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Moist Processes in the Atmosphere

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Abstract. Processes related to water in the atmosphere lead to severe uncertainties in weather forecasting and climate research. Atmospheric water vapor and cloud water strongly influence the Earth's energy budget through, *e.g.*, energy conversions associated with phase changes, fluid dynamical effects associated with buoyancy, and through their influence on radiative transfer properties of the atmosphere. Given the critical green-house effect of water vapor, it seems astounding that climate modellers cannot with certainty state whether the Earth's cloud system has a positive or negative influence on the global mean temperature. The formation of clouds involves small-scale processes currently unresolved by climate models, and thus cloud cover is one of the main sources of uncertainty. This large uncertainty has its roots in the extremely wide range of length and time scales associated with moist processes, which pose an equally wide range of challenges to mathematical and computational modelling.

New and innovative methods, modeling frameworks, efficient computational techniques, and complex statistical data analysis procedures as well as their mathematical analysis are urgently needed in order to make progress in this new field – from the mathematicians point of view. One of the main goals of this workshop is to show the path forward for current and future applied mathematical scientists, to work hand in hand across the disciplines of mathematics, physics, and atmospheric science, in order to tackle the complex problem of dynamical and thermodynamical processes associated with clouds and moisture, both from the theoretical and the applied view points.

Mathematics Subject Classification (2010): 35 Partial Differential Equations, 60 Probability Theory and Stochastic Processes, 65 Numerical Analysis, 76 Fluid Mechanics, 80 Classical Thermodynamics, Heat Transfer, 86 Geophysics.

Introduction by the Organizers

The workshop *Moist Processes in the Atmosphere*, organised by Boualem Khouider (Univ. of Victoria, Victoria, CA), Rupert Klein (Freie Univ. Berlin, Berlin, Germany) and Leslie Smith (Univ. of Wisconsin, Madison, USA), was well attended by mathematicians and physicists, meteorologists, and climate modellers. Its agenda deviated somewhat from Oberwolfach traditions in that all participants were invited to provide 15 min presentations with 5 min of discussion, rather than restricting to a smaller number of more extensive talks. This mode of operation, which was successfully exercised in previous related workshops in 2002, 2006, and 2010, enabled participants to gain a good impression of the broad range of challenges and the diversity of methodological approaches pursued in studies of moist atmospheric dynamics. The workshop was divided into eight separate morning and afternoon sessions, each covering a particular aspect of the workshop topic, and consisting of 5 to 6 talks. A period of up to one hour was reserved, at the end of each session, for open discussion and brainstorming. Allowing every participant to speak led to engaging and thought provoking discussions, covering the various angles and diverse approaches of each subject, and resulted in the involvement of everyone and especially of the junior participants.

Largely, the focus of presentations and discussions of the sessions was as follows. Monday's presentations addressed the principal inclusion of water phase transitions and transport of water constituents in mathematical models for the atmosphere. On Tuesday, we discussed the aggregated impact of moist processes on large-scale flow patterns, such as tropical and mid-latitude cyclones and fronts. Wednesday saw presentations on rigorous mathematical underpinnings of the moist aero-thermodynamic equations with emphasis on existence, uniqueness, and stability of solutions to various flow models, on non-uniqueness issues in the context of the recent results from convex integration theory for (very) weak solutions of partial differential equations, and on algebraic aspects of fluid dynamics in the Nambu formulation. Thursday's and Friday's presentations centred upon more general questions of mathematical and computational modelling, deterministic and stochastic parameterization schemes, and the role of entropy in this context.

Particularly intense discussions revolved around the meaning and importance of the concept of entropy in moist physics modelling. Mathematicians and meteorologists are interested in this concept for complementary reasons. While the concept promises to help analysts in selecting a unique solution from a multitude of possible weak solutions to one and the same problem, meteorologists demand consistency with the second law of thermodynamics, and developers of computational atmosphere models rely on entropy principles in the design of accurate *and* robust numerical schemes.

Water phase transitions are associated with high activation energies, high conversion rates, and nonlinear rate laws – typically with non-integer power laws – while at the same time substantically influencing the buoyancy budget of the atmosphere. This is known to give rise to multiscale phenomena, with organized cloud dynamics being observed on scales from one to thousands of kilometers. A variety of methodologies for handling this complexity, including stochastic modelling, adaptive numerical flow solvers, superparameterization / heterogeneous multiscale modelling, and multiple scales asymptotics were presented and discussed in the course of the week.

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Workshop: Moist Processes in the Atmosphere

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Abstracts

Separating physical impacts from natural variability using piggybacking (master-slave) technique

Wojciech W. Grabowski

In a chaotic system, like moist convection, it is difficult to separate the impact of a physical process from effects of natural variability. This is because modifying even a small element of the system physics typically leads to a different system evolution. For the modeling, the ensemble approach can be used. However, there is a simpler and less computationally demanding methodology referred to as the piggybacking or the master-slave approach. The idea is to use two sets of thermodynamic variables (the temperature, water vapor, and all aerosol, cloud, and precipitation variables) in a single cloud simulation. The two sets differ in a specific element of the physics, such as aerosol properties, microphysics parameterization, large-scale forcing, environmental profiles, etc. One thermodynamic set is coupled to the dynamics and drives the simulation, and the other set piggybacks the simulated flow, that is, thermodynamic variables are carried by the simulated flow but they do not affect it. By switching the sets (i.e. the set driving the simulation becomes the piggybacking one, and vice versa), the impact on the cloud dynamics can be evaluated. This presentation will provide details of the method, and it will discuss results of its application to such problems as the postulated deep convection invigoration in polluted environments, the impact of changes in environmental profiles (e.g., due to climate change) on convective dynamics, and the role of cloud-layer heterogeneities on shallow convective cloud field development.

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Investigating timescales of warm-rain bulk microphysics schemes

Juliane Rosemeier
(joint work with Peter Spichtinger and Manuel Baumgartner)

The presentation pursues the goal to provide a tool which enables the comparison of different cloud schemes. In particular, we pay attention to a certain type of warm-rain bulk microphysics schemes, so-called one-moment schemes, that can be found in weather prediction models. The cloud schemes which belong to this type are not derived from first principles and differ in their parameter choices. We point out that the decision for a certain cloud scheme within a weather forecasting model may have an impact on the model output. An extensive description of the ideas of the talk is given in [1].

In the presentation, the warm-rain schemes which can be found in the documentations of the weather prediction models COSMO [2] and IFS [3] as well as the research scheme proposed in [4] are examined in more detail. The cloud schemes are decoupled from the dynamics, especially we consider an air parcel which does not interact with neighboring air parcels. Thus we end up with a system of ordinary differential equations. The formulation of a generic cloud scheme which describes the time evolution of the mixing ratios of cloud droplets q_c and rain drops q_r is shown and provides a general representation of the warm-rain bulk microphysics schemes. The equations are as follows:

$$\begin{aligned} \dot{q}_c &= \underbrace{cSq_c}_{\text{condensation}} - \underbrace{a_1 q_c^{\gamma}}_{\text{autoconversion}} - \underbrace{a_2 q_c^{\beta_c)} q_r^{\beta_r}}_{\text{accretion}} \\ \dot{q}_r &= \underbrace{a_1 q_c^{\gamma}}_{\text{autoconversion}} + \underbrace{a_2 q_c^{\beta_c} q_r^{\beta_r}}_{\text{accretion}} + \underbrace{(e_1 q_r^{\delta_1} + e_2 q_r^{\delta_2})S}_{\text{evaporation}} - \underbrace{d q_r^{\zeta} + B}_{\text{sedimentation}} \end{aligned}$$

Parameterizations of the microphysical processes condensation, autoconversion, accretion, evaporation and sedimentation are incorporated. The cloud schemes in [2] and [3] as well as the scheme in [4] are special cases of the generic cloud scheme and can be obtained by a suitable choice of the parameters in the generic scheme. A fixed point and stability analysis is carried out to get insight how the systems in [2], [3] and [4] behave for $t \to \infty$. Additionally, asymptotic techniques are applied in order to derive reduced equations which are valid on certain timescales and verified by numerical simulations. Beside the reduced equations, the timescales of the microphysical cloud processes are identified by the asymptotic analysis.

The applied techniques reveal if the microphysical processes are parameterized similarly in the investigated schemes and give information about the qualitative behavior of the cloud systems. Our analysis showed differences in both the parameterizations and the qualitative behavior. It can be extended to two-moment schemes and schemes that include the ice phase. In these cases we end up with ODE systems of dimension > 2; so the qualitative behavior of the systems might be more complex.

