Mathematisches Forschungsinstitut Oberwolfach

Report No. 25/2010

DOI: 10.4171/OWR/2010/25

Geometry, Quantum Fields, and Strings: Categorial Aspects

Organised by
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June 6th – June 12th, 2010

Abstract. Currently, in the interaction between string theory, quantum field theory and topology, there is an increased use of category-theoretic methods. Independent developments (e.g. the categorificiation of knot invariants, bundle gerbes and topological field theories on extended cobordism categories) have put higher categories in the focus.

The workshop has brought together researchers working on diverse problems in which categorical ideas play a significant role.

Mathematics Subject Classification (2000): 81T, in particular 81T45 and 81T13.

Introduction by the Organisers

The workshop *Geometry, Quantum Fields, and Strings: Categorial Aspects*, organised by Peter Bouwknegt (Australian National University, Canberra), Dan Freed (University of Texas, Austin), and Christoph Schweigert (University of Hamburg) was held June 6th–June 12th, 2010. The meeting was attended by 52 participants from all continents.

18 talks of one hour each were contributed to the workshop. Moreover, young researchers were offered the possibility to present short contributions. On Monday and Wednesday evening a total of 11 short talks were delivered. We would like to stress the high quality and level of interest of these contributions. The two sessions have received much attention and have led to much additional scientific discussion about the work of younger participants. For this reason, these contributions are covered in these proceedings as well.

Another special event was a panel discussion on Tuesday evening on the topic "Whither the interaction of Geometry-QFT-String?". Chaired by Dan Freed and under the lively participation of the audience, Kevin Costello, Michael Douglas, Greg Moore and Tony Pantev exchanged their point of view on recent and present interactions between mathematics and physics in the area of quantum field theory. There was a broad agreement that the field is in rapid progress and presents many exciting challenges that necessitate the interaction of researchers of different background. Homotopical techniques and generalized cohomology theories can be expected to play an increasingly important role in the study of quantum field theories and string theories.

The much of the work presented during this workshop could be described as "mathematics inspired by string theory and quantum field theory". Most of the contributions to the workshop were related to the following three main topics that are strongly interrelated:

(1) (Higher) categorial descriptions for quantum field theories, in particular for topological quantum field theories and extended versions of quantum field theories.
(2) Structures related to moduli spaces.
(3) Higher categorial structures of string backgrounds.

We summarize the contributions to this workshop according to these three subfields.

A construction of topological field theories based on Fukaya categories has been explained by Chris Woodward. A three-category of chiral conformal field theories has been discussed in an operator algebraic approach by Arthur Bartels; Alexei Davydov explained how aspects of the classification of rational chiral conformal field theories can be captured in the definition of a Witt group of modular categories. In Michael Douglas' talk, the space of quantum field theories, in particular two-dimensional conformal field theories, has been addressed from a completely different point of view; in the form of defects, higher categorial structures have been central to this approach as well.

Generalizations of Knizhnik-Zamolodichikov equations for conformal blocks have been presented in the talk of Valerio Toledano Laredo. Constantin Teleman discussed some aspects of (extended) topological field theories in two dimensions related to Gromov-Witten invariants. Kevin Costello explained his notion of factorization algebra, an adaptation of vertex algebras to a smooth setting, and its applications to the Witten genus. Inspired by older work on anomalies in quantum field theories in a Hamiltonian framwork, Jouko Mickelsson proposed in particular applications of gerbal representations to quantum field theory.

In the young researchers' session, the contributions of Orit Davidovich on extended topological field theories from state sum models and by Konrad Waldorf on gauge anomalies in two-dimensional bosonic sigma models complemented this circle of topics.

Dennis Gaitsgory's and Craig Westerland's talk discussed different aspects of moduli spaces of bundles. Structured moduli spaces of curves were discussed in Ezra Getzler's contribution in a symplectic setting and Nathalie Wahl presented a graphical calculus to the Hochschild homology of structured algebras motivated by string topology.

The third topic about string backgrounds included in particular a careful discussion of orientifold backgrounds by Greg Moore in terms of differentially refined and twisted cohomology theories. Such theories are also important in the discussion of T-dualities: a perspective on T-duality using a Lagrangian formalism for sigma-models was given in Kentaor Hori's talk; Mathai Varghese discussed T-duality in the presence of background fluxes and explained the need to include non-commutative geometry and non-trivial associators in the picture. In the young researchers' session, this was complemented by contributions by Alexander Kahle (touching also aspects of differential refinements of cohomology theories) and Rishni Ratnam about non-commutative torus bundles.

Twisted K-theory and its relation to the Verlinde algebra was one topic of Igor Kriz' talk. Yan Soibelman presented an algebraic approach to motivic Donaldson-Thomas invariants based on Calabi-Yau categories. Ludmil Katzarkov finally discussed non-abelian mixed Hodge structures.

The contributions of Igor Bakovic on 2-stacks, of Dan Berwick-Evans on supersymmetric sigma-models, of Braxton Collier on categorial Lie algebras, of Thomas Nikolaus on algebraic methods for higher categories and Hisham Sati on the geometry of membranes in the young researchers' session added important complements to these topics.

Workshop: Geometry, Quantum Fields, and Strings: Categorial Aspects

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Abstracts

Functoriality for Lagrangian correspondences in Floer theory

Chris T. Woodward
(joint work with Katrin Wehrheim)

We study composition of Lagrangian correspondences in monotone and exact Lagrangian Floer theory. Following Donaldson and Fukaya, one associates to a compact monotone (or noncompact exact) symplectic manifold (M, ω) a category $\mathrm{Don}(M)$ whose objects are certain compact, oriented, relatively spin, monotone (or exact) Lagrangian submanifolds of (M, ω) (which we call *admissible*) and whose morphisms are Floer cohomology classes. We use a variation of the usual definition, which we denote $\mathrm{Don}^{\#}(M)$. Given two symplectic manifolds M_0 and M_1 of the same monotonicity type, an admissible Lagrangian correspondence $L_{01} \subset M_0^- \times M_1$ gives rise to a functor

$$\Phi(L_{01}) : \mathrm{Don}^{\#}(M_0) \to \mathrm{Don}^{\#}(M_1).$$

Given a triple M_0, M_1, M_2 of symplectic manifolds and admissible Lagrangian correspondences $L_{01} \subset M_0^- \times M_1$ and $L_{12} \subset M_1^- \times M_2$, the algebraic composition $\Phi(L_{01}) \circ \Phi(L_{12}) : \mathrm{Don}^{\#}(M_0) \to \mathrm{Don}^{\#}(M_2)$ is always defined. On the other hand, one may consider the geometric composition $L_{01} \circ L_{12}$ that was introduced by Weinstein. Under suitable transversality hypotheses, the restriction of the projection $\pi_{02} : M_0^- \times M_1 \times M_1^- \times M_2 \to M_0^- \times M_2$ to

$$L_{01} \times_{M_1} L_{12} := \big(L_{01} \times L_{12}\big) \cap \big(M_0^- \times \Delta_{M_1} \times M_2\big)$$

is an immersion, whose singular Lagrangian image we denote by

$$L_{01} \circ L_{12} \subset M_0^- \times M_2.$$

Our main result is that if $L_{01} \times_{M_1} L_{12}$ is a transverse (hence smooth) intersection and embeds by π_{02} into $M_0^- \times M_2$ then

$$\Phi(L_{01}) \circ \Phi(L_{12}) \cong \Phi(L_{01} \circ L_{12}). \tag{0.1}$$

In other words, "categorification commutes with composition". If M_1 is not spin, there is also a shift of relative spin structures on the right-hand side. The starting point for this functoriality is an elementary construction of a symplectic category consisting of symplectic manifolds and certain *sequences of Lagrangian correspondences.*

There is a slightly stronger version of this result, expressed in the language of 2-categories as follows. Let $\mathrm{Floer}^{\#}$ denote the *Weinstein-Floer* 2-category whose objects are symplectic manifolds, 1-morphisms are sequences of Lagrangian correspondences, and 2-morphisms are Floer cohomology classes; we denote composition of 1-morphisms in this category by $\#$. The maps above extend to a *categorification 2-functor* from $\mathrm{Floer}^{\#}$ to the 2-category of categories Cat. A refinement of the main result says that the concatenation $L_{01} \# L_{12}$ is 2-isomorphic to the geometric composition $L_{01} \circ L_{12}$ as 1-morphisms in Floer$^{\#}$; the main result follows by combining this result with the 2-functor axiom for 1-morphisms.

The stable topology of moduli spaces of principal bundles

Craig Westerland
(joint work with Jordan Ellenberg, Akshay Venkatesh)

We investigate the stable topology of Hurwitz spaces of branched covers of Riemann surfaces, with applications to questions in arithmetic geometry.

Let G be a finite group, $c < G$ a conjugacy class, and $n > 0$ an integer. We will write $Hur^c_{G,n}$ for the moduli space of branched covers $\Sigma \to \mathbb{C}$ with n (unordered) branch points, Galois group G, and monodromy around branch points in c. Our first result is a rational homological stability theorem for these spaces:

Theorem 1. Let G be a finite group and $c \subset G$ a conjugacy class which generates G, with the property that for any subgroup $H \leq G$, $c \cap H$ is either empty or a conjugacy class of H. Then there exist integers A, B, D such that $b_p(Hur^c_{G,n}) = b_p(Hur^c_{G,n+D})$ whenever $n \geq Ap + B$.

It is also possible to compute these stable homologies. The setting of Hurwitz spaces generalizes to the study of the moduli space $\mathcal{M}_{g,n}(X)$ of triples $(S, \underline{z}, f)$, where S is a compact Riemann surface of genus g with one boundary component, $\underline{z}$ is a configuration of n points in S, and $f : S \to X$ is a continuous function. This is a "weak" moduli space in the sense that X may be an arbitrary topological space, and we allow all continuous functions (not just holomorphic). In some settings the continuous and algebraic notions do coincide up to homotopy equivalence, most notably for Hurwitz spaces, which can be considered as a moduli of functions to the stack $[*/G]$. Consequently, $Hur^c_{G,n}$ is equivalent to the space $\mathcal{M}_{0,n}(BG)$.

Define $A(X)$ to be the pushout of the diagram

$$D^2 \times LX \xleftarrow{\supseteq} S^1 \times LX \xrightarrow{ev} X$$

where LX denotes the free loop space of X, $LX = Map(S^1, X)$, and $ev(t, f) = f(t)$. We prove the following:

Theorem 2. There are maps

$$\mathcal{M}_{g,n}(X) \to \mathrm{Map}_n((S, \partial), (A(X), X))_{h\mathrm{Diff}^+(S,\partial)}$$

which give an integral homology isomorphism in the limit $n \to \infty$.

In the case of Hurwitz spaces, the rational homology of this function space is surprisingly easy to compute: it breaks into a union of components, each of which has the rational homology of a circle. Together with Theorem 1 and the Grothendieck-Lefschetz fixed point theorem, we obtain a proof of an asymptotic version of the Cohen-Lenstra heuristics for imaginary quadratic function fields. In particular, we show:

Theorem 3. If A is a finite abelian ℓ-group ($\ell \neq 2$), and q is sufficiently large (depending upon A), the density of imaginary quadratic extensions K of $\mathbb{F}_q(t)$ for which the ℓ-part of the class group of K is isomorphic to A is given by

$$\left(\prod_{i>1}(1-\frac{1}{\ell^i})\right)\frac{1}{|\mathrm{Aut(A)}|}$$

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Spaces of quantum field theories

Michael R. Douglas

The concept of a "space of quantum field theories" (QFTs) or "theory space" was set out in the 1970's in work of Wilson, Friedan and others. This structure should play an important role in organizing and classifying QFTs, and in the study of the string landscape, allowing us to say when two theories are connected by finite variations of the couplings or by RG flows, when a sequence of QFTs converges to another QFT, and bounding the amount of information needed to uniquely specify a QFT, enabling us to estimate their number. As yet we do not have any definition of theory space which can be used to make such arguments.

We begin with an overview of physics definitions of QFT, examples, and some of the phenomena which must be taken into account in defining theory space. There is an important analogy to the problem of defining and studying spaces of manifolds carrying a Riemannian metric, whose foundations were laid in the 1970's by Gromov, Cheeger and other mathematicians.

We then report on work announced in [1]. There, we state two general conjectures about the space of two-dimensional conformal field theories (CFTs). One is technical, that a CFT is uniquely determined by the spectrum of all operators and operator product coefficients below a critical dimension Delta which grows linearly with the central charge c. The other is that every CFT can be realized by RG flow from a linear sigma model.

We also define a distance function on the space of CFTs, which gives a distance between any pair of theories, whether or not they are connected by varying moduli.

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Nonabelian Mixed Hodge structures and applications

Ludmil Katzarkov

In this talk, we have considered a new approach to a classical question in algebraic geometry: the rationality of algebraic varieties, e.g. of a generic four-dimensional cubic. We introduce new birational invariants: the spectrum of the birational type of a smooth projective projective variety and its gaps. We compute this in the case of the two-dimensional and three-dimensional cubics.

