one: $R = \mathcal{M} \otimes \mathcal{I}(F)$, describing the output state R of the local action of the map $\mathcal{M}$ on the input state F ($\mathcal{I}$ denotes the identity map). One calls the state F *tomographically faithful*[3] when the correspondence $\mathcal{M} \leftrightarrow R$ is one-to-one. Using such correspondence, one can perform the quantum tomography of the operation/channel $\mathcal{M}$ via a a joint tomography on the bipartite system at the output. The inversion formula from the output state to the map is $\mathcal{M}(\rho) = \mathrm{tr}_2[(I \otimes \rho^{\mathsf{T}})\mathcal{I} \otimes \mathcal{F}^{-1}(R)]$ where $\mathcal{F}$ is the map $\mathcal{F} = \mathrm{tr}_2[(I \otimes \rho^{\mathsf{T}})F]$ associated to the tomographically faithful state F.

The faithful state F establishes also the one-to-one correspondence between POVM's and ensembles of states: $p_n\rho_n = \mathcal{F}'(P_n)$ and $P_n = \mathcal{F}'^{-1}(p_n\rho_n)$,, where $\mathcal{F}'$ is the associated map $\mathcal{F}'(X) = \mathrm{tr}_2[(I \otimes X)F]$, p_n being the probability of the outcome n and ρ_n the corresponding conditioned state (to be determined tomographically). As an example of application, Ref.[4] presents a Monte Carlo simulation of an experiment of quantum calibration of a typical photodetector using a realistic homodyne tomography setup, and a twin beam from down-conversion of vacuum for the state F.

Quantum Programmability. The Choi one-to-one correspondence between channels and bipartite states is not only the basis of tomography of channels, but carries also a physical interpretation in terms of *probabilistic programmability* of channels. Here, however, we are interested in *deterministic programmability* of channels. We want to program the channel by a fixed device as follows $\mathcal{M}_{U,\sigma}(\rho) \doteq \mathrm{tr}_2[U(\rho \otimes \sigma)U^\dagger]$, with the system in the state ρ interacting with an ancilla in the state σ via the unitary operator U of the programmable device (the state of the ancilla is the *program*). For fixed U the above map can be regarded as a linear map from the convex set of the ancilla states $\mathscr{A}$ to the convex set of channels for the system. We will denote by $\mathscr{M}_{U,\mathscr{A}}$ the image of the ancilla states $\mathscr{A}$ under such linear map. According to the well known no-go theorem by Nielsen and Chuang it is impossible to program all unitary channels on the system with a single U and a finite-dimensional ancilla, namely the image convex $\mathscr{M}_{U,\mathscr{A}}$ is a proper subset of the whole convex of channels. This opens the following problem:

Problem 1 (The big U)**.** *For given dimension of the ancilla, find the unitary operators U that are the most efficient in programming channels, namely which minimize the largest distance of each channel $\mathcal{C} \in \mathscr{C}$ from the programmable set $\mathscr{M}_{\mathbf{Z},\mathscr{A}}$: $\varepsilon(U) \doteq \max_{\mathcal{C}\in\mathscr{C}} \min_{\mathcal{P}\in\mathscr{M}_{U,\mathscr{A}}} \delta(\mathcal{C},\mathcal{E})$.*

As a definition of distance one could consider any of those given in Ref.[8]. For POVM's we have a similar situation. In the following we will consider discrete spectrum and denote the POVM with the vector notation $\mathbf{P} \doteq (P_1, P_2, \ldots)$, P_i denoting the POVM elements. Here the deterministic programmability is represented by the extension map $\mathcal{M}_{\mathbf{Z},\sigma} \doteq \mathrm{tr}_2[(I\otimes\sigma)\mathbf{Z}] = \mathbf{P}$ from states to POVM's. A no-go theorem analogous to that of channels holds for POVM's[5], and this opens

the following problem (in the following $\mathscr{P}_N$ denotes the convex of POVM's with N outcomes)

Problem 2 (The big **Z**)**.** *For given dimension of the ancilla Hilbert space and cardinality of the POVM $N = |\mathbf{Z}| = |\mathbf{P}|$, find the joint observables* **Z** *that are the most efficient in programming POVM's, namely which minimize the largest distance of each POVM from the programmable set*

$$\mathcal{M}_{\mathbf{Z},\mathscr{A}} : \ \varepsilon(\mathbf{Z}) \doteq \max_{\mathbf{P}\in\mathscr{P}_N} \min_{\mathbf{Q}\in\mathscr{M}_{\mathbf{Z},\mathscr{A}}} \delta(\mathbf{P},\mathbf{Q}).$$

As a definition of distance we can use the physical distance $\delta(\mathbf{P},\mathbf{Q}) = \max_\rho \sum_i |\operatorname{tr}[\rho(P_i - Q_i)]|$. The solution of Problems 1 and 2 are unknown even for dimension $d = 2$ of the system. In Ref.[5] it is shown that using a joint observable **Z** of the form of a fixed local system observable evolved with a controlled-unitary interaction, one can program observables with polynomial growth of the dimension of the ancilla versus the accuracy ε^{-1}. For qubits one can even achieve linear growth.

Notice that if we pose restrictions on the set of programmable POVM's, then it maybe possible to program the full convex set exactly. This is the case of covariance under a unitary irreducible representation of a group, where the POVM density can be programmed by means of a fixed covariant Bell POVM density[5, 6] [the "seed" of the POVM is just the state of the ancilla, apart from a simple antilinear transformation]. This suggests that for ancilla having the same dimension of the system the observable **Z** should be Bell. Notice that the controlled-unitary form also occurs in connecting local to Bell observables[6]. Here another problem arises:

Problem 3 (The "Bellizing" U)**.** *Classify all unitary operators U that connect a fixed separable orthonormal basis to a Bell orthonormal basis.*

This problem needs the solution of another problem, namely that of the classification of Bell basis:

Problem 4 (Bell basis classification)**.** *Classify all orthonormal Bell basis, or, equivalently, classify all orthonormal basis of unitary operators.*

Regarding the last problem more material can be found on Ref.[9].

Processing of POVM's and the problem of Clean POVM's. If we precede a measuring apparatus by a quantum channel $\mathcal{E}$, the series of channel-measurement is equivalent to a new measurement, whose POVM is given by $\mathbf{Q} = \mathcal{E}(\mathbf{P})$. We call this pre-processing of the POVM (this is the case, e.g. of optical pre-amplification of photodetection or homodyning). The pre-processing scheme should be contrasted with that of post-processing, in which the output outcomes of the measurement are processed numerically, corresponding to an endomorphism of the probability space of the POVM (for discrete probability space this is just the composition of the POVM with a Markov matrix). Such post-processing is completely classical, whereas the pre-processing is quantum.

A quantum channel transforms POVM's into POVM's, generally irreversibly, thus loosing some of the information retrieved from the measurement. This poses the following problem:

Problem 5 (Clean POVM's)**.** *Which POVM's are "undisturbed", namely they are not irreversibly connected to another POVM via a channel?*

We will call such POVM *clean*. To define more precisely the problem, we introduce a pre-ordering relation, which we call *cleanness*, defined as follows: For two POVM's $\mathbf{P}$ and $\mathbf{Q}$ we define $\mathbf{P} \succ \mathbf{Q}$ iff there exists a channel $\mathcal{E}$ such that $\mathbf{Q} = \mathcal{E}(\mathbf{P})$. We will say that the POVM $\mathbf{P}$ is *cleaner* than the POVM $\mathbf{Q}$. We will say that $\mathbf{P} \simeq \mathbf{Q}$ if both $\mathbf{Q} \succ \mathbf{P}$ and $\mathbf{P} \succ \mathbf{Q}$ hold. We call a POVM $\mathbf{P}$ *clean* when for any POVM $\mathbf{Q}$ such that $\mathbf{Q} \succ \mathbf{P}$ one has $\mathbf{Q} \simeq \mathbf{P}$. Partial solutions to the problem are the following[7]: *1)* for $N < d$ outcomes there are no clean POVM's, and for $N = d$ the set of clean POVM's coincides with the set of observable; *2)* all rank-one POVM's are clean; *3)* for $d = 2$, $\mathbf{P} \simeq \mathbf{Q}$ iff $\mathbf{P}$ is unitarily equivalent to $\mathbf{Q}$; *4)* for $\mathbf{A}$ and $\mathbf{B}$ effects, $\mathbf{A} \succ \mathbf{B}$ iff $[\lambda_m(A), \lambda_M(A)] \supseteq [\lambda_m(B), \lambda_M(B)]$; *5)* if the POVM $\mathbf{Q}$ is infocomplete then every $\mathbf{P}$ such that $\mathbf{P} \succeq \mathbf{Q}$ is infocomplete; *6)* for infocomplete POVM's cleanness-equivalence is the same as unitary equivalence. One can easily see that generally *cleanness equivalence is different from unitary equivalence*. In fact it is possible to connect each other two unitarily inequivalent POVM's via two different channels (consider two effects with different spectrum and the same spectral interval). Moreover *cleanness is different from extremality in the POVM convex*. In fact, there are extremal POVM's that are not clean (e.g. any extremal POVM with $N < d$ outcomes, such as for $d = 3$, $\mathbf{P} = \{Z_0, Z_1\}$ with $Z_0 = |0\rangle\langle 0|, \qquad Z_1 = |1\rangle\langle 1| + |2\rangle\langle 2|$), and viceversa there are clean POVM's that are not extremal (e.g. any rank one POVM with $N > d^2$).

What does it mean that there are extremal POVM that are not clean? At a first sight this looks quite strange, since an extremal POVM is already perfect, in some sense. The answer is simply that sometimes we need to give-up some amount of information for the quality of the information. This is because maximizing the information is not necessarily compatible with the achievement of the minimal cost function in an optimization problem. Therefore, even though the channel decrease the information, this is the only way to achieve the minimal cost. On the other hand, once the measurement is performed, there is no classical post-processing that can achieve the same result of a quantum pre-processing, and achieving the full available amount of information is then useless. If we want to decide a posteriori the purpose of the measurement, then we need to use an informationally complete measurement, and the same amount of information is then available for each purpose.

Clearly, we can also define *cleanness for post-processing*, i.e. a POVM is cleaner than another when the latter can be obtained from the former via an irreversible classical processing. This classical case is very simple, since here cleanness is just equivalent to be rank-one. Therefore we conclude that rank-one POVM's are clean under both pre-processing and post-processing, On the other hand, both observables and rank-one informationally complete POVM's have all the following

properties: they are extremal, clean under post-processing, and clean under pre-processing.

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Beyond quantum cloning

M. Barbieri, F. De Martini, P. Mataloni, M. Ricci, and F. Sciarrino

In the last years a great deal of efforts has been devoted to the realization of the optimal approximations to the quantum cloning and flipping operations over an unknown qubit $|\phi\rangle$. It is well known that these two processes are unrealizable in their exact forms [1, 2], but they can be optimally approximated by the corresponding universal machines, i.e., by the universal optimal quantum cloning machine (UOQCM) and the universal-NOT (U-NOT) gate [3]. The UOQCM has been experimentally realized following several schemes, i.e. by exploiting the process of stimulated emission in a quantum-injected optical parametric amplifier (QI-OPA) [4, 5, 6, 7], by a quantum network [8] and by acting with projective operators over the symmetric subspaces of many qubits [9].

Since also the perfect cloning of subsets containing non orthogonal states is forbidden, recently *state dependent* cloning machines have been investigated which are optimal respect to any given ensemble [10]. It has been found in general that for *group-covariant* cloning, i.e. where the set of input states is the orbit of a given state under the action of a group of unitary transformations, the smaller is the group the higher is the optimal fidelity averaged over the input states [11]. The simplest and most relevant case is represented by the cloning covariant under the Abelian group U(1) of phase rotations, the so called "*phase-covariant*" quantum cloning machine (PQCM). There the information is encoded in the phase ϕ_i of the input qubit belonging to any equatorial plane i of the corresponding Poincare' sphere, e.g. the general state may be expressed as: $|\phi_i\rangle = 2^{-\frac{1}{2}}(|\psi_i\rangle + \exp(i\phi_i)\left|\psi_i^{\perp}\right\rangle)$ and $\left\{|\psi_i\rangle, \left|\psi_i^{\perp}\right\rangle\right\}$ is a convenient normalized basis,

$\left\langle \psi_i \mid \psi_i^{\perp} \right\rangle = 0$ [10]. We have reported the realization of a $1 \rightarrow 3$ PQCM by the implementation of a $1 \rightarrow 2$ UOQCM, followed by a spin flipper σ_i and the projection of the output qubits over the symmetric subspace Π_{sym} [12]. In the experiment the qubit to be cloned has been encoded into the polarization state $|\phi\rangle_{in} = 2^{-\frac{1}{2}}(|R\rangle + \exp(i\phi_Y)\,|L\rangle) = \alpha\,|H\rangle + \beta\,|V\rangle$ of a single photon, where $|H\rangle \equiv |0\rangle = 2^{-\frac{1}{2}}(|R\rangle + |L\rangle)$ and $|V\rangle \equiv |1\rangle = -i2^{-\frac{1}{2}}(|R\rangle - |L\rangle)$ stand for horizontal and vertical polarizations. We consider the optimal quantum cloning for $x-z$ equatorial qubits by taking linear polarization states as input, that is, the ones adopted in the $BB84$ cryptographic protocol. The UOQCM has been realized adopting a quantum-injected optical parametric amplifier (QI-OPA) [5], while the σ_Y operation and the Π_{sym} have been implemented with linear optics and post-selection techniques.

As a further contribution to the investigation of the quantum cloning process, we reported on a nearly decoherence-free all optical scheme based on the quantum-injected optical parametric amplification (QI-OPA) of a single photon in a quantum superposition state of polarization (π), i.e. a $\pi-$*encoded qubit* [4, 13]. Conceptually, the method consists of transferring the well accessible condition of quantum superposition characterizing a single-photon qubit, $N = 1$, to a *mesoscopic*, i.e. multi-photons amplified state $M >> 1$, here referred to as a "*multi-particle qubit*" (*M-qubit*). In quantum optics this can be done by injecting in the QI-OPA the single-photon *qubit,* $\alpha\,|H\rangle + \beta\,|V\rangle$, here expressed in terms of two mutually orthogonal $\pi-$states, e.g. horizontal and vertical linear $\pi's$: $|H\rangle$, $|V\rangle$. In virtue of the general *information preserving* property of the OPA, the generated multi-particle state is found to keep the *same* superposition character and the interfering capabilities of the injected qubit, thus realizing the most relevant and striking property of the *M-qubit* condition. Since the present scheme basically realizes the *deterministic* $1 \rightarrow M$ *universal optimal quantum cloning machine* (UOQCM), i.e. able to copy *optimally* any unknown input qubit into $M >> 1$ copies with the same fidelity, the output state will be necessarily affected by *squeezed-vacuum noise (SVN)* arising from the input vacuum field.

A different approach to increase the number of involved qubits exploits projections over symmetric subspaces. For instance the probabilistic $N \rightarrow M$ cloning process [14] it is based on the action of a projective operation on the symmetric subspace of the N input qubits and $(M-N)$ blank ancillas. This transformation assures the uniform distribution of the initial information into the overall system and guarantees that all output qubits are indistinguishable. In the last years some experiments has been carried out adopting linear optical set-ups realizing the projection over the symmetric subspace of two or three polarization encoded qubits. These are: teleportation of the universal NOT (U-NOT) gate; single qubits purification procedure; N→M universal and phase covariant cloning machine with N=1,2 M=2,3; 1→3 phase covariant cloning machine; measurement of the overlap between density matrices, entanglement enhanced capacity of a quantum channel with correlated noise, etc. [9, 12, 15, 16, 17, 18]. This symmetric subspace projective measurement (SSPM) was achieved exploiting the bosonic behavior of the

photons by stimulating a coalescence between the photon qubits to be projected by means of Hong-Ou-Mandel interferometers.

An alternative strategy for enlarging the dimension of the Hilbert space consists of manipulating the spatial degree of freedom of photons. A parametric source of polarization-entangled photon pairs with striking spatial characteristics is presented [19]. This consists of a high stability interferometer where the polarization entanglement arises from the superposition of two emission cones, bearing mutually orthogonal polarization, generated by SPDC under excitation of a thin Type I, NL crystal in two opposite directions by a UV laser beam. The distribution of the output electromagnetic k modes excited by spontaneous parametric down-conversion and coupled to the output detectors can be very broad. At least in principle, this source allows the coupling to the output detectors of the full set of optical modes carrying the particle pairs involved in the EPR measurement. In other words, all entangled pairs created over the entire set of wavevectors allowed by phase matching can virtually be detected. Since the detected emission process is entirely "quantum", i.e., not affected by any previous"classical" manipulation, such as wavelength of wavevector filtering, e.g., by filters and/or limiting pinholes, the new scheme allows in principle the realization of the necessary premises underlying the original formulation of the "EPR paradox" [20]. Using these states realized over a full entanglement ring output distribution, the nonlocal properties of the generated entanglement have been tested by standard Bell measurements.

The spatial features of this source allows to produce arbitrary pure and mixed states by exploiting a "mode-patchwork" technique, based on the quantum superposition principle. This is adopted to synthesize in a straightforward and reliable way any kind of mixed state, of large conceptual and technological interest in modern quantum information. Tunable Werner states and maximally entangled mixed states have indeed been created by this technique and investigated by quantum tomography. A study of the entropic and nonlocal properties of these states has been undertaken experimentally. Moreover, Werner states has been tested by means of the entanglement witness technique, which represents the most economical strategy for characterizing entanglement.

Furthermore, by the same source we have generated non maximally entangled states, by which Hardy's ladder theory has been verified up to the 20th step and the contradiction between the standard quantum theory and the local realism has been tested for 41% of entangled pairs [21].

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Quantum Entropies and Complexities

Fabio Benatti

(joint work with T. Krüger, M. Müller, R. Siegmund–Schultze and A. Szkoła)

In this talk we will review some basic facts concerning classical information theory and classical algorithmic complexity and relate them to existing counterparts in the quantum setting.