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Stochastic perturbations to the micro-physics parameterization in a convection-resolving weather model

Judith Berner

As weather prediction models are starting to resolve convection explicitly, the uncertainty implicit in the need for physical parameterization schemes will mostly lie he micro-physics (MP) and boundary layer parameterization schemes. Here, we are analyze the divergence of the trajectories of a convection-resolving ensemble system introduced by stochastic perturbations to the parameters in the Thompson micro-physics scheme. The time-dependent ensemble spread is used as a measure of forecast uncertainty and should be indicative of upscale error-growth for an reliable ensemble system.

We find that the spread introduced by the perturbations to the MP-physics scheme is comparable in amplitude and physical distribution to that obtained from white-noise perturbations to the temperature the boundary layer at initial time only. This suggest that unstable modes of the atmosphere (such as convection) govern the error-growth characteristic and that the exact nature of the perturbation matters little. This provides a likely explanation why ad hoc stochastic parameterization schemes have proven so successful in operational numerical weather prediction.

Structure formation in ice clouds - dynamical systems and beyond

Peter Spichtinger

Clouds constitute an important component of the Earth-Atmosphere system. They interact with radiation, thus leading to cooling or warming of the system and influence the hydrological cycle, e.g. by forming precipitation. Beside their role in the atmosphere and for weather and climate, clouds are interesting physical states, which are not completely understood. Clouds are complex multiple scale and multi phase systems, which can be described as open thermodynamic systems, driven by external forcings, namely local three dimensional motions. Such systems are known to form structures in an emergent way; small scale processes feedback to each other such that their interactions lead to the development of pattern on larger scales than cloud particle scales. Generally, these pattern cannot be predicted *a* *priori* from the knowledge of the particle system. This is a general feature of such complex open thermodynamic systems.

In this contribution, we focus on clouds in the upper troposphere, i.e. in the cold temperature regime ($T < 235\,\mathrm{K}$), which are consisting exclusively of ice crystals. The role of these clouds in the climate system is still unclear, since their radiative impact has not been determined yet. Cooling and warming effects are of comparable size, thus microphysical properties driven by key processes as formation, growth/evaporation and sedimentation of ice crystals play an important role. In addition, the local vertical motions are driving the system ice cloud in a crucial way, since the process ice nucleation depends on the local temperature and supersaturation, which is affected by vertical motions leading to adiabatic cooling.

Since a cloud consists of myriads of water particles, a description of structures on scales beyond the particle scale must rely on averaged quantities. In the case of ice clouds we use the bulk variables ice number and mass concentration, i.e. n_i and q_i, which would translate into zeroth and first general moment of an underlying particle distribution. In a first approach (so-called parcel model approach) we investigate the system in a distinct volume. The ice particles are homogeneously distributed in the box, but can fall through the bottom of the box due to gravitational acceleration (so-called sedimentation). The system is characterized by thermodynamic variables temperature and pressure, as well as saturation ratio with respect to ice, S_i, which controls formation of ice crystals (nucleation) as well as diffusion processes (growth or evaporation of particles). Since we investigate an air parcel, the system can be described using ordinary differential equations (ODEs); the system is externally driven by vertical motion, i.e. by an externally triggered expansion/cooling.

We want to characterize the quality of the system, therefore we have to investigate long term behaviour. For this purpose and for simplification of the system, we neglect vertical motions in the development of temperature and pressure, i.e. these variables are set to constant values, but cooling is regarded for the evolution of the supersaturation with respect to ice. Thus, the system can be reduced to a three dimensional ODE system for variables n_i, q_i and S_i

$$(1)\quad \dot{n}_i = \underbrace{a \cdot J(S_i,T)}_{\text{nucleation}} - \underbrace{b \cdot n_i^{1-\delta} q_i^{\delta}}_{\text{sedimentation}} + \underbrace{H(1-S_i) \cdot d \cdot (S_i - 1) \cdot n_i^{2-\alpha} q_i^{\alpha-1}}_{\text{evaporation}}$$

$$(2)\quad \dot{q}_i = \underbrace{a \cdot m_0 \cdot J(S_i,T)}_{\text{nucleation}} - \underbrace{c \cdot n_i^{-\delta} q_i^{1+\delta}}_{\text{sedimentation}} + \underbrace{d \cdot (S_i - 1) \cdot n_i^{1-\alpha} q_i^{\alpha}}_{\text{growth/evaporation}}$$

$$(3)\quad \dot{S}_i = \underbrace{e \cdot w(t) \cdot S_i}_{\text{adiabatic cooling}} - \underbrace{f \cdot (S_i - 1) \cdot n_i^{1-\alpha} q_i^{\alpha}}_{\text{growth/evaporation}}$$

with constants $a, b, c, d, m_0 > 0$, exponents $0 < \alpha, \delta < 1$, heaviside function $H(.)$, and a (time-dependent) function of vertical velocity $w(t)$. The right hand side of the system contains very steep terms (e.g. nucleation rate of the form $J(S_i,T) \sim \exp(C(S_i - S_0(T))))$ and stiff terms (for growth and sedimentation) due to very different time scales of the relevant processes. Thus, the analysis of the ODEs using theory of dynamical systems as well as the numerical integration of the equations is challenging.

For a first characterization we investigate scenarios of constant vertical updrafts (i.e. constant forcings $w(t) = w_0$), leading to an autonomous three dimensional ODE system. This system can be characterized as an externally driven, non-linear dissipative oscillator [1]. At fixed pressure value, we can distinguish two different regimes. The first regime is a stable focus, i.e. all trajectories tend asymptotically to a fixpoint. The second regime is a stable limit cycle in combination with an unstable focus. The regimes are separated by two Hopf bifurcations in the parameter space of vertical velocity vs. temperature.

In a second step, we investigate the behaviour of the system under oscillatory external forcing, i.e. the vertical velocity is now time dependent. The resulting non-autonomous ODE system must be investigated using numerical integration, since analytical investigations for the transformed autonomous 4D system are crucially limited. Similarly to classical investigations of non-linear oscillators driven by periodic forcings [2] we found very complicated behaviour of the system, leading to further bifurcations and, for specific parameters, to routes to chaos. Thus, the quite simple model already leads to complicated behaviour and time-dependent structures.

The ODE system is the basis for future investigations of structure formation. It is planned to extend the system by diffusion terms, leading to a system of reaction-diffusion equations, which can be investigated using classical tools.

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Reynolds-averaged bulk microphysics

Axel Seifert

Clouds in an atmospheric boundary layer (ABL) often develop through fluctuations in temperature and moisture. A simple model for the cloudy ABL has to include the variances of the liquid water potential temperature θ_l and the total water mixing ratio q_t where the latter is defined as the sum of the vapor mixing ratio q_v and the cloud water mixing q_c. In Reynolds-averaged form the evolution equations for those two variances given by [1]

$$\frac{\partial \overline{q_t'^2}}{\partial t} = -\underbrace{2\overline{q_t' w'}\frac{\partial \overline{q_t}}{\partial z}}_{\text{Production}} - \underbrace{\frac{\partial \overline{(w' q_t'^2)}}{\partial z}}_{\text{Transport}} + \underbrace{2\overline{q_t' Q_q}}_{\text{Source/Sinks}} - \underbrace{2\varepsilon_q}_{\text{Dissipation}}, \tag{1}$$

(2) $$\frac{\partial \overline{\theta_l'^2}}{\partial t} = -\underbrace{2\overline{\theta_l' w'}\frac{\partial \overline{\theta_l}}{\partial z}}_{\text{Production}} - \underbrace{\frac{\partial \overline{(w'\theta_l'^2)}}{\partial z}}_{\text{Transport}} + \underbrace{2\overline{\theta_l' Q_\theta}}_{\text{Source/Sinks}} - \underbrace{2\varepsilon_\theta}_{\text{Dissipation}},$$

contain a source/sink term due the microphysical processes Q_x as correlation between the scalar fluctuation and the corresponding microphysical process rate. For liquid clouds the source term is the sum of the microphysical processes autoconversion, accretion and evaporation

(3) $$Q_q = Q_{\text{evap}} + Q_{\text{accr}} + Q_{\text{auto}}.$$

With help of large-eddy simulation it can be shown that these correlation terms act as sinks and destroy variance [2]. In the cloud layer they replace the turbulent dissipation as the dominant dissipation mechanism. Thus, clouds dissipate by producing rain.

For a precipitating cloudy ABL the variances increase significantly compared to a non-precipitating regime [3]. The source of that variance is not easy to identify, because all explicit microphysical terms are either negative or small. It is postulated that the variance is produced by a slow positive feedback that includes the mean-gradient production term, which increases due to the evaporation of rain in the sub-cloud layer increasing the flux $\overline{q_t' w'}$. Whether the Reynolds-averaged equations are able to represent this feedback explicitly without the help of an additional stochastic sub-model remains an open question.