We show that if X is rational, then the gap equals one. We conjecture that for the 4-dimensional cubic the gap is strictly bigger than one.

Conformal nets and local field theory

Arthur Bartels
(joint work with Christopher L. Douglas, André Henriques)

Atiyah defined topological quantum field theories as symmetric monoidal functors

$$Z\colon \mathrm{Bor}^n_{n-1} \to \mathrm{Vect},$$

where Bor^n_{n-1} is the category whose objects are $n-1$-dimensional manifolds and whose morphisms are n-dimensional bordisms and Vect is the category of Vectorspaces over a fixed field, often the complex numbers. The category Bor^n_{n-1} can be delooped to an n-category Bor^n_0 whose objects are 0-dimensional manifolds, 1-morphisms are 1-dimensional manifolds with boundary and in general k-morphisms are k-dimensional manifolds with corners. (The precise formalism of higher categories is highly non-trivial, but was ignored in my talk.) A local field theory is the a symmetric monoidal functor

$$Z\colon \mathrm{Bor}^n_0 \to \mathcal{C},$$

where $\mathcal{C}$ is a symmetric monoidal n-category. According to the cobordism hypothesis established by Lurie and Hopkins such functors are determined by their value on the one-point manifold pt. Moreover, for each fully dualizable object C of $\mathcal{C}$, there is Z such that $Z(\mathrm{pt}) = C$. (More precisely, Bor^n_0 has to be replaced by the n-category of framed manifolds.) Thus to construct a local field theory it suffices to construct a symmetric monoidal n-category together with a fully dualizable object.

For $n = 2$ a good example is the 2-category of von Neumann algebras. Its objects are von Neumann algebras, morphisms between von Neumann algebras M and N are M-N-bimodules and 2-morphisms are bounded M-N-linear operators. The composition of morphism uses the Connes fusion product and the identity 1-morphism on a von Neumann algebra M is the standard form $L^2(M)$.

Conformal nets grew out of algebraic quantum field theory. They can be described as functors that associate to any interval a von Neumann algebra and to each embedding of intervals an embedding of von Neumann algebras. In addition a number of axioms (additivity, Haag duality, split property, Vaccum sector) are required to hold. There is a good notion of dimension for such nets, the μ-index. (This notion is closely related to the Jones index for subfactors of von Neumann algebras.)

The main result of my talk was that the 2-category of von Neumann algebras can be delooped to a 3-category CN whose objects are conformal nets of finite μ-index, and all these nets are fully dualizable in CN.

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The RR Charge of an Orientifold

Gregory W. Moore
(joint work with Jacques Dister, Daniel S. Freed)

This talk reviewed one aspect of an ongoing project with J. Distler and D. Freed aimed at establishing some firm mathematical foundations for the theory of orientifolds. A telegraphic summary of the point of view we advocate may be found in [2].

The focus of this talk was on the definition and partial computation of the RR charge of an orientifold fixed plane. There are three motivations for this work. First, much of the evidence for the alleged "landscape of four-dimensional string vacua with fixed moduli and N=1 supersymmetry," makes important use of orientifold constructions. This important claim should be put on a more solid mathematical footing, especially since the crucial tadpole constraints determining consistency of the models have not been checked at the K-theoretic level. (Thus far the state of the art has only allowed checking the "image" of these constraints under the Chern character). A second motivation is that the tension between the strong-weak coupling dualities of string theory and the K-theoretic classification of RR charge is sharpest in the orientifold examples. A third motivation is that this topic provides an interesting venue for applications of modern topology to theoretical physics.

The question "What is the RR charge of an orientifold?" is a complicated one. Most of the talk was devoted to explaining what is meant by an "orientifold" and what is meant by "RR charge."

Perturbative string theory is a theory of maps $\varphi : \Sigma \to X$ where Σ is a smooth Riemannian worldsheet and the spacetime is a smooth orbifold (in the mathematical sense of the word, as used, e.g. in [1]). To define an orientifold string theory we provide the extra datum of a double cover $X_w \to X$ where $w \in H^1(X; \mathbb{Z}_2)$. By definition, a perturbative string theory *orientifold* is a theory of maps from an unoriented worldsheet Σ to X subject to the constraint that $\varphi^*(w) = w_1(\Sigma)$. For spacetimes of the form $X = Y//\Gamma$ where Γ is a discrete group the orientifold data is equivalent to a disjoint decomposition $\Gamma = \Gamma_0 \amalg \Gamma_1$ where Γ_0 is an index two subgroup. Components of a fixed-point locus in Y of an element $g \in \Gamma_1$ is known as an "orientifold plane."

When thinking about the RR field it turns out to be very important to understand the proper mathematical nature of the so-called "B-field" - the abelian 2-form gauge potential for which strings are an electrical source. The B-field in an oriented bosonic string theory is a geometrical object whose gauge equivalence class is a class in differential cohomology $\check{H}^3(X)$. For bosonic string orientifolds the B-field is valued in the twisted differential cohomology $\check{H}^{3+w}(X)$. Surprisingly, for type II superstrings, one must choose a slightly different generalized cohomology theory $\check{\mathbb{B}}^{3+w}(X)$ which fits in an exact sequence

$$0 \to \check{H}^{3+w}(X) \to \check{\mathbb{B}}^{3+w}(X) \to H^0(X;\mathbb{Z}) \times H^1(X;\mathbb{Z}_2) \to 0. \tag{0.1}$$

After modding out by Bott periodicity the relevant generalized cohomology theory can be identified with a Postnikov truncation of connective KO theory: $ko\langle 0 \dots 4\rangle$. The need to place the B-field in this theory can be seen both from spacetime and worldsheet viewpoints, and the agreement between them is highly nontrivial. This talk focussed on the spacetime viewpoint. When $X = \wp//\mathbb{Z}_2$ is the quotient of a point $\wp$ there is a $\mathbb{Z}_8$ group of "universal B-fields" after modding out by Bott periodicity.

Let us now turn to RR charge. Type II string theory has abelian gauge fields known as RR fields. Several physical arguments show that the sources of these fields – RR currents – have gauge equivalence class in the twisted differential KR theory of X_w. The B-field is a geometrical twisting of the differential KR theory (this is the spacetime argument for (0.1)). Since a general theory of twisted equivariant differential generalized cohomology theory is not available we described a specific model for the twisting and the twisted classes.

A crucial aspect of the RR field is that it is a self-dual theory. We described the general theory of abelian self-dual theories quantized by a Poincaré-Pontryagin selfdual generalized cohomology theory $\mathcal{E}^\tau$ (where τ is a twisting), and explained that the crucial ingredient for a theory on n-dimensional spacetimes is a choice of quadratic functor from families of $(n+2)$-dimensional spaces over S equipped with currents to the Anderson dual $\check{I}^0(S)$. The center of the corresponding quadratic functor defined on families of $(n+1)$-dimensional spaces of the form $X \times S^1$ defines the background charge $\mu \in \mathcal{E}^\tau(X)$. In the case of the RR field of type II string theory we consider families $\mathcal{Z}$ of 12-manifolds and define a quadratic functor via

$$q(\check{j}) = \left[\int_{\mathcal{Z}/S} \kappa \check{j}\bar{\check{j}}\right]_\epsilon \tag{0.2}$$

Here $\check{j}\bar{\check{j}}$ is understood to be lifted to a twisted KO theory $KO^{\mathcal{R}(\tau)}(X)$ where $\mathcal{R}(\tau)$ is a KO twist which, under complexification, becomes the KR twist $\tau + \bar{\tau}$ up to a shift by the twist of the Bott element. In order for the integration to be well-defined we must provide an isomorphism of KO-twistings to the orientation twisting $\tau_{KO}(X)$. Such an isomorphism is called a *twisted spin structure* and constitutes an essential piece of the data needed to define a type II string theory orientifold. Once this data has been provided there is an invertible element κ which makes the integrand of equation (0.2) a proper density for KO-theory. The integrand is then valued in $\check{KO}_{\mathbb{Z}_2}^{-12}(S)$ which can be mapped to the representation ring $R(\mathbb{Z}_2) \cong \mathbb{Z}[\epsilon]/(\epsilon^2 - 1)$. The subscript ϵ in (0.2) indicates that we take the component of the sign representation ϵ. Having defined q as in (0.2) it follows from [4] that the background charge of an orientifold has been defined.

In the case that $X = Y//\mathbb{Z}_2$ is a global quotient one can localize the integrals with the multiplicative set $\{(1-\epsilon)^n\} \subset R(\mathbb{Z}_2)$. The result shows that once the prime 2 is inverted the background charge of an orientifold is localized (spatially) on the orientifold planes, and moreover one can give an explicit formula for this K-theoretic charge in terms of the topology of the orientifold plane and its embedding into spacetime. A special case of this formula (for the type I theory, in which $X_w = X \times \wp//\mathbb{Z}_2$, and the B-field is zero), was derived some time ago, using similar methods, in [3]. Taking the Chern character one finds a well-known formula in the physics literature:

$$\sqrt{\hat{A}(TX)}\mathrm{ch}(\mu) = \pm 2^{k-5}\iota_* \sqrt{\frac{\tilde{L}(TF)}{\tilde{L}(\nu)}} \tag{0.3}$$

where F is a component of the fixed point locus (an "orientifold plane"), $k = \dim F$, $\tilde{L}(V) = \prod \frac{x_i/4}{\tanh x_i/4}$ is a modification of the Hirzebruch genus, and ν is the normal bundle to $\iota : F \hookrightarrow X_w$. The sign is tricky, but after a year of hard work, we believe we have it completely under control. It depends on k and the B-field.

The existence of a twisted spin structure puts a nontrivial topological constraint on orientifold models relating the topology of the B-field to that of spacetime. As an example, on spacetimes admitting a lift of the involution on X_w to the Pin^- bundle (we call these spacetimes with pinvolution) the codimension r modulo 4 of the orientifold planes is well-defined (when nonempty) and the 8 universal B-fields are constrained by the codimension according to the table:

$r=0$	$KR^0(X_w)$	$KR^{\beta_2}(X_w)$
$r=1$	$KR^{1+\beta_1}(X_w)$	$KR^{1+\beta_3}(X_w)$
$r=2$	$KR^{\beta_1}(X_w)$	$KR^{\beta_3}(X_w)$
$r=3$	$KR^1(X_w)$	$KR^{1+\beta_2}(X_w)$

Here the 8 universal B-fields are presented as elements of $(\mathbb{Z} \oplus \mathbb{Z}_4)/\langle(2,1)\rangle$ via $d + \beta_\ell$ with $\ell \in \mathbb{Z}_4$.

Finally, some further consequences of this viewpoint, and some directions for future research were outlined.

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Twisted K-theory, branes, Verlinde algebras and related topics

Igor Kriz

In this talk, I described an investigation which started by looking at

$$K^*_{G,\tau}(G), \tag{0.1}$$

the equivariant twisted K-theory of a compact Lie group G acting on G by conjugation. In the famous papers [2], Freed, Hopkins and Teleman computed this group for non-degenerate twistings coming from a class in $H^4(BG, \mathbb{Z})$. In the case when G is connected and its fundamental group is free abelian, (0.1) can be identified canonically with the Verlinde algebra, which is, roughly speaking the representation ring of the rational vertex algebra obtained as a quotient of the affine vertex algebra associated with the Lie algebraa g by the maximal ideal, or the chiral WZW model. (Note: by new characterizations of vertex algebras and vertex tensor categories of their representations [3, 8], these structures can now be viewed as algebraic objects, i.e. can be considered for example over any field of characteristic 0 - in the case of vertex algebras, restriction to characteristic 0 is not even needed.) This connection between (0.1) and vertex algebras can be extended to the case of arbitrary G connected by the orbifold construction on vertex algebras. However, for the case of G not necessarily connected, the vertex algebra side is not canonically defined. For example, for G finite, there is no non-degeneracy restriction on the twisting. When the twisting is 0, (0.1) is actually the representation ring of the Drinfeld double of G; it is known [1] to be the Verlinde algebra of the orbifold vertex algebra of a holomorphic vertex algebra under a group action where all irreducible representations of G occur in the action.

In general, (0.1) is conjectured to be the Verlinde algebra of a *modular functor*. A modular functor in our sense is a holomorphic $1 + 1$-dimensional field theory valued in (finite-dimensional strongly dualizable) complex (super-) 2-vector spaces. (Note: when the central charge is 0, we have a topological field theory, although the modular functors corresponding to (0.1) tend to have non-integral rational central charge.)

Now it is not completely surprising that (0.1) should come from a modular functor: a modular functor can be always realized as a 1+1-dimensional topological field theory valued in strict modules over the E_∞ ring spectrum K. (A converse is not obvious; in fact, the K-theory realization seems to lose information about the central charge.) In any case, (0.1) can, rather than a group, be viewed as a strict K-module. Some of the operations of a topological field theory are in fact constructed in [2]. Concretely, the unit and product have completely topological constructions, and this can be extended by the same method to an action of the framed little 2-disk operad ([6]). In fact, it can be shown that these operations are compatible with the same operations in "string K-theory", which were considered in [10]. However, operations such as the augmentation or coproduct are more mysterious, and may contain information coming from outside of homotopy theory: certainly, these operations do *not* coincide with the corresponding operations in string K-theory: string K-theory (like all string topology) has no augmentation at all, and the coproduct is usually very nearly 0 ([10]).

