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Mini-Workshop: Mathematics of Dissipation – Dynamics, Data and Control (hybrid meeting)

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Abstract. Dissipation of energy — as well as its sibling the increase of entropy — are fundamental facts inherent to any physical system. The concept of dissipativity has been extended to a more general system theoretic setting via port-Hamiltonian systems and this framework is a driver of innovations in many of areas of science and technology. The particular strength of the approach lies in the modularity of modeling, the strong geometric, analytic and algebraic properties and the very good approximation properties.

Mathematics Subject Classification (2010): 35K90, 37J25, 41A25, 47D06, 49K20, 49K40, 65K05, 93D20, 94A12.

Introduction by the Organizers

This workshop was organized by Sara Grundel (MPI Magdeburg), Volker Mehrmann (TU Berlin), Jacquelien M.A. Scherpen (U of Groningen, Netherlands), and Felix L. Schwenninger (U Twente, Netherlands, and U Hamburg). The workshop topics represented, in particular, physics and data based modeling, with a strong focus in model order reduction, numerical simulation and control methods, for linear and nonlinear, finite and infinite dimensional problems. Applications included the distributed control of smart energy systems, such as the power grid or gas transport.

Dissipativity is pivotal for a variety of topics — including physics-oriented modeling (see, e.g., thermodynamics, conservation of energy), numerics, optimization,

and control. In the latter, it serves as the canvas of seminal concepts such as Lyapunov stability and passivity. In optimization it triggered the development of semi-definite programming to tackle linear matrix inequalities and in control theory it is at the forefront of approaches to robust design concepts pioneered by Kalman, Yakubovich and Popov. Whereas energy-based modeling has a long tradition, its framework character is key to taming the complexity of large-scale coupled systems. Indeed, the concept of dissipativity allows to easily interconnect various system components and to describe their characteristic properties. For instance, port-Hamiltonian systems leverage this advantageously. Yet, fundamental obstacles remain — prime examples of open problems are a unifying theory handling evolution equations in higher-dimensional domains analytically and numerically. This gap becomes even more pronounced for highly nonlinear and/or infinite-dimensional systems. Hence, deriving a mathematically sound abstraction formalism which paves the way towards unified methods in numerics, optimization, and control is of crucial importance. Here, preserving phenomena peculiar for infinite-dimensions requires profound theoretical tools from different mathematical fields including, e.g., complex analysis, operator theory, and partial differential equations.

Dissipation is also omnipresent in theoretical mathematical developments; such as contraction concepts playing a fundamental role in operator algebras, abstract differential equations, and the qualitative study of numerical schemes. The influential work by von Neumann continues to stimulate research in areas, as for instance the work on Crouzeix's conjecture. The latter strongly links Blaschke products as the building blocks of function theory to convergence rates for Krylov methods, hindered by the enigmatic properties of numerical ranges, which in turn relates to algebraic geometry. Further challenges arise when additional exogenous inputs are taken into account – either as controls or as disturbances. In this context, dissipativity has crucially influenced robust control design. Recently, strong links between properties of parametric problems in calculus of variations/optimal control and abstract dissipation inequalities have been established. The dissipativity approach proved to be pivotal in analyzing turnpike properties in optimal control problems. In turn, the turnpike property can also be leveraged for tailored numerical solution strategies for optimal control problems for partial differential equations.

It stands to reason that dissipativity is foundational for dynamic phenomena in thermodynamics, electrodynamics and beyond. That is, it appears in the equations of Maxwell and Navier-Stokes alike. On the other hand, the above described universal language offered through port-based modeling paves the way for elaborating multi-physics applications ranging from mechatronics (e.g. robotics) to coupled gas and energy networks. However, in the currently dawning age of an information rich world, the ubiquity of data, measurement devices, and embedded control units is in the process of becoming a presumption, if not a conditio sine qua non in manifold research activities in applied mathematics and beyond. While data-driven methods as such will lead to – and already have led to – seminal

progress in various application domains, this development carries the risk of unnecessarily distancing the analysis, design and control of dynamic systems at large from fundamental dissipation principles and the underlying mathematics. It will, and to a certain extend already is, a promising topic to leverage hybrid models including available data while not losing the physical knowledge gained.

To summarize, this mini-workshop investigated the mathematics of dissipation, which is a core concept to ensure robust modeling as well as robust numerical computation. In doing so, it addressed the gap between first principle-based approaches and the rapidly growing branch of data-driven methods.

This workshop had a focus in 1) Applications, 2) Control, Numerics and Optimization, 3) Data-Driven Methods 4) Infinite-Dimensional Systems, and 5) Model Order Reduction. This was also how the five days of the week were distributed initially. Each day had two to four talks and on Wednesday and Thursday we also had longer brainstroming sessions about research directions and open problems were discussed.

Discussed research directions: While the port-Hamiltonian framework in the setting of a linear ordinary differential equation is entirely clear, and extensions to nonlinear, infinite dimensional as well as the differential algebraic setting exists, some of those extensions are not yet developed in all generality. Aspects of that are, for instance, control design, delay equations, stochastic PDEs, discrete-time. We furthermore discussed the potential of control design in a pH network and also that we may have to rethink standard approaches in optimal control problems. Another topic was the link between gradients systems and pH systems.

Mini-Workshop (hybrid meeting): Mathematics of Dissipation – Dynamics, Data and Control

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Abstracts

The influence of the Hamiltonian in structure-preserving model reduction

Tobias Breiten

(joint work with Riccardo Morandin, Philipp Schulze, Benjamin Unger)

Computing reduced-order models for large-scale systems that arise, e.g., from a spatial semi-discretization of a partial differential equation has become one of the standard techniques for efficient control and simulation of complex dynamical processes. In the case that the original model has a particular structure, one often aims at a structured surrogate model. This is particularly true for the class of port-Hamiltonian systems for which several structure-preserving techniques have been developed recently, see, e.g., [4, 5].

However, basically all existing methods ensure structure-preservation by utilizing the Hamiltonian matrix within the projection step. Proceeding this way obviously reduces the degree of freedom for a Petrov-Galerkin projection as one of the matrices is pre-determined by the Hamiltonian. Moreover, the approximation qualitiy implicitly depends on the specifically chosen realization of the port-Hamiltonian system, raising the question of *existence of an optimal Hamiltonian* with regard to model order reduction.

As is well-known [1, 6, 7], by solving a Kalman-Yakubovich-Popov linear matrix inequality (KYP-LMI) every minimal and passive system can be represented as a port-Hamiltonian system. In [2], this relation has been used to improve the performance of a modified version of the Q-conjugated LQG balanced truncation from [8, 9]. In essence, the idea is to represent the port-Hamiltonian in coordinates that lead to the Hamiltonian being one of the extremal solutions of the KYP-LMI. Based on similar considerations, in [3], a novel passivity-preserving model reduction method has been suggested. Its main idea is to approximate certain spectral factors, thus performing implicit model reduction of the underlying system.

Interestingly enough, while being optimal for model reduction purposes, extremal solutions of the KYP-LMI are very sensitive with respect to perturbations, [1]. Consequently, finding appropriate port-Hamiltonian formulations may suffer from concurrent goals (e.g., sensitvity vs approximability) and should be investigated in more detail. In particular, finding appropriate representations for nonlinear port-Hamiltonian systems remain a largely open research field.