The fast development of quantum information, communication and computation [1] is indeed the main motivation behind all the attempts at seeking extensions to the so-called qubits of results and techniques so far applied to bit-strings emitted by classical stationary sources. A paradigmatic example is the quantum Shannon-Mc Millan theorem [3] which works for stationary quantum ergodic sources as much as its classical partner [2].

At its simplest, a quantum source $\mathcal{A}_{\mathbf{Z}}$ is the C^* inductive limit of local tensor products $\mathcal{A}^{\otimes n} = \bigotimes_{j=-n}^{n} (\mathcal{A})_j$ of 2×2 matrix algebras $(\mathcal{A})_j$ describing qubits at sites $j \in \mathbf{Z}$. Notice that by considering 2×2 diagonal algebras one has an algebraic description of classical sources.

The statistics of a stationary quantum source is fixed by a positive, normalized, shift-invariant expectation Ψ such that its restrictions to $\mathcal{A}^{\otimes n}$ are density matrices

$\rho^{(n)}$ with von Neumann entropy $S(\rho^{(n)}) := -\mathrm{Tr}\rho^{(n)} \log_2 \rho^{(n)}$ and entropy rate

$$h(\Psi) := \lim_{n\to\infty} \frac{1}{n} S(\rho^{(n)}) \ .$$

By restricting to diagonal matrix algebras, the above notions reduce to a shift-invariant probability measure μ, with Shannon entropy $H(\mu^{(n)})$ relative to bit-strings of length n, and entropy rate $h(\mu)$.

In classical information, $n\, h(\mu) \leq n$ gives the optimal number of bits that can be used to faithfully encode the information contained in the emitted strings of length n, for large n; namely, using less than $h(\mu)$ bits per input bit causes larger and larger errors [2].

In quantum information theory the same is true: one can compress the information contained in the local states $\rho^{(n)}$ by projecting onto states acting on roughly $2^{n\, h(\Psi)}$ dimensional Hilbert spaces, but not smaller, and still retrieve with high fidelity the information emitted by $\mathcal{A}_{\mathbf{Z}}$ [3, 4].

The classical entropy rate does describe how random a source is, but says nothing about the randomness of single strings $\mathbf{i}^{(n)} := i_1 i_2 \cdots i_n$, $i_j = 0, 1$.

It was an idea of Kolmogorow to associate the intuitive notion of randomness of $\mathbf{i}^{(n)}$ with the difficulty of describing it by means of a bit-program run by a Universal Turing Machine (UTM): if, for large n, the bits needed to have the UTM output $\mathbf{i}^{(n)}$ increase as n, then the string is patternless and cannot be compressed.

The *algorithmic complexity* of $\mathbf{i}^{(n)}$ is defined as the length $\ell(p)$ of the shortest program p that fed into any UTM $\mathfrak{U}$ reproduces it, $\mathfrak{U}(p) = \mathbf{i}^{(n)}$ [5]:

$$K(\mathbf{i}^{(n)}) := \min\Big\{ \ell(p) \ : \ \mathfrak{U}(p) = \mathbf{i}^{(n)} \Big\}$$

This is a machine-independent notion apart from an additive constant independent of $\mathbf{i}^{(n)}$ which disappears when looking at infinite strings and introducing the *complexity per symbol* as [6]

$$k(\mathbf{i}) := \limsup_{n\to+\infty} \frac{1}{n} K(\mathbf{i}^{(n)}) \ .$$

The intuitive idea that entropy rate and complexity per symbol should somehow be related is rigorously proved by Brudno's Theorem [7, 8] which states that for ergodic sources $h(\mu) = k(\mathbf{i})$ for μ-almost all $\mathbf{i}$.

In quantum computation, UTM are replaced by Universal Quantum Turing Machines (UQTM) which operate as probabilistic UTM, only their transition functions are characterized by probabilities amplitudes and not by probabilities, with the result that these machines operate unitarily on their inputs and create linear superpositions outputting quantum states [9].

Clearly, the issues at stakes are 1) whether it is possible to assign a degree of complexity to quantum states by quantifying how much it is possible to compress their description and 2) whether such a measure can be related to the quantum entropy rate via a Brudno-like relation.

While the Shannon entropy per bit has a natural extension to the von Neumann entropy per qubit, there is no unique quantum counterpart to the classical algorithmic complexity. Indeed, UQTM may be fed with classical or quantum descriptions of quantum objects; one must thus distinguish between a bit-based [10] and a qubit-based [11] algorithmic complexity.

The second choice declares that the complexity of a state ρ acting on a Hilbert space of dimension 2^n is measured by the $\log_2$ of the dimension of the smallest Hilbert space relative to an input quantum state σ, denoted by $\ell(\sigma)$, that operated upon by a UQTM $\mathfrak{U}^Q$ outputs a state closer in trace-norm to the target state than any computable degree:

$$QC^{\uparrow 1}(\rho) := \min\Big\{\ell(\sigma)\ :\ D(\mathfrak{U}^Q(\sigma,k),\rho) \leq \frac{1}{k}\,\forall k\in\mathbb{N}\Big\}\ ,$$

where $D(\rho_1,\rho_2) := \mathrm{Tr}|\rho_1-\rho_2|$ is the trace-distance of two density matrices.

Current research [12] indicates that the qubit-complexity is related to the von Neumann entropy in the following sense.

Quantum Brudno's relation: for any $\epsilon>0$ there exists N_ϵ such that for all $n\geq N_\epsilon$ there are typical subspaces $T_\epsilon^{(n)}\subseteq(\mathbb{C}^2)^{\otimes n}$ with orthogonal projections $P_\epsilon^{(n)}$ carrying most of the probability relative to the state Ψ of the quantum source, $\Psi(P_\epsilon^{(n)})\geq 1-\epsilon$, and such that all pure state projections $p\leq P_\epsilon^{(n)}$ have qubit-complexity

$$\frac{1}{n}QC^{\uparrow 1}(p)\in\big(h(\Psi)-\epsilon\,,\,h(\Psi)+\epsilon\big)\ .$$

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Distributed quantum dense coding

Dagmar Bruss
(joint work with G.M. D'Ariano, M. Lewenstein, C. Macchiavello, A. Sen(De), U. Sen)

The idea of so-called superdense coding, i.e. coding that uses quantum entanglement as a resource, was suggested in 1992 by Bennett and Wiesner [1]: a maximally entangled state of two qubits is shared between Alice and Bob. Alice applies one out of four operators (namely the three Pauli operators and the identity) on her qubit, and then sends it through a noiseless channel to Bob. Bob makes a measurement on the total state, which is one out of a set of four orthogonal states. In this way he receives two bits of classical information, although only one physical qubit was sent.

We study the generalisation of this scheme to a bipartite scenario in general finite dimensions $d_A \times d_B$, for a given state ρ^{AB}. Alice performs a *local* unitary transformation U_i with probability p_i on her part of ρ^{AB}, i.e. she transforms the state ρ^{AB} to the ensemble $\{p_i, \rho_i^{AB}\}$. She sends her state through a noiseless channel to Bob, who performs a measurement on the total state. An achievable upper bound on the accessible information on Bob's side is given by the Holevo bound [2]

$$I_{acc} = S(\overline{\rho}) - \sum_i p_i S(\rho_i^{AB}) \ . \tag{1}$$

Here $S(\varsigma) = -\mathrm{tr}(\varsigma \log_2 \varsigma)$ is the von Neumann entropy, and the average of the ensemble states is denoted as $\overline{\rho} = \sum_i p_i \rho_i^{AB}$. Thus, the capacity of dense coding is defined as $\chi = \max I_{acc}$, where the maximization is over all sets $\{U_i\}$ of unitaries performed by Alice, and all choices of probabilities $\{p_i\}$. Note that "capacity" here refers to the given quantum state ρ^{AB} as a resource for dense coding, rather than to a channel.

For bipartite systems the maximum is reached when Alice uses a complete set of orthogonal unitary operators $\{W_j\}$, to be chosen with equal probabilities, which satisfy the trace rule $\frac{1}{d_A^2} \sum_j W_j^\dagger \Xi W_j = \mathrm{tr}[\Xi] I$, for any operator Ξ. This choice can be easily seen to maximise the first term (the entropy of the ensemble average) in equation (1), while the second term is equal to $S(\rho^{AB})$ for any unitary encoding. Therefore, the capacity of dense coding for a given shared bipartite state ρ^{AB} is

$$\chi = \log_2 d_A + S(\rho^B) - S(\rho^{AB}), \tag{2}$$

see also [3]. The dense coding capacity is higher than in any classical protocol when $\chi > \log_2 d_A$, or $S(\rho^B) > S(\rho^{AB})$. The latter inequality never holds for separable states. Furthermore, it is also never satisfied for bound entangled states: $S(\rho^B) > S(\rho^{AB})$ implies distillability, and thus bound entanglement is not useful for bipartite dense coding. For dimension $d \times d$ this was pointed out in [4]. From equation (2) one sees immediately that dense coding is possible with any pure

entangled state. However, there are mixed entangled states, e.g. a Werner state with singlet fraction less than $\approx .7476$, that are not useful for dense coding.

We suggest a generalisation of the above scenario to dense coding in a distributed setting, i.e. for more than one sender and/or more than one receiver [5]. Suppose that there are $N-1$ Alices, say, $A_1, A_2, \ldots A_{N-1}$ and a single Bob (B). The Alices want to send (classical) information to Bob. The information of one Alice will in general be different from another Alice. To do this, they use a previously shared N-party state $\rho^{A_1 \ldots A_{N-1} B}$. The jth Alice A_j chooses the unitary tranformation $U_{i_j}^{A_j}$ with probability $p_{i_j}^{A_j}$ and applies it on her part of the total multipartite state. From the complete orthogonal set $\{W_{j_l}^{A_l}\}$ for A_l we can construct the set of local operators $\otimes_l W_{j_l}^{A_l}$ which is a complete and orthogonal set for the composite system of all Alices, whence the trace rule holds for their global Hilbert space. Then, the situation is equivalent to the previous case of a single Alice. We arrive at the capacity for distributed dense coding with a single receiver,

$$\chi^{A_1 \ldots A_{N-1} B} = \log_2 d_{A_1} + \ldots + \log_2 d_{A_{N-1}} + S(\rho^B) - S(\rho^{A_1 \ldots A_{N-1} B}). \tag{3}$$

We now consider the situation of several senders (called Alices, A_1, ..., A_{N-1}) and two receivers (called Bobs, B_1, B_2). If the receivers are distant and do not communicate, the corresponding dense coding capacities are simply additive. When the receivers are allowed to make global measurements, we are in the same situation as considered previously, for a single receiver. The interesting case is the one where the two receivers perform local operations and are allowed to use classical communication (LOCC). Here, some of the Alices, say $A_1, \ldots, A_k$, send their parts of the shared state $\rho^{A_1 \ldots A_{N-1} B_1 B_2}$ to B_1, while the rest of the Alices, $A_{k+1}, \ldots, A_{N-1}$, send their states to B_2. A Holevo-like universal upper bound for the accessible information I_{acc}^{LOCC} for this case was obtained in [6]. Its asymptotic version, maximized over all choices of unitaries and probabilities of the Alices, is the dense coding capacity χ^{LOCC}. We have shown that the same encoding as previously is again optimal in this scenario, and leads to the following upper bound for the capacity:

$$\chi^{LOCC} \leq \log_2 d_{A_1} + \ldots + \log_2 d_{A_{N-1}} + S(\rho^{B_1}) + S(\rho^{B_2}) - \max_{x=1,2} S(\rho^{(x)}), \tag{4}$$

where $\rho^{B_1} = \mathrm{tr}_{A_1 \ldots A_{N-1} B_2}\, \rho$, $\rho^{B_2} = \mathrm{tr}_{A_1 \ldots A_{N-1} B_1}\, \rho$, and $\rho^{(1)} = \mathrm{tr}_{A_{k+1} \ldots A_{N-1} B_2}\, \rho$, $\rho^{(2)} = \mathrm{tr}_{A_1 \ldots A_{k+1} B_1}\, \rho$.

We propose a classification scheme for given quantum states, based on their "dense-codeability" with the described protocols. For more than one receiver we introduce convex sets for global, LOCC and local dense codeability. These sets are non-empty and not of measure zero: example states are presented, namely a state that is globally, but not LOCC dense codeable; a state that is LOCC, but not locally dense codeable, and a state that is locally dense codeable.

There are several open questions related to multipartite dense coding: what is the capacity (or at least an upper bound) for the case of more than two receivers? Can multipartite bound entanglement be useful for distributed dense coding? Is

the set of globally dense codeable states convex, and how to prove it? Is the so-called W-state of four qubits in the LOCC dense codeable class? We hope that answering these questions will shed more light on the usefulness of entanglement for quantum information processing tasks.

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Test for entanglement: realignment criterion, entanglement witness and positive maps

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(joint work with Sergio Albeverio, Shao-Ming Fei and Ling-An Wu)

Quantum entangled states have showed remarkable applications and become one of the key resources in the rapidly expanding field of quantum information processing recently. In practice, we do not yet have a full understanding of the physical character and mathematical structure for entangled states. We even do not know completely whether a generic quantum state is entangled, and how much entanglement remained after some noisy quantum processes.

In this talk, we introduce a serial of operational methods to detect entanglement for quantum systems. A state of a composite quantum system is said to be *disentangled* or *separable* if it can be prepared in a "*local*" or "*classical*" way. A separable bipartite system can be prepared as an ensemble realization of pure product states $|\psi_i\rangle_A |\phi_i\rangle_B$ ($i = 1, ..., M$ for some positive integer M) occurring with a certain probability p_i:

$$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B,$$

where $\rho_i^A = |\psi_i\rangle_A \langle\psi_i|$, $\rho_i^B = |\phi_i\rangle_B \langle\phi_i|$, $\sum_i p_i = 1$ and $|\psi_i\rangle_A$, $|\phi_i\rangle_B$ are normalized pure states of the subsystems A and B, respectively [1]. If no convex linear combination exists for a given ρ_{AB}, the state is called "*entangled*" and includes quantum correlation.

The realignment criterion. Motivated by the Kronecker product approximation technique for a matrix, we developed a very simple method to obtain the realignment criterion in [2] (independently given in [3] named the greatest cross norm criterion). To recollect, the criterion says that, *for any separable $m \times n$ bipartite density matrix ρ_{AB}, the $m^2 \times n^2$ matrix $\mathcal{R}(\rho_{AB})$ should satisfy $||\mathcal{R}(\rho_{AB})|| \leq 1$, where $||\cdot||$ means the trace norm defined as $||G|| = Tr((GG^\dagger)^{1/2})$.*

Thus $||\mathcal{R}(\rho_{AB})|| > 1$ *implies the presence of entanglement in* ρ_{AB}. Here $\mathcal{R}(\rho_{AB})$ is just a realigned matrix of the original ρ_{AB} to be $\mathcal{R}(\rho_{AB})_{ij,kl} = \rho_{ABik,jl}$.

The generalized partial transposition criterion and generalized reduction criterion. Developing the realignment idea further, we obtain a multipartite version in [4]. A more generic criterion is obtained in [5] which include the Peres-Horodecki criterion [i.e., PPT (positive partial transposition) criterion][6, 7], the realignment criterion, the reduction criterion to be special cases. It one defines $\widetilde{\rho_{AB}} = ab\mathbb{I}_{mn} - a\mathbb{I}_m \otimes \rho_B - b\rho_A \otimes \mathbb{I}_n + \rho_{AB}$ where a, b are arbitrary complex numbers, it says

If a bipartite density matrix ρ_{AB} *defined on an* $m \times n$ *space is separable, then the generalized reduction version* $\widetilde{\rho_{AB}}$ *of* ρ_{AB} *should satisfy*

$$||\widetilde{\rho_{AB}}^{\mathcal{T}_\mathcal{Y}}|| \leq h_a h_b, \ \forall \mathcal{Y} \subset \{r_A, c_A, r_B, c_B\},$$

where $\mathcal{T}_{r_k}$ *or* $\mathcal{T}_{c_k}$ *(*$k = A, B$*) stands for transpositions with respect to the row or column for the subsystem* k*. Here* h_a, h_b *are simple functions depending on* a *and* b*, respectively.*

All of the above-mentioned criterion are powerful to detect most of the bound entangled stated appeared in the literatures, which can not be identified with the strong PPT criterion.

Universal construction of entanglement witnesses and positive maps Entanglement witness (EW) is another method to detect entanglement. It is a Hermitian operator that satisfies $Tr(W\rho_A \otimes \rho_B) \geq 0$ for any pure separable state $\rho_A \otimes \rho_B$, and has at least one negative eigenvalue. If a density matrix ρ satisfies $Tr(W\rho) < 0$, then ρ is an entangled state. We develop two methods to construct EW universally [8].

(1) *For any density matrix* ρ*, we can associate with it an EW defined as*

$$W = Id - (\mathcal{R}^{-1}(U^* V^T))^T,$$

where U, V *are the unitary matrices that yield the singular value decomposition (SVD) of* $\mathcal{R}(\rho)$*, i.e.,* $\mathcal{R}(\rho) = U\Sigma V^\dagger$.

(2) *Another EW can be of*

$$W = Id - (VU^\dagger)^{T_A},$$

where U, V *are unitary matrices that yield the SVD of* ρ^{T_A}*, i.e.,* $\rho^{T_A} = U\Sigma V^\dagger$.

When these matrices Ws are not Hermitian, we can choose EW to be $W^{'} = \frac{1}{2}(W + W^\dagger)$. Since entanglement witnesses are physical observables and may be measured locally our construction could be of great significance for future experiments.