The connection between (0.1) and string topology deserves a couple more comments. It is based on the equivalence

$$G \times_G EG \simeq LBG \tag{0.2}$$

where the left hand side is the Borel construction. On the right hand side, L denotes free loop space; BG is not a manifold, but behaves, in certain ways, like a manifold of dimension $-dimG$; this relationship is explained by inclusion of certain manifolds in BG, and duality ([10]). Applying the Borel construction to twisted K-theory results in a completion; these completions for G simple simply connected were computed explicitly in [7].

More can be said on the subject of completion. For example, M.Khorami [5] proved a beautiful theorem that

$$K_{\tau,*}X = K_*\widetilde{X} \otimes_{K_*\mathbb{C}P^\infty} K_*$$

where $\widetilde{X}$ is the principal $\mathbb{C}P^\infty$-bundle associated with the twisting. This can be pushed even further by considering, instead of $\widetilde{X}$, the S^1-gerbe corresponding to the twisting. In the case of (0.1), the twisting corresponds to a G-equivariant S^1-gerbe, to which there canonically corresponds a groupoid Γ. Using the methods of [2], one can prove [6] (at least when G is connected and $\pi_1(G)$ is free abelian, and the twisting is non-degenerate) that

$$K^*(\Gamma) \cong R(\widetilde{L}_\tau G) \tag{0.3}$$

where the right hand side denotes the representation ring of finite sums of lowest weight irreducible representation of the central extension of LG corresponding to the twisting. (Note: the correspondence of K-theory and representation twistings contains the dual Coxeter number; the representation ring considered here contains as a part of the information the action by bodily rotation of the loop.) Now applying a completion theorem to (0.3), one obtains a completion theorem for affine groups:

$$K^*B\widetilde{L}_\tau G \cong R(\widetilde{L}_\tau G)^\wedge_I$$

where I is the augmentation ideal in $R(S^1 \times G)$ ($R(\widetilde{L}_\tau G)$ has no appropriate augmentation).

Finally, a remark on the interpretation of (0.1) in terms of branes. The modular functor of a rational vertex algebra always has an interpretation as the algebra of Cardy brane charges. One mathematical rigorization of that statement was attempted in [4]. However, in the case of the WZW model, the correct algebra of charges is not (0.1), but the non-equivariant twisted K-theory group $K^*_\tau(G)$. G.Moore [11] clarified that these charges correspond to identification of branes using renormalization flow on the boundary CFT within a space of quantum field theories. The correspondence between modular functors and K-module valued field theories suggests yet another interpretation: one has, as K-modules,

$$K^*_\tau(G) = K^*_{\tau,G}(G) \wedge_{K^*_G(G)} K^*. \tag{0.4}$$

This can be interpreted as a two-sided bar construction

$$B(K^*_{\tau,G}(G), K^*_G(G), K^*) \tag{0.5}$$

in the category of K-modules. However, it turns out that (0.5) can be rigidified to a 2-sided bar construction of 2-vector spaces, and in fact chiral conformal field theories! This offers a mathematical formalism encoding the renormalization group flow in the simplicial coordinate. The author is particularly interested in the question whether there is an analogue of this construction for CFT backgrounds; in [9], the author discovered indications that certain deformations of $N = (2, 2)$ CFT's along marginal fields do not exponentiate in the framework of perturbative CFT, contrary to what is believed in physics. It would interesting to see if this phenomenon can be explained in terms of "simplicial" or "derived" background deformations.

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Gauged Topological Quantum Field Theories in 2 dimensions

Constantin Teleman

In this lecture, I review the notion of 'extended' topological quantum field theory in 2 dimensions, after Kontsevich, Costello, Hopkins-Lurie, and explain what it means to gauge such a theory for the action of a compact Lie group. When the underlying category is that of modules over a differential graded algebra, I present a concrete model for the generating category of the gauged TQFT, in terms of a 'curved Cartan model'.

TQFT's, as originally define by Atiyah, Segal and Witten, assign algebraic data to manifolds with structure (such as orientation) in adjacent dimensions. Thus, an n-dimensional TQFT Z assigns vector spaces $Z(M)$ to closed $(n-1)$-manifolds M and vectors $Z(N) \in Z(\partial N)$ to compact n-manifolds N with boundary. The assignment is 'symmetric monoidal', meaning it is multiplicative under disjoint unions, and satifies a 'sewgluinging condition' when two n-manifolds are glued along a connected component of their boundary. (This is sometimes rephrased by saying that Z is a symmetric monidal functor from the n-dimensional bordism category to that of vector spaces and linear maps.)

The idea behind extended TQFT's is to continue this assignment downwards in dimension, all the way to 0. Thus, an extended TQFT would be a functor from the n-dimensional bordism category of manifolds (with some structure on the tangent bundle) to some linear n-category; for $n = 2$, a target example could be the 2-category of linear differential graded categories, for instance, that of modules over a differential graded algebras. In this dimension, a characterization of the categories (Frobenius categories) that can appear as Z of a point was given (in slightly different versions) by Kontsevich, Costello, Hopkins-Lurie, who also showed that the full TQFT is determined by the target category. (Lurie then generalized this to arbitrary dimensions, calling this the 'cobordism hypothesis'.)

An example of 'structure' on a manifold is principal G-bundle, for a compact Lie group G. A TQFT defined on such manifolds is the mathematical definition of the physicist's 'classically gauged' TQFT (the principal bundle is a 'background gauge field'; in the topological case, it is not necessary to choose connections on the bundles). Quantizing the theory means integrating over gauge fields.

Given a category C generating a 2-dimensional TQFT, a classically gauged theory should arise whenever a Lie group G acts, in a suitable sense, on the category C. For finite groups, the obvious definition of an action is adequate, but there are compatibility constraints with the Frobenius structure. In this case, it is also quite easy to quantize the gauged theory, by summing over isomorphism classes of G-bundles. The quantized theory is in turn generated by a category, which is just the G-fixed point category C^G.

This talk will describe how the construction must be modified in the case when G is a compact Lie group. In this case, the appropriate notion of a G-action is one that comes with an infinitesimal trivialization, that is, one which has been factored through $G/\hat{G}$, where $\hat{G} \subset G$ is the formal subgroup. When C is the category of (dg) modules over a (differential graded) algebra A, the fixed-point category $C^{G/\hat{G}}$ is that of modules over some realization of the crossed product algebra $(G/\hat{G})\widetilde{\ltimes}A$. One version of this crossed product uses the algebra of (de Rham) chains on G, under convolution. It turns out that a preferable version is a Koszul dual model, in which the exterior algebra $\wedge\,\mathfrak{g}$ is replaced by the symmetric algebra $\mathrm{Sym}\,\mathfrak{g}^*$. We get an algebra version of the Cartan model for equivariant cohomology, $G\widetilde{\ltimes}\,(\mathrm{Sym}\,\mathfrak{g}^*\otimes A)$. A key feature of this model is the appearance of a *curvature*, that is, we obtain a curved algebra, where $d^2\neq 0$, but is a commutator.

When G is connected $A=\mathbb{C}$, the category of curved modules over this algebra is equivalent to that of graded modules over $(\mathrm{Sym}\,\mathfrak{g}^*)^G$ (or its completion at zero), which also has a topological interpretation as $H^*(BG)$. This is category is not quite Frobenius, because $\mathfrak{g}$ is not compact. An additional curvature (LAndau-Ginzburg superpotential) is necessary to 'compactify' the gauged theory and obtain a Frobenius category. The simplest compactifying potential is a non-degenerate, invariant quadratic function on the Lie algebra. In the case when $A=\mathbb{C}$, this semi-simplifies the category of $H^*(BG)$-modules into that of vector bundles over the dominant, integral, regular weights of $\mathfrak{g}$, and generates Witten's topological Yang-Mills theory.

Cohomology of the moduli space of bundles: from Atiyah-Bott to Tamagawa number

Dennis Gaitsgory
(joint work with Jacob Lurie)

Let X be a curve over a ground field k and G a split semi-simple simply connected group over k. Let Bun_G denote the moduli stack of G-bundles on X.

Assume first that the field k is a finite field $\mathbb{F}_q$. Although the set $\mathrm{Bun}_G(\mathbb{F}_q)$ is infinite, it carries a natural atomic measure (each point comes with a weight equal to the inverse of the cardinality of the group of its automorphisms). The volume of $\mathrm{Bun}_G(\mathbb{F}_q)$ with respect to this measure is a convergent series and the Tamagawa number formula (proved in this case by Harder) asserts that the volume of $\mathrm{Bun}_G(\mathbb{F}_q)$ equals

$$(*)\qquad q^{(g-1)\dim(G)}\cdot\prod_{X\in|X|}L_x,$$

where g is the genus of X, and L_x is a local factor equal to

$$\prod_{i=1,\dots,l}\frac{1}{1-q_x^{-e_i}},$$

where i runs through the set of exponents of G, e_i is the value of the corresponding exponent, and q_x is the cardinality of the residue field at a closed point $x\in X$. Thus, the product $\prod\limits_{X\in|X|}L_x$ can be viewed as a special value of a certain L-function. It is easy to see that the expression in (*) is the ratio between the atomic measure on $\mathrm{Bun}_G(\mathbb{F}_q)$ mentioned above and the canonical Tamagawa measure.

Let $\mathfrak{a}$ be the grade vector space (acted on by the Frobenius) equal to

$$\underset{i=1,\ldots,l}{\oplus} \mathbb{Q}_\ell[-2e_i](-e_i).$$

We have $H(BG, \mathbb{Q}_\ell) \simeq \mathrm{Sym}(\mathfrak{a})$, where BG the classifying space of G. Therefore, the factor L_x equals $\mathrm{Tr}(\mathrm{Frob}_x, H(BG, \mathbb{Q}_\ell)^\vee)$.

The goal of our project is to give a geometric proof of formula (*). The first step, carried out by Behrend, is to give a cohomological interpretation to $\mathrm{vol}(\mathrm{Bun}_G(\mathbb{F}_q))$. Namely, we consider $H_c(\mathrm{Bun}_G, \mathbb{Q}_\ell)$, and one shows that although Bun_G is of infinite type, the Grothendieck-Lefschetz trace formula is applicable, i.e., we have:

$$(**) \qquad \mathrm{vol}(\mathrm{Bun}_G(\mathbb{F}_q)) = \mathrm{Tr}(\mathrm{Frob}, H_c(\mathrm{Bun}_G, \mathbb{Q}_\ell))$$

(in particular, the right-hand side makes sense as an absolutely convergent series with vaues in $\mathbb{R}$). Let ω_{Bun_G} denote the dualizing sheaf of Bun_G. We obtain that formula (*) is equivalent to the equality

$$(***) \qquad \mathrm{Tr}(\mathrm{Frob}, H_c(\mathrm{Bun}_G, \omega_{\mathrm{Bun}_G})) = \prod_{X\in|X|} \mathrm{Tr}(\mathrm{Frob}_x, H(BG, \mathbb{Q}_\ell)^\vee).$$

The idea of the geometric proof of (***) relies on the notion of *factorization algebra*. Let $\mathrm{Ran}(X)$ denote the Ran space of X, as defined by Beilinson and Drineld. This is the space of finite non-empty collections of points in X. Although $\mathrm{Ran}(X)$ doesn't have a structure of ind-scheme, one can still consider the stable ∞-category of ind-ℓ-adic sheaves on it, which we denote by $D(\mathrm{Ran}(X))$. We also have a functor of global cohomology

$$H(\mathrm{Ran}(X), -) : D(\mathrm{Ran}(X)) \to D(\mathrm{Vect}).$$

For a point $x \in X$, we have a natural direct image functor

$$\iota_{x!} : D(\mathrm{Vect}) \to D(\mathrm{Ran}(X));$$

it admits left and right adjoints denoted ι_x^* and $\iota_x^!$, respectively.

We have a natural map

$$\mathrm{add} : \mathrm{Ran}(X) \times \mathrm{Ran}(X) \to \mathrm{Ran}(X),$$

given by the operation of union of subsets of X. By definition, a factorization algebra on X is an object $\mathcal{A} \in D(\mathrm{Ran}(X))$ equipped with an isomorphism

$$\mathrm{add}^*(\mathcal{A}) \simeq \mathcal{A} \boxtimes \mathcal{A}|_{(\mathrm{Ran}(X)\times\mathrm{Ran}(X))_{disj}},$$

where

$$(\mathrm{Ran}(X) \times \mathrm{Ran}(X))_{disj} \subset \mathrm{Ran}(X) \times \mathrm{Ran}(X)$$

is the open subset corresponding to pairs of disjoint collections of points of X.