References

[1] C. Beattie, V. Mehrmann, and P. Van Dooren, *Robust port-Hamiltonian representations of passive systems*, Automatica, 100 (2019), pp. 182–186.

[2] T. Breiten, R. Morandin, and P. Schulze, *Error bounds for port-Hamiltonian model and controller reduction based on system balancing*, ArXiv e-print 2012.15266, (2020).

[3] T. Breiten, and B. Unger, *Passivity preserving model reduction via spectral factorization*, ArXiv e-print 2103.13194, (2021).

[4] S. Gugercin, R. Polyuga, C. Beattie, and A. van der Schaft, *Structure-preserving tangential interpolation for model reduction of port-Hamiltonian systems*, Automatica, 48 (2012), pp. 1963–1974.
[5] R. Polyuga and A. van der Schaft, *Effort- and flow-constraint reduction methods for structure preserving model reduction of port-Hamiltonian systems*, Systems & Control Letters, 3 (2012), pp. 412–421.
[6] J. Willems, *Dissipative dynamical systems part I: General theory*, Archive for Rational Mechanics and Analysis, 45 (1972), pp. 321–351.
[7] J. Willems, *Dissipative dynamical systems part II: Linear systems with quadratic supply rates*, Archive for Rational Mechanics and Analysis, 45 (1972), pp. 352–393.
[8] Y. Wu, *Passivity preserving balanced reduction for the finite and infinite dimensional port Hamiltonian systems*, PhD thesis, Universite Claude Bernard - Lyon 1, 2016.
[9] Y. Wu, B. Hamroun, Y. Le Gorrec, and B. Maschke, *Structure-preserving reduction of port Hamiltonian systems using a modified LQG method*, in Proceedings of the 33rd Chinese Control Conference, Nanjing, China, 2014, pp. 3528–3533.

Optimization-based Control of Energy Grids and Dissipativity? Ideas and Open Problems

Timm Faulwasser

The crucial link between the system-theoretic notion of dissipativity and optimal control is already evident in the foundational work of Jan Willems [1] and in his seminal paper on least-squares optimal control [2]. In a nutshell, for finite-dimensional control systems

$$\dot{x} = f(x,u), \quad x(0) = x_0 \in \mathbb{R}^n \tag{1}$$

dissipativity implies the existence of a storage function $S : \mathbb{R}^n \to \mathbb{R}$ and a corresponding supply rate $\omega : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ such that the dissipation inequality

$$S(x(T)) - S(x_0) \leq \int_0^T \omega(x(t),u(t))\,\mathrm{d}t \tag{DI}$$

holds along all solutions of (1) driven by some control $u : [0,T] \to \mathbb{R}^m$.

Recently, there has been substantial interest in a specific dissipativity notion for optimal control which relates (DI), and specifically the supply rate ω, to the problem of minimizing

$$\int_0^T \omega(x(t),u(t)) - \bar{\omega}\,\mathrm{d}t \tag{2}$$

subject to (1) and further constraints, and where $\bar{\omega}$ is an appropriate normalization. When minimizing (2) subject to (1) the dissipation inequality (DI) directly implies a lower bound on the optimal value function. Moreover, strict variants of (DI), which hold with equality only on a subset of $\mathbb{R}^n$, imply the presence of a turnpike property in the open-loop optimal solutions, see [3] for a recent overview.

In this context we formulate re-dispatch problems for large-scale energy systems as receding-horizon optimal control which involves time-varying, and usually uncertain, predictions of future energy demands and renewable energy generation. We show how to handle the stationary description of electricity grids given by

the so-called power-flow equations, while the ramp constraints on generators and the dynamics of storages induce multi-stage coupling (i.e., discrete-time dynamics). Results on the dissipativity of the resulting discrete-time optimal control problem [4] motivate the formulation of open problems and ideas for future investigations. This includes:

(i) Classically, and especially in the context of passivity and port-Hamiltonian systems, dissipativity supply rates ω admit physical interpretations and motivations [5]. However, requiring that the objective (2) of an optimal control problem corresponds to the right hand side of the dissipation inequality (DI)—modulo normalization and see [3]—leads to the fundamental question of how both view points are related. First steps in this direction also discussed in [6].

(ii) Only little research has investigated the link of dissipativity and stochastic optimal control problems and corresponding stochastic turnpike notions. This is likely due to the fact that not much has been done beyond the in-expectation formulation of stochastic dissipation inequalities.

(iii) Despite substantial progress on the dissipativity-based analysis of optimal control problems, the vast majority of available results assume that inequality constraints on states and inputs are not active at the local steady-state minimizer of (2). Hence the problem of dissipativity-based turnpike analysis with active constraints appears to still open.

References

[1] J. C. Willems, *Dissipative dynamical systems part i: General theory*, Archive for Rational Mechanics and Analysis, 45 (1972), pp. 321–351.

[2] J. C. Willems, *Least squares stationary optimal control and the algebraic Riccati equation*, IEEE Transactions on Automatic Control, 16 (1971),pp. 621–634.

[3] T. Faulwasser and L. Grüne, *Turnpike properties in optimal control: An overview of discrete-time and continuous-time results*, 2021, https://arxiv.org/abs/arXiv:2011.13670.

[4] T. Faulwasser and A. Engelmann, *Toward economic NMPC for multistage AC optimal power flow*, Optimal Control Applications and Methods 41 (2020), pp. 107–127.

[5] A. Van Der Schaft and D. Jeltsema, *Port-Hamiltonian systems theory: An introductory overview*, Foundations and Trends in Systems and Control, 1 (2014), pp. 173–378.

[6] M. Schaller, F. Philipp, T. Faulwasser, K. Worthmann, and B. Maschke, *Control of port-Hamiltonian systems with minimal energy supply*, 2020, https://arxiv.org/abs/arXiv:2011.10296.

Model order reduction for phase field systems governed by the Cahn-Hilliard model

Carmen Gräßle
(joint work with Olena Burkovska)

Phase field systems are widely used in many applications to describe e.g. the process of phase separation and interface motion. We consider the Cahn-Hilliard model which can be formulated as a coupled system of equations

$$\partial_t u - \Delta w = 0, \tag{1a}$$

$$w = -\varepsilon^2 \Delta u + f'(u), \tag{1b}$$

subject to homogeneous Neumann boundary conditions and an initial condition $u(0) = u_0$. Here, u denotes the phase field variable, w is the chemical potential, $\varepsilon > 0$ is an interface parameter, which is related to the width of the interface, and f is a free energy potential with sufficient regularity. A common choice for f is, for instance, the smooth double-well potential

$$f(u) := \frac{1}{4}(u^2 - 1)^2, \tag{2}$$

see e.g. [7] for more details. The pure phases in the Cahn-Hilliard system correspond to the values $u = \pm 1$, whereas in the interface region steep transitions in the phase field variable occur. For the numerical implementations, this requires a high discretization resolution in the interface areas, whereas in the pure phases a coarse resolution suffices. This motivates to use adaptive finite elements. In order to reduce the computational times, a POD reduced-order approach is considered replacing the original system by a low-order surrogate model. To construct the reduced-order model, snapshots of the original system are collected in an offline phase and the combination of space-adapted snapshots with a POD reduced-order approach can be handled according to [9, 12].