Through the *Jamiołkowski isomorphism*

$$W = (Id_A \otimes \Lambda)P_+^m,$$

where $P_+^m = |\Phi\rangle\langle\Phi|$ and $|\Phi\rangle = \frac{1}{\sqrt{m}} \sum_{i=1}^m |ii\rangle$ is the maximally entangled state in $\mathcal{H}_A \otimes \mathcal{H}_A$, one can have a corresponding positive map (PM) $\Lambda : (|i\rangle\langle j|) \to \langle i|W|j\rangle$. From [7], one knows that ρ is separable iff for any positive map Λ the inequality $(Id_A \otimes \Lambda)\rho \geq 0$ holds. Thus any negative eigenvalue of $(Id_A \otimes \Lambda)\rho$ will signal existence of entanglement.

We find that the constructed EWs, their optimized versions and corresponding PM are always more powerful than the PPT criterion, the realignment criterion and their generalization versions. Our method also gives a new method to find a large family of positive but non-completely positive maps of arbitrary high dimensions, which can detect bound entangled states systematically. All these results can significantly expand our ability to recognize directly the entanglement.

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Projected Entangled–Pair States: Properties and Applications

Ignacio Cirac

(joint work with Frank Verstraete)

The description of many-body quantum states is, typically, very hard. The reason is that the number of parameters needed to characterize the quantum state of N d–level systems scales as d^N, so that even for qubits ($d = 2$) already for $N > 40$ it is impossible to store all the corresponding coefficients. Furthermore, if one wants to determine the expectation value of any observable one needs to perform a number of basic operations which also scale exponentially with the number of particles. However, in Nature, only some particular states appear, and thus it may happen that different ways of parametrizing quantum states are much more efficient and do not require an exponential scaling. In this talk I presented a new characterization of quantum states, what we call Projected Entangled-Pair States (PEPS). This characterization is based on constructing pairs of maximally entangled states in a Hilbert space of dimension D^2, and then projecting those states

in subspaces of dimension d. In one dimension, one recovers the familiar matrix product states, whereas in higher dimensions this procedure gives rise to other interesting states. We have used this new parametrization to construct numerical algorithms to simulate the ground state properties and dynamics of certain quantum-many body systems in two dimensions. The results are very encouraging, since we have been able to simulate 20×20 spin 1/2 lattices interacting with the Heisenberg nearest neighbor Hamiltonian, as well as with other frustrated Hamiltonians.

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Classification of Quantum States under Local Unitary Transformations

Shao-Ming Fei

(joint work with S. Albeverio, L. Cattaneo, N.H. Jing, X.H. Wang, W.L. Yang)

As the nonlocal properties and the entanglement of a quantum system remain invariant under local unitary transformations (LUT), it is of importance to classify the quantum states under LUT. In principle one could judge wether two quantum states are equivalent or not under LUT by computing all the invariants of LUT. The method developed in [1, 2] allows one to compute all such invariants, though it is generally not operational. An explicit set of invariants are calculated only for some special cases, e.g., two and three qubits [3, 4]. We study the equivalence of bipartite mixed states from both the invariants approach and the matrix tensor product decomposition approach. The results are used in investigating the case of tripartite pure states.

Let H be an N-dimensional complex Hilbert space, with $|i\rangle$, $i = 1, ..., N$, as an orthonormal basis. A general pure state on $H \otimes H$ is of the form

$$|\Psi\rangle = \sum_{i,j=1}^{N} a_{ij}|i\rangle \otimes |j\rangle,\ a_{ij} \in \mathbb{C} \tag{1}$$

with the normalization $\sum_{i,j=1}^{N} a_{ij}a_{ij}^* = 1$. Let A denote the matrix given by $(A)_{ij} = a_{ij}$. The following quantities are invariants under LUT [5]:

$$I_\alpha = Tr(AA^\dagger)^\alpha,\ \alpha = 1, ..., N; \tag{2}$$

Two pure bipartite states $|\Psi\rangle$ and $|\Psi'\rangle$ are equivalent under LUT, $|\Psi'\rangle = U_1 \otimes U_2|\Psi\rangle$, if and only if $I_\alpha = I'_\alpha$, where U_1 and U_2 are unitary matrices on H.

Two bipartite density matrices ρ and ρ' are said to be equivalent under LUT if

$$\rho' = (U_1 \otimes U_2)\rho(U_1 \otimes U_2)^\dagger. \tag{3}$$

Let ρ be a density matrix on $H \otimes H$ with $rank(\rho) = n \leq N^2$, decomposed according to its eigenvalues and eigenvectors: $\rho = \sum_{i=1}^{n} \lambda_i |\nu_i\rangle\langle\nu_i|$. The eigenvector $|\nu_i\rangle$ has the form

$$|\nu_i\rangle = \sum_{k,l=1}^{N} a_{kl}^i |k\rangle \otimes |l\rangle,\ a_{kl}^i \in \mathbb{C},\ \sum_{k,l=1}^{N} a_{kl}^i a_{kl}^{i*} = 1,\ i = 1, ..., n.$$

Let A_i denote the matrix given by $(A_i)_{kl} = a_{kl}^i$. We introduce $\{\rho_i\}$, $\{\theta_i\}$,

$$\rho_i = Tr_2 |\nu_i\rangle\langle\nu_i| = A_i A_i^\dagger,\ \theta_i = (Tr_1 |\nu_i\rangle\langle\nu_i|)^* = A_i^\dagger A_i,\ i, j = 1, ..., n, \tag{4}$$

Tr_1 and Tr_2 stand for the traces over the first and second Hilbert spaces respectively. Let $\Omega(\rho)$ and $\Theta(\rho)$ be two "metric tensor" matrices, with entries given by

$$\Omega(\rho)_{ij} = Tr(\rho_i \rho_j),\ \Theta(\rho)_{ij} = Tr(\theta_i \theta_j),\ \text{for } i, j = 1, ..., n, \tag{5}$$

and $\Omega(\rho)_{ij} = \Theta(\rho)_{ij} = 0$, for $N^2 \geq i, j > n$. We call a mixed state ρ *generic* one if the corresponding "metric tensor" matrices Ω, Θ satisfy $det(\Omega(\rho)) \neq 0$, and $det(\Theta(\rho)) \neq 0$. Similarly we also introduce $X(\rho)$ and $Y(\rho)$ as $X(\rho)_{ijk} = Tr(\rho_i \rho_j \rho_k)$, $Y(\rho)_{ijk} = Tr(\theta_i \theta_j \theta_k)$, $i, j, k = 1, ..., n$. For generic states we have the following conclusion [6]:

[Theorem 1]. Two generic density matrices are equivalent under LUT if and only if there exists an ordering of the corresponding eigenstates such that the following invariants have the same values for both density matrices:

$$J^s(\rho) = Tr_2(Tr_1 \rho^s),\ s = 1, ..., N^2;\ \Omega(\rho),\ \Theta(\rho),\ X(\rho),\ Y(\rho). \tag{6}$$

In fact if two density matrices ρ and ρ' are equivalent under LUT, they have the same set of eigenvalues λ_i, $i = 1, ..., n$. Let X and Y be the unitary matrices that diagonalize two density matrices ρ and ρ' respectively, $\rho = X \Lambda X^\dagger$, $\rho' = Y \Lambda Y^\dagger$, where $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_{MN})$.

[Theorem 2]. Set

$$V = X \begin{pmatrix} d_1 I_{n_1} & 0 & \cdots & 0 \\ 0 & d_2 I_{n_2} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & d_r I_{n_r} \end{pmatrix} Y^\dagger, \tag{7}$$

where n_i, $i = 1, 2, ..., r$, stands for the geometric multiplicity of the eigenvalue λ_i of ρ, $\sum_1^r n_r = N^2$, $d_i = e^{i\theta_i}$ for $\theta_i \in I\!R$. Then two non-degenerate states ρ and ρ' are equivalent under local unitary transformations if the rank $r(\tilde{V}) = 1$, where $\tilde{V}$ is the realigned matrix of V [7].

The results for bipartite mixed states can be used to study the equivalence of pure tripartite states under LUT. Let H_A resp. H_B resp. H_C be K resp. M resp. N dimensional complex Hilbert spaces. We denote by $\{|e_i\rangle\}_{i=1}^K$, $\{|f_i\rangle\}_{i=1}^M$ and $\{|h_i\rangle\}_{i=1}^N$ the orthonormal bases in H_A, H_B and H_C respectively. A general pure

state on $H_A \otimes H_B \otimes H_C$ is of the form

$$|\Phi\rangle = \sum_{i=1}^{K}\sum_{j=1}^{M}\sum_{k=1}^{N} a_{ijk}|e_i\rangle \otimes |f_j\rangle \otimes |h_k\rangle,\ a_{ijk} \in \mathbb{C} \tag{8}$$

with the normalization $\sum_{i=1}^{K}\sum_{j=1}^{M}\sum_{k=1}^{N} a_{ijk}a^*_{ijk} = 1$. $|\Phi\rangle$ can be regarded as a state on the bipartite systems A-BC, The following quantities are invariants under LUT with respect to the A-BC system, $J_\alpha = Tr(Tr_1|\Phi\rangle\langle\Phi|)^\alpha,\ \alpha = 1, 2, \cdots, S$, where $S = min\{K, M, N\}$. We have [8, 9]:

[Theorem 3]. For two tripartite states $|\Phi\rangle$ and $|\Phi'\rangle$, if they have the same values of the invariants J_α, there are unitary matrices U_1 on H_A and V_1 on $H_B \otimes H_C$ such that $|\Phi'\rangle = U_1 \otimes V_1|\Phi\rangle$. $|\Phi\rangle$ and $|\Phi'\rangle$ are equivalent under local unitary transformations: 1) if the V_1 satisfies $r(\tilde{V_1}) = 1$; or 2) the reduced bipartite mixed states $\rho = Tr_1|\Phi\rangle\langle\Phi|$ and $\rho' = Tr_1|\Phi'\rangle\langle\Phi'|$ are equivalent under LUT, $\rho' = (U_2 \otimes U_3)\rho(U_2 \otimes U_3)^\dagger$ for some unitary matrices U_2 on H_B and U_3 on H_C.

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Optimizing Bell Experiments

Richard Gill

(joint work with Toni Acín, Nicolas Gisin, Peter Grünwald, Wim van Dam)

Van Dam, Gill and Grünwald (2005) propose to compare existing Bell-type experiments (CHSH, Hardy, GHZ, ...) and to optimally design or tune new ones (CGLMP,...) by the statistical strength of the experiment: how many runs are required to obtain a pre-specified (large) degree of confidence that "local realism" (local hidden variables) is violated. The better the experiment, the fewer the number of runs. Statistical strength is quantified by $D(q; \mathcal{P}) = \inf_{p\in\mathcal{P}} D(q : p)$, with

$D(q:p) = \sum_i q_i \log(q_i/p_i)$ being the relative entropy between the actual probability distribution q of the overall result i of one run (a choice of joint settings and resulting joint outcomes), and a hypothetical probability distribution p from the set $\mathcal{P}$ of all distributions allowed by local realism. If one experiment has twice as large a value of D as another, then half the number of runs are equally convincing statistical evidence against local realism.

The experimenter fixes a point $q \in \mathcal{Q}$, the set of all probability distributions allowed by quantum mechanics, by choice of a multipartite quantum state, measurement settings for each of the parties, and a probability distribution over "joint settings", ie, over each party's choice of measurement. So we are interested in computing $D(\widehat{q}:\widehat{p}) = \sup_{q\in\mathcal{Q}} \inf_{p\in\mathcal{P}} D(q:p)$.

To be more specific, let us take for the moment the number of parties, the number of measurements (or measurement settings) per party, and the number of outcomes per measurement as fixed; together I call this the *format* of the experiment. Given the format, let us also fix a joint probability distribution σ of the measurement settings, for a given format. Given the format and the distribution σ, we consider the set of all local realistic probability distributions $\mathcal{P}$, the set of all probability distributions $\mathcal{Q}$ allowed by quantum mechanics, and the set $\mathcal{R}$ containing all the probability distributions which do not violate causality: the distribution of the outcome of one party's measurement should not be influenced by the settings of the other parties. It is well known that $\mathcal{P} \subseteq \mathcal{Q} \subseteq \mathcal{R}$, both inclusions being strict. $\mathcal{P}$ and $\mathcal{R}$ are closed, convex polytopes; $\mathcal{R}$ is just closed and convex. $\mathcal{P}$ is called the local polytope, and $\mathcal{R}$ the non-signalling polytope. The intermediate set $\mathcal{Q}$ is called the quantum body. The three sets are contained in an affine subspace determined by the normalization constraints on the probabilities (the setting distribution σ is fixed), and of a dimension determined by the format of the experiment.

As the dimensions involved increase, the number and types of faces of the local polytope rapidly increases. Each face of $\mathcal{P}$ which is not simultaneously a boundary of $\mathcal{Q}$ is a Bell inequality. Very little is known about the structure of $\mathcal{P}$ and $\mathcal{Q}$; open questions abound, such as: are all faces of $\mathcal{P}$, which are *not* contained in faces of $\mathcal{R}$, Bell inequalities? Is the boundary of $\mathcal{Q}$ covered by probability distributions coming from projective measurements on pure states with minimal dimension Hilbert space (thus all projectors having rank one)?

Van Dam, Gill and Grünwald (2005) made use of results from mathematical statistics (computation of $\widehat{p}$) and from game theory (minimax theorem) to rigorously compute the strength of some celebrated experiments. It turns out for instance that the GHZ experiment is about 9 times more powerful than CHSH. Taking account of the way the GHZ experiment is conventionally performed, this figure has to be divided by 2 exactly three times. Conclusion: GHZ is hardly better than CHSH despite many claims to the contrary.

In recent and as yet unpublished work, Acín, Gill and Gisin (2005) have been further searching for good experiments. Under some plausible conjectures they are able to compute the strength of the best $2 \times 2 \times d$ format experiment, for d up

to several thousand. One of the conjectures being used here, is that *all non-trivial faces of the* $2 \times 2 \times d$ *local polytope are versions of the CGLMP inequality*, a recent generalization of CHSH (Collins, Gisin, Linden, Massar and Popescu, 2002). The best measurements are found numerically to be the same as certain measurements involving the Quantum Fourier Transform, which have turned up in several quite different contexts recently, but also, only as a result of numerical optimization. Amazingly, the best state for the experiment turns out to be far from maximally entangled. A corollary of the findings is that it pays off, for large d to use formats with more than two measurements per party, which so far was not known.

A tantalizing open problem is to find out what is the mathematical link between the CGLMP inequality and QFT.

A new derivation of the CGLMP was presented, which might aid this quest. Suppose that X_1, X_2, Y_1 and Y_2 take values in $\{0, \ldots, d-1\}$. Note that $(a+b) \bmod d \leq a \bmod d + b \bmod d$. We have

$$(X_1 - Y_1) + (Y_1 - X_2) + (X_2 - Y_2) + (Y_2 - X_1 - 1) = -1.$$

Therefore,

$$[(X_1 - Y_1) + (Y_1 - X_2) + (X_2 - Y_2) + (Y_2 - X_1 - 1)] \bmod d = d - 1$$

from which it follows that

$$(X_1 - Y_1) \bmod d + (Y_1 - X_2) \bmod d + (X_2 - Y_2) \bmod d + (Y_2 - X_1 - 1) \bmod d \geq d - 1.$$

Taking expectation values,

$$\langle X_1 - Y_1 \bmod d \rangle + \langle Y_1 - X_2 \bmod d \rangle + \langle X_2 - Y_2 \bmod d \rangle + \langle Y_2 - X_1 - 1 \bmod d \rangle \geq d - 1,$$

equivalent to the CGLMP inequality (which looks rather more complicated and takes much more hard work to derive).

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A complete set of local invariants for a class of mixed states.

Debashish Goswami

(joint work with S. Albeverio, S.M. Fei)

Quantum entangled states are playing very important roles in quantum information processing and quantum computation [1]. The properties of entanglement for multipartite quantum systems remain invariant under local unitary transformations on the subsystems. Hence the entanglement can be characterized by all the invariants under local unitary transformations. A complete set of invariants gives

rise to the classification of the quantum states under local unitary transformations. Two quantum states are locally equivalent if and only if all these invariants have equal values for these states. In [2, 3], a generally non-operational method has been presented to compute all the invariants of local unitary transformations. In [4], the invariants for general two-qubit systems are studied and a complete set of 18 polynomial invariants is presented. In [5] the invariants for three qubits states are also discussed. In [6] a complete set of invariants for generic density matrices with full rank has been presented.

In the present talk we investigate the invariants for arbitrary (finite-) dimensional bipartite quantum systems. We present a complete set of invariants for a class of quantum mixed states and show that two of these density matrices are locally equivalent if and only if all these invariants have equal values for these density matrices [7].

Let us consider a general mixed state ρ in a bi-partite $n \times n$ system $H \otimes H$ ($n \geq 2$), with a given orthonormal basis $\{e_1, ..., e_n\}$ of H. Let ρ have the eigen-decomposition

$$\rho = \sum_{l=0}^{N} \mu_l |\xi_l >< \xi_l|,$$

where the rank of $r(\rho)$ is $N+1$ ($N \geq 1$), μ_l are eigenvalues with the eigenvectors $|\xi_l >= \sum_{ij} \xi_{ij}^{(l)} |ij >$ (and $|\xi_l >< \xi_l|$ denotes, as usual, the projector onto $|\xi_l >$). Let $A_l := (\xi_{ij}^{(l)})$. We call a matrix "multiplicity free" if each of its singular values has multiplicity one. Let F denote the class of states ρ for which A_0's is multiplicity free. We shall find a complete set of local invariants for the class $\mathcal{F}$.