To determine the microphysical process rates and, hence, close the system, it is necessary to first calculate the cloud fraction C and the cloud water mixing ratio q_c. This can be done by integrating over the probability density functions (PDFs) of the moisture and temperature fields. To avoid the use of a joint PDF it is convenient to linearize the system at the saturation point and introduce the extended liquid water mixing ratio [4, 5]

(4) $$s = \frac{q_r - q_s(T_l)}{1 + \frac{L}{c_p}\left.\frac{\partial q_s}{\partial T}\right|_{T=T_l}}$$

where $q_s(T)$ is the saturation mixing ratio and T_l is the liquid water temperature. The cloud fraction and the cloud water mixing ratio are then given by

(5) $$C = \int_0^\infty P(s)ds, \qquad q_c = \int_0^\infty sP(s)ds.$$

where $P(s)$ is the probability density function of s. The choice of $P(s)$ is crucial for the representation of different cloud regimes like stratocumulus and shallow convection and various suggestions have been made over the last decades starting from simple Gaussian distributions, to Gamma and Beta distributions and, more recently, double-Gaussian distributions. The latter is a five parameter distribution, but using some additional empirical relations it can be reduced to a three parameter distribution, which seems be appropriate for most cloud regimes [6, 7]. Obviously the variance equations are thus insufficient to close the system and we require either a diagnostic relation for the skewness of s or a prognostic equation that needs to be solved alongside Eqs. (1) and (2). Such a prognostic skewness equation is discussed in [1]. An even more advanced approach is the CLUBB model ('Cloud Layers Unified By Binormal'), which solves for the full joint PDFs of the cloudy boundary layer [8, 9].

Joint PDFs are necessary and unavoidable for the parameterization of some of the microphysical processes like accretion or evaporation, which need correlation between at least two variables [10]. For the rain water mixing ratio a log-normal distribution seems to be appropriate, while the relative humidity necessary for evaporation is part of the double-Gaussian distribution of s.

This approach can only be applied to liquid clouds in such a straightforward way because only those are close to thermodynamic equilibrium, whereas non-equilibrium metastable states occur frequently for ice clouds. In addition, ice clouds form due to various different nucleation mechanisms, e.g. heterogeneous and homogeneous ice nucleation, which leads to additional complications. Hence, ice clouds would require a much more general and probably stochastic model.

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Homogeneous Nucleation from an Asymptotic Point of View

Manuel Baumgartner
(joint work with Peter Spichtinger)

Cirrus clouds in the Earth's atmosphere consist exclusively of ice crystals. Although these clouds are a commonly observed phenomenon, the details of the formation of ice crystals remain unclear. It is known that the major formation pathway of ice crystals in the low temperature regime below 235 K is homogeneous nucleation, i.e. spontaneous freezing of pre-existing solution droplets, in contrast to heterogeneous nucleation where foreign substances aid the freezing to occur at higher temperatures [2]. Moreover, homogeneous freezing is very sensitive to the environmental temperature T and humidity, quantified by the saturation ratio $S_i = \frac{p_v}{p_{\mathrm{sat},i}(T)}$ with respect to ice, comparing the actual water vapor pressure p_v to the ice saturation vapor pressure $p_{\mathrm{sat},i}(T)$ at temperature T. The environmental temperature defines a so-called critical saturation ratio $S_{i,\mathrm{crit}} = S_{i,\mathrm{crit}}(T)$. If an air parcel is vertically lifted, its temperature decreases due to adiabatic cooling and causes the saturation ratio to increase. Eventually the saturation ratio S_i becomes comparable to the critical value $S_{i,\mathrm{crit}}$ and the pre-existing solution droplets freeze within a very short time interval, causing the number of ice crystals to increase explosively. After the solution droplets are frozen, the newly formed ice crystals grow by water vapor diffusion and cause the saturation ratio to decrease well below the critical value and inhibit another nucleation event. The goal of the study [1] is to model and analyze this process mathematically.

The nondimensional mathematical model is given by the ODE system

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = J_0 \exp\left(A(S_i - S_{i,\mathrm{crit}})\right), \tag{1a}$$

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = \delta(S_i - 1)N_i, \tag{1b}$$

$$\frac{\mathrm{d}S_i}{\mathrm{d}t} = \alpha S_i - \gamma(S_i - 1)N_i \tag{1c}$$

where N_i is the number density of the ice crystals, q_i the mass mixing-ratio and J_0, A, δ, α, γ denote nondimensional constants. The first equation is based on the homogeneous nucleation rate [4]. Assuming the case of no pre-existing ice crystals, numerical investigations of (1) reveal, that the mass mixing-ratio q_i of the ice crystals is virtually constant during the nucleation event itself. Consequently, we neglect equation (1b) and analyze the reduced system consisting of the equations (1a) and (1c).

In order to analyze the system (1a), (1c) and capture the sharp transition, we use the technique of matched asymptotics [5] inspired by combustion theory [6, 7]. The Distinguished Limit is chosen as

$$J_0 = \frac{J_0^*}{\varepsilon}, \quad A = \frac{A^*}{\varepsilon} \tag{2}$$

with J_0^*, A^*, δ, α, $\gamma \in \mathcal{O}(1)$ as $\varepsilon \to 0$. This choice encodes that the constants J_0, A are both asymptotically large compared to the other constants.

As indicated above, we distinguish two states of the system. The first state is characterized by $S_i - S_{i,\text{crit}} = \mathcal{O}(1)$ as $\varepsilon \to 0$ and encodes that the saturation ratio is not comparable to the critical saturation ratio. This state is active before the nucleation occurs or after the nucleation event is completed. The main result is that the ice crystal number density N_i is constant in leading and first order, whereas the saturation ratio increases or decreases exponentially in leading and first order due to the adiabatic cooling or the diffusional growth, respectively.

The second state of the system is the short Nucleation Zone, characterized by $S_i - S_{i,\text{crit}} = \mathcal{O}(\varepsilon)$ as $\varepsilon \to 0$ where the saturation ratio is comparable to its critical value. From the equation, we find that the sharp transition in the number density N_i is accomplished according to a hyperbolic tangent and it is possible to explicitly write down the exact solution of the leading order problem.

The general solutions for the individual states contain integration constants and a subsequent matching procedure is needed to determine all missing constants and therefore arrive at a uniformly valid composite asymptotic approximation to the exact solution on the whole time interval. Moreover, the matching procedure gives rise to an expression for the (approximated) time instant of the nucleation event and to the number of homogeneously nucleated number of ice crystals. Inspecting the expression for the number of nucleated ice crystals reveals, that only the coefficient A from (1a) contributes, but not J_0. This implies that mainly the steepness (through coefficient A) of the nucleation rate determines the number of ice crystals, but not its precise values (through coefficient J_0), which only should be large such that the Distinguished Limit (2) is valid.

The constructed asymptotic solution is an approximation to the exact solution of the equations (1a), (1c) and therefore suggests itself to be used as a parameterization for homogeneous nucleation in the context of numerical models of the atmosphere. The widely used parameterization of this process is described in [8] and relates the number of homogeneously nucleated ice crystals to the model-calculated updraft velocity. Translating in the context of our study [1], the vertical velocity is contained in the nondimensional coefficient α in (1c) and the asymptotic approximation provides the number of nucleated ice crystals. A comparison between the predicted number of homogeneously nucleated ice crystals in [8] and our study shows, that the approach using asymptotics is able to reproduce the results. However, since we neglected equation (1b) for the mass mixing-ratio q_i, a mean mass of the ice crystals enters the nondimensional coefficient γ and the results are very sensitive to the precise values of this coefficient. Therefore, we need to carefully estimate a meaningful mean mass. The remedy to this shortcoming would be to include the neglected equation (1b) into the asymptotic analysis. As a by-product, we would also overcome another limitation of the current approach: after the nucleation event has occured, the newly nucleated ice crystals change the (mean) mass due to diffusional growth, thus the reduced system (1a), (1c) is no longer appropriate to represent this behavior.

It is intended to extend the asymptotic analysis in a future study to the full system and also investigate other possible scalings of the nondimensional constants. The latter seems promising since the precise value of δ in (1b) is strongly affected by temperature and therefore gives the possibility to study the nucleation event within different temperature regimes.

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Energy Decompositions for Moist Boussinesq and Anelastic Equations with Phase Changes

Sam Stechmann

(joint work with David Marsico, Leslie Smith)

To define a conserved energy for an atmosphere with phase changes of water (such as vapor and liquid), motivation in the past has come from generalizations of dry energies—in particular, from gravitational potential energy, $\rho g z$. Here another definition of moist energy is introduced, and it generalizes another form of dry potential energy, proportional to θ^2,which is valuable since it is manifestly quadratic and positive definite. This moist potential energy here is piecewise quadratic, and it is quadraticwithin each phase. The potential energy can be decomposed into three parts, proportional to $b_u^2 H_u$, $b_s^2 H_s$, and $M^2 H_u$, which represent, respectively, buoyant energies and a moist latent energy that is released upon a change of phase. The H_u and H_s are Heaviside functions that indicate that unsaturated and saturated phases, respectively. The M^2 energy is also associated with an additional eigenmode that arises for a moist atmosphere but not a dry atmosphere. Both the Boussinesq and anelastic equations are considered, and similar energy decompositions are shown in both cases, although the anelastic energy is not quadratic. Extensions that include cloud microphysics are also discussed, such as the Kessler warm-rain scheme. As an application, empirical orthogonal function (EOF) analysis is considered, using the piecewise quadratic moist energy as a weighted energy in contrast to the standard L^2 energy. By incorporating information about phase changes into the energy, the leading EOF modes become fundamentally different and capture the variability of the cloud layer rather than the dry sub-cloud layer.