Under certain explicit technical assumptions on an augmented factorization algebra $\mathcal{A}$, one can establish the following version of the Grothendieck-Lefschetz formula:

The expression

$$\mathrm{Tr}(\mathrm{Frob}, H(\mathrm{Ran}(X), \mathcal{A}))$$

makes sense (i.e., is given by an absolutely convergent series) and equals the (absolutely convergent) product

$$\prod_{x\in|X|} \mathrm{Tr}(\mathrm{Frob}_x, \iota_x^*(\mathcal{A})).$$

Thus, to prove (***) we would like to find a factorization algebra $\mathcal{A}$ with the following two properties:

(I) $\iota_x^*(\mathcal{A}) \simeq H(BG, \mathbb{Q}_\ell)^\vee$.

(II) $H_c(\mathrm{Bun}_G, \omega_{\mathrm{Bun}_G}) \simeq H(\mathrm{Ran}(X), \mathcal{A})$.

Note that both these properties are geometric in the sense that they make sense over an arbitrary ground field k. It is easy to see that the algebra satisfying (I) exists and is essentially unique. For this algebra, the cohomology $H(\mathrm{Ran}(X), \mathcal{A})$ can be easily computed to be isomorphic to

$$\mathrm{Sym}(\mathfrak{a}^* \otimes H(X, \mathbb{Q}_\ell)),$$

which coincides with the Atiyah-Bott formula for $H_c(\mathrm{Bun}_G, \omega_{\mathrm{Bun}_G})$ (i.e., the homology of Bun_G). Thus, what we want is essentially to reprove the Atiyah-Bott formula in a way that would work over an arbitrary ground field.

Let Gr_G be the affine Grassmannian of G, and let $\mathrm{Gr}_{G,\mathrm{Ran}}$ be its Ran version. We have the natural maps

$$\mathrm{Bun}_G \xleftarrow{p} \mathrm{Gr}_{G,\mathrm{Ran}} \xrightarrow{\pi} \mathrm{Ran}(X).$$

Our main geometric result is the following:

Theorem. *The natural map*

$$p_!(\omega_{\mathrm{Gr}_{G,\mathrm{Ran}}}) \to \omega_{\mathrm{Bun}_G}$$

is an isomorphism.

One could reformulate this theorem by saying that the fibers of the map p are contractible.

Thus, we obtain isomorphisms:

$$H_c(\mathrm{Bun}_G, \omega_{\mathrm{Bun}_G}) \simeq H_c(\mathrm{Bun}_G, p_!(\omega_{\mathrm{Gr}_{G,\mathrm{Ran}}})) \simeq$$
$$\simeq H_c(\mathrm{Gr}_{G,\mathrm{Ran}}, \omega_{\mathrm{Gr}_{G,\mathrm{Ran}}}) \simeq H(\mathrm{Ran}(X), \pi_!(\omega_{\mathrm{Gr}_{G,\mathrm{Ran}}})).$$

We conclude the argument by the proving the isomorphism of factorization algebras

$$\pi_!(\omega_{\mathrm{Gr}_{G,\mathrm{Ran}}}) \simeq \mathcal{A},$$

which is a well-known result in topology.

Hochschild homology of structured algebras and TCFT's

Nathalie Wahl
(joint work with Craig Westerland)

Let $\mathcal{A}_\infty$ be the linear monoidal category with objects the natural numbers and with the property that symmetric monoidal functors $\phi : \mathcal{A}_\infty \to Comp$ correspond exactly to A_∞-algebra structures on $\phi(1)$.

Given a monoidal functor $i : \mathcal{A}_\infty \to \mathcal{E}$, for $\mathcal{E}$ some linear category, we define the Hochschild complex C as an operator on functors $\phi : \mathcal{E} \to Comp$, with the property that, for a symmetric monoidal functor ϕ, its Hochschild complex $C(\phi)$ evaluated at 0 is the usual Hochschild complex of the $\mathcal{A}_\infty$-algebra $\phi(1)$. Our main theorem says that, if the iterated Hochschild complexes of the representable functors $\mathcal{E}(e, \_)$ admit an action of some category $\mathcal{D}$

$$C^n(\mathcal{E}(e, \_)) \otimes \mathcal{D}(n, m) \longrightarrow C^m(\mathcal{E}(e, \_))$$

naturally in e, then the Hochschil complex $C(\phi)(0)$ of any monoidal functor $\phi : \mathcal{E} \to Comp$ is a homotopy $\mathcal{D}$-module, that is we have maps

$$C(\phi)(0)^{\otimes n} \otimes \mathcal{D}(n, m) \longrightarrow C(\phi)(0)^{\otimes m}$$

satisfying the coherences of an action up to homotopy. Moreover, this action is natural with respect to maps $\mathcal{E} \to \mathcal{E}'$ and $\mathcal{D} \to \mathcal{D}'$.

Applied to $\mathcal{E} = \mathcal{O}$, the open string cobordism category, this recovers a theorem of Costello [1] and Kontsevich-Soibelmann [2] saying that the homology of the moduli spaces of surfaces acts on the Hochschild homology of "A_∞-Frobenius algebras". Applied to $\mathcal{E} = H_0(\mathcal{O})$, we get an action of the homology of compactified Sullivan diagrams on the Hochschild homology of strict Frobenius algebras, recovering a result of Tradler-Zeinalian [3]. Our naturality statement says that, if an A_∞-Frobenius algebra happens to be strict, then the action of the moduli space of Riemann surfaces on its Hochschild homology factors through an action of compactified Sullivan diagram. Over the rationals, this can be used to explain the triviality of many operations in string topology.

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3-dimensional Calabi-Yau categories and their Donaldson-Thomas invariants

Yan Soibelman
(joint work with Maxim Kontsevich)

My talk was devoted to motivic Donaldson-Thomas invariants introduced in a series of papers joint with Maxim Kontsevich (see [1, 2, 3]). Basic idea is the following. A $3CY$ category is a triangulated A_∞-category $\mathcal{C}$ endowed with the Serre pairing $Hom(E,F) \otimes Hom(F,E) \to \mathbf{C}[-3]$ (we assume for simplicity that the ground field is $\mathbf{C}$). Then one can define a potential, which is (roughly speaking) a function $W(E,\alpha)$, which is locally regular along the locus of objects $E \in Ob(\mathcal{C})$ and formal with respect to $\alpha \in Ext^1(E,E)$. These properties of W require some assumptions on the stack of objects of $\mathcal{C}$, e.g. it is an ind-Artin stack. Using properly defined cohomology of the ind-Artin stack of objects with coefficients in the sheaf of vanishing cycles of W, we introduced the so-called *cohomological Hall algebra* (COHA), which is graded by the K-theoretical lattice. Graded components of COHA can be thought of as elements of the tensor category EMHS of *exponential mixed Hodge structures*. Hence they define elements of the commutative ring, the K_0-ring of the category EMHS. The generating series of the K_0-classes of components of COHA with values in an appropriate quantum torus is called *motivic DT-series* of the category $\mathcal{C}$.

After a choice of stability structure, motivic DT-series factorizes into a product over all possible slopes of the central charge. Each slope factor is defined in terms of the equivariant cohomology of the stack of semistable objects of this slope. This allows us to introduce numerical DT-invariants, both ordinary and refined.

Motivic and numerical DT-invariants enjoy many nice properties, e.g. they are integers. Conjecturally motivic DT-invariants give in the "classical limit" generalized DT-inavariants of Joyce and Song. We proved a general wall-crossing formula, which shows how the invariants jump as we cross a real codimension one "wall of marginal stability" in the space of stability conditions. One application of our results is a new "motivic" invariant of 3-dimensional manifolds, given in terms of Chern-Simons theory with complex group. Our results can be thought of as a mathematically rigorous treatment of the notions of BPS invariants and refined BPS invariants in quantum phyiscs.

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A Filtration of Open/Closed Topological Field Theory

Ezra Getzler

A well-known theorem states that there is an equivalence between two-dimensional topological field theories and commutative Frobenius algebras. This theorem, and its equivariant generalization (Turaev [14]), are implied by a result contained in an appendix to a paper of Hatcher and Thurston [4]. Using Morse theory, they prove that any pair of handle decompositions of a surface S is joined by a sequence of moves of two types, respectively associated to embeddings in S of a surface of genus 0 with four holes, and of a surface of genus 1 with one hole.

Moore and Seiberg [11] categorify this result: they give a presentation of modular functors, that is, two-dimensional field theories taking values in a symmetric monoidal bicategory. They prove that on adjoining 2-cells associated to open embeddings in S of a surface of genus 0 with five holes, and of genus 1 with two holes, to Hatcher and Thurston's graph, one obtains a simply connected 2-dimensional cell complex.

In this talk, we present a point of view on these results which allows us to obtain higher categorifications of them, as well as generalizing them to the open/closed setting (along with orientifolds).

The genus zero sector of open topological field theory is equivalent to the theory of A_∞-algebras. There are two well-known formalisms for these. One approach, due to Segal [12], amounts to representing them as weak monoidal functors from the category of totally ordered finite sets to a homotopy category, such that the coherence morphisms

$$\Phi_{\mathbf{m},\mathbf{n}} : F(\mathbf{m}+\mathbf{n}) \to F(\mathbf{m}) \otimes F(\mathbf{n})$$

are homotopy equivalences. A second approach, due to Stasheff [13], represents them as algebras for an operad A_∞ whose space $A_\infty(n)$ of n-fold operations is a polyhedron of dimension $n-2$.

The filtration of these polyhedra by the dimension of their faces yields a filtration

$$F_0 A_\infty \subset F_1 A_\infty \subset \dots$$

of the operad A_∞ such that the inclusion $F_k A_\infty \hookrightarrow A_\infty$ is k-connected; that is, A_∞ is obtained from $F_k A_\infty$ by gluing cells of dimension greater than k. It is this picture that we wish to generalize to topological field theory.

We start with the closed case. Let S be a compact oriented surface with marked points $\mathbf{z}$ such that $\chi(S \setminus \mathbf{z}) < 0$ and each component of S contains at least one point of $\mathbf{z}$, let $\mathcal{M}(S,\mathbf{z})$ be the moduli space of hyperbolic metrics on S with cusps at the marked point, and let $\widehat{\mathcal{M}}(S,\mathbf{z})$ be its Harvey compactification. This moduli space is a real analytic manifold (or rather, orbifold) with corners, homotopy equivalent to $\mathcal{M}(S,\mathbf{z})$, and hence a classifying stack for the mapping class group $\Gamma(S,\mathbf{z}) = \pi_0(\mathrm{Diff}_+(S,\mathbf{z}))$. If S is connected of genus g and $|\mathbf{z}| = n$, write $\widehat{\mathcal{M}}_{g,n}$ for $\widehat{\mathcal{M}}(S,\mathbf{z})$. Define the **closed signature** $\alpha(S,\mathbf{z}) \geq 0$ of the surface $(S,\mathbf{z})$ by the formula

$$\alpha(S,\mathbf{z}) = \begin{cases} n-3, & g=0, \\ 2g-2+n, & g>0, \end{cases}$$

if S is connected, and as a sum over the components of S, in general.

A **configuration of curves** in $(S,\mathbf{z})$ is a disjoint collection of closed embedded curves in $S \setminus \mathbf{z}$, defined up to isotopy, cutting it into components of negative Euler characteristic. Configurations of curves form a partially ordered set $\mathcal{C}(S,\mathbf{z})$, ordered by inclusion, whose maximal elements are the generalized pants decompositions, which decompose $S \setminus \mathbf{z}$ into pieces of Euler characteristic -1. Pairs consisting of a configuration of curves together with a choice of generalized pants decomposition refining it form a partially set $\widetilde{\mathcal{C}}(S,\mathbf{z})$. There is a natural action of the mapping class group on the partially ordered sets $\mathcal{C}(S,\mathbf{z})$ and $\widetilde{\mathcal{C}}(S,\mathbf{z})$.

The orbifold $\widehat{\mathcal{M}}(S,\mathbf{z})$ has an atlas whose charts are indexed by elements $\mathbf{c}$ of the partially ordered set $\widetilde{\mathcal{C}}(S,\mathbf{z})/\Gamma(S,\mathbf{z})$: the coordinates of the chart are the length and angle coordinates associated to the generalized pants decomposition, while the curves in the configuration indicate those length coordinates which are permitted to go to 0 in the chart. The corner in such a chart in which all of the length coordinates vanish is a stratum of $\widehat{\mathcal{M}}(S,\mathbf{z})$, denoted by $\widehat{\mathcal{M}}(S,\mathbf{z},\mathbf{c})$. If S is connected, $\widehat{\mathcal{M}}(S,\mathbf{z})$ has dimension $6g-6+2n$, and the dimensions of its strata lie between $3g-3+n$ and $6g-6+2n$.