Let $(\psi_1, ...\psi_n)$. $(\eta_1, ..., \eta_n)$ be orthonormal bases such that

$$A0 = \sum_i \lambda_i |\psi_i >< \eta_i|$$

be the singular value decomposition of A_0, where $\lambda_1 > ... > \lambda_n$ denote the singular values arranged in a decreasing order. Let $b_{ij}^{(l)} :=< \psi_i | \rho_l \eta_j >$ for $l = 1, 2, ..., N$, and for positive integers $k, r \geq 1$, and multi-indices $\underline{i} = (i_1, ...i_{k+1})$, (with i_p's all distinct), $\underline{j} = (j_1, ..., j_{r+1})$ (j_q's distinct) where $i_p, j_q \in \{1, ..., n\}$ $\forall p, q$, $\underline{l} = (l_1, ..., l_k)$, $\underline{m} = (m_1, ...m_r)$ ($l_t, j_s \in \{1, ..., N\}$) with $i_1 = j_1$, $i_{k+1} = j_{r+1}$, we define

$$I^\rho(\underline{i}, \underline{j}, \underline{l}, \underline{m}) := \frac{b_{i_1 i_2}^{(l_1)} ... b_{i_k i_{k+1}}^{(l_k)}}{b_{j_1 j_2}^{(m_1)} ... b_{j_r j_{r+1}}^{(m_r)}} \tag{1}$$

whenever the denominator in the above formula is nonzero. Let Σ^ρ be the set of $(\underline{i}, \underline{j}, \underline{l}, \underline{m})$ such that $I^\rho(\underline{i}, \underline{j}, \underline{l}, \underline{m})$ is well defined.

We are now in a position to state the main result :

[Theorem]. A complete set of local invariants for the class of states $\mathcal{F}$ is given by:

$$
\begin{array}{l}
\{\mu_l, l = 0, ..., N\}, \\
\{| < \psi_i|\rho_l\eta_j > |,\ i, j = 1, ..., n;\ l = 1, ..., N\}, \\
\{< \psi_i|\rho_l\eta_i >,\ i = 1, ..., n-1;\ l = 0, ..., N\}, \\
\{I^\rho(\underline{i}, \underline{j}, \underline{l}, \underline{m}),\ (\underline{i}, \underline{j}, \underline{l}, \underline{m}) \in \Sigma^\rho\}.
\end{array} \tag{2}
$$

Two states in $\mathcal{F}$ are locally equivalent if and only if all these invariants (2) have equal values in these two states. For instance, the Werner state [8] $\rho_w = (1-p)I_{4\times4}/4 + p|\Psi_- >< \Psi_-|$, where $0 \leq p \leq 1$, $I_{4\times4}$ is the 4×4 identity matrix and $|\Psi_- >= \frac{1}{\sqrt{2}}(|01> - |10>)$, gives the same values to all the invariants (2) as does the isotropic state [9] $\rho_{iso} = (1-p)I_{4\times4}/4 + p|P_+ >< P_+|$, where $P_+ = \frac{1}{\sqrt{2}}(|11> + |00>)$.

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Decoherence in the Algebraic Framework

Mario Hellmich

(joint work with Philippe Blanchard, Piotr Ługiewicz and Robert Olkiewicz)

In my talk I reviewed some recent work of Ph. Blanchard (Bielefeld), and R. Olkiewicz and P. Ługiewicz (Wrocław) on decoherence in the algebraic framework of quantum theory.

The question why the objects surrounding us obey the laws of classical physics, despite the fact that our most fundamental physical theory, quantum theory, when directly applied to these objects, results in contradictions to what is observed, is a fundamental one. The most promising answer is given by the program of decoherence [1, 2]. It asserts that quantum theory is universally valid and that classicality is due to the unavoidable interaction of quantum systems with their environment. The algebraic approach allows a rigorous description of both finite and infinite quantum systems (i. e. quantum fields or many body systems) as well

as classical systems and is therefore well suited for the discussion of the emergence of classical properties.

Consider a quantum system S described by a von Neumann algebra $\mathfrak{M}$ acting on a Hilbert space $\mathscr{H}_{\mathrm{E}}$. The total system S + E consisting of S and its environment E is described by a von Neumann algebra $\mathfrak{N}$ acting on $\mathscr{H}$. Since S + E is considered as a closed system, its time evolution is reversible and given by a group of *-automorphisms $\{\alpha_t\}_{t\in\mathbb{R}}$ on $\mathfrak{N}$. To study decoherence we consider the reduced dynamics of S. In the Heisenberg picture it is given by a family of completely positive normal contractive and unital maps $T_t = E \circ \alpha_t \circ i$, $t \geq 0$, on $\mathfrak{M}$; here $i : \mathfrak{M} \longrightarrow \mathfrak{N}$ describes the inclusion of S in S + E and $i \circ E$ is a conditional expectation onto $i(\mathfrak{M})$. In models one frequently has $\mathfrak{N} = \mathfrak{M} \bar{\otimes} \mathfrak{M}_{\mathrm{E}}$, where $\mathfrak{M}_{\mathrm{E}}$ is the algebra of the environment, acting on $\mathscr{H}_{\mathrm{E}}$, thus $\mathscr{H} = \mathscr{H}_{\mathrm{S}} \otimes \mathscr{H}_{\mathrm{E}}$. In this case the reduced dynamics is given by

$$T_t(x) = E_{\omega_{\mathrm{E}}}\left[\mathrm{e}^{\mathrm{i}tH}(x \otimes 1)\mathrm{e}^{-\mathrm{i}tH}\right], \qquad x \in \mathfrak{M}, \tag{1}$$

where $E_{\omega_{\mathrm{E}}}$ is a conditional expectation onto $\mathfrak{M}$ with respect to the reference state ω_{E} of the environment and H is the total Hamiltonian. By duality the Schrödinger picture time evolution becomes $T_{t,*}(\rho) = \mathrm{tr}_{\mathrm{E}}\left[\mathrm{e}^{-\mathrm{i}tH}(\rho \otimes \omega_{\mathrm{E}})\mathrm{e}^{\mathrm{i}tH}\right]$ for any density matrix ρ of S, where tr_{E} denotes the partial trace with respect to the environment E. Generally the operators $\{T_t\}_{t\geq 0}$ satisfy a complicated integro-differential equation, however, in many models they can be approximated by a dynamical semigroup, e. g. by using a weak or singular coupling limit.

Now it is possible to discuss the emergence of classical properties of the system S. It was shown [3, 7] that for any dynamical semigroup $\{T_t\}_{t\geq 0}$ on $\mathfrak{M}$ satisfying a number of technical assumptions the following decomposition exists:

$$\mathfrak{M} = \mathfrak{M}_1 \oplus \mathfrak{M}_2, \tag{2}$$

where $\mathfrak{M}_1$ is a T_t-invariant von Neumann subalgebra of $\mathfrak{M}$ such that the restriction $T_t|_{\mathfrak{M}_1}$ is a *-automorphism on $\mathfrak{M}_1$, and $\mathfrak{M}_2$ is a linear T_t-invariant subspace such that for any state ρ of S and $x \in \mathfrak{M}_2$ we have $\langle T_t(x), \rho\rangle \to 0$ as $t \to \infty$, i. e. all expectation values for observables in $\mathfrak{M}_2$ tend to 0 as time becomes large and are thus beyond experimental resolution after the decoherence time. We call $\mathfrak{M}_1$ the *algebra of effective observables,* it describes the system after the decoherence time. More generally, also in case $\{T_t\}_{t\geq 0}$ does not satisfy a semigroup law we say that *environmental decoherence* takes place if the splitting (2) exists with $\mathfrak{M}_2 \neq \{0\}$. According to the structure of $\mathfrak{M}_1$ we can identify the following scenarios of environmental decoherence [4]: If $\mathfrak{M}_1$ is noncommutative and its center is nontrivial we speak of *environment induced superselection rules.* If $\mathfrak{M}_1$ is commutative and $T_t|_{\mathfrak{M}_1}$ is trivial, we speak of *environment induced pointer states.* Both superselection rules and pointer states may either be discrete or continuous. If $\mathfrak{M}_1 = \mathsf{L}^\infty(\Omega)$ is commutative and $T_t|_{\mathfrak{M}_1}$ is given by a nontrivial flow on the configuration space Ω we speak of an *environment induced classical dynamical system* (see [4] for an example). Finally if $\mathfrak{M}_1$ is noncommutative with a trivial

center then $\mathfrak{M}_1$ again describes a quantum system without any classical properties, but possibly with *different quantum properties.* In this way a transition from an infinite quantum system to a finite system is possible.

We now present a simple model illustrating the emergence of continuous pointer states [5] and of new quantum properties [6], see also [7]. The system S consists of an infinite array of spins at positions labeled by $n = 1, 2, \ldots$, each spin is described by the 2×2-matrix algebra M_2, thus $\mathfrak{M}$ is given by the weak closure of $\pi_0\big(\bigotimes_{n=1}^{\infty} M_2\big)$, where π_0 is a faithful representation of the Glimm algebra $\bigotimes_{n=1}^{\infty} M_2$ on a Hilbert space $\mathscr{H}_{\mathrm{S}}$. Remark that $\mathfrak{M}$ is a factor of type II_1. The environment E consists of an infinitely extended thermal system of phonons of a linear crystal, the Hilbert space of a single phonon is given by $\mathscr{H}_1 = \mathsf{L}^2(\mathbb{R}, \mathrm{d}k)$. The algebra of the environment $\mathfrak{M}_{\mathrm{E}}$ is the weak closure of $\pi_{\omega_{\mathrm{E}}}(\Delta(\mathscr{H}_1))$, where we denote the CCR algebra over one-particle space $\mathscr{H}_1$ by $\Delta(\mathscr{H}_1)$, and $\pi_{\omega_{\mathrm{E}}}$ is the GNS-representation, acting on $\mathscr{H}_{\mathrm{E}}$, of the quasifree gauge invariant thermal state ω_{E} with two-point function $\omega_{\mathrm{E}}(a^*(f)a(g)) = \int \rho(k) f(k) \bar{g}(k)\, \mathrm{d}k$, $f, g \in \mathscr{H}_1$, and with thermal equilibrium distribution of the phonon energy given by $\rho(k) = 1/(\mathrm{e}^{\beta\omega(k)} - 1)$ with respect to the dispersion relation $\omega(k) = |k|$. The joint system S + E evolves unitarily with Hamiltonian $H = H_{\mathrm{S}} \otimes 1 + 1 \otimes H_{\mathrm{E}} + H_{\mathrm{I}}$, where H_{E} describes the free phonon evolution and $H_{\mathrm{S}} = 0$. We choose the interaction as $H_{\mathrm{I}} = \lambda Q \otimes \phi(g)$, where $Q = \pi_0\big(\sum_{n=1}^{\infty} 2^{-n} \sigma_n^3\big)$, and σ_n^3 denotes the third Pauli matrix of site n, $\phi(g)$ is a field operator corresponding to the representation $\pi_{\omega_{\mathrm{E}}}$ with a suitably chosen test function g. Using the singular coupling limit the reduced dynamics can be approximated by a dynamical semigroup $T_t = \mathrm{e}^{tL}$ with generator $Lx = \mathrm{i}b[Q^2, x] + 2\pi\lambda b \beta^{-1}(QxQ - \frac{1}{2}\{Q^2, x\})$, with b depending on the test function g.

Theorem 1. *The decomposition* (2) *exists where* $\mathfrak{M}_1$ *is a continuous and commutative algebra of functions on the configuration space* $\Omega = \{(i_1, i_2, \ldots) : i_n = \pm 1\}$ *of the one-dimensional Ising model. The dynamics* T_t *restricted to* $\mathfrak{M}_1$ *is trivial.*

We identify each $x \in \mathfrak{M}_1$ with the function $\Omega \ni \eta \mapsto x(\eta)$. If μ_0 is a probability measure on $\{+1, -1\}$ with the values $\frac{1}{2}$ at $\{\pm 1\}$, and if $\mu = \bigotimes_{n=1}^{\infty} \mu_0$ is the product measure on Ω, for all $x \in \mathfrak{M}_1$ we have $\mathrm{tr}x = \int x(\eta)\, \mathrm{d}\mu(\eta)$. Thus for any $\alpha \in [0, 1]$ there exists a self-adjoint $x \in \mathfrak{M}_1$ with $\mathrm{tr}x = \alpha$. In this sense the pointer states correspond to a pointer with continuous readings.

Now suppose that system S and environment E are as above, but we assume that the spin chain is exposed to a magnetic field. We choose the free Hamiltonian as $H_{\mathrm{S}} = \pi_0\big(\sum_{n=1}^{\infty} -g\mu_{\mathrm{B}} H(n) \sigma_n^3\big)$, where g is the Landé factor and μ_{B} the Bohr magneton, we assume that the magnetic field $H(n)$ at site n decreases like $H(n) \sim 2^{-n}$. The interacting Hamiltonian is again chosen as $H_{\mathrm{I}} = \lambda Q \otimes \phi(g)$ with $Q = \pi_0\big(\sum_{n=1}^{\infty} a_n \sigma_n^1\big)$, where σ_n^1 is the first Pauli matrix of site n and the coupling constants decrease like $a_n \sim 2^{-n}$, so that Q is a bounded operator. If the test function g is suitably chosen, the reduced dynamics of S may again be approximated by a Markovian one, i. e. $T_t = \mathrm{e}^{tL}$ with generator $Lx = \mathrm{i}[H_{\mathrm{S}} - bQ^2, x] + 2\pi\lambda\beta^{-1}(QxQ - \frac{1}{2}\{Q^2, x\})$.

Theorem 2. *The decomposition* (2) *exists and* $\mathfrak{M}_1 = \mathbb{C}1$. *Moreover, if the first site does not interact, i. e.* $a_1 = 0$, *then* $\mathfrak{M}_1 = M_2$ *and* $T_t(x) = \mathrm{e}^{\mathrm{i}tH(1)\sigma^3} x \mathrm{e}^{-\mathrm{i}tH(1)\sigma^3}$ *for all* $x \in \mathfrak{M}_1$.

We see the algebra of effective observables is again a noncommutative factor and thus after the decoherence time S behaves effectively like a system with a pure quantum character. Albeit being physically unsatisfactory (but mathematically nontrivial), this example shows that the interaction with an environment may not only induce classical properties but also new quantum properties without introducing any classicality.

To discuss in a systematic way Boson systems with finitely or infinitely many degrees of freedom we developed a formalism to construct dynamical semigroups on CCR algebras by using perturbed convolution semigroups of promeasures. For details see [7]. A current effort is to extend these to the von Neumann algebras generated by representations of the CCR algebras. For example in the case of infinite Boson systems in thermal equilibrium this would yield a class of dissipative dynamics on type III von Neumann algebras.

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Additivity: from finite to infinite dimensions

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Recently P. Shor [7] gave arguments which show that conjectured additivity properties for several quantum information quantities are in fact equivalent. In this talk we first give several equivalent formulations of the additivity conjecture for finite-dimensional constrained channels, which formally is substantially stronger than the unconstrained additivity [2]. To this end a characteristic property of the optimal ensemble for such a channel is derived, generalizing the maximal distance

property [4]. It is shown that the additivity conjecture for constrained channels holds true for certain nontrivial classes of channels. After giving an algebraic formulation for the Shor's channel extension, its main asymptotic property is proved. It is then used to show that additivity for two constrained channels can be reduced to the same problem for unconstrained channels, and hence, "global" additivity for channels with arbitrary constraints is equivalent to additivity without constraints.

Then we pass to the systematic study of the classical capacity (more precisely, closely related entropic quantities: the output entropy, its convex closure and their difference, called the χ-capacity), of infinite dimensional quantum channels. While major attention in quantum information theory up to now was paid to finite dimensional systems, there is an important and interesting class of Gaussian channels which act in infinite dimensional Hilbert space. Although many questions for Gaussian Bosonic systems with finite number of modes can be solved with finite dimensional matrix techniques, a general underlying Hilbert space operator analysis is indispensable.

There are two important features essential for channels in infinite dimensions. One is the necessity of the input constraints (such as mean energy constraint for Gaussian channels) to prevent from infinite capacities (although as we have shown considering input constraints is quite useful also in the study of the additivity conjecture for channels in finite dimensions). Another is the natural appearance of infinite, and, in general, "continuous" state ensembles understood as probability measures on the set of all quantum states.

Moreover, it was observed recently that Shor's proof of global equivalence of different forms of the famous additivity conjecture is related to weird discontinuity of the χ-capacity in the infinite dimensional case. It is also well known that quantum entropy has some pathological properties in infinite dimensions [8]. All this calls for a mathematically rigorous treatment involving specific results from operator theory and measure theory.

By using compactness criteria from probability and operator theory we can show that the set of all generalized ensembles with the average in a compact set of states is itself a compact subset of the set of all probability measures eqipped with the weak topology. With this in hand we give a sufficient condition for existence of an optimal generalized ensemble for a constrained quantum channel. This condition can be verified in particular in the case of Bosonic Gaussian channels with constrained mean energy [3].

In [5] it is shown that additivity of χ-capacity for all finite-dimensional channels without constraints implies additivity for infinite-dimensional channels with arbitrary constraints.

For a general survey of current state of art in the additivity/multiplicativity problems in quantum information theory see [1].

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Environmental localization of matter waves

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(joint work with Markus Arndt)

Recent interference experiments with fullerene matter waves allow one to study the natural decoherence phenomena affecting massive particles with a complex internal structure [1, 2]. These effects, which are due to the interaction with unobserved, environmental degrees of freedom, lead to the loss of coherence of the initially delocalized center-of-mass states in an interferometer. Specifically, we focus on the theoretical description of decoherence due to collisions with a thermal background gas (of small mass) and on the emission of heat radiation by the particle.

In both cases it is appropriate and necessary to use a Markovian master equation which treats the interaction non-perturbatively. Rather than by a weak coupling calculation, they are obtained by evaluating the rate of decoherence events (such as a collision or the emission of a thermal photon) and the effect of a single event individually.

The change of the state due to a single scattering-type interaction is obtained by a trace over the environmental degree of freedom. In the studied cases, this leads to a modification of the particle's center-of-mass statistical operator $\hat{\rho}$ which is multiplicative in the position representation,

$$\rho'(\mathbf{r}_1, \mathbf{r}_2) = \rho(\mathbf{r}_1, \mathbf{r}_2)\, \eta(\mathbf{r}_1 - \mathbf{r}_2).$$

A careful calculation of the scattering process based on a convex decomposition of the environmental state in terms of localized wave packets yields the decoherence function η [3]. It satisfies the natural requirements $\eta(0) = 1$, $\lim_{|\mathbf{R}|\to\infty}(\mathbf{R}) = 0$.