Note that the choice (2) permits pure phases, but allows the phase field variable u to take unphysical values $|u| > 1$. For this reason, the non-smooth double obstacle free energy can be considered, given by

$$f(u) := \begin{cases} f_0(u) & \text{if } |u| \le 1 \\ +\infty & \text{if } |u| > 1 \end{cases} = f_0(u) + I_{[-1,1]}(u), \tag{3}$$

where f_0 is a smooth, non-convex part of the potential, which is often chosen as $f_0(u) = \frac{1}{2}(1 - u^2)$ and $I_{[-1,1]}$ is the convex indicator function

$$I_{[-1,1]}(u) := \begin{cases} 0 & \text{if } u \in [-1, 1], \\ +\infty & \text{otherwise.} \end{cases}$$

With this non-smooth choice for the potential f, the derivative of the potential in (1) should be understood by means of a generalized subdifferential $\partial f(u) =$

$f_0'(u)+\partial I_{[-1,1]}(u)$, where $\partial I_{[-1,1]}(u)$ is the subdifferential of the indicator function $I_{[-1,1]}(u)$. This leads to the Cahn-Hilliard model involving a variational inequality:

$$(4)\quad \begin{cases} \partial_t u - \Delta w = 0, \\ (w, v-u) \le \varepsilon^2(\nabla u, \nabla(v-u)) + (f_0'(u), v-u), \quad \forall v \in H^1(\Omega), |v| \le 1, \end{cases}$$

together with $|u| \le 1$ a.e., see e.g. [3, 4] for more details. In order to solve the discretized problem the primal-dual active set method [11] can be applied, see also, e.g., [2]. In [5], we introduce a parametrized version of the Cahn-Hilliard problem (4), for instance by considering a parametrization of the initial condition or by parametrization of the problem by model parameters.

To construct low-dimensional spaces following the reduced basis method, we consider a problem in a saddle point form. We extend some of the previously developed model order reduction techniques for elliptic and parabolic variational inequalities with unilateral constraints [6, 10], cf. also [1, 8, 13]. Possible transport phenomena of the interfaces as well as the non-smoothness of the potential pose additional challenges for the model reduction.

References

[1] A. Benaceur, A. Ern, V. Ehrlacher, *A reduced basis method for parametrized variational inequalities applied to contact mechanics*, Int. J. Numer. Methods. Eng. **121** (2020), 1170–1197.

[2] L. Blank, M. Butz, H. Garcke, *Solving the Cahn-Hilliard variational inequality with a semi-smooth Newton method*, ESAIM:COCV **17**(4) (2011), 931–954.

[3] J.F. Blowey, C.M. Elliott, *The Cahn-Hilliard gradient theory for phase separation with non-smooth free energy. I. Mathematical analysis*, Eur. J. Appl. Math. **2**(3) (1991), 233–280.

[4] J.F. Blowey, C.M. Elliott, *The Cahn-Hilliard gradient theory for phase separation with non-smooth free energy. II. Numerical analysis*, Eur. J. Appl. Math. **3**(2) (1992), 147–179.

[5] O. Burkovska, C. Gräßle, *Model order reduction for the Cahn-Hilliard model with a double obstacle potential*, in preparation (2021).

[6] O. Burkovska, B. Haasdonk, J. Salomon, B. Wohlmuth, *Reduced Basis Methods for Pricing Options with the Black–Scholes and Heston Models*, SIAM J. Finan. Math. **6**(1) (2015), 685–712.

[7] C.M. Elliott, *The Cahn-Hilliard Model for the Kinetics of Phase Separation*, In:Rodrigues J.F. (eds) Mathematical Models for Phase Change Problems, Internat. Ser. Numer. Math. **88**, Birkhäuser Basel (1989), 35–73.

[8] S. Glas, K. Urban, *On Noncoercive Variational Inequalities*, SIAM J. Numer. Anal. **52**(5) (2014), 2250–2271.

[9] C. Gräßle, M. Hinze, *POD reduced-order modeling for evolution equations utilizing arbitrary finite element discretizations*, Adv. Comput. Math. **44**(6) (2018), 1941–1978.

[10] B. Haasdonk, J. Salomon, B. Wohlmuth, *A Reduced Basis Method for Parametrized Variational Inequalities*, SIAM J. Numer. Anal. **50**(5) (2012), 2656–2676.

[11] M. Hintermüller, K. Ito, K. Kunisch, *The Primal-Dual Active Set Strategy as a Semismooth Newton Method*, SIAM J. Optim. **13**(3) (2002), 865–888.

[12] S. Ullmann, M. Rotkvic, J. Lang, *POD-Galerkin reduced-order modeling with adaptive finite element snapshots*, J. Comput. Phys. **325** (2016), 244–258.

[13] Z. Zhang, E. Bader, K. Veroy, *A slack approach to reduced-basis approximation and error estimation for variational inequalities*, Comptes Rendus Mathematique, **354**(3) (2016), 283–289.

Optimal Control Design for Fluid Mixing: from Open-Loop to Closed-Loop

Weiwei Hu

1. Abstract

The question of what velocity fields effectively enhance or prevent transport and mixing, or steer a scalar field to a desired distribution, is of great interest and fundamental importance to the fluid mechanics community. In this work, we mainly discuss the problem of optimal mixing of an inhomogeneous distribution of a scalar field via active control of the flow velocity, governed by the Stokes or the Navier-Stokes equations. Specifically, we consider that the velocity field is steered by a control input which acts tangentially on the boundary of the domain through the Navier slip boundary conditions. This is motivated by mixing within a cavity or vessel by rotating or moving walls. Our main objective is to design a Navier slip boundary control for achieving optimal mixing. Non-dissipative scalars governed by the transport equation will be of our main focus. In the absence of molecular diffusion, mixing is purely determined by the flow advection. This essentially leads to a nonlinear control and optimization problem. A rigorous proof of the existence of an optimal open-loop control and the first-order necessary conditions for optimality will be addressed. Moreover, a feedback law for the closed-loop system will be also constructed utilizing the optimal control approach. Finally, numerical experiments will be presented to demonstrate our ideas and control designs.

2. Problem Statement

Consider an inhomogeneous scalar field advected by an incompressible flow in an open bounded and connected domain $\Omega \subset \mathbb{R}^2$ with a smooth boundary Γ. The scalar field is described by the transport equation

$$\partial_t \theta + v \cdot \nabla \theta = 0, \quad \nabla \cdot v = 0 \quad \text{in} \quad \Omega \times [0, \infty), \tag{1}$$

with initial condition $\theta(0) = \theta_0$, where θ stands for the mass distribution or scalar concentration, $\kappa > 0$ is the thermal diffusivity, and v is the velocity of an incompressible fluid flow. Here we consider that the flow velocity is governed by the incompressible Stokes equations

$$\partial_t v - \nu \Delta v + \nabla p = 0, \quad \nabla \cdot v = 0 \quad \text{in} \quad \Omega \times [0, \infty), \tag{2}$$

or the incompressible Navier-Stokes equations

$$\partial_t v + v \cdot \nabla v + \nabla p = \nu \Delta v + f(\theta) \quad \text{and} \quad \nabla \cdot v = 0 \quad \text{in} \quad \Omega \times [0, \infty), \tag{3}$$

where $\nu > 0$ is the viscosity, p is the pressure, and $f(\theta)$ stands for the local force (such as buoyancy) imposed on the velocity field in the case of active transport. Moreover, the Navier slip boundary conditions are employed [12], that is,

$$v \cdot n|_\Gamma = 0 \quad \text{and} \quad (2\nu n \cdot \mathbb{D}(v) \cdot \tau + \alpha v \cdot \tau)|_\Gamma = g \cdot \tau, \tag{4}$$

where n and τ denote the outward unit normal and tangential vectors with respect to the domain Ω, and $\mathbb{D}(v) = (1/2)(\nabla v + (\nabla v)^T)$. The friction between the fluid and the wall is proportional to $-v$ with the positive coefficient of proportionality α. The nonhomogeneous boundary term g with $g \cdot n|_\Gamma = 0$, is the control input depending on both space and time, which is applied to generate the velocity field for mixing. The initial condition is given by $(\theta(0), v(0)) = (\theta_0, v_0)$.