The moduli space $\widehat{\mathcal{P}}(S,\mathbf{z})$ is defined in a similar way to $\widehat{\mathcal{M}}(S,\mathbf{z})$, but with n additional coordinates, the angle parameters at the marked points: $\widehat{\mathcal{P}}(S,\mathbf{z})$ carries a free torus action which rotates these angles, whose quotient is the orbifold $\widehat{\mathcal{M}}(S,\mathbf{z})$. In particular, $\dim \widehat{\mathcal{P}}(S,\mathbf{z}) = 6g-6+3n$, and its strata, which are torus bundles over the strata of $\widehat{\mathcal{M}}(S,\mathbf{z})$, have dimension between $3g-3+2n$ and $6g-6+3n$. The manifolds with corners $\widehat{\mathcal{P}}(S,\mathbf{z})$ assemble to form a modular operad (Getzler and Kapranov [2], Kimura, Stasheff and Voronov [5]) in the $U(1)$-equivariant category.

Definition. A two-dimensional field theory is an $U(1)$-equivariant algebra for $\widehat{\mathcal{P}}$.

The modular operad $\widehat{\mathcal{P}}$ has a filtration

$$F_0\widehat{\mathcal{P}} \subset F_1\widehat{\mathcal{P}} \subset F_2\widehat{\mathcal{P}} \subset \dots$$

Let $\widehat{\mathcal{P}}(S,\mathbf{z},\mathbf{c})$ be a stratum of $\widehat{\mathcal{P}}(S,\mathbf{z})$. Cutting along the curves of the configuration $\mathbf{c}$ and contracting the new boundary components to marked points, we obtain a possibly disconnected surface $(S[\mathbf{c}],\mathbf{z}[\mathbf{c}])$; let $\alpha(S,z,\mathbf{c})$ equal $\alpha(S[\mathbf{c}],\mathbf{z}[\mathbf{c}])$. The filtrand $F_k\widehat{\mathcal{P}}(S,\mathbf{z})$ is a union of those strata $\widehat{\mathcal{P}}(S,\mathbf{z},\mathbf{c})$ such that $\alpha(S,\mathbf{z},\mathbf{c}) \le k$. Equivalently, $F_k\widehat{\mathcal{P}}(S,\mathbf{z})$ is the union of strata $\widehat{\mathcal{P}}(S,\mathbf{z},\mathbf{c})$ for which no fewer than $2g+n-2-k$ components of $S[\mathbf{c}]$ have genus 0. For example, $F_0\widehat{\mathcal{P}}(S,\mathbf{z})$ is a union of tori each of which is labelled by a generalized pants decomposition of $S \setminus \mathbf{z}$.

Theorem. *The inclusion $F_k\widehat{\mathcal{P}} \hookrightarrow \widehat{\mathcal{P}}$ is k-connected.*

For $k = 0$, the theorem says that $\pi_0(F_0\widehat{\mathcal{P}}(S,\mathbf{z})) \to \pi_0(\widehat{\mathcal{P}}(S,\mathbf{z}))$ is surjective, in other words, every surface with marked points has a generalized pants decomposition. Of course, this property was already used in the construction of the Harvey compactification.

For $k > 0$, the theorem implies that $\pi_0(F_k\widehat{\mathcal{P}}(S,\mathbf{z})) \cong \pi_0(\widehat{\mathcal{P}}(S,\mathbf{z}))$, and thus that $F_k\widehat{\mathcal{P}}(S,\mathbf{z})$ is connected. For $k = 1$, this is precisely the result of Harer and Thurston which we mentioned in the first paragraph.

Finally, the theorem says that for any choice of basepoint $x \in F_k\widehat{\mathcal{P}}(S,\mathbf{z})$, the morphism

$$\pi_i(F_k\widehat{\mathcal{P}}(S,\mathbf{z}),x) \to \pi_i(\widehat{\mathcal{P}}(S,\mathbf{z}),x)$$

is an isomorphism for $0 < i < k$, and surjective for $i = k$. In particular, $\pi_1(F_2\widehat{\mathcal{P}}(S,\mathbf{z}),x)$ is isomorphic to the mapping class group $\Gamma_{g,n} = \pi_1(\widehat{\mathcal{P}}(S,\mathbf{z}))$: this is the main result of Moore and Seiberg. More generally, it follows that two-dimensional topological field theories in a symmetric monoidal k-category are the same thing as $U(1)$-equivariant algebras for the modular suboperad $F_k\widehat{\mathcal{P}}$: this is a k-categorification of the original theorem of Moore and Seiberg.

The theorem is proved in the following steps (cf. Mondello [9]).

(1) It is a theorem of Harer [3] that if $g > 0$, the pair $(\widehat{\mathcal{M}}_{g,1}, \partial\widehat{\mathcal{M}}_{g,1})$ is a relative cellular complex with no cells in dimension $< 2g - 1$.
(2) By induction on n, one sees that if $g > 0$, $(\widehat{\mathcal{M}}_{g,n}, \partial\widehat{\mathcal{M}}_{g,n})$ is a relative cellular complex with no cells in dimension $< 2g - 2 + n$.
(3) On the other hand, $\widehat{\mathcal{M}}_{0,3}$ is a point, so the analogous induction shows that $(\widehat{\mathcal{M}}_{0,n}, \partial\widehat{\mathcal{M}}_{0,n})$ is a relative cellular complex with no cells in dimension $< n - 3$.

There is a generalization of this result to the open/closed setting, or, as we prefer to think of it, in the **real** category: this is Atiyah's term for the category of spaces (X,σ) with involution. The homotopy type is now replaced by the real homotopy type, made up of the homotopy types of the space X and its fixed-point set X^σ.

Thus, we now consider surfaces S with an orientation-reversing involution σ which preserves the set of marked points $\mathbf{z}$: the world-sheet of open/closed topological field theory is identified with the quotient of S by the action of σ, and the fixed-point set σ is identified with its boundary. Points of $\mathbf{z}$ invariant under the involution correspond to points on the boundary of the world sheet, while the free orbits correspond to points in its interior. It is important to allow the surface S to be disconnected, in order to permit the inclusion of the purely closed sector in the open/closed theory.

Denote the number of components of S^σ by h; let h_0 be the number of these components which contain no points of $\mathbf{z}^\sigma$, and let m be the number of points in $\mathbf{z}^\sigma$. Define the **open signature** $0 \leq \beta(S,\mathbf{z},\sigma) \leq \alpha(S,\mathbf{z})$ of the surface $(S,\mathbf{z},\sigma)$ by the formula

$$\beta(S,\mathbf{z},\sigma) = \begin{cases} m-3, & g=0,\ h=1,\ m=n, \\ m+2h_0-1, & g=0,\ h=1,\ m<n, \\ 1, & g=0,\ h=0, \text{ and } \sigma \text{ preserves } S, \\ m+2h_0-2, & g=1,\ h=2, \\ m+2h_0, & \text{otherwise,} \end{cases}$$

if S is connected, and as a sum over the components of S in general. In terms of the associated open/closed worldsheet, the exceptional cases correspond to a disk with no marked points in the interior, a disk with at least one marked point in the interior, a real projective plane $\mathbb{RP}^2$, and a cylinder.

The involution σ on S induces an involution σ on the moduli spaces $\widehat{\mathcal{M}}(S,\mathbf{z})$ and $\widehat{\mathcal{P}}(S,\mathbf{z})$; denote the resulting real spaces by $\widehat{\mathcal{M}}(S,\mathbf{z},\sigma)$ and $\widehat{\mathcal{P}}(S,\mathbf{z},\sigma)$. In this way, we obtain the real, $U(1)$-equivariant, modular operad $\widehat{\mathcal{P}}$. (Here, $U(1)$ has its usual real structure, induced by complex conjugation.) Real $U(1)$-equivariant algebras for this modular operad are open/closed unoriented topological field theories; a modification of this yields a definition of open/closed oriented topological field theories.

In the presence of the involution σ, the filtration $F_k\widehat{\mathcal{P}}(S,\mathbf{z})$ has a refinement

$$F_{k,0}\widehat{\mathcal{P}}(S,\mathbf{z},\sigma) \subset F_{k,1}\widehat{\mathcal{P}}(S,\mathbf{z},\sigma) \subset \cdots \subset F_{k,k}\widehat{\mathcal{P}}(S,\mathbf{z},\sigma).$$

The filtrand $F_{k,\ell}\widehat{\mathcal{P}}(S,\mathbf{z},\sigma)$ is the union of those strata $\widehat{\mathcal{P}}(S,\mathbf{z},\mathbf{c})$ of $\widehat{\mathcal{P}}(S,\mathbf{z},\sigma)$ such that $\alpha(S,\mathbf{z},\mathbf{c}) < k$, or $\alpha(S,\mathbf{z},\mathbf{c}) = k$ and $\beta(S,\mathbf{z},\mathbf{c}') \le \ell$ for some $\mathbf{c}' \le \mathbf{c}$. (The small complication in the definition is due to the possibility that, unlike in the case of the closed signature α, the open signature β can increase as one moves to strata on the boundary of a given stratum.)

Theorem. *The inclusion* $\big(F_{k,\ell}\widehat{\mathcal{P}}\big)^\sigma \hookrightarrow \widehat{\mathcal{P}}^\sigma$ *is* $\frac{1}{2}(k+\ell)$*-connected.*

The proof follows a similar pattern to that in the closed case.

This analysis immediately yields the analogue for open/closed topological field theories of the identification of closed topological field theories with commutative Frobenius algebras. (See for example, Alexeevski and Natanzon [1], Lauda and Pfeiffer [6], Moore and Segal [10]; Lewellen [7] did pioneering work on this subject in a non-topological setting.) It also yields a generalization of the theorem of Moore and Seiberg to the open/closed setting.

Acknowledgements. I wish to thank Kentaro Hori, Eduard Looijenga and Greg Moore for numerous useful suggestions during the lengthy gestation of this project.

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Witt group of modular categories

Alexei Davydov
(joint work with Michael Müger, Dmitri Nikshych, Victor Ostrik)

Modular categories [7] are important for mathematical physics due to their appearance as categories of representations of chiral algebras in a rational conformal field theory. There are several mathematical constructions (or classes) of modular categories. The first (and the most simple) is the class of pointed modular categories (a category is pointed if all its simple objects are invertible with respect to the tensor product). They correspond to so-called lattice conformal field theories.The second is the class of affine modular categories. These categories appear as positive energy representations of loop groups and correspond to so-called WZW models. Finally due to a theorem of M. Müger [4] monoidal (or Drinfeld) centre of any spherical category is modular. Physical examples of monoidal centres come from subtheories of holomorphic theories.

In this paper we propose a way of "taming the zoo" of modular categories by introducing an equivalence relation on the set of modular categories, which makes all monoidal centers equivalent to the trivial modular category (the category of vector spaces). Due to another theorem of M. Müger [5] the set of equivalence classes is an abelian group with respect to the Deligne product of modular categories. We call this group the *Witt group of modular categories.* The choice of the name is motivated by the fact that classes of pointed categories form a subgroup isomorphic to the Witt group of finite abelian groups with quadratic forms.

Since the end of eighties there is a common believe among physicists that all rational conformal field theories come from lattice and WZW models via coset and orbifold (and perhaps chiral extension) constructions (see [3]). Analogous statement for modular categories would imply that the Witt group of modular categories is generated by affine classes (labelled by a Dynkin diagram and a natural number). We discuss relations between these classes coming from conformal embeddings [1, 6] and coset presentations of the minimal series [2].

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The rational and trigonometric Casimir connections

Valerio Toledano Laredo

1. The (rational) Casimir connection

Let $\mathfrak{g}$ be a complex, semisimple Lie algebra, $\mathfrak{h} \subset \mathfrak{g}$ a Cartan subalgebra, $\Phi \subset \mathfrak{h}^*$ the corresponding root system and $W \subset GL(\mathfrak{h})$ the Weyl group of $\mathfrak{g}$. Fix a non–degenerate invariant bilinear form $(\cdot,\cdot)$ on $\mathfrak{g}$.

The *Casimir connection* ∇_C of $\mathfrak{g}$ is a flat, W–equivariant connection on $\mathfrak{h}$ with logarithmic singularities on the root hyperplanes and values in any $\mathfrak{g}$–module V given by

$$\nabla_C = d - \hbar \sum_{\alpha\in\Phi_+} \frac{d\alpha}{\alpha}\kappa_\alpha$$

where the sum ranges over the positive roots of $\mathfrak{g}$, $\kappa_\alpha = \dfrac{(\alpha,\alpha)}{2}(e_\alpha f_\alpha + f_\alpha e_\alpha)$ is the truncated Casimir operator of the $\mathfrak{sl}_2$–subalgebra of $\mathfrak{g}$ corresponding to α and $\hbar \in \mathbb{C}$ is a deformation parameter. This connection was discovered in [8, 11] and independently by De Concini around 1995 (unpublished) and by Felder *et al.* [4].

Set $\mathfrak{h}_{\mathrm{reg}} = \mathfrak{h} \setminus \bigcup_\alpha \mathrm{Ker}(\alpha)$, and let $B_W = \pi_1(\mathfrak{h}_{\mathrm{reg}}/W)$ be the generalised braid group of type W. The monodromy of ∇_C yields a one–parameter family of representations of B_W on V.