The rates, on the other hand, can be characterized by an appropriately simplified description of the complex interior structure of the particles in terms of their bulk properties (such as the frequency dependent polarizability or the dielectric

function). This way realistic quantitative predictions can be obtained for complex molecules. Moreover, scaling assumptions on the bulk properties allow one to predict decoherence rates in the macroscopic domain.

The master equations obtained by combining rate and effect read, for particles of large mass m,

$$\partial_t \hat{\rho}_t = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}_t] - \int \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}' \gamma_t(\mathbf{r}-\mathbf{r}')\rho_t(\mathbf{r},\mathbf{r}')|\mathbf{r}\rangle\langle\mathbf{r}'|.$$

In the case of collisional interactions the localization rate is given by [3]

$$\gamma_t(\mathbf{R}) = n_t \int_0^\infty \mathrm{d}q\, \nu_t(q) \frac{q}{m} \int \frac{\mathrm{d}\mathbf{n}\mathrm{d}\mathbf{n}'}{4\pi} \left(1 - e^{iq(\mathbf{n}-\mathbf{n}')\mathbf{R}/\hbar}\right) |f(q\mathbf{n}', q\mathbf{n})|^2$$

(and differs by a factor from earlier results [4, 5]). Here, n_t and ν_t are the gas density and its (typically thermal) momentum distribution, respectively. f is the scattering amplitude and $\mathrm{d}\mathbf{n}$ gives the element of solid angle associated with the unit vector $\mathbf{n}$. The limit of a large mass ratio between scattered and colliding particle taken here is compatible with a detailed asymptotic analysis of the collision dynamics [6, 7]. Moreover, we note that the structure of this equation is compatible with the approximate master equation for finite mass ratios in terms of the dynamic structure factor obtained by Vacchini [8].

The localization rate due to isotropic heat radiation reads

$$\gamma_t(\mathbf{R}) = \int_0^\infty R(\lambda; T_t) \left[1 - \mathrm{sinc}\left(2\pi \frac{|\mathbf{R}|}{\lambda}\right)\right] \mathrm{d}\lambda,$$

where $R(\lambda; T_t)$ is the temperature-dependent spectral photon emission rate at wavelength λ. In practice this rate differs from the Planck law of a macroscopic black body even for mesoscopic particles with many hundreds of internal degrees of freedom, but it can be characterized by the absorption cross section and the heat capacity of the object.

When treating the effect of decoherence in an interferometer it is advantageous to avoid solving the time dependent master equation. Instead, one integrates an equivalent differential equation for the stationary state. It describes the change of coherence in the state at the interferometer output with an increasing region where decoherence takes place [9]. In the case of a near-field Talbot-Lau interferometer this can be done analytically and one obtains a closed expression for the reduction of the interference visibility.

The result of decoherence is particularly simple in the case of a background gas at room temperature, where a single collision suffices to localize the particle below the observable scale. As a result, the contrast is expected to fall exponentially with increasing gas pressure (unlike the case of simple absorption or loss, where the particle flux would fall exponentially at constant visibility). The decay is determined by the characteristic pressure $p_\mathrm{d} = k_\mathrm{B}T/(2L\sigma_\mathrm{eff})$ where L is the distance between the gratings in a Talbot-Lau setup and σ_eff is the effective scattering cross

section in the thermal gas at particle velocity v,

$$\sigma_{\rm eff}(v) = \int_0^\infty {\rm d}q\, \nu(q) \int \frac{{\rm d}\mathbf{n}}{4\pi v} \sigma(|q\mathbf{n} - mv\mathbf{n}_{\rm p}|) \left| \frac{q\mathbf{n}}{m_{\rm g}} - v\mathbf{n}_{\rm p} \right| .$$

The situation is a bit more complicated in the case of heat radiation, when the wave lengths of the thermal photons are comparable to the separation between the interfering paths. The resulting reduction of the interference visibility V is then given by an integration over all possible wavelengths λ and longitudinal positions vt in the interferometer,

$$V = V_0 \exp\left(- \int_0^{2L/v} {\rm d}t \int_0^\infty {\rm d}\lambda\ R(\lambda, T_t) \left[1 - {\rm sinc}\left(\frac{2\pi d}{\lambda} \frac{L - |vt - L|}{L_{\rm T}} \right) \right] \right) .$$

The argument of the sinc function involves the grating constant d and the Talbot length $L_{\rm T} = mvd^2/h$. It gives the ratio between the effective path separation at the various possible emission positions to the photon wavelength. Note that the time dependence of the particle temperature T allows for the effect of cooling in the interferometer, which can be important in practice.

The comparison with the experimental data for fullerenes with various background gases and different temperatures shows that the observed loss of interference can be fully understood in the described framework [1, 2]. Specifically, the fullerenes turn gradually from a pure quantum behavior to being indistinguishable from classical particles as their internal temperature increases from 2000 to 3000 K in the experiment.

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Tensor product decomposition and Separability

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(joint work with Shao-Ming Fei)

Since the beginning of the study of quantum information, quantum entanglement has occupied a central position in quantum information processing due to its important applications in quantum teleportation, quantum cryptography, quantum dense coding, quantum error correction and parallel computation [1, 2, 3]. However the theory of quantum entanglement itself is far from satisfaction.

A mixed state on $H_1 \otimes H_2$ is described by a density matrix ρ: ensembles of state $|\Psi_i\rangle$ with probabilities p_i. The quantum state is called *separable* if there exists a decomposition such that

$$\rho = \sum_i p_i \rho_i^1 \otimes \rho_i^2, \tag{1}$$

where ρ_i^1 and ρ_i^2 are rank one density matrices on H_1 and H_2 respectively. It is a challenge to find a decomposition like (1) or proving that it does not exist for a generic mixed state ρ [4, 5, 6]. With considerable effort in analyzing the separability, there have been some (necessary) criteria for separability in recent years, for instance, the Bell inequalities [7], PPT (positive partial transposition)[8] (which is also sufficient for the cases 2×2 and 2×3 bipartite systems [9]), reduction criterion[10, 11], majorization criterion[12], entanglement witnesses [9] and [13, 14], realignment [15, 16, 17] and generalized realignment [18], as well as some necessary and sufficient criteria for low rank density matrices [19, 20, 21] and also for general situations but not easy to apply or not operational [9].

In this work we use the idea of matrix approximation to study separability. The basic idea is to use appropriate tensor products to approximate the given density matrix. In [22] the minimum distance (in the sense of matrix norm) between a given matrix and some other matrices with certain rank is studied. In [23] and [24], for a given matrix A, the minimum of the Frobenius norm like $||A - \sum_i B_i \otimes C_i||_F$ is investigated. We see that if this minimum is zero, then the matrix A has a decomposition of the form $A = \sum_i B_i \otimes C_i$, which looks similar to the separable decomposition of a density matrix (1). To achieve our goal we need to put constraints on the components B_i and C_i. We have developed systematically how to decompose a given Hermitician matrix into a sum of Hermitian tensor products. By dealing with the Hermitian condition as higher dimensional real constraints, an explicit construction of density matrices on $H_1 \otimes H_2$ according to the sum of the tensor products of Hermitian matrices as well as real symmetric matrices on $H_1 \otimes H_2$ is presented. The results are generalized to the multipartite case. The separability problem is discussed in terms of these tensor product expressions. We also define a special quantity called *separability indicator* and proved that the given density matrix is separable if and only if the separability indicator is non-negative.

Let A be a matrix, and $||A|| := \sqrt{tr(AA^\dagger)}$ be the Hilbert-Schmidt norm (real case called Frobenius norm). Our first result is obtained via the singular value decomposition. We first study the problem of approximation of A by tensor product of symmetric matices, i,e. we are looking for $\sum_i B_i \otimes C_i$ to minimize the norm $||A - \sum_i B_i \otimes C_i||$ where B_i, C_i are real symmetric matrices.

[Theorem 1]. Let A be an $mn \times mn$ real symmetric matrix on $H_1 \otimes H_2$, where $dim(H_1) = m$, $dim(H_2) = n$. The minimun of the Frobenius norm $||A - \sum_i^r B_i \otimes C_i||_F$ is obtained for some $m \times m$ real symmetric matrix B_i on H_1 and $n \times n$ real symmetric matrix C_i on H_2, given by

$$vec(B_i) = Q_1 \begin{pmatrix} 0 \\ \hat{\mathcal{B}}_i \end{pmatrix}, \; vec(C_i) = Q_2 \begin{pmatrix} 0 \\ \hat{\mathcal{C}}_i \end{pmatrix}. \tag{2}$$

For $i \neq j \in \{1, \cdots, m\}$, let q_{ij} be the column vector of dimension m^2 with all entries zero except that the ij entry is $1/\sqrt{2}$ and ji entry is $-1/\sqrt{2}$, here we arranged the numbers $1, \cdots, m^2$ lexicographically. Let Q_s be the following block matrix of size $m^2 \times \frac{m(m-1)}{2}$

$$[q_{12}, q_{13}, \cdots, q_{1m}, q_{23}, \cdots, q_{2m}, \cdots, q_{m-1,m}]$$

We consider now the tensor product decompositions of Hermitian matrices.

Let A be a given $mn \times mn$ Hermitian matrix on $H_1 \otimes H_2$. As any Hermitian matrix A can be uniquely expressed as $A = B + iC$, where B and C are real symmetric and skew-symmetric matrices respectively. It is easy to show that any Hermitian matrix of size $n_1 n_2 \times n_1 n_2$ can always be expressed as a summation of tensor product of $n_1 \times n_1$ Hermitian matrices and $n_2 \times n_2$ Hermitian matrices. However the explicit decomposition needs special treatment. We give a systematic method for such a decomposition.

Let $\tilde{A}$ be the realignment of A, and set

$$Q_1^t \tilde{A} Q_2 = \begin{pmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{pmatrix}. \tag{3}$$

where Q_1 and Q_2 are the matrices defined above for the space H_1 and H_2 respectively. Let u_i (resp. v_i) be the eigenvectors of the matrix $\hat{A}_{22}\hat{A}_{22}^\dagger$ (resp. $\hat{A}_{22}^\dagger \hat{A}_{22}$). Let r and λ_i, $i = 1, 2, ..., r$, be the rank and eigenvalues of $\hat{A}_{22}^\dagger \hat{A}_{22}$ respectively. According to the singular value decomposition we have $\hat{A}_{22} = \sum_{i=1}^r \lambda_i u_i v_i^t$. Set $\hat{\mathcal{B}}_i = \lambda_i u_i$, $\hat{\mathcal{C}}_i = v_i$.

[Theorem 2]. Let A be an $mn \times mn$ Hermitian matrix on $H_1 \otimes H_2$, where $dim(H_1) = m$, $dim(H_2) = n$. The minimum of the Hilbert-Schmidt norm $||A - \sum_i^r B_i \otimes C_i||_{HS}$ is obtained for some $m \times m$ Hermitian matrix B on H_1 and $n \times n$ Hermitian matrix C on H_2, if $\hat{A}_{22} = \sum_{i=1}^r \hat{\mathcal{B}}_i \hat{\mathcal{C}}_i^t$, where $\hat{A}_{22}$ is defined by (3), $B_i = \mathrm{B}_i + i\mathcal{B}_i$, $C_i = \mathrm{C}_i + i\mathcal{C}_i$, are given by the relations

$$\begin{pmatrix} vec(\mathrm{B}_i) \\ -vec(\mathcal{B}_i) \end{pmatrix} = Q_1 \begin{pmatrix} 0 \\ -\hat{\mathcal{B}}_i \end{pmatrix}, \; \begin{pmatrix} vec(\mathrm{C}_i) \\ -vec(\mathcal{C}_i) \end{pmatrix} = Q_2 \begin{pmatrix} 0 \\ -\hat{\mathcal{C}}_i \end{pmatrix}. \tag{4}$$

Let $m(A)$ and $M(A)$ denote the smallest and the largest eigenvalues of a matrix A. For a decomposition of $A = \sum_i B_i \otimes C_i$ we can transform the decomposition into another decomposition such that each factor has the smallest eigenvalue zero as follows,

$$\begin{aligned} A &= \sum_{i=1}^{r} (B_i - m(B_i)I_m) \otimes (C_i - m(C_i)I_n) \\ &+ \left[\sum_{i=1}^{r} m(C_i)(B_i - m(B_i)I_m) - m\left(\sum_{i=1}^{r} m(C_i)(B_i - m(B_i)I_m) \right) \right] \otimes I_n \\ &+ I_m \otimes \left[\sum_{i=1}^{r} m(B_i)(C_i - m(C_i)I_n) - m\left(\sum_{i=1}^{r} m(B_i)(C_i - m(C_i)I_n) \right) \right] \\ &+ q_{B,C} I_m \otimes I_n \,, \end{aligned}$$

where I_m and I_n stand for $m \times m$ and $n \times n$ identity matrices and the coefficient of $I_m \otimes I_n$ is

$$q_{B,C} \equiv m\left(\sum_{i=1}^{r} m(C_i)B_i \right) + m\left(\sum_{i=1}^{r} m(B_i)C_i \right) - \sum_{i=1}^{r} m(B_i)m(C_i)$$

In general the $q_{B,C}$ associated with $A = \sum_{i=1}^{r} B_i \otimes C_i$ is not necessary positive. We define the *separability indicator* $S(A)$ to be the maximum of $q_{B,C}$ for all possible decomposition of $A = \sum_i B_i \otimes C_i$.

[Theorem 3]. Let A be a Hermitian matrix with tensor product decompositions of Hermitian matrices like $A = \sum_{i=1}^{r} B_i \otimes C_i$. A is separable if and only if the separability indicator $S(A) \geq 0$. Moreover $S(A)$ satisfies the following relations:

$$\begin{aligned} S(A) &\leq m(A), \\ S(A) &\geq \frac{1}{2} \sum_{i=1}^{r} [M(B_i)m(C_i) + M(C_i)m(B_i) \\ &\quad - |m(B_i)|(M(C_i) - m(C_i)) - |m(C_i)|(M(B_i) - m(B_i))], \\ S(A) &\geq m(A) - \sum_i M(B_i)M(C_i). \end{aligned}$$

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The Bogoliubov transformation and its effect in phase space

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(joint work with Norbert Lütkenhaus, Ulrik L. Andersen)

Intense light pulses with non-classical properties are used to implement protocols for quantum communication. Most of the elements in the tool box needed to assemble the experimental set-ups for these protocols are readily described by Bogoliubov transformations corresponding to Gaussian transformations that map Gaussian states onto Gaussian states. This seemingly linear field transformation also accounts for squeezing of light in phase space. One particularly interesting application is quantum interferometry [1] where e.g. phase resolution can be improved. A closely related protocol is quantum dense coding [2]. Quantum key distribution with continuous variables does require a quantum state but not necessarily one which belongs to the class of non-classical states [see QKD with coherent states: [3],[4]. For recent experiments see [5],[6]. Furthermore, it was realized that a higher secret bit rate can be attained by measuring the Q-function at the receiving end as oppose to the standard switched-basis measurement strategy [6],[7]. Among the quantum protocols which have been realized in our laboratory, based on Bogoliubov transformations, are quantum cryptography, quantum erasing, quantum cloning and quantum purification. Due to the fact that these protocols can be described by Bogoliubov transformations, only Gaussian operations were required. However, for some more advanced continuous variable quantum protocols (such as quantum computing and entanglement distillation) non-Gaussian operations,

which cannot be described by Bogoliubov transformations, are required. This involves Hamiltonians which are third or higher order in the field operators, an example being the Kerr effect. Intense light pulses propagating in standard fibres experiences the Kerr effect. An interesting aspect is that the Bogoliubov transformations leading to squeezing and to optical amplification are formally closely related. While the amplifier can be readily implemented using just linear optical elements [8], we found no way to do the same for squeezing. For a proposal to implement a cubic phase gate based on squeezing and detection see page 15-18 in [9].

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Optimal phase covariant transformations in arbitrary dimension

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(joint work with F. Buscemi, G.M. D'Ariano and P. Perinotti)

We present some recent results related to the optimality of phase covariant quantum transformations for quantum systems with arbitrary finite dimension. We will study some tasks of interest in quantum information, where the information itself is encoded into relative phases of quantum systems in dimension d. This kind of analysis is motivated by the fact that relative phases play a fundamental role in quantum information theory: for example, it was shown that most existing quantum algorithms can be viewed as multi-particle interferometers where the result of the computation is encoded in a relative phase [1], or also the latest generation of atomic clocks is based on the measurement of a phase shift in atomic systems [2]. The interest in looking at quantum systems in dimension higher than two is triggered by the fact that some quantum information tasks can be advantageous by increasing the dimension: for example, it was shown that the robustness of quantum cryptographic protocols increases [3]. Moreover, experimental achievements have been recently reported in the generation, manipulation and detection of quantum systems with higher dimension[4].

We address the issues of phase covariant cloning, multi-phase estimation procedures and transposition maps. We will optimise these procedures for the set of

equatorial states for d-dimensional quantum systems, defined as

$$(1) \qquad |\psi(\{\phi_j\})\rangle = \frac{1}{\sqrt{d}}(|0\rangle + e^{i\phi_1}|1\rangle + e^{i\phi_2}|2\rangle + ... + e^{i\phi_{d-1}}|d-1\rangle) \,,$$

where $\{|0\rangle, |1\rangle, |2\rangle ... |d-1\rangle\}$ represents a basis for the system under consideration and $\{\phi_j\}$ denotes a set of $d-1$ independent phase-shifts ($\phi_j \in [0, 2\pi]$).