The presence of the feedback f couples the transported scalar to the velocity. In contrast, a passive scalar has no dynamical effect on the flow, i.e., $f(\theta) = 0$, and hence the velocity determines the properties of the scalar. Although passive and active scalars considered in this proposal are governed by the same transport equation, their nature is essentially distinct. The complexity of the two-way coupling between the scalar and the flow gives rise to a major challenge in analysis. The understanding of the active transport is far behind that of its passive counterpart. In the study of active scalars, the current work only focuses on mixing in the two dimensional buoyancy-driven flow modeled by the Boussinesq approximation with zero diffusivity, which arises naturally in certain geophysical situations [5, 13, 15]. In this case, $f(\theta) = \theta e_2$ and $e_2 = (0, 1)^T$ is a unit vector in the direction of buoyancy.

To quantify mixing, a classical measure is the spatial variance of the concentration of the scalar [1]. However, this measurement fails in the case of zero diffusivity since it is unable to quantify pure stirring effects [11]. Recently, the mix-norm and negative Sobolev norms have been adopted to quantify mixing based on ergodic theory, which are sensitive to both stirring and diffusion [11, 14]. In fact, any negative Sobolev norm H^{-s}, for $s > 0$, can be used as a mix-norm thanks to the property of weak convergence [14]. Since a general open and bounded domain will be considered in this project, without imposing any additional boundary conditions other than no-penetration on the velocity field, the negative Sobolev norm will be replaced by the norm of the dual space $(H^s(\Omega))'$ of $H^s(\Omega)$ with $s > 0$ as in our previous work [6–10]. Without loss of generality, $(H^1(\Omega))'$ will be used to quantify mixing in this work.

2.1. **Formulation of the Optimal Control Problem.** We formulate the optimal control problem as follows: for a given $T > 0$, find a control g minimizing the cost functional

$$J(g) = \frac{1}{2}\|\theta(T)\|^2_{(H^1(\Omega))'} + \frac{\gamma}{2}\|g\|^2_{U_{\mathrm{ad}}} - \frac{\zeta}{2}\int_0^T \|\nabla \times v\|^2_{L^2}\, dt, \qquad \text{(P)}$$

where U_{ad} is the set of admissible controls and $\gamma > 0$ is the control weight parameter, which is chosen to establish the relative weight depending on the first and the third term. Here $\nabla \times v = \partial_1 v_2 - \partial_2 v_1$ stands for the vorticity and $\zeta \geq 0$ is the regularization parameter for vorticity, which is set to be zero for the Stoke flow problem. Note that the long-time dynamics may be dominated by strong coherent vortices that can possibly slow down mixing, thus ζ may be used to test the sensitivity of mixing rate with respect to vorticity. Since the mapping $g \mapsto (\theta, v)$

is nonlinear, problem (P) is non-convex and hence the optimal solution may not be unique in general.

3. Main Results and Ongoing Work

Problem (P) has been well-studied in our recent work [6–10], subject to Stokes or the Navier-Stokes equations. Specifically, the existence and uniqueness of an optimal solution are addressed, where the uniqueness is obtained under certain small conditions on the system data. The first-order optimality conditions are derived for charactering and solving the optimal solution. Some preliminary numerical results for mixing via Stokes flows have been presented in [10]. However, to solve the resulting optimality system, one has to solve the governing system forward in time, coupled with the adjoint system backward in time together with a nonlinear optimality condition. Straightforward use of this theory can result in extremely to impossibly high computational costs.

Our ongoing work is to establish the closed-loop feedback controls and the corresponding feasible computational methods for optimization and stabilization of both passive and active transport and mixing. In particular, the idea of instantaneous control design [2–4] is employed, which leads to a suboptimal feedback law. In fact, such a feedback law can be possibly obtained by a direct approximation of the optimality system using appropriate numerical schemes. Furthermore, we are also interested in understanding the long-time behavior of the nonlinear closed-loop system, identifying the explicit mixing decay rate, and establishing the relation between the mixing decay rate and the control actuation. These are rather challenging problems that require fine analysis and merit continuing investigation in our future work.

References

[1] P. V. Danckwerts, *The definition and measurement of some characteristics of mixtures*, Applied Scientific Research, Section A **3** (1952), no. 4, 279–296.
[2] M. Hinze and K. Kunisch, *Control strategies for fluid flows-optimal versus suboptimal control*, In H.G.Bock *et al.*, editor, ENUMATH 97 (1997), 351–358.
[3] M. Hinze, *Optimal and instantaneous control of the instationary Navier-Stokes equations* (2000).
[4] M. Hinze and S. Volkwein, *Analysis of instantaneous control for the Burgers equation*, Nonlinear Analysis **50** (2002), no. 1, 1–26.
[5] J. R. Holton and G. J. Hakim, *An introduction to dynamic meteorology*, Vol. 88, Academic Press, 2012.
[6] W. Hu, *Boundary control for optimal mixing by Stokes flows*, Applied Mathematics & Optimization **78** (2018), no. 1, 201–217.
[7] W. Hu, *An Approximating Control Design for Optimal Mixing by Stokes Flows*, Applied Mathematics & Optimization, https://doi.org/10.1007/s00245-018-9535-4.
[8] W. Hu and J. Wu, *Boundary Control for Optimal Mixing via Navier–Stokes Flows*, SIAM Journal on Control and Optimization **56** (2018), no. 4, 2768–2801.
[9] W. Hu and J. Wu, *An approximating approach for boundary control of optimal mixing via Navier-Stokes flows*, Journal of Differential Equations **267** (2019), no. 10, 5809–5850.
[10] W. Hu and X. Zheng, *Numerical implementation of boundary control for optimal mixing via flow advection*, Preprint.

[11] G. Mathew, I. Mezić, and L. Petzold, *A multiscale measure for mixing*, Physica D: Nonlinear Phenomena **211** (2005), no. 1, 23–46.
[12] C.-L. Navier, *Mémoire sur les lois du mouvement des fluides*, Mémoires de ÍAcadémie Royale des Sciences de ÍInstitut de France **6** (1823), 389–440.
[13] R. Salmon, *Lectures on geophysical fluid dynamics*, Oxford University Press, 1998.
[14] J.-L. Thiffeault, *Using multiscale norms to quantify mixing and transport*, Nonlinearity **25** (2012), no. 2, R1.
[15] G. K. Vallis, *Atmospheric and oceanic fluid dynamics*, Cambridge University Press, 2017.