Let now $U_\hbar\mathfrak{g}$ be the quantum group corresponding to $\mathfrak{g}$. The *quantum Weyl group operators* of $U_\hbar\mathfrak{g}$ defined by Lusztig, Kirillov–Reshetikhin and Soibelman give rise to an action of the braid group B_W on any integrable $U_\hbar\mathfrak{g}$–module $\mathcal{V}$ [7]. The following is an analogue for the Casimir connection of the Kohno–Drinfeld theorem [2].

Theorem ([11, 12])**.** *The quantum Weyl group action of B_W on an integrable $U_\hbar\mathfrak{g}$–module $\mathcal{V}$ is equivalent to the monodromy of the connection ∇_C on the $\mathfrak{g}$–module $\mathcal{V}/\hbar\mathcal{V}$.*

2. The trigonometric Casimir connection

We describe next an extension of the Casimir connection which gives monodromy representations of the *affine* braid group $\widehat{B}$ corresponding to $\mathfrak{g}$ on finite–dimensional representations of the *Yangian* $Y(\mathfrak{g})$.

2.1. **The connection.** Let $P \subset \mathfrak{h}^*$ be the weight lattice and $H = \operatorname{Hom}_{\mathbb{Z}}(P, \mathbb{C}^*)$ the complex algebraic torus with Lie algebra $\mathfrak{h}$ and ring of regular functions given by the group algebra $\mathbb{C}P$. Set

$$H_{\text{reg}} = H \setminus \bigcup_{\alpha\in\Phi} \{e^\alpha - 1\}$$

where $e^\lambda \in \mathbb{C}[H]$ is the function corresponding to $\lambda \in P$. The Weyl group W acts freely on H_{reg} and the fundamental group $\pi_1(H_{\text{reg}}/W)$ is the affine braid group $\widehat{B}$.

The Yangian $Y(\mathfrak{g})$ is a deformation of the enveloping algebra $U(\mathfrak{g}[t])$ over the ring $\mathbb{C}[\hbar]$. Let $\nu : \mathfrak{h} \to \mathfrak{h}^*$ be the isomorphism determined by the inner product $(\cdot,\cdot)$ and set $t^i = \nu(\lambda_i^\vee)$ where $\lambda_1^\vee, \ldots, \lambda_n^\vee$ are the fundamental coweights of $\mathfrak{g}$ relative to Φ_+. We shall think of the t^i as linear coordinates on $\mathfrak{h}$ and their differentials dt_i as translation–invariant one–forms on H. Let $T_{i,r}$, $r \in \mathbb{N}$, $i = 1, \ldots, n$, be the Cartan loop generators of $Y(\mathfrak{g})$ in Drinfeld's new realisation [3].

Theorem ([13])**.** *The $Y(\mathfrak{g})$–valued connection on H_{reg} given by*

$$\widehat{\nabla}_\kappa = d - \hbar \sum_{\alpha\in\Phi_+} \frac{d\alpha}{e^\alpha - 1}\,\kappa_\alpha + \sum_{i=1}^n dt^i \left(2T_{i,1} - \frac{\hbar}{2}\,T_{i,0}^2\right)$$

is flat and W–equivariant.

We call $\widehat{\nabla}$ the *trigonometric Casimir connection* of $\mathfrak{g}$. Its monodromy yields an action of the affine braid group $\widehat{B}$ on any finite–dimensional $Y(\mathfrak{g})$–module.

2.2. **Monodromy.** Let now $L\mathfrak{g} = \mathfrak{g}[s, s^{-1}]$ be the loop algebra of $\mathfrak{g}$ and $U_\hbar L\mathfrak{g}$ the corresponding quantised enveloping algebra. It is known that the Yangian $Y(\mathfrak{g})$ and the quantum loop algebra $U_\hbar L\mathfrak{g}$ have the same finite–dimensional representation theory (see [14] and [5]).

By analogy with Theorem 1 we make the following

Conjecture ([13]). *The monodromy of the trigonometric Casimir connection is equivalent to the quantum Weyl group action of the affine braid group $\widehat{B}$ on finite–dimensional $U_\hbar(L\mathfrak{g})$–modules.*

We are currently working on this conjecture in collaboration with S. Gautam [6].

2.3. **Relation to Quantum cohomology.** If $\mathfrak{g}$ is simply–laced, finite–dimensional $Y(\mathfrak{g})$–modules may be realised geometrically via Nakajima's quiver varieties $\mathcal{M}(v,w)$ [9]. Specifically, for any $w \in \mathbb{N}^n$, the direct sum of the equivariant cohomologies

$$\bigoplus_{v\in\mathbb{N}^n} H^*_{G_w\times\mathbb{C}^\times}(\mathcal{M}(v,w))$$

carries an action of the Yangian $Y(\mathfrak{g})$.

The corresponding *quantum* equivariant cohomology carries a flat connection known as the quantum differential equation.

Theorem ([1]). *The equivariant quantum differential equation on*

$$\bigoplus_{v} QH^*_{G_w\times\mathbb{C}^\times}(\mathcal{M}(v,w))$$

coincides with the trigonometric Casimir connection.

Theorem 2.3 is consistent with the results of Nekrasov–Shatashvili according to which the spetrum of the operators of quantum multiplication is described by the Bethe ansatz equations for the Yangian $Y(\mathfrak{g})$ [10].

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D-branes, T-duality, and Index Theory, Part II

Kentaro Hori

Finding T-duality transformation of D-branes is an important problem. D-branes were discovered as the T-dual images of the Neumann boundary condition in the first place! Solutions of different levels of preciseness and generality had been obtained in the past. I (the speaker) demonstrated in 1999 [1] that T-duality is a differential geometric version of Fourier-Mukai transform, and used it to derive the T-duality transformation of D-brane charges, i.e., the T-duality isomorphism of K-theory. This is for the case where the spacetime is a direct product of a torus and another manifold and the H-field is zero. The K-theory level T-duality was extended by Bouwknegt et al in 2003 [2] to the case of non-trivial torus fibration and with non-zero H-field, which was followed by several important works. More recently, T-duality at the level of differential K-theory (in the general set-up as in [2]) was obtained by Kahle and Valentino [3], as was presented in the Gong Show of Tuesday night. Concerning the level of preciseness, this may be close to that achieved in [1].

One thing I would like to emphasize is that a proposal is complete only after it is *derived* to be equal to what T-duality does. In the 1999 paper [1], I used D-brane probes to demonstrate that T-duality is indeed the differential geometric version of Fourier-Mukai transform. Recently, I found a simpler derivation of the same transformation, based on an elementary worldsheet analysis [4]. The new derivation has several advantages, and I expect that it will lead to further development. The talk was an outline of this work.

I started the talk with the path-integral derivation of T-duality, which extends Buscher's work in 1987-88 [5] to worksheets with boundary. (This was first outlined in my paper in 2000 [6], to the best of my knowledge.) Next, I reviewed how superconnections can be used to specify D-branes, by explicitly writing down the boundary interactions. Applying the path-integral dualization to such boundary interactions, we obtain the T-duality transform of superconnections. This indeed takes the form of differential geometric version of Fourier-Mukai transform.

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C^*-algebras in tensor categories

Varghese Mathai
(joint work with P. Bouwknegt, K. Hannabuss)

1. Concrete nonassociative C^*-algebras

Here we briefly outline the theory of nonassociative C^*-algebras, viewed as C^*-algebras in tensor categories in [2], which appeared when trying to construct T-duals of compactified spacetimes with background H-flux in [1]. We refer the reader to these papers for details.

Let $\mathsf{G} = \mathbb{R}^n$ and consider the space of all bounded operators $\mathcal{B}(L^2(\mathsf{G}))$ on the Hilbert space $L^2(\mathsf{G})$. We will argue that $T \in \mathcal{B}(L^2(\mathsf{G}))$ determines a unique tempered distribution k_T on G^2. That is, there is a canonical embedding, $\mathcal{B}(L^2(\mathsf{G})) \hookrightarrow \mathcal{S}'(\mathsf{G}^2)$, which will be used, for instance to give the algebra $\mathcal{B}(L^2(\mathsf{G}))$ a nonassociative product, that has the advantage of being rather explicit.

Recall that there is a scale of Hilbert spaces $H^s(\mathsf{G})$, $s \in \mathbb{R}$, called Sobolev spaces, which are defined as follows: the Fourier transform on Schwartz functions $\mathcal{S}(\mathsf{G})$ on G is a topological isomorphism, $\widehat{\ }\colon \mathcal{S}(\mathsf{G}) \to \mathcal{S}(\mathsf{G})$, where we identify G with its Pontryagin dual group. It extends uniquely to an isometry on square integrable functions on G, $\widehat{\ }\colon L^2(\mathsf{G}) \to L^2(\mathsf{G})$.

Moreover, by duality, the Fourier transform extends to be a topological isomorphism on tempered distributions on G, $\widehat{\ }\colon \mathcal{S}'(\mathsf{G}) \to \mathcal{S}'(\mathsf{G})$. Then for $s \in \mathbb{R}$, $H^s(\mathsf{G})$ is defined to be the Hilbert space of all tempered distributions Q such that $(1 + |\xi|^2)^{s/2}\widehat{Q}(\xi)$ is in $L^2(\mathsf{G})$, with inner product given by $\langle Q_1, Q_2\rangle_s = \langle (1 + |\xi|^2)^{s/2}\widehat{Q}_1(\xi), (1 + |\xi|^2)^{s/2}\widehat{Q}_2(\xi)\rangle_0$, where $\langle\ ,\ \rangle_0$ denotes the inner product on $L^2(\mathsf{G})$.

The following are some basic properties of the scale of Sobolev spaces, which are established in any basic reference on distribution theory. For $s < t$, $H^t(\mathsf{G}) \subset H^s(\mathsf{G})$ and moreover the inclusion map $H^t(\mathsf{G}) \hookrightarrow H^s(\mathsf{G})$ is continuous. Also one has $\mathcal{S}(\mathsf{G}) = \bigcap_{s\in\mathbb{R}} H^s(\mathsf{G})$ and $\mathcal{S}'(\mathsf{G}) = \bigcup_{s\in\mathbb{R}} H^s(\mathsf{G})$. It follows that the inclusions $\iota_s\colon \mathcal{S}(\mathsf{G}) \hookrightarrow H^s(\mathsf{G})$ and $\kappa_s\colon H^s(\mathsf{G}) \hookrightarrow \mathcal{S}'(\mathsf{G})$ are continuous for any $s \in \mathbb{R}$. Recall also the Schwartz kernel theorem says that a continuous linear operator $T\colon \mathcal{S}(\mathsf{G}) \to \mathcal{S}'(\mathsf{G})$ determines a unique tempered distribution k_T on G^2, and conversely.

Lemma. *There is a canonical embedding,*

$$\mathcal{B}(L^2(\mathsf{G})) \hookrightarrow \mathcal{S}'(\mathsf{G}^2)\,, \tag{1.1}$$

whose image is contained in the subspace of composable tempered distributions.

Proof. Suppose that $T \in \mathcal{B}(L^2(\mathsf{G}))$. Then in the notation above, the composition

$$\kappa_0 \circ T \circ \iota_0\colon \mathcal{S}(\mathsf{G}) \to \mathcal{S}'(\mathsf{G})\,, \tag{1.2}$$

is a continuous linear operator. By the Schwartz kernel theorem, it determines a unique tempered distribution $k_T \in \mathcal{S}'(\mathsf{G}^2)$. Suppose now that $S \in \mathcal{B}(L^2(\mathsf{G}))$.

Then $ST \in \mathcal{B}(L^2(\mathsf{G}))$ and

$$k_{ST}(x,y) = \int_{z\in\mathsf{G}} k_S(x,z)k_T(z,y)\; dz\,, \tag{1.3}$$

where $\int_{z\in\mathsf{G}} dz$ denotes the distributional pairing. □

We can now define a new product on $\mathcal{B}(L^2(\mathsf{G}))$ making it into a nonassociative C*-algebra.

Definition. Let $\phi \in C(\mathsf{G}\times\mathsf{G}\times\mathsf{G})$ be an antisymmetric tricharacter on G. For $S,T \in \mathcal{B}(L^2(\mathsf{G}))$, define the tempered distribution $k_{S\star T} \in \mathcal{S}'(\mathsf{G}^2)$ by the formula,

$$k_{S\star T}(x,y) = \int_{z\in\mathsf{G}} k_S(x,z)k_T(z,y)\phi(x,y,z)\; dz\,. \tag{1.4}$$

Then for all $\xi,\psi \in L^2(\mathsf{G})$, the linear operator $S\star T$ given by the prescription,

$$\langle \xi, S\star T\psi\rangle_0 = \int_{x,y\in\mathsf{G}} k_{S\star T}(x,y)\bar{\xi}(x)\psi(y)\; dxdy\,, \tag{1.5}$$

defines a bounded linear operator in $\mathcal{B}(L^2(\mathsf{G}))$, which follows from the observation that $S\star T$ is an adjointable operator.

Definition. We denote by $\mathcal{B}_\phi(L^2(\mathsf{G}))$ the space $\mathcal{B}(L^2(\mathsf{G}))$ endowed with the nonassociative product $\star$. This definition extends that of the twisted compact operators $\mathcal{K}_\phi(L^2(\mathsf{G}))$, so in particular $\star$ defines a nonassociative product on $\mathcal{B}(L^2(\mathsf{G}))$ which agrees with the nonassociative product on the twisted compact operators.