We first consider phase covariant cloning for qudits. A cloning map can be viewed as a special kind of *quantum channel*, i. e. a trace-preserving completely positive (CP) map. We consider the general case of an arbitrary number of input copies N and an arbitrary number of output copies M. We impose the phase covariant condition on the cloning map C_{NM}, given by

$$(2) \qquad C_{NM}(U_{\{\phi_j\}}^{\otimes N} \, \rho_N \, U_{\{\phi_j\}}^{\dagger \otimes N}) = U_{\{\phi_j\}}^{\otimes M} \, C_{NM}(\rho_N) \, U_{\{\phi_j\}}^{\dagger \otimes M} \,,$$

where $U_{\{\phi_j\}} = \exp(i \sum_{j=1}^{d-1} \phi_j |j\rangle\langle j\,|)$ is the unitary phase rotation operator acting on a qudit and ρ_N is the state of the N input qudits. Such a condition guarantees that all equatorial input states are treated in the same way.

The optimal phase covariant cloning maps, which are optimised by maximising the single output copy fidelity with respect to the input, can be easily derived for any value of N and for values of M related to N and d as $M = kd + N$ [5], where k can take any positive integer value. The most interesting aspect in the case $M = kd + N$ is that the optimal phase covariant cloning map can be realised in an "economical" way, without the need of auxiliary qudits in addition to the M output copies. This is in contrast to the case of universal cloning [6] and to the case of $1 \to 2$ phase covariant cloning [7, 8], where auxiliary qubits are needed to achieve the optimal transformation.

The issue of phase covariant cloning is closely related to the issue of optimally estimating the $d-1$ phases, assumed to be completely unknown, for equatorial states (1). This problem was studied in [9] following the framework of quantum estimation theory, and the optimal positive operator valued measurement (POVM) was derived for arbitrary dimension and arbitrary number of available input copies N. It can be seen that the optimal phase estimation fidelity is equal to the optimal phase covariant cloning fidelity for an infinite number of output copies. This result is an interesting generalisation to arbitrary dimension of the same correspondence between estimation and cloning procedures proved to hold for the case of qubits [10, 11].

Another interesting quantum map is the transposition of the density operator. This operation has recently attracted much interest: for example, when transposition is performed on a subsystem of a composite system it leads to the well known partial transpose criterion for separability of mixed states [12]. As in the case of ideal cloning, transposition of a density operator is not a unitary transformation and can just be appriximated by physical processes. Approximate universal transposition transformations, i.e. transformations whose fidelity is the same for any input state, have been studied in the case of qubits for an arbitrary number of input and out copies (they are unitarily related to the U-NOT gate [13], which

corresponds to the inversion of the Bloch vector) and for a single copy in arbitrary dimension [14]. In both cases it was shown that the optimal universal transposition can be achieved by optimally measuring the input state and then prepare the transposed of the estimated state according the result of the classical measurement step. This feature is what is usually referred to as the *classicality* of the universal map.

We study the case of phase covariant transposition map for equatorial qudits. We prove that for a single copy the optimal fidelity is given by the simple expression $F = 2/d$. We also derive "economical" realisations for the case of $M = kd - N$, with $k \geq N$ [15]. It is interesting to notice that in the case of phase covariant transposition the optimal fidelity is always higher than the optimal fidelity of phase estimation with the same number of input copies. This means that, in contrast to the universal case, the transposition operation can be approximated optimally for equatorial states only in a fully quantum fashion, with the exclusion of a classical measurement/preparation step. This situation is particularly trivial in the case of qubits, for which it is possible to perfectly transpose all equatorial states by means of a σ_x operation (and actually $F = 1$ for $d = 2$), while the phase can never be measured exactly with finite resources.

Finally, we report some preliminary results achieved for mixed states. All the procedures presented above hold for input pure states. When the input states become mixed, such as for example after the action of some noise process, the derivation of optimal covariant maps becomes highly non trivial. We have recently studied the problem of optimal phase estimation for mixed qubit states, namely for a set of input qubits all in the same mixed state $\rho = (I + \vec{r} \cdot \vec{\sigma})/2$, with $|r| \leq 1$ [16]. We have derived the optimal estimation procedure, while the asymptotic behaviour of its efficiency for a large number of available copies is presently under study. Of course it is a very interesting and still open problem the relation between optimal estimation procedures and other kinds of quantum transformations, such as cloning and transposition, for mixed states.

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Quantum String Flipping

Serge Massar

Coin Tossing is the cryptographic task in which two parties who do not trust each other want to generate a random bit. If the parties can only communicate through a classical communication channel this task is impossible. But if they can communicate through a quantum channel it is possible to guarantee some degree of randomness to the bit. However using quantum communication, perfect randomness is impossible when a single random bit must be generated.

Here we consider the task, called string flipping, of generating a string of n bits. We describe a simple protocol for doing so using quantum communication. We show that for this protocol it is possible to guarantee that all but a vanishing fraction of the bits are random [1][2]. More precisely we show that it is possible to ensure that the total entropy of the string if one of the parties is dishonest and the other is honest is $H = n - cn^{-\alpha}$ where $c > 0$ and $0 < \alpha < 1$ are constants. Thus much better randomness is possible than when tossing a single coin.

We then describe an experimental realization of quantum string flipping in which the randomness of the string is provably higher than could be achieved classically, even when all imperfections, such as limited optical visibilities, detector inefficiency, detector dark counts are taken into account[3]. This experiment uses light pulses at 1550nm propagating in optical fibers. It is based on the "plug and play" scheme developed by N. Gisin and coworkers for long distance Quantum Key Distribution. Hence it is suitable for long distance implementation.

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Quantum Information Processing with Neutral Atoms

Dieter Meschede

Introduction Neutral atoms are one of the most promising candidates for realising quantum information processing devices. Sophisticated methods, widely applied in atomic clocks, have been developed over many decades to coherently manipulate the internal quantum states of an atom. Their charge neutrality protects them well from external perturbations, leading to a reduced decoherence of the quantum dynamics. Neutral atoms in optical lattices are unique for quantum

information purposes, as they are so far the only physical system, in which both an outstanding degree of single particle control exists, while simultaneously large scale qubit systems can be realized.
This contribution is focused on the state of art of controlling individual neutral atoms in 1D optical lattices and outline the path for future challenges in this field.

Trapping atoms in optical lattices. Optical lattices are formed by interfering several laser beams to form a perfectly periodic intensity pattern of light in space. The simplest (1D-) optical lattice can in fact be created by just superposing two counterpropagating laser beams, such that an optical standing wave is created. The optical standing wave consists of dark and bright stripes with a period of half an optical wavelength. The interfering light pattern, the standing wave, is completely defect free and forms a perfectly periodic spatial structure. Typically the wavelength of the light fields used to form the optical lattice are very far detuned from an atomic resonance transition but the oscillating electric field of the laser light nevertheless induces an oscillating atomic dipole moment within the atom which is in-phase for red detuning and 180° out-of-phase for blue detuning of trapping laser field and atomic resonance frequency. This oscillating electric dipole then interacts with the external oscillating electric field of the laser which causes the internal energy of the atom to decrease for red and increase for blue detuning. In inhomogeneous, patterned light fields, the internal energy shift results in an effective potential which is used to trap neutral atoms. For example, when the frequency of the laser light is below an atomic transition frequency, atoms are pulled into the intensity maxima of the laser field, whereas they are repelled from it in the opposite case. This force is very feeble and only extremely slow 'cold' atoms can be influenced by this force. The advent of efficient laser cooling methods was thus necessary before such experiments could start.

Storing, observing, and transporting single atoms.[1]–[3] A small but exact number of neutral atoms is initially prepared in a so called magneto-optical trap, where atoms from the residual gas of the experimental chamber are slowed by radiation pressure forces and detected by fluorescence. With sensitive photon counters the number of atoms is inferred from the step like fluorescence.

These atoms are then transferred to the 1D lattice which is formed from two counterpropagating laser beams. If the frequency of one of the two counterpropagating laser beams is slightly lowered or increased the interference pattern walks in the direction of the laser beams. allowing transport of the atoms over macroscopic distances (cm). A photon counting camera allows to record small movies showing such controlled motion of a group of atoms when the optical lattice is operated as the optical conveyor belt. The 'optical conveyor belt' opens the potential to bring atoms in a controlled way from one functional site to another one. For applications the quantum state of atoms can for example be prepared at one point and then transported into the electromagnetic field of a Fabry-Perot type optical microcavity where interactions with single photons can take place.

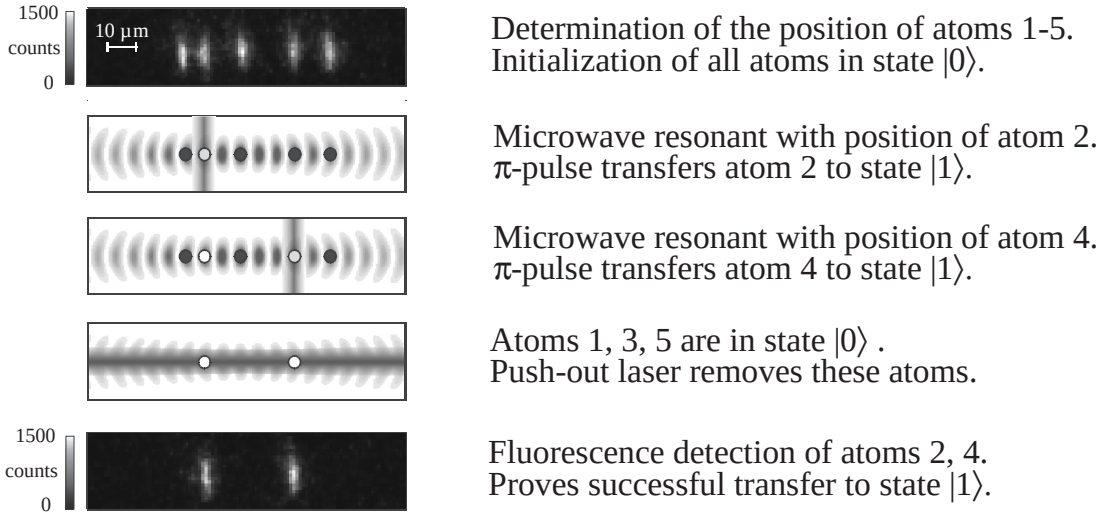

FIGURE 1. *Quantum Register with 5 Caesium Atoms. The images were taking with a photon counting camera. Resolution is about 1 µm per pixel.*

Controlling individual atom qubits in optical lattices: a neutral atom quantum register. [4] With neutral atoms, qubits can in principle be realized with any two levels of their quantum structure, external states of atomic motion as well as internal states of the atomic electron and spin. For this article we concentrate on magnetic sublevels of the internal atomic quantum state which allow convenient manipulation by external radiofrequency fields. In an experiment, two specific magnetic sublevels of the electronic atomic ground state are selected for the representation of the qubit. The qubit may be modelled by a sphere where the north- and southpole correspond to the logical '0' and '1'state, respectively. The state of the qubit can be indicated by an arrow locating the state on the surface of the sphere. Application of a resonant microwave pulse causes rotation of the arrow, and the rotation angle depends on the duration of the pulse. For instance, a so called 'π-pulse' rotates the angle from the north- to the south-pole and vice versa, thus it is equivalent to a logical inversion operation. A quantum register is obviously a combination of several qubits, which can be manipulated one by one, in our case atom by atom.

Operation of a register, whether of classical and quantum nature, requires realization of several functional steps: In the first step, the register must be physically prepared and initialised. In the second step, information must be written into the individual bits or qubits. In the final step, the full information must be retrieved bit by bit or qubit by qubit. For a quantum register one must demonstrate in addition to the case of a classical register that coherent superposition states of the two qubit states can be generated and is maintained for extended periods of time.

The first step is straightforward once a string of neutral atom is prepared as described in the previous section: Identical spin states (more precisely pseudo spin states) of all atomic qubits are prepared by 'optical pumping' which by a series

of absorption-emission cycles prepares all atoms to the same quantum state, for example the state corresponding to the logical '0' .

To achieve selectivity in the second step, an inhomogeneous magnetic field is applied which renders the magnetic resonance condition for the microwave pulse rotating the qubit state valid for a single qubit site only. Thus application of the microwave field causes rotation at this site only, even though the microwave field fills all space. At a measured spatial resolution of 2.5 μm this method could nominally allow operation of a 400 qubit register in a 1mm standing wave light field. Note that a closely related method is used to obtain spatial resolution in magnetic resonance imaging procedures (MRI).

For information retrieval the quantum state of the atomic qubit is unambiguously detected through quantum state projection by methods of laser spectroscopy which can discriminate between two atomic levels with excellent contrast. With this method for example the time evolution of the coherent superposition of the two qubit states can be observed as a function of the microwave pulse duration. The sinusoidal oscillation of the qubit state between the '0' and the '1'state is called 'Rabi-oscillation'İt proves the coherent nature of the superposition and thus the quantum property of the neutral atom quantum register.

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Positive linear maps and entanglement: an application

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(joint work with Fabio Benatti, Roberto Floreanini)

Entanglement appears to be a basic resource in the fields of quantum information and quantum computation (see [1, 2] and references therein). A state ρ_{AB} of a finite dimensional bipartite system A+B is entangled if can not be written as [3]:

$$\sum_i c_i \, \rho_{\mathrm{A}}^{(i)} \otimes \rho_{\mathrm{B}}^{(i)} \ , \quad c_i \geq 0 \ , \quad \sum_i c_i = 1, \tag{1}$$

with every $\rho_{\mathrm{A(B)}}^{(i)}$ a state of system A(B).

There are different results in the literature regarding the classification of states. One of the more interesting [4, 5] is based on the use of linear maps which are positive (P) but not completely positive (CP) [6, 7, 8, 9]: we shall refer to them as PnCP maps. A map is P if it transforms any state into another positive operator. A map Λ is CP if also the map id $\otimes \Lambda$, i.e. the action of Λ on a part of a composed system, is P. In the case of a bipartite system, a state is entangled if

and only if there exists a PnCP map such that the operator obtained acting with the map on only one of the two subsystems is not positive any more. The simplest example of PnCP map is the operation of transposition T (with respect to a given basis). The action of transposition on one of the subsystems is called *partial transposition* (PT). Because of the structure of the set of positive maps [10, 11], in the 2×2 and 2×3 dimensional cases PT can "detect" all the entangled states: states that remain positive under PT (PPT states) are separable; states that develop negative eigenvalues under PT (NPT states) are entangled. Unfortunately in higher dimensions PT is not a "complete" test any more and there are PPT states which are entangled [12].

The PnCP approach to the problem of entanglement characterization can also give information about the distillability of the state (see [13] for a review). A state is said to be distillable if, having at disposal a large number of copies of the state, it is possible to obtain some maximally entangled states, under the constraint of using only local operations and classical communication. It turns out that a PPT entangled state (PPTES) can not be distilled, so that its entanglement can be considered "bound" [14]; however it can still be useful for tasks that it would be impossible to perform classically [15]. In order to identify this bound entanglement it is necessary to use PnCP that are not decomposable, that is which can not be written as $\Lambda^1_{\rm CP} + \Lambda^2_{\rm CP} \circ T$, with $\Lambda^i_{\rm CP}$, $i = 1, 2$, CP maps, T the transposition operation and $\circ$ the symbol for composition.

It is therefore clear that the study of P maps is strictly related to the study of entanglement, the link being provided by the Choi-Jamiołkowsky isomorphism [6, 16]. Inspired by the study of properties of factorized semigroups [17, 18, 19], we contribute to the phenomenology of positive maps [20, 21, 22] giving some general methods to construct classes of PnCP maps acting on operators of a bipartite $d_1 \times d_2$ dimensional system [23].

We denote by $M_d(\mathbb{C})$ the set of $d \times d$ matrices with complex entries. Let Λ_i be maps acting on $M_{d_i}(\mathbb{C})$, $i = 1, 2$, in the following way

$$\Lambda_i[X] = \sum_{\mu=0}^{d_i^2-1} \lambda_\mu^{(i)} \, F_\mu^{(i)} \, X \, F_\mu^{(i)}, \tag{2}$$

with

$$F_\mu^{(i)} = \left(F_\mu^{(i)}\right)^\dagger \tag{3}$$

for all $\mu = 0, \ldots, d_i^2 - 1$, $i = 1, 2$, and $\mathrm{tr} F_\mu^{(i)} F_\nu^{(i)} = \delta_{\mu\nu}, \quad i = 1, 2$. If all the coefficients $\lambda_\mu^{(i)} \in \mathbb{R}$ are positive apart from one, let us say $\lambda_k^{(2)} = -|\lambda_k^{(2)}|$, and all the positive coefficients are greater or equal to $|\lambda_k^{(2)}|$, then the map $\Lambda : M_{d_1 \times d_2}(\mathbb{C}) \to M_{d_1 \times d_2}(\mathbb{C})$,

$$\Lambda = \Lambda_1 \otimes \mathrm{id}_{d_2} + \mathrm{id}_{d_1} \otimes \Lambda_2 \tag{4}$$

is positive.

In particular, we consider the case in which $d_1 = 2^m$, $d_2 = 2^n$, so that Λ acts on the state space of $N = m + n$ qubits, i.e. N two-level systems. We test the

decomposability of Λ by finding at the same time examples of PPT (and therefore bound) entangled states of $N+N$ qubits.