Controllability and Riesz bases of infinite-dimensional port-Hamiltonian systems

Birgit Jacob
(joint work with Julia T. Kaiser and Hans Zwart)

The location of the spectrum, the Riesz basis property and controllability of infinite-dimensional linear port-Hamiltonian systems on a 1D spatial domain are studied. It is shown that these systems with full boundary control are exactly controllable and the Riesz basis property is equivalent to the fact that system operator generates a strongly continuous group. Moreover, in this situation the spectrum consists of eigenvalues only, located in a strip parallel to the imaginary axis and they can decomposed into finitely many sets having each a uniform gap.

More precisely, we consider first order linear port-Hamiltonian systems on a one-dimensional spatial domain of the form

$$\begin{aligned} \frac{\partial x}{\partial t}(\zeta,t) &= \left(P_1\frac{\partial}{\partial \zeta}+P_0\right)(\mathcal{H}(\zeta)x(\zeta,t)),\\ x(\zeta,0) &= x_0(\zeta),\\ u(t) = W_B\begin{pmatrix} f_\partial(t)\\ e_\partial(t)\end{pmatrix},&\quad y(t)=W_C\begin{pmatrix} f_\partial(t)\\ e_\partial(t)\end{pmatrix}\end{aligned} \tag{1}$$

where $\zeta \in [0,1]$ and $t \geq 0$. Equation (1) describes a special class of port-Hamiltonian systems, which cover in particular the wave equation, the transport equation and the Timoshenko beam as well as coupled systems. For more information we refer to [4–7].

In this article we make the following assumptions: The $d \times d$ Hermitian matrix P_1 is invertible, P_0 is a $d \times d$ skew-symmetric matrix, W_B and W_C are full row rank $d \times 2d$-matrix such that $\left[\begin{smallmatrix} W_B\\ W_C\end{smallmatrix}\right]$ is invertible and $\mathcal{H}(\zeta)$ is a positive $d \times d$ Hermitian matrix for a.e. $\zeta \in (0,1)$ satisfying $\mathcal{H}, \mathcal{H}^{-1} \in L^\infty(0,1;\mathbb{C}^{d\times d})$. Thus, the matrix $P_1\mathcal{H}(\zeta)$ can be diagonalized as $P_1\mathcal{H}(\zeta) = S^{-1}(\zeta)\Delta(\zeta)S(\zeta)$, where $\Delta(\zeta)$ is a diagonal matrix and $S(\zeta)$ is an invertible matrix for a.e. $\zeta \in (0,1)$. We suppose the technical assumption that S^{-1}, S, $\Delta : [0,1] \to \mathbb{C}^{d\times d}$ are continuously differentiable. Boundary effort e_∂ and boundary flow f_∂ are defined by

$$\begin{pmatrix} f_\partial\\ e_\partial\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} P_1 & -P_1\\ I & I\end{pmatrix}\begin{pmatrix}[\mathcal{H}x](1)\\ [\mathcal{H}x](0)\end{pmatrix},$$

and the Hamiltonian (energy) of the port-Hamiltonian system is given by

$$H(x(\cdot,t)) = \frac{1}{2}\int_0^1 x(\zeta,t)^*\mathcal{H}(\zeta)x(\zeta,t)\,d\zeta.$$

An easy calculation shows that

$$\frac{d}{dt}H(x(\cdot,t)) = \frac{1}{2}\left[[\mathcal{H}x]^*(\zeta,t)P_1\left[\mathcal{H}x\right](\zeta,t)\right]_0^1 = e_\partial(t)^* f_\partial(t).$$

In the following we assume that the port-Hamiltonian system *impedance passive*, that is,

$$\frac{d}{dt}H(x(\cdot,t)) \le u(t)^* y(t).$$

This is satisfied if and only if

$$\begin{pmatrix} W_B\Sigma W_B^* & W_B\Sigma W_C^* \\ W_C\Sigma W_B^* & W_C\Sigma W_C^* \end{pmatrix}^{-1} \le \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},$$

where $\Sigma = \left[\begin{smallmatrix} 0 & I \\ I & 0 \end{smallmatrix}\right]$. As *state space* we choose $X := L^2((0,1);\mathbb{C}^d)$ equipped with the (energy) norm $\langle x,y\rangle_X := \frac{1}{2}\int_0^1 x(\zeta)^*\mathcal{H}(\zeta)y(\zeta)\,d\zeta$, which is equivalent to the standard L^2-norm. The system operator $A : \mathcal{D}(A) \subset X \to X$ is defined by

$$Ax := (P_1\frac{d}{d\zeta} + P_0)(\mathcal{H}x), \qquad x \in \mathcal{D}(A), \tag{2}$$

$$\mathcal{D}(A) := \left\{x \in X \mid \mathcal{H}x \in H^1((0,1);\mathbb{C}^d) \text{ and } W_B\left[\begin{smallmatrix} f_\partial \\ e_\partial \end{smallmatrix}\right] = 0\right\}. \tag{3}$$

The following theorem summarizes results concerning existence of solutions of the port-Hamiltonian system (1).

Theorem 1 ([7]). *If the port-Hamiltonian system impedance passive, then*

1. *The operator A, given by* (2)-(3) *generates a contraction semigroup on X.*
2. *There are $t_0, m_{t_0} > 0$: Every classical solution of* (1) *satisfies $\|x(t_0)\|_X^2 + \int_0^{t_0}\|y(t)\|^2 dt \le m_{t_0}\left[\|x(0)\|_X^2 + \int_0^{t_0}\|u(t)\|^2 dt\right]$.*
3. *For every initial condition $x_0 \in X$ and input function $u \in L^2_{loc}([0,\infty),\mathbb{C}^d)$ the system has an unique (mild) solution $x \in C([0,\infty),X)$ and $y \in L^2_{loc}([0,\infty),\mathbb{C}^d)$.*

Definition 2. *We call the port-Hamiltonian system* (1) exactly controllable*, if there exists a time $\tau > 0$ such that for all $x_1 \in X$ there exists a control function $u \in L^2([0,\tau],\mathbb{C}^d)$ such that the corresponding mild solution satisfies $x(0) = 0$ and $x(\tau) = x_1$.*

Our first main result is as follows:

Theorem 3 ([2]). *Every impedance passive port-Hamiltonian system* (1) *is exactly controllable.*

We note that the operator A, given by (2)-(3), is closed and that its resolvent operator is compact, see [1]. This immediately implies that the spectrum of A, denoted by $\sigma(A)$, consists of isolated eigenvalues with finite multiplicity only. Thus

$$\sigma(A) = \{\lambda_k\}_{k\in\mathbb{N}}.$$

As the port-Hamiltonian system is impedance passive, we have $\sup_{k\in\mathbb{N}} \mathrm{Re}\lambda_k \leq 0$.

By $E(\lambda_k)$ we denote the spectral projections on the spectral subset $\{\lambda_k\}$ is defined as

$$E(\lambda_k) = \frac{1}{2\pi i}\int_{\Gamma_k} (s-A)^{-1} ds,$$

where Γ_k is a closed Jordan curve containing λ_k and no point of $\sigma(A)\setminus\{\lambda_k\}$. If $W_B\Sigma W_B^* = 0$, then it is easy to see that the normalized eigenvectors of A form an orthonormal basis. In general there are port-Hamiltonian systems with $\sigma(A) = \emptyset$, for example the operator $Ax = \frac{d}{d\zeta}x$ with $D(A) = \{x \in X \mid x' \in X, x(1) = 0\}$ has empty spectrum.