There is an involution $k_{S^*}(x,y) = \overline{k_S(y,x)}$ for all $S \in \mathcal{B}_\phi(L^2(\mathsf{G}))$, and norm on $\mathcal{B}_\phi(L^2(\mathsf{G}))$ is the usual operator norm. The following are obvious from the definition: $\forall\, \lambda \in \mathbb{C}$, $\forall\, S_1,S_2 \in \mathcal{B}_\phi(L^2(\mathsf{G}))$,

$$\begin{array}{rcl} (S_1+S_2)^* &=& S_1^*+S_2^*\,, \\ (\lambda S_1)^* &=& \bar{\lambda}S_1^*\,, \\ S_1^{**} &=& S_1\,. \end{array} \tag{1.6}$$

The following lemma can be proved as in Section 5 in [1]

Lemma. $\forall\, S_1,S_2 \in \mathcal{B}_\phi(L^2(\mathsf{G}))$,

$$(S_1\star S_2)^* = S_2^*\star S_1^*\,. \tag{1.7}$$

What appears to missing for the deformed bounded operators $\mathcal{B}_\phi(L^2(\mathsf{G}))$ is the so called C*-identity,

$$||S_1^*\star S_1|| = ||S_1^*S_1|| = ||S_1||^2. \tag{1.8}$$

However, we will continue to call $\mathcal{B}_\phi(L^2(\mathsf{G}))$ a nonassociative C*-algebra and this prompts the following concrete definition of a general class of nonassociative C*-algebras.

Definition. A nonassociative C^*-subalgebra $\mathcal{A}$ of $\mathcal{B}_\phi(L^2(\mathsf{G}))$, is defined to be a $\star$-subalgebra of $\mathcal{B}_\phi(L^2(\mathsf{G}))$ that is closed under taking adjoints and also closed in the operator norm topology.

In particular, $\mathcal{A}$ satisfies the earlier identities. Besides the examples of $\mathcal{B}_\phi(L^2(\mathsf{G}))$ and $\mathcal{K}_\phi(L^2(\mathsf{G}))$, another is the nonassociative torus, described in the next section.

2. Nonassociative tori

The following proposition can be proved.

Theorem ([2])**.** *The group* G *acts on the twisted algebra of bounded operators* $\mathcal{B}_\phi(L^2(\mathsf{G}))$ *by natural* $*$*-automorphisms*

$$\theta_x[k](z,w) = \phi(x,z,w)k(zx,wx), \tag{2.1}$$

and $\theta_x\theta_y = ad(\sigma(x,y))\theta_{xy}$ *where* $ad(\sigma(x,y))[k](z,w) = \phi(x,y,z)k(z,w)\phi(x,y,w)^{-1}$ *comes from the multiplier* $\sigma(x,y)(v) = \phi(x,y,v)$.

Let Γ be a lattice in G. Then by the theorem above, it acts on $\mathcal{B}_\phi(L^2(\mathsf{G}))$ and also the ideal $\mathcal{K}_\phi(L^2(\mathsf{G}))$. Then we define as in [1],

Definition. The crossed product $\mathcal{K}_\phi(L^2(\mathsf{G}))\rtimes\Gamma$ is defined to be the nonassociative torus A_ϕ.

The application to T-duality is encapsulated in the following:-

Theorem (rank n case, [1])**.** *Let*

$$\begin{array}{ccc} \mathbb{T}^n & \xrightarrow{\;i\;} & E \\ & & \Big\downarrow{\scriptstyle p} \\ & & M \end{array}$$

be a principal torus bundle over M, $[H] \in H^3(E,\mathbb{Z})$. *Now suppose that the restriction,* $i^*([H]) \neq 0 \in H^3(\mathbb{T}^n,\mathbb{Z})$.

Then the $\mathbb{R}^n$ *action on* E *lifts to a twisted* $\mathbb{R}^n$ *action on* $CT(E,H)$, *viewed as a* C^**-algebra in the tensor category with associator equal to* $i^*(H)$, *and the T-dual of* (E,H) *is defined to be the twisted crossed product* $CT(E,H)\rtimes_{twist}\mathbb{R}^n$ *which is a nonassociative algebra, or what we call a* C^**-algebra in the same tensor category. It is in general a continuous field of hybrids of noncommutative tori & nonassociative tori.*

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From gauge anomalies to gerbes and gerbal representations: Categorified representation theory

Jouko Mickelsson

In this talk I will explain relations between on one hand the recent discussion on 3-cocycles and categorical aspects of representation theory, [2], and on the other hand gauge anomalies, gauge group extensions and 3-cocycles in quantum field theory, [1].

The set up for categorical representation theory consists of an abelian category C, a group G, and a map F which associates to each $g \in G$ a functor F_g in the category C such that for any pair $g, h \in G$ there is an isomorphism

$$i_{g,h} : F_g \circ F_h \to F_{gh}.$$

For a triple $g, h, k \in G$ we have a pair of isomorphisms $i_{g,hk} \circ i_{h,k}$ and $i_{gh,k} \circ i_{g,h}$ from $F_g \circ F_h \circ F_k$ to F_{ghk} :

They are not necessarily equal; one can have a *central extension* (with values in an abelian group)

$$i_{g,hk} \circ i_{h,k} = \alpha(g,h,k) i_{gh,k} \circ i_{g,h}$$

with $\alpha(g,h,k) \in \mathbb{C}^\times$ a 3-cocycle.

The smooth loop group LG (G compact,simple) has a central extension defined by a (local) 2-cocycle. According to Frenkel and Zhu, increase the cohomoligal degree by one unit by going to the double loop group $L(LG)$. They do this algebraically, utilizing the idea of A. Pressley and G. Segal by embedding the loop group LG (actually, its Lie algebra) to an appropriate universal group $U(\infty)$ (or its Lie algebra). The point of this talk is to show how this is done in the smooth setting, globally, and connecting to the old discussion of QFT anomalies in the 1980's.

Following [3], let $\mathcal{B}$ be an associative algebra and G a group. Assume that we have a group homomorphism $s : G \to \mathrm{Out}(\mathcal{B})$ where $\mathrm{Out}(\mathcal{B})$ is the group of outer automorphims of $\mathcal{B}$, that is, $\mathrm{Out}(\mathcal{B}) = \mathrm{Aut}(\mathcal{B})/\mathrm{In}(\mathcal{B})$, all automorphims modulo the normal subgroup of inner automorphisms.

If one chooses any lift $\tilde{s} : G \to \mathrm{Aut}(\mathcal{B})$ then we can write

$$\tilde{s}(g)\tilde{s}(g') = \sigma(g,g') \cdot \tilde{s}(gg')$$

for some $\sigma(g,g') \in \mathrm{In}(\mathcal{B})$. From the definition follows immediately the cocycle property

$$\sigma(g,g')\sigma(gg',g'') = [\tilde{s}(g)\sigma(g',g'')\tilde{s}(g)^{-1}]\sigma(g,g'g'')$$

Prolongation by central extension

Let next H be any central extension of $\mathrm{In}(\mathcal{B})$ by an abelian group a. That is, we have an exact sequence of groups,

$$1 \to a \to H \to \mathrm{In}(\mathcal{B}) \to 1.$$

Let $\hat{\sigma}$ be a lift of the map $\sigma : G \times G \to \mathrm{In}(\mathcal{B})$ to a map $\hat{\sigma} : G \times G \to H$ (by a choice of section $\mathrm{In}(\mathcal{B}) \to H$). We have then

$$\hat{\sigma}(g, g')\hat{\sigma}(gg', g'') = [\tilde{s}(g)\hat{\sigma}(g', g'')\tilde{s}(g)^{-1}]\hat{\sigma}(g, g'g'') \cdot \alpha(g, g', g'') \text{ for all } g, g', g'' \in G$$

where $\alpha : G \times G \times G \to a$.

Here the action of the outer automorphism $s(g)$ on $\hat{\sigma}(*)$ is defined by $s(g)\hat{\sigma}(*)s(g)^{-1}$ = the lift of $s(g)\sigma(*)s(g)^{-1} \in \mathrm{In}(\mathcal{B})$ to an element in H. One can show that α is a 3-cocycle

$$\alpha(g_2, g_3, g_4)\alpha(g_1g_2, g_3, g_4)^{-1}\alpha(g_1, g_2g_3, g_4)$$
$$\times\alpha(g_1, g_2, g_3g_4)^{-1}\alpha(g_1, g_2, g_3) = 1.$$

Remark If we work in the category of topological groups (or Lie groups) the lifts above are in general discontinuous; normally, we can require continuity (or smoothness) only in an open neighborhood of the unit element.

The above situation appears in gauge theory. The algebra $\mathcal{B}$ is realized as the C^* algebra of fermionic anticommutation relations for fermions on a circle and in the simplest case the outer automorphism as the group of functions on an interval with values in a compact Lie group, the inner automorphisms as the loop group LG (elements of which are implemented up to projective factor as operators in the fermionic Fock space). The central extension comes automatically when lifting the 1-particle operators to operators in the Fock space. The group 3-cocycle can be computed but is complicated. Instead, the corresponding Lie algebra 3-cocycle is simple and equal to

$$\frac{1}{4\pi i}\mathrm{tr}\, X[Y, Z],$$

where X, Y, Z are elements of the Lie algebra of G and the trace is computed in an appropriate representation of G.

This construction can be generalized to gauge theory in higher dimensions. The loop group LG is then replaced by a group $Map(M, G)$ of G-valued functions on a compact space G and the central extension by an abelian extension induced by renormalization effects in quantum field theory, [4]. For further details see [5].

Back to the double loop group $L(LG)$

Next we can replace the group G by $\mathcal{G} = L(LG)$. Assuming G connected, simply connected, the group $\mathcal{G}$ is connected and we can again go through the same steps as in the case of G earlier, except that now for $L\mathcal{G}$ the representation has to be understood in the sense of groupoid central extension or in other words, as Hilbert cocycle. The groupoid here is actually the natural transformation groupoid coming from the gauge action of $L\mathcal{G}$ on gauge connections A on a 3-torus. The cocycle is then a function of the parameter A.

As before, one can compute the 3-cocycle for the double loop group. The corresponding Lie algebra 3-cocycle is obtained by transgression from the Lie algebra 2-cocycle for $L\mathcal{G}$, [4, 5]. Explicit expressions are given as

$$c_2 = const. \int_{T^3} \operatorname{tr} A[dX, dY]$$

with $X, Y : T^3 \to \mathfrak{g}$, transgressing to

$$c_3 = const. \int_{T^2} \operatorname{tr} X[dY, dZ]$$

with now $X, Y, Z : T^2 \to \mathfrak{g}$.

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Some remarks on holomorphic Chern-Simons theory and the Witten genus

Kevin Costello

Let X be a complex manifold. A beautiful result of Fedosov [6] and Bressler-Nest-Tsygan [1] explains how one can see the Todd class of X by thinking about the Hochschild homology of the algebra of differential operators on X.

This talk described an analogous result where the Witten class appears in place of the Todd class. This result is explained in detail in [2] and [3].

In this result, the algebra of differential operators on X is replaced by the chiral (or factorization) algebra of chiral differential operators on X. Instead of considering Hochschild homology, we consider chiral homology along an elliptic curve. Then, the Witten class of X evaluated at that elliptic curve appears in the same way that the Todd class appears in the results of Fedosov and Bressler-Nest-Tsygan.

This result is proved using the approach to quantum field theory developed in [5], applied to a quantum field theory of maps from an elliptic curve to X.

These quantum field theory calculations are then translated into the language of factorization algebras using the results of [4].

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State Sums in G-equivariant 2-dimensional Extended Topological Field Theories

Orit Davidovich

Our goal is to compute state sums in a 2-dimensional extended topological field theory,

$$\mathcal{F} : \text{Bord}_2 \longrightarrow \mathcal{C}$$

where both Bord_2 and $\mathcal{C}$ are symmetric monoidal $(\infty, 2)$-categories. Given a closed surface Σ endowed with a polygonal subdivision (e.g. a triangulation), one can derive a state sum formula, which computes $\mathcal{F}(\Sigma)$, by combining the $\mathcal{F}$-invariants of elements of the subdivision. This requires incorporating polygonal subdivisions into Bord_2, considered as a 2-fold complete Segal space in [1]. To that end, we describe a series of blow-ups of elements of the subdivision of Σ. From the resulting blown-up surface we extract an element of $(\text{Bord}_2)_{2,1}$ which captures both the topological type of Σ and its subdivision. Its $\mathcal{F}$-invariant can be computed once we know what value $\mathcal{F}$ assigns to the saddle. This procedure can be generalized to surfaces with extra structure such as an orientation or a principal G-bundle, or to surfaces with boundary. We test it in the case of G-equivarient theories,

$$\mathcal{F} : \text{Bord}_2^{\text{ori},G} \longrightarrow \text{Alg}$$

where Alg denotes the 2-category of algebras, bi-modules and inter-twiners, and manifolds in $\text{Bord}_2^{\text{ori},G}$ are equipped with orientation and principal G-budles (G is assumed finite). By the cobordism hypothesis of [1], $\mathcal{F}$ is determined by a choice of a $G \times SO(2)$-invariant, fully-dualizable object of Alg. Such an object gives rise to a biangular G-algebra (see [2] for terminology). Our state sum calculation in this case reproduces Turaev's formula in [2].