For the sake of concreteness, we give here some results in the case of the lowest dimension, $m=n=1$, $N=2$. Given σ_0, the identity in $M_2(\mathbb{C})$, and the Pauli matrices σ_a, $a=1,2,3$, we define $\hat{\sigma}_\alpha = \sigma_\alpha/\sqrt{2}$, $\alpha=0,1,2,3$. Then we consider a map Λ_B as in (4) with

$$\Lambda_1[X] = \sum_{\alpha=0}^{3} \hat{\sigma}_\alpha X \hat{\sigma}_\alpha, \quad \Lambda_2[X] = \sum_{\alpha=0}^{3} \varepsilon_\alpha \hat{\sigma}_\alpha X \hat{\sigma}_\alpha, \quad \varepsilon_\alpha = (-1)^{\delta_\alpha 2}, \tag{5}$$

that, according to the result just stated, is positive; it is moreover non-CP. Let us define the maximally entangled state of 2+2 qubits

$$|\psi_+\rangle_{\mathrm{AB}} = \frac{1}{2} \sum_{a,b=1}^{2} |a\otimes b\rangle_{\mathrm{A}} \otimes |a\otimes b\rangle_{\mathrm{B}} \tag{6}$$

and the orthogonal maximally entangled states

$$|\psi_{\alpha\beta}\rangle = \big(\mathbb{1}_4 \otimes (\sigma_\alpha \otimes \sigma_\beta)\big)|\psi_+\rangle, \quad \alpha,\beta = 0,1,2,3. \tag{7}$$

We consider mixed states

$$\rho_I = \frac{1}{\#I} \sum_{(\alpha,\beta)\in I} |\psi_{\alpha\beta}\rangle\langle\psi_{\alpha\beta}\,|, \tag{8}$$

with I a subset of $\{(\alpha,\beta)|\alpha,\beta=0,1,2,3\}$. The properties of a state ρ_I depend only on I. The condition of positive partial transposition is easily checked for such states. In particular one finds that the state corresponding to the choice

$$I = \tilde{I} = \{(\alpha,\beta)|\alpha,\beta=1,2,3\} \cup \{(0,2)\} \tag{9}$$

is PPT. Moreover, $(\mathrm{id}_{\mathrm{A}} \otimes \Lambda_{\mathrm{B}})[\rho_{\tilde{I}}] \not\geq 0$ so that one can conclude that $\rho_{\tilde{I}}$ is a PPT bound entangled state and Λ_B is non-decomposable. Such a result is quite easily generalized to construct PPT bound entangled states of $N+N$ qubits [23].

There are some open questions: can the family of positive maps (4) be generalized someway, for example relaxing condition (3)? What are the properties of states like $\rho_{\tilde{I}}$? In particular, may their entanglement be detected by other criteria?

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Two and multi-particle entanglement manipulation under positive-partial-transpose preserving operations

Martin B. Plenio

Constraints and resources are intimately related in physics. If we impose a constraint on a physical setting then certain tasks become impossible. A resource must be made available to overcome the restrictions imposed by the constraints. By definition such a resource cannot be created employing only the constrained set of operations but it may be manipulated and transformed under these operations. That the amount of resource does not increase under any operation satisfying the constraint emerges then as a fundamental law, for example in entanglement theory.

One example of particular importance is the restriction to local quantum operations and classical communication (LOCC). The resource that is implied by this constraint are non-separable states and in particular pure entangled states such as singlet states, neither of which can be created by LOCC alone. This setting gives rise to a theory of entanglement as a resource under LOCC operations.

Any such theory of entanglement as a resource will generally aim to provide mathematical structures to allow answers to three questions, namely (1) the characterization of entanglement, (2) the manipulation of entanglement and (3) the quantification of the entanglement resource [1, 2, 3] under the given constraint. Of particular interest is the question of how many inequivalent types of entanglement exist within such a theory. In the limit of infinitely many identically prepared copies of bipartite pure states, entanglement can be inter-converted reversibly [4] and it is reasonable to say that there is only one type of pure bipartite entanglement. Even for pure states, the situation changes dramatically when we consider the single copy setting. It has been shown that the Schmidt rank of bipartite pure states cannot be increased by LOCC [5, 6, 7, 8, 9]. At the single copy level, the convertibility of bi-partite entanglement is then characterized by the Schmidt-rank [10]. For finite dimensional systems a state can be converted to another one with finite probability exactly if the Schmidt-number of the target state is not larger than that of the initial state. In a tripartite setting the situation is more complicated. Here it is well-known for example that a GHZ state cannot be transformed to a W state and vice versa [10]. These states are then said to be incomparable. It can be shown that there are two incomparable types of tripartite entanglement in three qubits systems. The situation is even more complicated in multipartite settings composed by many parties [11] or infinite dimensional bipartite systems [12], where there are many (possibly infinitely many) incomparable types of entanglement.

A different setting is presented by the concept of partial time reversal or partial transposition [13]. For two qubits, states that remain positive under partial transposition (denoted as PPT-states) are exactly the separable states [14] but for higher dimensions this is generally not the case as there are PPT-states that are inseparable [15]. This motivates the definition of the set of positive-partial-transpose-preserving operations (PPT-operations), defined as operations that map any PPT-state into another PPT-state [16]. In this case, the resource are states that are not PPT (denoted as NPT-states).

The emerging theory of entanglement under ppt-operations still possesses the property that in an asymptotic setting pure state transformations are reversible and that consequently there is only one type of pure state entanglement. In the mixed state setting examples for reversible state transformations have been discovered [17] and supported by further numerical evidence it has been **conjectured** that in this setting all entanglement reduces to only one type, in stark contrast to the LOCC setting where reversible mixed entanglement transformations are known only in trivial cases [18, 19]. The additional power afforded by ppt operations as compared to LOCC operations becomes transparent both in the mixed state and the multi-party setting. In the non-asymptotic setting for pure state it has been shown that both under ppt operations [17] and under LOCC operations supported by ppt-bound entanglement [20] state transformations become possible that are impossible under LOCC. Indeed, it has been shown that on the single copy level we can use trace preserving completely positive ppt-operations

to transform for example a GHZ state $|GHZ\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ into a W-state $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$ with a maximal success probability of $x\frac{1}{4}(-2 + (18 - 6\sqrt{3})^{1/3} + (18 + 6\sqrt{3})^{1/3})$, ie approximately 75% [21]. This is in marked contrast to the situation under LOCC where this transformation has a zero success probability [10]. The surprisingly large success probability and the proven existence of some asymptotically reversible state transformations under ppt-operations in the bi-partite setting suggests that a theory of entanglement under ppt-operations might have a simpler structure than that under the LOCC constraint. Motivated by this the MREGS problem under the more general setting of ppt-preserving operations has been considered in [22]. Unfortuantely, it was found that in this asymptotic setting GHZ and W states remain asymptotically inequivalent even under ppt-operations. In fact, the the structure of MREGS appears to be as complicated as that under LOCC.

More work is required to achieve a full understanding of the structure of quantum entanglement under various possible constraints and the possible relations of such theories to the structures underlying other resource theories such as entanglement.

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Complementarity, Entanglement and Schrödinger cat states: Rabi oscillation in a new light.

Jean-Michel Raimond

Manipulating single quantum systems in a carefully controlled environment, we can now perform some of the gedankenexperiments used by the founders of quantum mechanics to assess their interpretation of the formalism. On the one hand, these experiments shed light onto intimate quantum features, such as entanglement and decoherence. On the other hand, they are prototypes of quantum information processing networks.

In this context, Cavity Quantum Electrodynamics (CQED), using circular Rydberg atoms and superconducting cavities, is a particularly fertile ground [1]. The strong atom-field coupling and the weak relaxation of both systems makes it possible to reach the 'strong coupling regime', in which the coherent atom-field coupling dominates dissipative processes. The simplest situation is then the 'vacuum Rabi oscillation', periodic energy exchange between an initially excited atom and the empty cavity. The photon emitted by the atom is trapped in the cavity and absorbed again. This 'oscillatory spontaneous emission' produces long-lived atom-field entangled states and provides elementary stitches to knit complex quantum logics manipulations [2]. This abstract is devoted to CQED experiments illustrating directly complementarity, entanglement, decoherence and the deep links between them.

The first experiment [3] is an implementation of the 'moving slit' Young's interferometer discussed by Bohr in the early days of quantum mechanics. In this device, the interfering particle kicks the moving slit when crossing it. The final slit's motion reveals the path followed by the particle in the interferometer. For a macroscopic slit, as found in usual interferometers, this kick is very small. The slit does not record any which-path information and fringes are observed. A microscopic slit, on the other hand, is set in motion by the particle. It gets entangled with it, and fringes are blurred by this entanglement. In intermediate, mesoscopic situations, the which-path information is partial and the fringes have a reduced contrast.

We have implemented a variant of this experiment, using a Ramsey atomic interferometer. Two $\pi/2$ pulses induced by a resonant field drive the atom from the upper state to the lower one. A transition may occur in one pulse or in the other, and interferences result from the superposition of the associated amplitudes. The atom, while making the transition, leaves a photon in one of the pulses. For the large fields used in standard Ramsey interferometers (atomic clocks for instance), the Ramsey fields involve very many photons. The single photon added by the atom does not provide an unambiguous which-path information and interferences are observed.

When one of the Ramsey fields is a mesoscopic coherent state stored in the cavity, it is noticeably modified by the addition of a single photon. In the limiting case in which the cavity is initially empty, the Ramsey pulse is produced by the vacuum Rabi oscillation. The final sates corresponding to the interfering paths (zero or one photon Fock state) are then orthogonal. The atom and the cavity field are maximally entangled and no fringes show up. We have observed the progressive washing out of the interferences when the photon number is reduced, which plays the role of the slit's mass in Bohr's 'experiment'.

Which-path information can also be gathered by an external detector that observes the interfering particle on its trajectory through the interferometer. We have realized an experiment of this kind [4], using a mesoscopic cavity field as a detector. The atom undergoes a Ramsey interference process between two classical fields sandwiching the cavity. It interacts dispersively with the cavity field. It thus cannot exchange energy with the mode, but behaves as a piece of transparent dielectrics that transiently shifts the cavity mode frequency. The shifts corresponding to the two atomic levels have opposite signs. The final classical phase of the field thus 'measures' the atomic state. We have observed, accordingly, a cancelation of the fringes contrast when the phase kick produced by the atom is larger than the initial quantum uncertainty on the field phase.

The final cavity state after atomic detection is particularly interesting. It is a quantum superposition of two coherent fields with noticeably different classical phases, a close analogue of the famous 'Schrödinger cat's' state. Using a second atom as a probe, we have measured the progressive decoherence of this state towards a mere statistical mixture [4]. The decoherence time scale is much shorter than the cavity energy damping time, and gets shorter and shorter when the distance in phase space between the two field components increases. This is an essential feature of decoherence, which explains why macroscopic quantum state superpositions are never observed.

This first exploration of the quantum/classical boundary opens interesting perspectives for fundamental decoherence studies, having an obvious interest for understanding the limitations of quantum information processing. It is essential, thus, to generate larger and larger cats. We have recently prepared large cat states via the resonant interaction between a mesoscopic cavity field and a single atom. This seemingly trivial situation leads to an interesting atom-field entanglement. The Rabi oscillation results from a quantum interference process between

two atomic states, in phase or π-out-of-phase with two coherent field components. These two fields rotate slowly in phase space in opposite directions. At most times, they are distinguishable. They then carry a which-path information about the atomic interference signal. The Rabi oscillations show up only when these two field components overlap. This complementarity argument explains, in simple terms, the quantum collapse and revival of the Rabi oscillations.

Using a field phase distribution measurement technique, we have observed the gradual splitting of the initial coherent field into two components [5]. Using an echo technique, reminiscent of the spin echoes of RMN, we have also assessed the coherence of the whole process. The evolution is time-reversed at some point and the two components merge back, erasing the which-path information and restoring the Rabi oscillation [6].

These experiments shed light onto fundamental quantum processes. They illustrate the deep links between complementarity, entanglement and decoherence. They also open the way to more sophisticated decoherence studies. The ability to measure directly the Wigner function of the cavity field [7] is particularly promising for monitoring, in 'real time', the detailed features of a Schrödinger cat state decoherence. Moreover, a two-cavity set-up, under construction, should allow us soon to realize non-local cat states, shared by two radiation modes. Studying the decoherence of their non-local properties is a very exciting perspective.

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Mathematical Model for Decoherence Induced by Scattering

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(joint work with R. Adami, R. Figari, D. Finco)

We consider a quantum system in $\mathbb{R}^3$ composed by one heavy plus N light particles described by the hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2M}\Delta_R + U(R) + \sum_{j=1}^{N}\left(-\frac{\hbar^2}{2m}\Delta_{r_j} + \alpha_0 V(r_j - R)\right) \tag{1}$$

Notice that each light particle is assumed to interact only with the heavy particle through the potential $\alpha_0 V$, while the heavy particle is also subject to the one-particle potential U.
To simplify the notation we fix $\hbar = M = 1$, $m = \epsilon$ and then we rescale the coupling constant $\alpha_0 \to \alpha = \epsilon\alpha_0$. The hamiltonian now reads

$$\hat{H}(\epsilon) = -\frac{1}{2}\Delta_R + U(R) + \frac{1}{\epsilon}\sum_{j=1}^{N}\left(-\frac{1}{2}\Delta_{r_j} + \alpha V(r_j - R)\right) \tag{2}$$

Moreover we consider an initial state in the product form

$$\Psi_0(R, r) = \phi(R)\prod_{j=1}^{N}\chi_j(r_j) \qquad R \in \mathbb{R}^3, r \in \mathbb{R}^{3N} \tag{3}$$

where ϕ and χ_j are the initial states of the heavy particle and and the j-th light particle respectively.
Our main result concerns the asymptotic behaviour for $\epsilon \to 0$ (and α kept fixed) of the solution $\Psi^\epsilon(t)$ of the Schrödinger equation associated to (2),(3), i.e.

$$\Psi^\epsilon(t) = e^{-it\hat{H}(\epsilon)}\Psi_0 \tag{4}$$

It is clear that the problem has two distinct time scales, one slow for the heavy particle and the other one fast for the light ones. In a rather crude zero-th order appoximation (see [5], [4]) one can describe the interaction through the instantaneous transition

$$\phi(R)\prod_{j=1}^{N}\chi_j(r_j) \to \phi(R)\prod_{j=1}^{N}(\Omega_+^{-1}(R)\chi_j)(r_j) \tag{5}$$

where $\Omega_+(R)$ is the wave operator for each light particle with the heavy one in the fixed position R. In formula (5) the evolution in time of the system is neglected in the sense that time zero for the heavy particle corresponds to infinite time for the light ones.

In order to restore the time evolution and to "see" the motion of the particles a first order approximation is required. The result is summarized in the following theorem.

Theorem 1. Assume
1) V, U "smooth" and decaying "sufficiently fast" at infinity
2) 0 is not eigenvenvalue nor resonance for $-\frac{1}{2}\Delta + \alpha V$
3) ϕ,χ_j "smooth".
Then for $0 < t < T < \infty$

$$\|\Psi^\epsilon(t) - \Psi^a(t)\| \leq C\sqrt{\epsilon} \tag{6}$$

$$\Psi^a(R, r; t) = \int dZ e^{-itX}(R, Z)\phi(Z) \prod_{j=1}^{N} \left(e^{-i\frac{t}{\epsilon}h_{0,j}}\Omega_+^{-1}(Z)\chi_j\right)(r_j) \tag{7}$$

where

$$X = -\frac{1}{2}\Delta_R + U(R)$$

$$h_{0,j} = -\frac{1}{2}\Delta_{r_j}, \Omega_+(Z) = s - \lim_{\tau\to\infty} e^{i\tau[-\frac{1}{2}\Delta+\alpha V(\cdot - Z)]}e^{-i\tau[-\frac{1}{2}\Delta]}$$

$$Z \in \mathbb{R}^3.$$

Moreover the dependence of the constant C on $T, N, \alpha, \Psi_0, V, U$ is explicitely given.

For the proof see [2] (see also [3], [1] for the analysis of the two-particle case).
The result given in theorem 1 can be used for a derivation of the decoherence effect on the heavy particle due to the scattering of the light ones, starting from first principles, i.e. from the Schrödinger equation for the entire $N+1$-particle system. In fact, we can define the reduced density matrix for the heavy particle

$$\hat{\rho}_t^\epsilon(R, R') \equiv \int dr \overline{\Psi^\epsilon}(R', r, t)\Psi^\epsilon(R, r, t) \tag{8}$$

and from theorem 1 we easily get

$$\lim_{\epsilon\to 0} Tr|\hat{\rho}_t^\epsilon - \hat{\rho}_t^a| = 0 \tag{9}$$

where

$$\hat{\rho}_t^a(R,R') \equiv \int dr \overline{\Psi^a}(R',r,t)\Psi^a(R,r,t)$$

$$(10) = \int dZdZ' e^{-itX}(R-Z)e^{itX}(R'-Z')\phi(Z)\overline{\phi}(Z') \prod_{j=1}^{N} \left(\Omega_+^{-1}(Z')\chi_j, \Omega_+^{-1}(Z)\chi_j\right)$$

Using the asymptotic reduced density matrix $\hat{\rho}_t^a$ it is now possible to see that the interference terms of a superposition state are reduced as a consequence of the scattering of the light particles.
In fact, let us choose $\phi(R) = \frac{1}{\sqrt{2}}\left(f^+(R) + f^-(R)\right)$ where

$$f^+(R) = \frac{1}{\sigma^{3/2}} f\left(\frac{R+R_0}{\sigma}\right) e^{iP_0\cdot R},$$

$$f^-(R) = \frac{1}{\sigma^{3/2}} f\left(\frac{R-R_0}{\sigma}\right) e^{-iP_0\cdot R} \text{ and}$$

$$f \in C_0^\infty(\mathbb{R}^3), R_0, P_0 \in \mathbb{R}^3.$$

Moreover let us assume $2|R_0| \gg \sigma$ and $\alpha\sigma \ll 1$. Then it is easily seen that

$$\begin{aligned}
\hat{\rho}_t^a(R,R') &= \frac{1}{2}\left(e^{-itX}f^+\right)(R)\left(e^{itX}\overline{f^+}\right)(R') \\
&+ \frac{1}{2}\left(e^{-itX}f^-\right)(R)\left(e^{itX}\overline{f^-}\right)(R') \\
&+ \frac{\Lambda}{2}\left(e^{-itX}f^+\right)(R)\left(e^{itX}\overline{f^-}\right)(R') \\
&+ \frac{\overline{\Lambda}}{2}\left(e^{-itX}f^-\right)(R)\left(e^{itX}\overline{f^+}\right)(R') + \mathcal{E} \qquad (11)
\end{aligned}$$

where $\Lambda = \prod_{j=1}^{N}\left(\Omega_+^{-1}(-R_0)\chi_j, \Omega_+^{-1}(R_0)\chi_j\right)$ and $\mathcal{E}$ is a computable small error.
Formula (11) shows that the diagonal terms remain unchanged while the non-diagonal terms are reduced by the factor Λ.
Taking into account that for N large $|\Lambda| \ll 1$ one concludes that the non-diagonal terms in $\hat{\rho}_t^a$ are negligible.
This means that the interaction with the light particles produces a decoerence effect on the heavy one and the corresponding reduced density matrix is close to a (classical) statistical mixture of the two pure states f^+ and f^-.