Definition 4. *The operator A is called a* discrete Riesz spectral operator*, if*

(1) *for every $k \in \mathbb{N}$ there exists $N_k \in L(X)$ such that*

$$AE_k = (\lambda_k + N_k)E_k,$$

(2) *the sequence of closed subspaces $(E_k(X))_{k\in\mathbb{N}}$ is a* Riesz basis of subspaces *of X, that is,* $\mathrm{Span}(E_k(X))_{n\in\mathbb{N}}$ *is dense and there exists an isomorphism $T \in L(X)$, such that $(TE_k(X))_{n\in\mathbb{N}}$ is system of pairwise orthogonal subspaces of X.*

(3) *$N := \sum_{k\in\mathbb{N}} N_k$ is bounded and nilpotent.*

Our second main results reads:

Theorem 5 ([3])**.** *Let $\begin{bmatrix} W_1 & W_0 \end{bmatrix} := W_B \frac{1}{\sqrt{2}} \begin{bmatrix} P_1 & -P_1 \\ I & I \end{bmatrix}$, $Z^-(1)$ denotes the span of the eigenvectors of $P_1\mathcal{H}(1)$ corresponding to its negative eigenvalues and $Z^+(0) :=$ the span of the eigenvectors of $P_1\mathcal{H}(0)$ corresponding to its positive eigenvalues. Then the following are equivalent:*

(1) *A is a discrete Riesz spectral operator.*
(2) *$-A$ is the generator of a C_0-semigroup.*
(3) *A is the generator of a C_0-group.*
(4) *$W_1\mathcal{H}(1)Z^-(1) \oplus W_0\mathcal{H}(0)Z^+(0) = \mathbb{C}^d$.*

If A is a discrete Riesz spectral operator, then the eigenvalues (counted according to the algebraic multiplicity) can be decomposed into finitely many sets each having a uniform gap, i.e., $\inf_{k\neq m} |\mu_k - \mu_m| > 0$.

References

[1] B. Augner, *Stabilisation of Infinite-dimensional Port-Hamiltonian System*, PhD thesis, University of Wuppertal, 2016.

[2] B. Jacob and J. T. Kaiser, *On exact controllability of infinite-dimensional linear port-Hamiltonian systems*, IEEE Control Systems Letters, 3 (2019), pp. 661–666.

[3] B. Jacob, J. T. Kaiser an H. Zwart, *Riesz bases of port-Hamiltonian systems*, Submitted, arXiv:2009.08521.

[4] B. Jacob and H. J. Zwart, *Linear port-Hamiltonian Systems on Infinite-Dimensional Spaces*, vol. 223 of Operator Theory: Advances and Applications, Birkhäuser/Springer Basel AG, Basel, 2012.
[5] B. Jacob and H. Zwart, *An operator theoretic approach to infinite-dimensional control systems*, GAMM-Mitt., 41 (2018), pp. e201800010, 14.
[6] Y. Le Gorrec, H. Zwart, and B. Maschke, *Dirac structures and boundary control systems associated with skew-symmetric differential operators*, SIAM J. Control Optim., 44 (2005), pp. 1864–1892.
[7] H. Zwart, Y. Le Gorrec, B. Maschke, and J. Villegas, *Well-posedness and regularity of hyperbolic boundary control systems on a one-dimensional spatial domain*, ESAIM Control Optim. Calc. Var., 16 (2010), pp. 1077–1093.

Recent progresses on Implicit Port Hamiltonian Systems defined with respect to reciprocal differential operators

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(joint work with Arjan van der Schaft)

Recently Port Hamiltonian systems [3,9] have been extended to an implicit definition of the energy function which is defined in terms of a Lagrangian submanifold. This is classically used in the definition of Hamiltonian systems [1, chap. 5.3] and has been adapted to *finite-dimensional* linear Port Hamiltonian systems in [7,8]. In this contribution, we elaborate on the recent extension of [7] to infinite-dimensional linear Port Hamiltonian systems where the Lagrangian subspace has been defined with respect to differential operators of first order [6].

First, let us briefly recall the definition of *Boundary Port Hamiltonian Systems* [10]. They are defined by a Hamiltonian system

$$\frac{\partial x}{\partial t} = \mathcal{J}\,\mathcal{Q}\,x \tag{1}$$

where $x \in H^N([a,b],\mathbb{R}^n)$, $\mathcal{Q} \in L^\infty([a,b],\mathbb{R}^{n\times n})$ is symmetric positive and $\mathcal{J}$ is a Nth-order matrix differential *Hamiltonian operator*

$$\mathcal{J} = \sum_{k=0}^{N} \frac{\partial^k}{\partial z^k} J_k \tag{2}$$

where $J_k \in \mathbb{R}^{n\times n}$ is symmetric if k is odd and $J_k \in \mathbb{R}^{n\times n}$ is skew-symmetric if k is even, augmented with the *pair of conjugated boundary port variables*

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} \tilde{\Lambda} & -\tilde{\Lambda} \\ I_k & I_k \end{pmatrix} \mathrm{tr}\left(\tilde{L} \begin{pmatrix} e^\top & \frac{\partial e^\top}{\partial z} & \cdots & \frac{\partial^{N-1} e^\top}{\partial z^{(N-1)}} \end{pmatrix} \right) \tag{3}$$

where tr denotes the trace operator,

$$e = \mathcal{Q}\,x \tag{4}$$

and the matrices $\tilde{\Lambda}$, $\tilde{L}$ are matrices chosen in such a way that the operator $\mathcal{J}$ and (3) defines a Stokes-Dirac structure [5,10].

We have extended the constitutive relation defining the co-energy variables (4) to the differential constitutive relations

$$\mathcal{S}^* x - \mathcal{P}^* e = 0 \tag{5}$$

defined by two $(n \times n)$ matrix differential operator of differential order m, denoted by $\mathcal{P}$ and $\mathcal{S}$

$$\mathcal{P} = \sum_{i=0}^{M} P_i \frac{\partial}{\partial z}^i \, , \mathcal{S} = \sum_{i=0}^{M} S_i \frac{\partial}{\partial z}^i \, , \; M \in \mathbb{N}, \; P_i, \, S_i \in \mathbb{R}^{n \times n}$$

and satisfying the *formal reciprocity condition*

$$\mathcal{S}^* \mathcal{P} - \mathcal{P}^* \mathcal{S} = 0 \tag{6}$$

We have then defined the class of *Boundary Port Hamiltonian systems with power and energy port variables*

$$\frac{\partial}{\partial t} \mathcal{P} \xi = \mathcal{J} \mathcal{S} \xi \tag{7}$$

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} \tilde{\Lambda} & -\tilde{\Lambda} \\ I_k & I_k \end{pmatrix} \mathrm{tr} \left(\tilde{L} \, \mathcal{S} \xi \right) \tag{8}$$

$$\begin{pmatrix} \phi_\partial \\ \psi_\partial \end{pmatrix} = \Sigma \mathrm{tr} \left(\tilde{\Pi} \, \xi \right) \tag{9}$$

where $\xi \in H^M \left([a, \, b] \, , \mathbb{R}^n \right)$ and the matrix $\tilde{\Pi}$ is chosen in such a way that (5) and the *energy port variables* (9) define a *Lagrangian subspace* associated with the total energy of the system.

We have illustrated this definition on different examples of elasto-dynamical systems and have shown that it encompasses non-local constitutive relations. We have also discussed the relation with the *Descriptor Port Hamiltonians systems* suggested in [2, 4].