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Homogeneous Nucleation from an Asymptotic Point of View: History & Future Prospects

Mitchell W. Moncrieff

Impressive advances have been made in our understanding the physics and dynamics of organized moist convection, notably mesoscale convective systems (MCS). Nevertheless, the parameterization of organized convection in global climate models (GCMs) has languished. Being neither resolved nor represented by parameterizations, organized convection is absent from contemporary GCMs. This unfortunate situation raises issues that include but are certainly not restricted to mesoscale extremes, severe weather, and the distribution, type, and intensity of precipitation. In a warmer world such issues cannot be adequately projected by GCMs without serious attention to organized convection. A new paradigm called mesoscale convective system parameterization a.k.a. multiscale coherent structure parameterization (MCSP) treats organized convection as coherent dynamical structures embedded in a turbulent environment. The transport modules are dynamical models based on fundamental nonlinear Lagrangian conservation principles. For the first time, the large-scale effects of organized convection are quantified simply by the difference between simulations with and without MSCP. A minimalist MCSP prototype implemented in the NCAR Community Atmosphere Model (CAM) provides a reasonable proof-of-concept. For instance, its effects are consistent with Tropical Rainfall Measurement Mission (TRMM) satellite-based precipitation analyses. Moreover, the upscale effects of MCSP generate large-scale precipitation patterns in the tropical warm-pool and maritime continent regions, improve the intertropical Convergence Zone, and have beneficial effects on convectively coupled equatorial wave modes and the MJO.

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A Dynamical Theory for the Motion of Cloud Edges

Dave Muraki

Spatial patterns in convectively-stable and non-precipitating cloud are shown to be influenced by the fluid dynamics of gravity waves. An inspirational illustration is evidenced by the rare example of a holepunch cloud — a growing circular hole in a thin cloud layer. A Boussinesq-type of scaling analysis is designed to derive disturbance dynamics from a background atmosphere that is exactly at saturation. Hence, small wave motions can condense or evaporate cloud, and the implied buoyancy changes then affect the wave propagation. A first example of the shaping of cloud by winds is illustrated by the generation of cloud by orographic uplifting.

Towards understanding the dynamics of spin up in Emanuel's tropical cyclone model

Roger K. Smith
(joint work with Michael T. Montgomery)

We seek to understand the mechanism of vortex spin up in Emanuel's 2012 axisymmetric theory for tropical-cyclone intensification in physical coordinates, starting from first principles. It is noted that, while spin up of the maximum tangential wind must occur at low levels, within or at the top of the friction layer, this spin up is unconstrained by a radial momentum equation in this layer. Instead, the spin up is controlled by a parameterization of turbulent mixing in the upper tropospheric outflow layer, which, as is shown, determines indirectly the rate of inward movement of the absolute angular momentum surfaces. Nevertheless, the physics of how upper-tropospheric mixing leads to spin up in the friction layer are unclear and, as discussed, may be irrelevant to spin up in Emanuel's model.

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On the hypothesized outflow control of tropical cyclone intensification

Michael T. Montgomery
(joint work with John Persing, Roger K. Smith)

In the talk I briefly reviewed the motivation for this study. The aim was to test the premise of a revised theory of tropical cyclone intensification proposed by Emanuel, 2012. The premise is that small-scale turbulence in the upper tropospheric outflow layer determines the thermal stratification of the outflow and, in turn, an amplification of the system-scale tangential wind field above the boundary layer.

Compared to the control experiment in which the small-scale, shear-stratified turbulence is parameterized in the usual way based on a Richardson number criterion, the vortex in a calculation without a parameterized representation of vertical mixing above the boundary layer has similar evolution of intensity. The experiments do not support the premise on which the new theory is based. The results would appear to have ramifications for recent studies that invoke the new theory.

Asymptotics: bulk microphysics, convective towers, and tropical cyclones

Rupert Klein
(joint work with Tom Dörffel, Sabine Hittmeir)

In [1, 2] the author has introduced a unified approach to the modelling of scale-dependent processes in the atmosphere based on methods of asymptotic analysis. Up until recently, the incorporation of moist process representations within that framework remained somewhat unsatisfactory, [3], as it violated one basic scaling relation for the stability of the atmospheric stratification adopted in [2] that seemed quite universally valid otherwise.

The first part of the lecture reported on recent work with S. Hittmeir, [4], in which we identify two distinguished limits for the parameters of a bulk microphysics closure scheme both of which are now entirely compatible with the general modelling framework.

An interesting initial result, derived in the same publication as a sample application of the new regime, was discussed in the second part of the lecture. It concerns the time evolution of towers of deep convection as often encountered in the tropics. According to this theory, in a narrow deep convective tower that develops on the characteristic time scale of deep convection ($\sim 30\,\text{min}$) buoyancy should be zero to leading order. Since in this regime, buoyancy involves additive contributions from entropy (potential temperature) perturbations, from the mass fraction of water vapor, and from the weight of condensed water in the form of rain, this leads to an algebraic constraint for these three variables. Since water vapor adopts its local only height-dependent saturation value, the constraint ties entropy perturbations and rain water content to each other algebraically. As both these quantities must satisfy independent vertical advection equations with the same vertical velocity as the carrier speed, the net result is an algebraic relation for the vertical velocity as a function of the rain amount. Re-inserted into the rain water transport equation, this generates a new Hamilton-Jacobi equation for this quantity that seems not to have been discussed in the literature so far. Numerical solutions of this equation reveal a mechanism by which a convective tower can sustain itself over substantially longer times than would be expected on the basis of just the vertical advection or terminal rain fall velocities.

The third part of the lecture wasx concerned with how the new knowledge on deep convective motions can be incorporated in the theory for developing tropical storms from Päsche et al. [5]. This theory describes atmospheric vortices that are nearly axisymmetric in every horizontal slice, but with height and time dependent centers of symmetry. It covers the self-induced motion of the vortex by providing an evolution equation for its centerline, coupled to a partial differential equation for the time evolution of the primary circulation velocity as a function of time, height and distance from the centerline. The theory also allows for prescription of external heat sources and sinks and one of its key conclusions was that a heating dipole aligned with the tilt vector would attenuate, while an anti-aligned heating dipole would amplify the vortex' primary circulation. These conclusions were corroborated recently in [6] through three-dimensional simulations set up to closely match the underlying assumptions of the asymptotic theory.

Incorporating the effects of ensembles of narrow convective towers in lieu of the externally prescribed vortex-scale heating patterns is surprisingly straightforward. The influence of external heating in the theory of Päsche et al. manifests itself through the impact of heating on the vertical velocity. All expressions for the centerline motion and the time evolution of the primary circulation that involve the heating pattern can all be cast in terms of just the heating-induced vertical velocity without accessing the explicit form of the heating functions. Now, with heating replaced by ensembles of deep convective towers, these expressions for the heating-induced vertical velocity perturbations are replaced with the average of the vertical velocity conditioned on being in side a convective tower.

Ongoing research work addresses the influence of heating/convection on the vortex tilt and the effects of the near-surface boundary layer on convection.

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Intensification of tropical cyclones by asymmetric diabatic heating

Tom Dörffel

(joint work with Ariane Papke, Rupert Klein, Piotr K. Smolarkiewicz)

Rapid Intensification of tropical cyclones (TC), i.e. the acceleration of wind speed by 15 knots per 24 hours, still is not fully understood and a large uncertainty in predicting TC strength at landfall. As incipient TCs exhibit strong vertical tilts and asymmetric convection patterns we examine the role of an asymmetric intensification mechanism which arises from the asymptotic analysis of the underlaying equations [1]. With the help of numerical simulations we show that the leading-order dynamics of governing equations is well captured by the asymptotic analysis and intensification by purely asymmetric heating can lead to intensification [2].