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Mathematical characterization and physical examples of translation-covariant Markovian master equations

Bassano Vacchini

The notion of covariance under a given symmetry group has proved to be very powerful in characterizing different mathematical structures relevant for quantum mechanics and especially for quantum information and computation, such as positive operator-valued measures and channels. In the same spirit, more recent work has been devoted to exploit the notion of covariance in order to point out interesting structures of generators of quantum-dynamical semigroups, which describe the Markovian dynamics of an open system.

In particular Holevo has given a full characterization of possible generators of quantum-dynamical semigroups for the case of covariance under translations, relying on a non-commutative generalization of the Lévy-Khintchine formula [1]. These results provide a natural setting to look in a unified way at different master-equations used for the description of decoherence of the center of mass degrees of freedom. Since one generally has to deal also with unbounded operators, the general strategy has been to characterize the so-called form-generators, which essentially amounts to provide a formal operator expression and an invariant domain. The generator may be expressed in the Heisenberg picture as

$$\mathcal{L}[\hat{X}] = i[H(\hat{\mathbf{p}}), \hat{X}] + \mathcal{L}_G[\hat{X}] + \mathcal{L}_P[\hat{X}]$$

putting into evidence a Gaussian and a Poisson component

$$\mathcal{L}_G[\hat{X}] = i\left[\hat{\mathbf{y}}_0 + \frac{1}{2i}\sum_{k=1}^{3}(\hat{\mathbf{y}}_k L_k(\hat{\mathbf{p}}) - L_k^\dagger(\hat{\mathbf{p}})\hat{\mathbf{y}}_k), \hat{X}\right]$$
$$+ \sum_{k=1}^{3}\left[(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}}))^\dagger \hat{X}(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}})) - \frac{1}{2}\left\{(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}}))^\dagger(\hat{\mathbf{y}}_k + L_k(\hat{\mathbf{p}})), \hat{X}\right\}\right]$$

$$\mathcal{L}_P[\hat{X}] = \int \sum_{j=1}^{\infty} \left[L_j^\dagger(\mathbf{q},\hat{\mathbf{p}}) \hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) L_j(\mathbf{q},\hat{\mathbf{p}}) - \frac{1}{2} \left\{ L_j^\dagger(\mathbf{q},\hat{\mathbf{p}}) L_j(\mathbf{q},\hat{\mathbf{p}}), \hat{X} \right\} \right] d\mu(\mathbf{q})$$
$$+ \int \sum_{j=1}^{\infty} \Big[\omega_j(\mathbf{q}) L_j^\dagger(\mathbf{q},\hat{\mathbf{p}}) (\hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) - \hat{X})$$
$$+ (\hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) - \hat{X}) L_j(\mathbf{q},\hat{\mathbf{p}}) \omega_j^*(\mathbf{q}) \Big] d\mu(\mathbf{q})$$
$$+ \int \sum_{j=1}^{\infty} \left[\hat{U}^\dagger(\mathbf{q}) \hat{X} \hat{U}(\mathbf{q}) - \hat{X} - i \frac{[\hat{X}, \mathbf{q}\cdot\hat{\mathbf{x}}]}{1+|\mathbf{q}|^2} \right] |\omega_j(\mathbf{q})|^2 d\mu(\mathbf{q})$$

with $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ position and momentum operators, $\hat{\mathbf{y}}_j = \sum_{i=1}^{3} a_{ji} \hat{\mathbf{x}}_i, a_{ji} \in \mathbf{R}$ $(j = 0, \ldots, 3)$, $\hat{U}(\mathbf{q}) = e^{i\mathbf{q}\cdot\hat{\mathbf{x}}}$, and the other functions depending on the system considered.

A general structure of master-equation for the description of both dissipation and decoherence of the center of mass degrees of freedom of a quantum system interacting through collisions with a homogeneous fluid has been obtained in [2] providing a physical realization of the Poisson component according to

$$\mathcal{L}[\hat{\rho}] = -i \left[H_0(\hat{\mathbf{p}}), \hat{\rho} \right]$$
$$+ \frac{2\pi}{\hbar} (2\pi\hbar)^3 n \int d^3\mathbf{q} \, |\tilde{t}(q)|^2 \left[\hat{U}(\mathbf{q}) \sqrt{S(\mathbf{q},\hat{\mathbf{p}})} \hat{\rho} \sqrt{S(\mathbf{q},\hat{\mathbf{p}})} \hat{U}^\dagger(\mathbf{q}) - \frac{1}{2} \left\{ S(\mathbf{q},\hat{\mathbf{p}}), \hat{\rho} \right\} \right]$$

corresponding to the Schödinger picture, with $S(\mathbf{q},\mathbf{p})$ a two-point correlation function known as dynamic structure factor, here appearing operator-valued, n the particle density in the fluid, $\tilde{t}(q)$ the Fourier transform of the interaction potential. Neglecting the dependence on the momentum operator, which is responsible for the dissipative effects, one recovers the typical structure of master-equation recently used in the quantitative experimental assessment of collisional decoherence [3]. Considering furthermore the limit of small momentum and energy transfer one obtains a quantum description of Brownian motion [4]

$$\mathcal{L}[\hat{\rho}] = -i \left[H_0(\hat{\mathbf{p}}), \hat{\rho} \right]$$
$$- \eta \sum_{i=1}^{3} \left\{ \frac{i}{2\hbar} \left[\hat{\mathbf{x}}_i, \{ \hat{\mathbf{p}}_i, \hat{\rho} \} \right] + \frac{\Delta p_{\text{th}}^2}{\hbar^2} \left[\hat{\mathbf{x}}_i, [\hat{\mathbf{x}}_i, \hat{\rho}] \right] + \frac{\Delta x_{\text{th}}^2}{4\hbar^2} \left[\hat{\mathbf{p}}_i, [\hat{\mathbf{p}}_i, \hat{\rho}] \right] \right\}$$

with η a microscopically determined friction coefficient, $\Delta p_{\text{th}}^2 = M/\beta$ and $\Delta x_{\text{th}}^2 = \beta\hbar^2/4M$, used in the recoilless approximation in order to estimate decoherence effects and giving a physical realization of the Gaussian component.

It thus appears that all these different master-equations used in the physical literature share the common feature of translation covariance, which strongly characterizes their structure.

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Quantum Information as Private Information

Reinhard F. Werner

1. No measurement without disturbance

Quantum information theory deals with the kind of information carried by systems described by quantum theory. Among the characteristic differences between these systems and classical ones is their great sensitivity to perturbations, demonstrated, for example in Heisenberg's Uncertainty Relations.

Another sharp formulation of this fundamental fact is the Theorem summarized as "No measurement without disturbance". It refers to a general kind of measurement, by which some classical data are obtained from a quantum system, leaving the system in a typically changed state for further experimentation. The Theorem considers measurements introducing *no* disturbance, in the sense that all statistical experiments with the output particles (without selecting according to the measurement outcomes) give exactly the same expectations as the corresponding experiment on the input particles. The conclusion is that in this case the measured outcomes are independent of the input, i.e., the whole measurement can effectively be replaced by a classical random generator producing "outcomes" completely unrelated to the quantum system. In other words, such a device does not measure anything.

This guarantees privacy of transmitted information in a very strong way: if sender and receiver operating the channel T can verify that their channel is ideal, then they can be sure that nothing whatsoever can be learned from observing the environment of the channel: no tapping of wires, and no "receive, read and resend" (or "man in the middle") attack has a chance. Of course, verifying that the channel is ideal is itself a statistical problem, so there is a subtle tradeoff

between monitoring the channel and sending messages, and hence between the level of security and the usable transmission rate. This tradeoff is the subject of quantum cryptography.

For a formal statement of the principle, recall that the statistical properties of quantum systems are characterized by spaces of operators on a Hilbert space $\mathcal{H}$: The possible preparations are given by "density operators", i.e., by positive operators ρ with trace 1, whereas the yes/no measurements are given by operators F with $0 \leq F \leq \mathbf{1}$. The probability for finding F on systems prepared according to ρ is then $\mathrm{tr}(\rho F)$. We will denote the space of bounded operators by $\mathcal{B}(\mathcal{H})$, and consider finite dimensional Hilbert space only, for which all operators are bounded anyhow.

The possible operations on quantum systems are given by normalized completely positive maps. When $\mathcal{H}_{\mathrm{in}}$ is the Hilbert space for the input systems, and the outputs are described in the space $\mathcal{H}_{\mathrm{out}}$, we need a map $T : \mathcal{B}(\mathcal{H}_{\mathrm{out}}) \to \mathcal{B}(\mathcal{H}_{\mathrm{in}})$. The interpretation is that, for a measurement $F \in \mathcal{B}(\mathcal{H}_{\mathrm{out}})$, the image $T(F) \in \mathcal{B}(\mathcal{H}_{\mathrm{in}})$ describes the measurement on the input systems consisting of first doing the operation and then measuring F. The property of complete positivity [1] is, by definition, that $F \geq 0$ implies $T(F) \geq 0$, even if the operation is applied to only a part of the system, i.e., if the operation is applied with an innocent, but possibly correlated bystander. Normalization means that $T(\mathbf{1}) = \mathbf{1}$. The possible measurement outcomes compatible with the overall state change T correspond to a decomposition of T into likewise completely positive terms T_x, i.e., $T = \sum_x T_x$. Then $\mathrm{tr}(\rho T_x(F))$ is the probability, in an experiment on systems prepared according to ρ, first the measurement result x and then a positive answer on the yes/no measurement described by F.

Now the structure of such measurements is completely described by the Stinespring dilation theorem, which asserts that T can be represented as $T(F) = V^*(F \otimes \mathbf{1})V$, where $V : \mathcal{H}_{\mathrm{in}} \to \mathcal{H}_{\mathrm{out}} \otimes \mathcal{K}$ is an isometry, and $\mathcal{K}$ is some auxiliary Hilbert space. These objects are uniquely determined up to unitary equivalence, provided the set $\{(A \otimes \mathbf{1})V\phi |\, \phi \in \mathcal{H}_{\mathrm{in}}, A \in \mathcal{B}(\mathcal{H}_{\mathrm{out}})\}$ generates the whole Hilbert space $\mathcal{H}_{\mathrm{out}} \otimes \mathcal{K}$, which we will assume. Moreover, any decomposition of $T = \sum_x T_x$ into completely positive summands is of the form

$$T_x(F) = V^*(F \otimes G_x)V, \tag{1}$$

where the operators $G_x \in \mathcal{B}(\mathcal{K})$ satisfy $G_x \geq 0$ and $\sum_x G_x = \mathbf{1}$.

The interpretation is that $\mathcal{K}$ represents the environment, and that T (or, equivalently, V) transforms the input states ρ into a state $V\rho V^*$ on $\mathcal{H}_{\mathrm{out}} \otimes \mathcal{K}$, which is correlated between the output and the environment. Due to the essential uniqueness of the Stinespring dilation $(\mathcal{K}, V)$, we can associate with each quantum channel T a *companion channel* $T^\sharp$, in which the roles of environment and output are interchanged.

The principle in the section title can now be obtained very simply: If T is the identity, its Stinespring decomposition is with a 1-dimensional environment $\mathcal{K}$, and $V = \mathbf{1}$. Hence only trivial measurements $G_x = p_x\mathbf{1}$ on the environment are possible.

2. No disturbance without measurement

The principle of the previous section has a converse: If we can be sure in that particular, strong way that no information can be obtained from observing the channel and its environment, we can conclude that the channel must be, in some sense, an ideal quantum channel. More precisely, all the input quantum information must be retrievable from the output. Or put in a way explaining the title of this section: any disturbance, which cannot be corrected, is accompanied by some information flow to the environment, so that a suitable measurement on the environment gives non-trivial information about the input state.

In formal terms this means that if the companion channel $T^\sharp$ of a channel is a depolarizing channel, i.e., $T^\sharp(F) = \mathrm{tr}(\sigma F)\mathbf{1}$, for some density operator σ, or, equivalently if the output state of the companion channel is σ, irrespective of the input state, then the channel has a right inverse, i.e., there is a decoding channel $D : \mathcal{B}(\mathcal{H}_{\mathrm{in}}) \to \mathcal{B}(\mathcal{H}_{\mathrm{out}})$ such that TD is the identity on $\mathcal{B}(\mathcal{H}_{\mathrm{in}})$. This is readily shown by computing the Stinespring dilation of the depolarizing channel and connecting it with the isometry V, which also gives a decomposition of $T^\sharp$, although the minimality condition ("$\mathcal{H} \otimes \mathcal{K}$ spanned by $(\mathbf{1} \otimes X)V\phi$") need not be satisfied. This gives an isometry connecting the two decompositions, from which the inverse channel D is readily constructed.

In another guise, this is a fundamental result on quantum error correcting codes: the Knill-Laflamme condition for the existence of a decoding operation for a given quantum code is precisely of this form.

Summarizing our conclusions so far, we can say that transmitting quantum information perfectly is equivalent to establishing a communication link which is absolutely (physically) safe against eavesdropping.

3. Doing it approximately

Such a conclusion is worth very little, however, if it only pertains to perfect channels: Nothing in the world is ideal. So in order to really support the conclusion we must show that it is stable under small modifications of the channels involved: having a nearly ideal channel should be equivalent to a channel which is nearly safe against intrusion, with explicit bounds making sense of the two occurrences of "nearly" in this sentence.

Since we are dealing with finite dimensional systems such bounds can be given, by just making estimates for every step of the Stinespring construction. However, bounds obtained in this way usually depend very strongly not only on the size of errors, but also on the dimensions $\dim \mathcal{H}_{\mathrm{in}}$ and $\dim \mathcal{H}_{\mathrm{out}}$. This may be tolerable for some applications with a fixed single channel. But it is desastrous for applications in the theory of channel capacity, where one considers asymptotically many parallel uses of the channel, and hence an exponentially exploding sequence of dimensions.

The main (apparently new) result of this talk is just such a dimension independent estimate securing the continuity of the Stinespring dilation. This is much more than the cases needed (i.e., the neighbourhoods of the ideal and depolarizing channels). Surprisingly, it turns out that the bounds are of exactly the same form

as bounds known for the case of *states*, i.e., the special case with 1-dimensional in put Hilbert space $\mathcal{H}_{\text{in}}$. Therefore, we will also use terminology familiar from that case.

Consider two channels T_1, T_2 with the same input and output Hilbert spaces. Since these $T_i : \mathcal{B}(\mathcal{H}_{\text{out}}) \to \mathcal{B}(\mathcal{H}_{\text{in}})$ are operators between normed spaces, there is a natural norm, $||T_1 - T_2|| = \sup_X ||T_1(X) - T_2(X)||/||X||$ to quantify the difference. From the statistical interpretation this is directly linked to the largest difference of probabilities, between two experiments of the form "preparation-channel-detection", differing only by substituting T_2 for T_1. However, for many applications it is better to allow more general experiments, namely also experiments in which the two channels are only applied to a subsystem. This leads to the so called *norm of complete boundedness*, or "cb-norm" for short. It is defined as

$$\|T_1 - T_2\|_{\text{cb}} = \sup_n ||\text{id}_n \otimes (T_1 - T_2)||,$$

where id_n denotes the identity channel on an n-dimensional Hilbert space, and the norm on the right hand side is the ordinary Banach space norm discussed previously.

On the other hand, we would like to have a description of how close the two Stinespring isometries V_1, V_2 for these channels are. Since the dilating isometries are only unique up to unitary equivalence in the first place, and the dilation spaces $\mathcal{K}_1, \mathcal{K}_2$ are a priori unrelated, we cannot directly look at the norm difference between V_1 and V_2. What we rather want to know is how close together these operators can be chosen. Therefore we just define the *Bures Distance* [2] of the channels T_1, T_2 as

$$\beta(T_1, T_2) := \inf ||V_1 - V_2||, \tag{2}$$

where the infimum is over all Hilbert spaces $\mathcal{K}$ and all operators $V_i : \mathcal{H}_{\text{in}} \to \mathcal{H}_{\text{out}} \otimes \mathcal{K}$ such that $T_i(F) = V_i^*(A \otimes \mathbf{1}_{\mathcal{K}})V_i$. This is a generalization of the corresponding definition for states, where the dilating isometries become "purifying vectors". The cb-norm difference in that case is equal to the trace norm difference of the density operators. Then the following is a direct generalization of the known bounds for states:

Theorem 1. *For any two channels T_1, T_2 with the same finite dimensional input and output spaces,*

$$\beta(T_1, T_2)^2 \leq \|T_1 - T_2\|_{\text{cb}} \leq 2\beta(T_1, T_2). \tag{3}$$

The proof (written out for the first time at the Oberwolfach workshop) uses the result for states, but in addition an application of von Neumann's Minmax Theorem. Some parts were also inspired by bounds for fidelity-like quantities defined by D'Ariano et al. [3].

The non-trivial part of this result is the lower bound. It says, quite generally, that if two channels are close in cb-norm, their dilating isometries can be chosen to be close in norm. Consequently, the companion channels (which share) the dilating isometries are also close to each other in cb-norm. Hence if a channel T

is nearly ideal in cb-norm, its companion $T^\sharp$ must be close to the companion of the ideal channel, which is depolarizing. So small disturbance means almost no information leakage to the environment.

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