References

[1] R. Abraham and J. E. Marsden. *Foundations of Mechanics*. Benjamin Cummings Publ. Comp., Reading, MA, U.S.A., ii edition, 1987. ISBN 0-8053-0102-X.

[2] C. Beattie, V. Mehrmann, H. Xu, and H. Zwart. Linear port-Hamiltonian descriptor systems. *Mathematics of Control, Signals, and Systems*, 30(4):17, Oct 2018.

[3] V. Duindam, A. Macchelli, S. Stramigioli, and H. eds. Bruyninckx. *Modeling and Control of Complex Physical Systems - The Port-Hamiltonian Approach*. Springer, Sept. 2009. ISBN 978-3-642-03195-3.

[4] H. Heidari and H. Zwart. Port-Hamiltonian modelling of nonlocal longitudinal vibrations in a viscoelastic nanorod. *Mathematical and Computer Modelling of Dynamical Systems*, 0(0):1–16, 2019.

[5] Y. Le Gorrec, H. Zwart, and B.M. Maschke. Dirac structures and boundary control systems associated with skew-symmetric differential operators. *SIAM J. of Control and Optimization*, 44(5):1864–1892, 2005.

[6] B. Maschke and A.J. van der Schaft. Linear Boundary Port Hamiltonian systems defined on Lagrangian submanifolds. *IFAC-PapersOnLine*, 53(2):7734–7739, 2020. 21th IFAC World Congress.

[7] A. van der Schaft and B. Maschke. Generalized Port-Hamiltonian DAE systems. *Systems & Control Letters*, 121:31–37, 2018.
[8] A. van der Schaft and B. Maschke. Dirac and Lagrange algebraic constraints in nonlinear Port-Hamiltonian systems. *Vietnam Journal of Mathematics*, 48(4):929–939, 2020.
[9] A.J. van der Schaft and B.M. Maschke. The Hamiltonian formulation of energy conserving physical systems with external ports. *Archiv für Elektronik und Übertragungstechnik*, 49(5/6):362–371, 1995.
[10] A.J. van der Schaft and B.M. Maschke. Hamiltonian formulation of distributed parameter systems with boundary energy flow. *J. of Geometry and Physics*, 42:166–174, 2002.

Optimal control of port-Hamiltonian systems with minimal energy supply

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(joint work with Timm Faulwasser, Bernhard Maschke, Friedrich Philipp, Karl Worthmann)

Port-Hamiltonian systems are a class of dynamical systems stemming from network-based modeling [1] which in their linear form are given by

$$\begin{aligned} \dot{x}(t) &= (J-R)Qx(t) + Bu(t) \\ y(t) &= B^\top Qx(t), \end{aligned} \tag{pHs}$$

where $J = -J^*$, $R = R^* \geq 0$ and $Q = Q^* \geq 0$ are square matrices of appropriate dimension and B is an input matrix.

A central tool is the energy-based analysis and modeling and in particular the energy balance equation

$$H(x(T)) - H(x(0)) = \int_0^T y(t)^\top u(t) - \|R^{\frac{1}{2}}Qx(t)\|^2 \, dt, \tag{1}$$

where $H(x) := \frac{1}{2}x^\top Qx$ is the Hamiltonian and $\int_0^T y(t)^\top u(t)\,dt$ is the energy supplied to the system over the time period $[0,T]$. In modeling of electrical circuits for example, the input u is typically be given by the voltage, whereas the output y is the resulting current, leading to the product of y and u being the electrical power.

A natural task for optimal control is then the following: Given an initial state x^0 and a desired terminal state x_T, find a control $u \in L_1(0,T;[-1,1]^m)$ ($m \in \mathbb{N}$ being the input dimension) that drives the dynamical system (pHs) from x^0 to x_T with *minimal energy supply.* Such a control is given by a solution of the optimal control problem

$$\begin{aligned} \min_{u \in L^1(0,T;[-1,1]^m)} & \int_0^T y(t)^\top u(t)\,dt \\ \text{s.t.} \quad & \text{(pHs)} \\ & x(0) = x^0, \quad x(T) = x_T. \end{aligned} \tag{pHOCP}$$

A key property of this optimal control problem is the fact that the cost functional is not quadratic. This means, that standard techniques for existence of solutions,

Riccati theory or turnpike theory can not be applied to characterize the optimal control. However, due to the close connection of the cost functional and the dynamics via the energy balance relation (1), one can show various qualitative and quantitative properties of the optimal control problem [2, 4]:

i) Despite the singularity of the optimal control problem, i.e., the cost not being coercive in the control, one can show (assuming $\ker(RQ) \cap \operatorname{im}(B) = \{0\}$) that the optimal control is completely determined by the optimal state and the corresponding adjoint state [2, Theorem 8].
ii) Using the energy balance equation one can prove a subspace turnpike property towards the conservative subspace of the port-Hamiltonian dynamics, i.e., $\ker RQ$, see [2, Lemma 15] . Qualitatively speaking this means that solutions of (pHOCP) stay close to the kernel of the dissipation operator for the majority of the time, cf. Figure 1.
iii) Under controllability assumptions and reformulating the OCP via the energy balance equation (1), one can prove that the adjoint states show a turnpike behavior towards zero.
iv) The optimal control problem (pHOCP) is strictly dissipative with respect to the kernel of the dissipation operator, see [2, Theorem 18].

We presented first steps towards quantitative and qualitative analysis of optimal control problems with port-Hamiltonian systems with the goal of minimal energy supply. Many of the above presented properties can also be shown in the context of differential-algebraic or infinite dimensional port-Hamiltonian systems, cf. [3, 4].

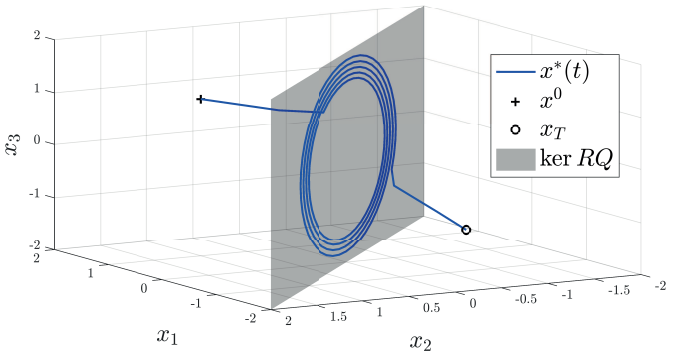

FIGURE 1. Subspace turnpike behaviour of the optimal state.

References

[1] V. Duindam, A. Macchelli, S. Stramigioli, and H. Bruyninckx, *Modeling and Control of Complex Physical Systems - The Port-Hamiltonian Approach*, Springer, 2009.

[2] M. Schaller, F. Philipp, T. Faulwasser, B. Maschke, K. Worthmann, *Control of port-Hamiltonian systems with minimal energy supply*, to appear in European Journal of Control.

[3] F. Philipp, M. Schaller, T. Faulwasser, B. Maschke, K. Worthmann, *Minimizing the energy supply of infinite-dimensional linear port-Hamiltonian systems*, 2021, submitted, https://arxiv.org/abs/2105.03873.

[4] T. Faulwasser, B. Maschke, F. Philipp, M. Schaller, K. Worthmann, *Optimal control of port-Hamiltonian descriptor systems with minimal energy supply*, 2021, submitted.