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Equatorial Waves and the Skill of NCEP and ECMWF Numerical Weather Prediction Systems

Juliana Dias

Despite decades of research on the role of moist convective processes in large-scale tropical dynamics, tropical forecast skill in operational models is still deficient when compared to the extratropics, even at short lead times. In this presentation, we present an overview of the current state of operational tropical and Northern Hemisphere (NH) model performance in the NCEP Global Forecast System (GFS) and ECMWF Integrated Forecast System (IFS). Our analysis reveal that, in general, initial conditions are reasonably well estimated in both forecast systems, as indicated by relatively good skill scores for the 6-24-h forecasts. However, overall, tropical precipitation forecasts in both systems are not considered useful by typical metrics much beyond 4 days. To quantify the relationship between precipitation and dynamical skill, space-time spectra and coherence of rainfall and divergence fields are calculated. It is shown that while tropical variability is too weak in both models, the IFS is more skillful in propagating tropical waves for longer lead times. In agreement with past studies demonstrating that extratropical skill is partially drawn from the tropics, a comparison of daily skill in the tropics versus NH suggests that in both models NH forecast skill at lead times beyond day 3 is enhanced by tropical skill in the first couple of days. This study indicates that the differences in physics used in each system, in particular, how moist convective processes are coupled to the large-scale flow through these parameterizations, appear as a major source of tropical forecast errors.

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A simple stochastic model of tropical waves

Scott Hottovy

There are a number of identifiable tropical atmospheric waves on various length and time scales. Some examples of these waves are the Kelvin, equatorial Rossby, inertial gravity, and Madden-Julian Oscillation (MJO) waves. Currently it is a challenge to capture all of these waves in one model. Here, a new stochastic linear model is presented which unifies the convectively coupled equatorial waves (Kelvin, Rossby, and IG) as well as the MJO. The keys to the model are two vertical levels of moisture, middle and lower tropospheric, which have different convective adjustment time scales and eddy diffusion of moisture. Furthermore, this model is used to infer the mechanism of anti-resonance as a possible explanation for the shaping of the tropical power spectrum. For example, anti-resonance may be the cause for the absence of Eastern inertial gravity waves.

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A Dynamical Stochastic Skeleton Model for the MJO and ENSO

Sulian Thual

(joint work with Andrew J. Majda , Nan Chen)

A simple dynamical stochastic model for the tropical ocean-atmosphere is proposed for major intraseasonal to interannual processes including the El Niño Southern Oscillation (ENSO) as well as the Madden-Julian Oscillation (MJO) and associated wind bursts. As compared to usual simple to intermediate models for the ENSO, the MJO and wind bursts are here solved dynamically instead of being prescribed or parameterized which provides their upscale contribution to the interannual flow as well as their modulation in return in a more explicit way. Such a model serves as a prototype "skeleton" for General Circulation Models (GCMs) that solve similar dynamical interactions across several spatio-temporal scales but usually show common and systematic biases in representing tropical variability as a whole. The most salient features of the ENSO, the wind bursts and the MJO are captured altogether including their overall structure, evolution and fundamental interactions in addition to their intermittency, diversity and energy distribution across scales. This includes a realistic onset of El Niño events with increased wind bursts and MJO activity starting in the Indian ocean to western Pacific and expanding eastward towards the central Pacific, as well as significant interannual modulation of the characteristics of intraseasonal variability. The model developed here also should be useful to diagnose, analyze and help eliminate the strong tropical biases which exist in current operational models.

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Moist effects on large-scale atmospheric dynamics

Mike Cullen

The talk covered two separate topics.

The first topic was a illustration of the effects of moisture on the large-scale behaviour of a comprehensive atmospheric model, the Met Office Unified Model (UM). This work is described fully by [3]. Theory shows that the large-scale behaviour of the governing equations, and thus of the UM, is accurately described by semi-geostrophic theory. In the tropics, geostrophic theory degenerates to the weak temperature gradient approximation. The pressure tendency and ageostrophic wind can then be calculated from the pressure fields and the forcing terms from the model physics, which include the effects of moisture. In the tropics, the resulting ageostrophic wind is that required to maintain the weak temperature gradient above the boundary layer. This balance is discussed in [4]. The calculation is linear in the forcing, so that the effects of individual physical processes can be determined. The response to forcing depends on the large-scale state. In particular, the response is greater in areas of weak stability.

More insight into the effects of moisture can then be obtained by incorporating moist effects into the calculation of the response to forcing. Latent heating now reduces the stability of the large-scale state, so can be inferred as part of the response to forcing. Deep convection can also be regarded as part of the response to forcing, and then the vertical motion calculated from the theory is replaced by a convective mass flux as illustrated in [5], section 3. These illustrations show that the effective moist stability is the critical parameter for estimating the response of a stable moist atmosphere to forcing. The deep convection estimate depends on the instability of the profile to parcel ascent, and is then determined by quasi-equilibrium.

The second topic was a rigorous mathematical analysis of the moist rearrangement problem. This is a simple model of deep convection in a single column, and assumes that the convective timescale is much faster than that of the driving forcing. In the dry case, there is a unique stable rearrangement of a column of air with given potential temperature. This is not true in the moist case, as moisture only matters when there is saturation. However, the evolution of a moist column subjected to bulk ascent can be calculated in a well-defined way by applying standard parcel theory. Particular features are that the effect of convective inhibition controls the convective mass flux, and that the solution is fundamentally probabilistic where moist parcels which have convected to their level of neutral buoyancy mix with the pre-existing dry parcels. There is no way of controlling the scale of the mixing. A rigorous version of this theory is given by [1], and a more descriptive version with computations by [2].

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A potential vorticity perspective on the influence of latent heating on midlatitude cyclones

Stephan Pfahl

(joint work with Dominik Büeler)

Diabatic processes, in particular the release of latent heat (LH) during cloud formation, can strongly influence the dynamics of weather systems in the midlatitude storm tracks. Future changes in the atmospheric moisture content and associated LH may thus also lead to changes in the dynamical properties of these systems, but the magnitude of this effect is currently not well known. Here we discuss how the potential vorticity (PV) framework can be useful to investigate and quantify such diabatic effects on weather systems. LH in ascending air streams induces positive PV anomalies in the lower and middle troposphere, which can contribute to the intensification of midlatitude cyclones. Recently, a simplified diagnostic method has been introduced that quantifies this contribution of LH to cyclone dynamics (Bueler and Pfahl, 2017). To this end, the two leading terms in the PV tendency equation, diabatic PV modification and vertical advection, have been used to derive a diagnostic equation to explicitly calculate the fraction of a cyclone's positive lower-tropospheric PV anomaly caused by LH. Here we show that this theory provides a useful framework to understand the increasing importance of LH for cyclone intensification in idealized "aquaplanet" simulations of warmer climates. In these simulations, cyclone intensity increases with warming due to the continuous increase in LH, reaches a maximum in climates warmer than present-day, and decreases beyond a certain warming once the increase of LH is overcompensated by the counteracting reduction in mean available potential energy. Because of their substantially stronger increase in LH, the most intense cyclones reach their maximum intensity in warmer climates than moderately intense cyclones with weaker LH. This suggests that future projections of the extreme tail of the storm tracks might be particularly sensitive to a correct representation of LH. Altogether, our results show that the PV framework can yield novel insights into the increasing importance of diabatic processes for midlatitude weather system dynamics in a warming climate.

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Developing atmospheric models on emerging architectures

Frank Giraldo
(joint work with Jeremy Kozdon, Lucas Wilcox)

The last time I visited Oberwolfach was in 2010 (MFO Report 34) where I gave one of the first talks on the Nonhydrostatic Unified Model of the Atmosphere [1] (NUMA, an NWP model) during its very nascent stage. 9 years later we are embarking on a new project called CLIMA (the Climate Modeling Alliance – the paper outlining this project is [4] and the project website is `https://github.com/climate-machine`) with the goal to build atmospheric and oceanic components and subgrid-scale parameterizations that are accurate and performant on emerging computer hardware architectures such as CPU/GPU hybrids. We are currently living in a golden era of atmospheric model development. Many operational NWP and climate centers are rewriting codes due to: 1) moving from hydrostatic to nonhydrostatic equations and 2) the changing high-performance computing (HPC) landscape. So the question that we need to concern ourselves with is how do we write these new models that will allow us to harness the power of these new computers? Do we use standard application programming interfaces or APIs (e.g., OpenACC) or do we rely on domain specific languages or DSLs (e.g., Gridtools - see `http://eth-cscs.github.io/gridtools`). For now, we propose to venture forward using Julia as our programming language with a combination of MPI and CUDA because the current hardware on the fastest machines in the U.S. are Nvidia GPU machines. However, to insulate ourselves from having to refactor too much code in the future, we should strive to build compute-kernels that work (as much as possible) on CPUs and GPUs and perhaps future hardware. To do so, we follow the model of separating local compute-intensive operations with lean communication stencils. The model for this approach can be described quite easily with the discontinuous Galerkin method that requires local compute-intensive volume integrals and nearest neighbor communication flux integrals. If this approach is maintained throughout the code then, in principle, it will be straightforward to adopt these codes to future hardware (such approaches can be found in [2, 3]).