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Lie n-algebras, supersymmetry and division algebras

John Huerta

There is a relationship between normed division algebras and certain supersymmetric theories of physics which lies at the heart of the following pattern:

- The only normed division algebras are $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ and $\mathbb{O}$. They have dimensions $k = 1$, 2, 4 and 8.
- The classical superstring makes sense only in spacetimes of dimension $k + 2 = 3$, 4, 6 and 10.
- The classical super-2-brane makes sense only in spacetimes of dimension $k + 3 = 4$, 5, 7 and 11.

I will sketch how to use the normed division algebras to prove the spinor identities necessary for the existence of the classical superstring and 2-brane theories. Then I will describe how *exactly the same mathematics* implies the existence of certain higher structures, namely:

- In the superstring dimensions $k + 2 = 3$, 4, 6 and 10, we can use the normed division algebras to construct a Lie 2-superalgebra $\mathfrak{superstring}$ which extends the Poincaré Lie superalgebra in these dimensions.
- In the super-2-brane dimensions $k + 3 = 4$, 5, 7 and 11, we can use the normed division algebras to construct a Lie 3-superalgebra 2-$\mathfrak{brane}$ which extends the Poincaré Lie superalgebra in these dimensions.

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Algebraic models for higher categories

Thomas Nikolaus

I present the theory of algebraic Kan complexes and, more generally, of algebraic fibrant objects in a general model category.

First I give a short review of simplicial sets, including the aspect of Kan complexes as a model for weak ∞-groupoids. Kan complexes have several problems: the lack of fixed composition- and coherence-cells and the fact that limits and colimits in the category of Kan complexes do not necessarily exist. As a solution to these problems I propose the notion of an algebraic Kan complex. The main results about the category AlgKan of algebraic Kan complexs are:

(1) AlgKan is complete and cocomplete.

(2) AlgKan is monadic over simplicial sets.
(3) AlgKan admits a combinatorial model structure Quillen equivalent to simplicial sets and a further equivalence

$$\Pi_\infty : \mathrm{Top} \to \mathrm{AlgKan}.$$

(4) All objects in AlgKan are fibrant.

Generalizing this construction, I introduce for any model category $\mathcal{C}$ (satisfying some technical conditions) the category Alg$\mathcal{C}$ of algebraic fibrant objects which also admits a model structure and is Quillen equivalent to $\mathcal{C}$. Among applications of this general procedure are algebraic quasi-categories as an algebraic model for $(\infty, 1)$-categories and algebraic simplicial presheaves as an algebraic model for ∞-stacks.

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Global Gauge Anomalies in two-dimensional Bosonic Sigma Models

Konrad Waldorf

In my talk I gave a quick overview about the article [1] written in collaboration with Krzysztof Gawędzki and Rafał Suszek. The first objective of the paper is to define a general framework for gauged sigma models. The target space of such a sigma model is a differentiable stack obtained as a quotient of a smooth manifold M by an action of a Lie group H. The fields are triples (Σ, P, ϕ) consisting of a (closed, oriented) surface Σ, a principal H-bundle $p : P \to \Sigma$ and a smooth, H-equivariant map $\phi : P \to M$. The B-field is an H-equivariant gerbe $\mathcal{G}$ over M with a pseudo-connection. The Feynman amplitudes of the model are defined by the formula

$$\mathcal{A}(\Sigma, P, \phi) := \mathrm{Hol}_\Sigma(p_*(\phi^*\mathcal{G} \otimes \mathcal{I}_A)).$$

Here, A is a connection on P, $\mathcal{I}_A$ is a topologically trivial gerbe over P with connection defined by A, p_* is the pushforward of gerbes provided by the equivariant structure on $\mathcal{G}$, and Hol_Σ denotes the surface holonomy of the pushed gerbe around Σ. Anomalies aries when the amplitudes $\mathcal{A}$ depend on gauge transformations of connection A, i.e. they are not gauge invariant.

The second objective of the paper is to use our formalism in order to detect anomalies and „discrete torsion“ in gauged Wess-Zumino-Witten models. The latter arises from different choices of equivariant structures on the same gerbe. In my talk I discussed the case of $SU(2) \times SU(2)$ at level $(k, 2)$ with the adjoint action of diag$(SU(2))$/diag$(SU(2))$, considered by Hori [2]. There, we can explain a sign ambiguity of the partition function found by Hori by detecting two different $SO(3)$-equivariant structures on the relevant gerbe.

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T-Duality and Differential *K*-theory

Alexander Kahle

In this talk I state the main theorem in [4], refining topological T-Duality (in the sense of [1, 2]) to an isomorphism in twisted *differential* K-theory. The basic datum is a *T-duality pair* $(P, \hat{P}, \sigma)$ consisting of a principal torus bundle with connection $(P, \nabla) \to X$, a dual torus bundle with connection $(\hat{P}, \hat{\nabla}) \to X$, and a trivialisation $\sigma : 0 \to P \cdot \hat{P}$ in $\mathcal{H}^4(X; \mathbb{Z})$, where $\mathcal{H}^\bullet(-; \mathbb{Z})$ is the geometric groupoid associated to the differential cohomology of a space (described, e.g. in [3] pg. 9). From this datum canonical twists $\tau \in \mathcal{H}^3(P; \mathbb{Z})$, $\hat{\tau} \in \mathcal{H}^3(\hat{P}; \mathbb{Z})$ of the differential K-theory of P and $\hat{P}$ respectively are constructed. One may then construct a canonical homomorphism of twisted differential K-theory groups $T : \check{K}^{\tau+\bullet}(P) \to \check{K}^{\hat{\tau}+\bullet-\dim P/X}(\hat{P})$, which may be seen as a Fourier-Mukai transform in the setting of differential K-theory. The theorem presented in this talk states that when restricted to suitably defined invariant subgroups, this homomorphism is in fact an isomorphism.

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The 0|2-Sigma Model Computes Euler Characteristic

Daniel Berwick-Evans

I will begin by surveying the emerging picture in the Stolz-Teichner program that connects supersymmetric field theories and generalized cohomology theories. Roughly, we expect that "good" supersymmetric field theories over a manifold will give cocycles for a generalized cohomology theory on that manifold. Under this correspondence, pushforwards can be understood as quantizations and certain physically interesting examples (e.g. nonlinear sigma models) turn out to be interesting classes in cohomology. The known examples are provided in the table below, where the super dimension of the theory is denoted by $d|\delta$ and conjectures are marked by question marks. All theories listed have a special choice of geometry on the worldsheet.

$d\|\delta$	cohomology theory	Class the $\Sigma-$Model Represents
$0\|1$	$HP^*_{dR},\ H^*_{dR}$	0 if $\dim(M) > 0,\ \chi$ else
$1\|1$	$K^*,\ KO^*$	$\hat{A}$
$2\|1$	TMF^*?	Wit?
$0\|2$	HP^*?	χ
$1\|2$	?	χ, σ?

Results in dimension 0|1 are due to [1], and the state of the art in 1|1 and 2|1 is summarized in [2]. I will present my recent results in dimension 0|2. In particular, I will describe how the Euler characteristic arises from quantizing the 0|2-sigma model and some interesting counterexamples furnished by 0|2-dimensional field theories.

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Classification of noncommutative torus bundles

Rishni Ratnam

In 2005 Bouwknegt, Hannabuss and Mathai [1] proposed that the curvature classes of noncommutative torus bundles arising as T-duals of commutative torus bundles should be classified by a group arising as the target of an "integration over the fibres" map in a dimensionally reduced Gysin sequence. Their paper however was restricted to the image of integer cohomology in de Rham cohomology, and therefore omitted torsion.

Somewhat earlier, Packer, Raeburn and Williams [2], using the theory of group actions on continuous trace algebras, had written a version of the Gysin sequence that includes torsion, but was restricted to the case where the H-flux on the commutative bundle had at most "one leg" in the fibres.

Using the groupoid cohomology of Tu [3] we construct an integer cohomology version of the Gysin sequence that agrees with both of these results and extends to the case where the H-flux on the commutative bundle has at most "two legs" in the fibres. This sequence therefore provides a group that classifies noncommutative torus bundles that are T-dual to commutative ones.

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Twisted String/Fivebrane structures and geometry of M-branes

Hisham Sati

The goal of my talk is to identify (higher) geometric and topological structures associated to M-branes. I mainly outline ideas from [2].

One way to study M-theory is through topology. Our strategy is to view anomalies in physics as corresponding to obstructions to having some (higher) structure as a bundle over spacetime and/or for having an orientation with respect to some generalized cohomology theory. When applied to the quantization condition of the C-field in M-theory this leads to degree four twisted String structure [3].

From a generalized cohomology point of view, a String structure provides an orientation for TMF. My expectation that a twisted String structure corresponds to an orientation in twisted TMF is confirmed in [1]. The C-field provides the twist. I also argue from various angles, including the equation of motion and S-duality, that it is also essentially what is being twisted. This suggests that the M-branes carry such twisted structure and hence have charges in twisted TMF [2]. Making this precise is the subject of current investigation.

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Stacks and étale 2-spaces

Igor Baković

(joint work with Branislav Jurčo)

For any topological space X, there is a well known pair of adjoint functors between a category $Bun(X)$ of bundles over X, and a category $Set^{\mathcal{O}(X)^{op}}$ of presheaves over X, which restricts to an adjoint equivalence between a category $Sh(X)$ of sheaves over X, and a category $Et(X)$ of étale spaces over X. The right adjoint is a cross-section functor which assigns to every étale space over X a sheaf of its cross-sections, and the left adjoint is a stalk functor which assigns to every presheaf over X its étale space of germs. Stalks of stacks were less familiar so far, since they appeared as filtered bicolimits over (an opposite of) a category $\mathcal{O}_x(X)$ of open neighborhoods of a fixed point x in X. Stalks defined in a such way were categories with too many objects and it was not possible to introduce a sensible topology on them. We introduce a new notion of a stalk of a stack, using the smallest equivalence relation generated by a restriction relation on objects, which allows to introduce topology on both objects and morphisms. This construction corresponds to a filtered pseudocolimit over a category $\mathcal{O}_x(X)$, and its universal property is defined up to an isomorphism of categories, unlike the case of filtered bicolimits, whose universal property is defined up to an equivalence of categories. In this way, we extend above pair of adjoint functors to a pair of biadjoint 2-functors between a 2-category $Fib(X)$ of fibered categories over X, and a 2-category $2Bun(X)$ of 2-bundles over X, which restricts to an adjoint biequivalence between a 2-category $St(X)$ of stacks over X, and a 2-category $2Et(X)$ of étale 2-spaces over X.

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What can we learn from infinite-dimensional Lie groups about Lie 2-groups (and vice versa)?

Christoph Wockel

In this talk we emphasize the perspective that locally convex Lie groups are groups endowed with locally smooth group operations. Under some mild requirements, these locally smooth group operations determine the Lie group structure completely [1, Th. II.2.1].

Most definitions of smooth 2-groups or Lie 2-groups are either too restrictive or rather complicated. Taking the above perspective over to 2-groups (categorical groups) leads to a quite general (but yet easy to understand) notion of Lie 2-groups. In particular, the String 2-group can be understood in those terms [2, Ex. IV.10.]. On the other hand, 2-groups can solve non-integrality problems occurring in the prequantization of infinite-dimensional Lie groups. In particular, étale Lie 2-groups serve as natural integrating objects for Banach-Lie algebras (cf. [2, Th. VI.5.]) in cases that Banach-Lie algebras fail (cf. [1, Ex. VI.1.16]):

Theorem. If $\mathfrak{g}$ is a Banach-Lie algebra, then there exists an étale Lie 2-group such that its Lie algebra is isomorphic to $\mathfrak{g}$.

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Infinitesimal Symmetries of Dixmier-Douady Gerbes

Braxton Collier

Given a Dixmier-Douady gerbe $\mathcal{C}$ over a manifold M, we construct a *generalized Atiyah sequence*

$$0 \longrightarrow B\mathbb{R}_M \longrightarrow \mathcal{L}_\mathcal{C} \longrightarrow \chi(M) \longrightarrow 0. \tag{0.1}$$

The term $\mathcal{L}_\mathcal{C}$ has the structure of what we call a *Lie algebra category*, and we interpret it as the category of lifts of vector fields on M to vector fields on $\mathcal{C}$. The left hand term is the category of principal $\mathbb{R}$-bundles over M. If in addition $\mathcal{C}$ has a connective structure, we can define the more structured notion of a *connective lift*. We explain how the strongly homotopy Lie algebra associated to the exact Courant algebroid constructed from a gerbe with connective structure can be interpreted very naturally in our framework.

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