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Dynamics of Atmospheric Models and Thermodynamics

Matthias Hieber

Many atmospheric models are described by a set of equations for the density ϱ, velocity u, temperature θ and moisture q of the underlying fluid. The evolution of q is often described by a saturation term involving the Heaviside function, yielding then numereous difficulties in the analysis of these equations. Aiming for thermodynamical consistent models, we transfer techniques from modeling of two-phase flows with phase transitions to the situation of atmospheric models. The thermodynamical consistency of a model may in particular be used as a selection mechanism for those atmospheric models for which analytical well-posedness is being expected.

On solvability of gas dynamics equations

Eduard Feireisl

We present several examples of ill–posed problems in the context of the full Euler system of gas dynamics. These involve the Euler system driven by a potential force and the Euler system driven by a stochastic bulk force. In both cases, the problem admits infinitely many solutions for a large class of initial data.

Solution semiflow to the isentropic Euler system

Martina Hofmanová

It is nowadays well understood that the multidimensional isentropic Euler system is desperately ill–posed. Even certain smooth initial data give rise to infinitely many solutions and all available selection criteria fail to ensure both global existence and uniqueness. We propose a different approach to well–posedness of this system based on ideas from the theory of Markov semigroups: we show the existence of a Borel measurable solution semiflow. To this end, we introduce a notion of dissipative solution which is understood as time dependent trajectories of the basic state variables - the mass density, the linear momentum, and the energy - in a suitable phase space. The underlying system of PDEs is satisfied in a generalized sense. The solution semiflow enjoys the standard semigroup property and the solutions coincide with the strong solutions as long as the latter exist. Moreover, they minimize the energy (maximize the energy dissipation) among all dissipative solutions. A joint work with Dominic Breit and Eduard Feireisl.

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The equations of the multi-phase humidity atmosphere with instantanenous vapor-to-cloud water conversion

Yining Cao

(joint work with M. Hamouda, R. Temam, J. Tribbia and X. Wang)

We propose in [CHTTW18] a new formulation of the equations of the humid atmosphere with a multi-phase saturation, generalizing the models studied in earlier works by M. Coti-Zelati, J. Tribbia, R. Temam and others (see [CFTT13], [CHKTZ15], [CT12], [TT16], [TWa16] and [TWu15]). More precisely, we consider the situation where the humid quantities in the clouds comprise the water vapor, the cloud-condensates, and rain water with relative mass densities q_v, q_c and q_r, respectively. Meanwhile, the saturation vapor concentration q_{vs} is a diagnostic variable depending itself on the state (i.e., the temperature T and pressure p). The model equations for q_v, q_c, q_r and for the temperature T (or more precisely the difference $\theta^{'}$ between the potential temperature θ and a reference temperature θ_h, $\theta^{'} = \theta - \theta_h$) are based on [KW78] and [MP74]. When considering the parameterization of the condensation of water vapor to cloud water and the inverse evaporation process, it is often assumed, in particular for warm clouds, that the vapor-to-cloud water conversion is instantaneous, see e.g., [Gra98]. Accordingly, the water vapor mass ratio q_v satisfies the constraints $q_v \leq q_{vs}$, as the air can not be supersaturated in general when cloud water is formed instantaneously. In view of the above assumption and the constraint, we are led, from the mathematical point of view, to introduce and handle a system of equations and inequations involving some *quasi-variational inequalities.* By penalization techniques and careful energy estimates, we prove the global existence of solutions satisfying the constraint $q_v \leq q_{vs}$ with compactness argument. In the meantime, a different model for multi-species humid atmosphere was introduced in [KM06] and studied from the mathematical viewpoint in the recent article [HKLT17]. In that model, the authors do not assume the limiting instantaneous vapor-to-cloud water conversion behavior from the outset and demonstrate how it may be derived in a consistent asymptotic framework given large but finite condensation rates. This is the main deviation of the bulk microphysics description in [KM06] from the scheme we study here, but the mathematical routes are quite different.

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On algebraic aspects of fluid dynamics based on Nambu mechanics

Annette Müller

The Nambu formulation for three-dimensional vortex dynamics is introduced to gain a comprised, algebraic description of vortices. Nambu mechanics can be seen as a generalization of Hamilton's formulation. The equations of motion are represented in terms of two constitutive conserved quantity: the energy and the helicity. By formulating the Helmholtz vorticity equation in terms of Nambu mechanics the algebraic structure for incompressible, inviscid fluids can be analyzed. To explore the algebraic structure a matrix representation of the Lie algebra, respectively Lie group, for three-dimensional vortex dynamics is represented. In this way, incompressible, inviscid vortex dynamics can be regarded from a different perspective. As an example of the applicability of the corresponding vortex Lie group to atmospheric phenomena the onset of splitting storms is analyzed.

Energy cycle of convection

Jun-Ichi Yano

A general formulation is outlined, and a simplified version originally presented by Arakawa and Schubert (1974, JAS) in context of the convection parameterization problem is presented (Yano and Plant 2012a, b, QJ, JAS). A basic point that convection is driven by buoyancy is emphasized (Yano et al. 2005 QJ). The energy–cycle system consists of a set of equations for the kinetic energy and the cloud work function. The latter may furthermore be reduced to the convective available potential energy (CAPE) under certain approximations. The convective kinetic–energy equation is carefully derived in Yano (2015 DAO), and that for the cloud work function is derived in the Appendix B of Arakawa and Schubert (1974).

When only a single convective mode is considered, this system can simulate a sudden onset of convection under a cycle of recharge and discharge without invoking any specific triggering mechanism (such as cold pool) nor stochasticity. Rather, a nonlinearity of the buoyancy–driven kinetic–energy generation simply leads to a sudden onset of convection (Yano and Plant 2012 QJ). Implications from this system is discussed in Yano and Ouchtar (2017 QJ).

When the analysis is extended to two modes consisting of shallow and deep convection, a qualitative difference between these two types of convection becomes important (Yano and Plant 2012 JAS, Plant and Yano 2013 DAO): deep convection stabilizes the atmosphere by environmental adiabatic descent that warms the atmosphere, thus both shallow and deep convection are damped. On the other hand, shallow convection, due to its only weakly precipitating nature, moists the environment by detraining its cloudy air into the environment. The detrained cloudy air re-evaporates in the environment, and the resulting the evaporating cooling tends to destabilizes the atmosphere, and tend to enhance both shallow and deep convection. Thus, when two type of convection interacts each other, shallow convection tends to promote both shallow and deep convection: destabilization tendency of shallow convection naturally induces deep convection, that provides a simple explanation for the transition from the former to the latter.

Towards a Stochastic Relaxation for the Quasi-Equilibrium Theory of Cumulus Parameterization: Multicloud Instability, Multiple Equilibria and Chaotic Dynamics

Boualem Khouider
(joint work with Etienne Leclerc)

The representation of clouds and organized tropical convection remains one of the biggest sources of uncertainties in climate and long-term weather prediction models. Some of the most common cumulus parameterization schemes, namely mass-flux schemes, rely on the quasi-equilibrium (QE) closure, which assumes that convection consumes the large scale instability and restores large scale equilibrium instantaneously (Arakawa and Schubert 1974). However the QE hypothesis has been challenged both conceptually and in practice (Bechtold et al. 2004). In existing work, the QE assumption was relaxed and instead prognostic equations for the cloud work function (CWF) and the cumulus kinetic energy (CKE) were derived and used (Pan and Randall, 1998). It was shown that when the CWF kernel merely acts to decrease the CWF, the prognostic closure system leads to damped oscillations on a time scale of a few hours, giving parameterized cumulus clouds enough memory to interact with each other, with the environment, and with stratiform anvils in particular. Here we take a few steps forward and show that when cloud-cloud interactions are reintroduced into the CWF-CKE equations, the coupled system becomes unstable. More importantly, a refinement for the CWF-CKE prognostic closure including their coupling to the mean field equations, from a stochastic multicloud model (SMCM; Khouider, 2004), for the cloud area fraction (CAF) is proposed. Qualitative analysis and numerical simulations show that in the case of a single cloud type, the CKE-CWF-CAF equations exhibit interesting dynamics including multiple equilibria, limit cycles, and chaotic behaviour both when the large-scale forcing is held fixed and when it oscillates with various frequencies, representative of cumulus convection unresolved variability.

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Convective Momentum Transport and Multiscale Organization in Simulated Shear Parallel Mesoscale Convective Systems

Ying Han
(joint work with Boualem Khouider)

The way in which moist convection interacts with large-scale flows is a major contemporary research issue. Organized mesoscale systems are, in particular, a medium through which small scale convective cells interact with the ambient shear. Here we present numerical simulations of mesoscale systems evolving in a background shear using the Research and Weather Forecasting (WRF) model. We are particularly interested in the long time integration, allowing the systems to repeatedly develop and die and effectively interact with the background shear. Starting with a typical African and equatorial jet shear, the simulated solution goes through various phases or stages. First, a transient state, consisting of scattered squall -like systems that are aligned perpendicular to the background shear, develops and then evolves into a regime of multiscale mesoscale systems with large stratiform anvils. During the latter period the background wind changes substantially through the effect of both up scale and down scale convective momentum transport (CMT), resulting in a first baroclinic flow structure. At this stage, the mesoscale systems turn around and become aligned parallel to the wind shear, with elongated stratiform anvils in which meso-beta scale convective cells evolve and propagate parallel to the shear direction, with much slower speeds compared to the main stratiform envelope, which moves at a much faster speed consistent with an upper level steering level wind. These results are reminiscent of the development of shear parallel mesoscale convective systems observed for instance in the Eastern Pacific ITCZ and corroborate recent theoretical results obtained with a simple multi-cloud model; as such they have important implications for the parameterization of CMT in climate models.

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The second law of thermodynamics as constraint for atmospheric modeling

Almut Gassmann

Atmospheric models should obey several laws such as mass conservation, energy conservation and the second law of thermodynamics. The ability to which current models fulfill the second law of thermodynamics in generating internal entropy is scrutinized. Atmospheric moist flow models include several subgrid-scale fluxes, such as the momentum flux, the heat flux, the turbulent and diffusive (sedimentation) fluxes and phase changes. The contributions of those fluxes to internal entropy production must be independently positive for different tensor orders of the fluxes. This is checked for state of the art model formulations. It results that momentum fluxes are well represented with a nonlinear Smagorinsky tensor formulation. Turbulent water vapour fluxes must be formulated depending on gradients of partial pressures of water vapour and dry air instead of being downgradient with respect to the specific water vapour content. The difference between those two mentioned formulations is small and current modeling is about to be right. The phase changes are well represented in numerical models. Rain or other sedimentation fluxes lead to positive entropy production. For reflecting this process correctly in models, the prognostic velocity must be barycentric and the non-sedimenting constituents must have a diffusive velocity upwards such that the mass flux control condition $\sum_i \mathbf{J}_i = 0$ (i summing all constituents including the sedimenting ones and the non-sedimenting ones) is fulfilled.

The most problematic flux formulation is the heat flux formulation. The second law requires this flux downgradient with respect to temperature. At unstable stratification, the current formulation which is downgradient with respect to potential temperature is suitable. But at stable stratification, this downgradient formulation leads to internal entropy destruction instead of entropy production. Indeed, a downgradient potential temperature flux leads to erroneous amplifying wave solutions at stable stratification. Nevertheless, the physical intention to diffusive potential temperature instead of temperature is correct. Potential temperature diffusion reflects both, a heat (temperature) diffusion and a buoyancy production or loss (subgrid-scale work) term. Subgrid-scale turbulent kinetic energy (tke) is generated from internal energy at unstable stratification. Both energies (tke and internal energy) are subgrid-scale at unstable or slightly stable stratification. Therefore contemporary downgradient fluxes of potential temperature with a slight countergradient contribution are correct. But the related temperature diffusion coefficient must not become negative. If buoyancy is lost when the air is truly stably stratified, grid-scale(!) kinetic energy is lost and transformed into internal energy. This is likewise associated with a subgrid-scale downgradient potential temperature flux, but then, the diffusion coefficient depends on the kinetic energy of vertical motions which can be transferred into heat. The described formulation of the heat flux at stable stratification has been implemented into a model and it could be shown that spurious amplifying solutions do no longer appear.

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Uncertainty Quantification in Cloud Flows

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(joint work with Alina Chertock, Alex Kurganov, Peter Spichtinger, Bettina Wiebe)

We have developed a new computational model for solving moist three-dimensional flow dynamics including clouds, see [1], [7]. The model is based on the dynamic core, governed by the Navier-Stokes equations for *weakly compressible flows*, which is coupled to the recently developed new cloud model with a consistent representation of cloud processes [9]. The cloud model for pure water clouds includes the condensation (C), evaporation (E), autoconversion (A_1), accretion (A_2) and sedimentation:

$$
\begin{aligned}
\partial_t \rho' + \nabla \cdot (\rho \mathbf{u}) &= 0, \\
\partial_t (\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + p' \mathrm{Id} - \mu_m \rho \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right)\right) &= -\rho' g \mathbf{e_3}, \\
\partial_t (\rho\theta)' + \nabla \cdot (\rho \theta \mathbf{u} - \mu_h \rho \nabla \theta) &= S_\theta,
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\partial_t(\rho q_v) + \nabla \cdot (\rho q_v \mathbf{u} - \mu_h \rho \nabla q_v) &= \rho(-C + E), \\
\partial_t(\rho q_c) + \nabla \cdot (\rho q_c \mathbf{u} - \mu_h \rho \nabla q_c) &= \rho(C - A_1 - A_2), \\
\partial_t(\rho q_r) + \nabla \cdot (-v_q \rho q_r \mathbf{e_3} + \rho q_r \mathbf{u} - \mu_h \rho \nabla q_r) &= \rho(A_1 + A_2 - E).
\end{aligned} \tag{2}
$$

Here ρ is the density, ρ' is the density perturbation, i.e. $\rho = \overline{\rho} + \rho'$, where $\overline{\rho}$ is the underlying (hydrostatic) equilibrium density. Analogous notation holds for the moist potential temperature θ and the pressure p. Furthermore, $\mathbf{u}$ denotes the velocity vector, g the acceleration due to gravity, μ_m the dynamic viscosity, μ_h the thermal conductivity and $\mathbf{e_3} = (0,0,1)^T$. The mass densities of water vapor, cloud water and rain are denoted by q_v, q_c and q_r, respectively.

Our numerical method for (1), (2) is based on a higher order finite volume method for space discretization. In order to efficiently resolve multiscale dynamics we split the whole nonlinear system in a stiff linear part governing the acoustic and gravitational waves as well as diffusive effects and a non-stiff nonlinear part that models nonlinear advection effects. We use a stiffly accurate *second order* IMEX scheme for time discretization to approximate the stiff linear operator implicitly and the nonstiff nonlinear operator explicitly. Fast microscale cloud physics is approximated by small scale subiterations. For the latter the *third order* explicit Runge-Kutta method, the so-called DUMKA 3 method [8] with enlarged stability region, has been applied. Explicit coupling between the weakly-compressible Navier-Stokes system (i.e. sound-proof model) and the cloud model is realized through the source term S_θ in the equation for potential energy that expresses the released or absorbed latent heat. Such a well-chosen combination of modern discretization techniques improves stability properties and thus yields an efficient simulation tool for further investigations, see [7]. In order to analyse the influence of uncertain parameters or initial/boundary data we have applied to our 3D cloud model the so-called generalized polynomial chaos stochastic Galerkin method, see [2]. The main idea of the *stochastic Galerkin method* lays in the extension of the classical space-time Galerkin-type approximation to the stochastic space by means of the spectral element method. In our recent paper [2] we have applied the polynomial chaos expansion and the stochastic Galerkin method to the convection-diffusion-reaction equations (2) for the mass densities of water vapor, cloud water and rain q_v, q_c and q_r. Consequently, the latter are the functions of time t, space x and a random variable, denoted by ω, i.e. $(\rho q_i)(x,t,\omega)$ for

$i \in \{v, c, r\}$ and the system (2) now reads

$$
\begin{aligned}
(3) \qquad & \partial_t(\rho q_v)(\omega) + \nabla \cdot (\rho q_v(\omega)\mathbf{u} - \mu_h \rho \nabla q_v(\omega)) = \rho(-C(\omega) + E(\omega)) \\
& \partial_t(\rho q_c)(\omega) + \nabla \cdot (\rho q_c(\omega)\mathbf{u} - \mu_h \rho \nabla q_c(\omega)) = \rho(C(\omega) - A_1(\omega) - A_2(\omega)) \\
& \partial_t(\rho q_r)(\omega) + \nabla \cdot (\rho q_r(\omega)(-v_q(\omega)\mathbf{e_3} + \mathbf{u}) - \mu_h \rho \nabla q_r(\omega)) \\
& \qquad\qquad = \rho(A_1(\omega) + A_2(\omega) - E(\omega)).
\end{aligned}
$$

To the best of our knowledge, this is the first study where the gPC-SG method has been applied and generalize for atmospheric models. The stochastic Galerkin method allows us to take into account the uncertainty of (initial, boundary) data or model parameters and omit myriads of Monte-Carlo-type simulations for ensembles. It offers an interesting alternative for explicit evolution of the model uncertainties and their quantification.

Another topic of interest is the *rigorous convergence analysis* of “classical” finite volume or finite element methods for multidimensional compressible flows, such as the Navier-Stokes-Fourier system or the Euler system. In general, this question has been open for decades and just recently we were able to show rigorously schemes’ convergence by using the concept of dissipative measure-valued solutions. We have derived suitable stability and consistency estimates, which imply that the Young measure generated by numerical solutions represents a dissipative measure-valued solution of the limit system. In particular, using the dissipative measure-valued–strong uniqueness principle we obtain that the numerical solutions converge strongly to a strong (i.e. classical, smooth) solution of the limit system as long as the latter exists [3–6].

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Latest approaches of improving moist process parameterization in high resolution climate model

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Prediction of spatio-temporal distribution of Indian Summer Monsoon is crucial for the economy and growth of the country. While significant improvement has been done in monsoon research, realistic prediction of extreme rain spells and the intraseasonal variability still remains a challenge. One of the key processes that controls the evolution of space-time distribution of monsoon rain is the moist processes vis-à-vis the cloud and convective processes. We have recently made several attempts including the super- parameterization (Goswami et al. 2015) and stochastic multi-cloud parameterization (Goswami et al. 2017) where we have improved the representation of moist processes related to cloud and convection. Apart from making climate free run, we have made reforecast with the modified moist processes which have showed significant improvement in rainfall skill of CFSv2 model. Along with the improvement in physics (Abhik et al. 2017), we have made suitable modification in dynamic core of the model and resolution of the model (Mukhopadhyay 2019), through which we have intended to make a holistic improvement in the monsoon rainfall forecast.

A comprehensive improvement of monsoon rainfall skill has been achieved through the improved representation of sub-grid scale and grid-scale moist processes in the CFSv2 model.

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Stochastic Multi-cloud Model (SMCM) in the Climate Forecast System Model (version 2)

Bidyut Bikash Goswami

(joint work with R. Phani, P. Mukhopadhyay, B. Khouider, A. Majda)

General circulation models (GCM) show limitations of various sorts in their representation of synoptic and intra-seasonal variability associated with tropical convective systems. This systematic deficiency is believed to be due to inadequate treatment of organized convection by the underlying cumulus parameterizations.

Here, we discuss the implementation of the stochastic multicloud model (SMCM) in NCEP Climate Forecast System model version-2 (CFSv2) through the use of a simple parametrization of adiabatic heating and moisture sink due to cumulus clouds based on their observed vertical profiles. The SMCM mimics the interactions between the three cloud types, congestus, deep, and stratiform, that are observed to play a central role across multiple scales in the dynamics and physical structure of tropical convective systems. It is based on a stochastic lattice model, overlaid over each GCM grid box, where an order parameter taking the values 0,1,2,3 at each lattice site according to whether the site is clear sky or occupied by a congestus, deep, or stratiform cloud, respectively. As such the SMCM mimics the unresolved variability due to cumulus convection and the interactions across multiple scales of organized convective systems, following the philosophy of superparameterization.

A 17 year simulation showed tremendous improvements in the ability of the CFSsmcm model to represent synoptic and intraseasonal variability associated with organized convection as well as a few minor improvements in the simulated climatology when compared to the control CFSv2 model which is based on the widely used simplified Arakawa-Schubert parameterization.

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Regional Superparameterization with LES

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(joint work with Pier Siebesma, Fredrik Jansson, Gijs van den Oord, Inti Pelupessy, Johanna Gronqvist, Maria Chertova)

Superparamaterization (SP) is a type of multiscale modeling framework in atmospheric science in which local high-resolution Cloud Resolving Models (CRMs) are nested inside the model columns of a global General Circulation Model (GCM). By using the CRMs, the use of traditional parameterizations can be avoided. SP is computationally very expensive but well-suited for massive parallelization.

We are developing a new SP set-up in which SP can be aplied in a selected region, making it possible to use high-resolution 3-dimensional Large Eddy Simulation (LES) models as CRMs that fill the entire GCM column [1]. This is in contrast with previous work on SP, in which CRMs are nested in every GCM column, however the CRMs are simplified (e.q. 2-dimensional) or have very small grids in order to reduce the computational cost. By applying SP only in selected columns, and using the standard parameterizations in the other columns, we can use full-blown high-resolution 3-dimensional LES on large grids as CRMs. We have implemented our set-up using the Dutch Atmospheric LES (DALES) model as CRM and the OpenIFS model from ECMWF as GCM.

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Coupling a neural network parameterization to a GCM

Noah D. Brenowitz

One of the most challenging problems in climate and weather prediction is how to represent unresolved processes, such as rainfall and clouds, in coarse-resolution atmospheric models. In the past, scientists constructed these parameterizations based on conceptual models, an approach which cannot easily incorporate new insights from cloud resolving models and observational datasets. At the same time, these datasets can be readily exploited by machine learning techniques, such as neural networks. In this talk, I present such a parametrization trained to represent all unresolved and untreated physics. The biggest challenge is to ensure that this scheme remains numerically stable when coupled to atmospheric fluid dynamics. We do this by 1) training the scheme to minimize the error accumulated over several time steps, and 2) removing physically implausible connections between inputs/outputs.

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Power-Law Behavior of Hourly Precipitation

Christian L. E. Franzke

(joint work with Lichao Yang)

Precipitation is an important climatic variable and critical for weather risk assessments. For instance, intense short precipitation events can lead to flash floods and land-slides. Most modeling studies assume that the occurrence of precipitation events is based on a Poisson process with exponentially distributed waiting time and precipitation intensity described by a gamma distribution or a mixture of two exponential distributions. Here, we use hourly precipitation data over the United Sates to show that the waiting time between precipitation events is non-exponentially distributed and best described by a fractional Poisson process. A systematic model selection procedure reveals that the hourly precipitation intensities are best represented by a two-distribution model for about 90% of all stations. The two-distribution model consists of (i) a Generalized Pareto Distribution (GPD) model for bulk precipitation event sizes and (ii) a power-law distribution for large and extreme events. Finally, we analyze regional climate model output to evaluate how the climate models represent the high-frequency temporal structure of United States precipitation. Our results reveal that these regional climate models fail to accurately reproduce the power-law behavior of intensities and severely underestimate the long durations between events [1]. We show evidence that the power-law distributed waiting times might be due to atmospheric persistent regime behavior [2].

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Impact of Diurnal Warm Layers on air-sea fluxes and convection in a GCM: The CINDY/DYNAMO case.

Xavier Perrot

(joint work with H. Bellenger, J.P. Duvel , L. Guez , Y. Zhang, S. Xie)

Due to the lack of vertical mixing, shallow thermo-haline stable stratifications, corresponding to changes up to few degrees and several psu within the first tens of centimeters, forms when the wind is weak. These changes are induced by the absorption of solar radiation in the case of Diurnal Warm Layers (DWL) and by the addition of cool and fresh rainwater in the case of Cool Freshwater Lenses (CFL). By changing the ocean skin temperature, these phenomena can perturb the heat exchanges at the ocean interface and thus have an impact on the atmospheric boundary layer structure and on convection. In particular, DWL of 1 to 3°C are frequently observed in the deep tropics where they can destabilize the atmosphere and trigger convection in the afternoon. Less is known however about CFL characteristics and on their impact on atmosphere processes. The international CINDY/DYNAMO field campaign took place in the equatorial Indian Ocean in winter 2011-2012 to observe the complete lifecycle of Madden-Julian Oscillation (MJO) in the region where it usually originate. Three MJO events were documented. During this campaign, several large DWL and some CFL were observed from the R/V Revelle situated on the equator at 80°E. ECMWF analysis constrained by R/V Revelle precipitation radar measurements are used to derived large-scale forcing terms necessary to force single column models and limited area models. A single column version of the LMDZ atmospheric general circulation model coupled to a simple model for DWL and CFL is then used to study the impact of representing DWL and CFL. In particular, we focus on their impact on the simulated atmospheric tendencies in temperature and moisture associated with different atmospheric processes: boundary layer turbulence, boundary layer thermals (shallow convection) and deep convection. The first results for the DWL show a good accordance between the diurnal heat fluxes evolution in the single column simulation and the data from the zone for the days with DWL. In addition we observe an interesting link between the tendencies for the boundary layer and the thermals as the day evolve in case of DWL.

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Coherent set analysis for Lagrangian trajectory data using diffusion maps

Robert Malte Polzin
(joint work with Péter Koltai)

Dynamical systems often exhibit the emergence of finite-time coherent sets. These are regions in state space that keep their geometric integrity to a high extent. A method for extracting coherent sets from possibly sparse Lagrangian trajectory data for embedded domains is presented. The challenge is that if parts of a domain of interest are distorted by the dynamics in a way that is inconsistent with our view of coherence. Examples in geophysical systems are ocean eddies, ocean gyres and atmospheric vortices. The method can be seen as an extension of diffusion maps to trajectory space. It allows to construct dynamical coordinates. These reveal the intrinsic low-dimensional organization of the data with respect to transport. The only a priori knowledge about the dynamics that we require is a locally valid notion of distance, which renders our method highly suitable for automated data analysis. Its potential on a two-dimensional example is presented.

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