Passivity-based modeling and control of multi-producer district heating systems

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(joint work with Juan E. Machado and Michele Cucuzzella)

1. Introduction

District Heating (DH) refers to a network of insulated pipes carrying thermal power from heating stations (producers) towards clusters of consumers within a neighborhood, town center or city [1]. Prospective DH systems would promote a more sustainable heating sector by substantially increasing the share of renewable energy sources (e.g., geothermal or solar thermal), waste heat sources (e.g., from industrial buildings) and thermal storage units, featuring as consequence system topologies with *multiple* (and distributed) heat producers connected to a common distribution network (see, e.g., [2]).

In this research [3] (see also [4]) we propose an ODE-based thermo-hydraulic model of a multi-producer DH system and establish conditions under which hydraulic and thermal layers of the model are *shifted passive*.

2. System Model

Consider a water-based DH system with n_{pr} heat producers and n_{c} consumers connected to a common distribution network. Producers, consumers and distribution network are composed of elementary hydraulic devices, namely, valves, pipes and pumps. Producers are assisted by hydraulic pumps to deliver thermal power to the system by circulating and heating water through heat exchangers (viewed here as pipes): cold water is continuously drawn from the return layer of the distribution network which is then heated and injected back into the supply layer. The operation mode of consumers is analogous to that of producers (see [8] for details). Storage devices are not considered in this abstract due to space constraints.[1]

We view the DH system as a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where the nodes $\mathcal{N}$ are the system junctions and the edges $\mathcal{E}$ are all two terminal devices (pumps,

[1] It is assumed that water is incompressible and that its density ρ is constant. All system pipes are assumed to be cylindrical. For simplicity gravitational forces are neglected.

pipes and valves). Each edge is assumed to have an arbitrary and fixed orientation and this is codified through the node-edge incidence matrix $\mathcal{B}_0$. For any edge $i \in \mathcal{E}$, $T_{\mathrm{E},i}$ and $q_{\mathrm{E},i}$ denote temperature and volumetric flow rate; for any node $k \in \mathcal{N}$, $T_{\mathrm{N},k}$ and $p_{\mathrm{N},k}$ are temperature and pressure. The cardinalities of $\mathcal{E}$ and $\mathcal{N}$ are denoted by n_{E} and n_{N}, respectively. For simplicity, we view edges and nodes as lumped elements and assume that water occupies the whole interior of these for all time; their volumes are $V_{\mathrm{E},i}$ and $V_{\mathrm{N},j}$, for each $i \in \mathcal{E}$, $j \in \mathcal{N}$.

EDGES: Under a number of simplifying assumptions, the equations for mass, momentum and energy balance of each edge $i \in \mathcal{E}$ can be written as follows:

$$q_{\mathrm{E},i} = q_{\mathrm{E},i}^{\mathrm{in}} = q_{\mathrm{E},i}^{\mathrm{out}} \tag{1a}$$

$$p_{\mathrm{E},i}^{\mathrm{in}} - p_{\mathrm{E,i}}^{\mathrm{out}} = J_{\mathrm{E},i}\dot{q}_{\mathrm{E},i} + \theta_{\mathrm{E},i}|q_{\mathrm{E},i}|q_{\mathrm{E},i} - w_{\mathrm{E},i} \tag{1b}$$

$$\rho c_{\mathrm{sh}} V_{\mathrm{E},i}\dot{T}_{\mathrm{E},i} = \rho c_{\mathrm{sh}}|q_{\mathrm{E},i}|\left(T_{\mathrm{E},i}^{\mathrm{in}} - T_{\mathrm{E},i}\right) + P_{\mathrm{pr},i} - P_{\mathrm{c},i} \tag{1c}$$

where $q_{\mathrm{E},i}^{\mathrm{in}}$, $q_{\mathrm{E},i}^{\mathrm{out}}$ and $p_{\mathrm{E},i}^{\mathrm{in}}$, $p_{\mathrm{E},i}^{\mathrm{out}}$ are the pipe's inlet-outlet flow and pressure pairs. If $i \in \mathcal{E}$ is a pipe, then $J_{\mathrm{E},i} = (\rho\ell_{\mathrm{E},i})/A_{\mathrm{E},i} > 0$, where $\ell_{\mathrm{E},i}$ and $A_{\mathrm{E},i}$ are the pipe's length and cross-section area; also $\theta_{\mathrm{E},i} > 0$ depends on the pipe's friction factor and diameter. If i is a valve, then $J_{\mathrm{E},i} = V_{\mathrm{E},i} = w_{\mathrm{E},i} = 0$ and $\theta_{\mathrm{E},i} > 0$. The latter parameter is constant and represents the valve's friction coefficient. If i is a pump, then $J_{\mathrm{E},i} = V_{\mathrm{E},i} = 0$ and $w_{\mathrm{E},i}$ is the pressure difference produced between its terminals. If i is associated to the heat exchanger of a producer (consumer), then $P_{\mathrm{pr},i}$ ($P_{\mathrm{c},i}$) is the heat injection (extraction) into (from) the DH system by the producer (consumer). In this work $P_{\mathrm{pr},i}$ and $w_{\mathrm{E},i}$ are viewed as control inputs.

NODES: Let $\mathfrak{S}_k$ and $\mathfrak{T}_k$ denote the set of edges whose streams source from or target node $k \in \mathcal{N}$, respectively. Then, we impose the following nodal constraints

$$\dot{V}_{\mathrm{N},k} = 0 = \sum_{i\in\mathfrak{T}_k} |q_{\mathrm{E},i}| - \sum_{i\in\mathfrak{S}_k} |q_{\mathrm{E},i}| \tag{2a}$$

$$p_{\mathrm{N},k} = p_{\mathrm{E},i}^{\mathrm{in}},\ i \in \mathfrak{S}_k,\ p_{\mathrm{N},k} = p_{\mathrm{E},i}^{\mathrm{out}},\ i \in \mathfrak{T}_k,\ k \in \mathcal{N} \tag{2b}$$

$$0 = \rho c_{\mathrm{sh}} \sum_{i\in\mathfrak{T}_k} |q_{\mathrm{E},i}|T_{\mathrm{E},i} - \rho c_{\mathrm{sh}} \left(\sum_{i\in\mathfrak{S}_k} |q_{\mathrm{E},i}|\right) T_{\mathrm{N},k},\ k \in \mathcal{N}. \tag{2c}$$

Equations (2a) and (2b) guarantee mass balance and pressure consistency at each junction. The energy balance at $k \in \mathcal{N}$ is given by (2c), where we have assumed that $V_{\mathrm{N},k}$ is negligibly small and that any number of streams targeting k are perfectly mixed in it, then, any stream leaving k will have a temperature $T_{\mathrm{N},k}$. An additional constraint is $T_{\mathrm{N},k} = T_{\mathrm{E},i}^{\mathrm{in}}$, for all $i \in \mathfrak{S}_k$, $k \in \mathcal{N}$.

3. Propositions

Proposition 1 (Hydraulic layer)**.** There exists a collection $\mathcal{C} \subset \mathcal{E}$ of n_{ch} pipes whose flows $q_{\mathrm{ch},i}$ are independent variables. All system flows can be computed as $q_{\mathrm{E}} = \mathcal{F}^{\top} q_{\mathrm{ch}}$, where $\mathcal{F}$ is the (full rank, properly ordered) fundamental loop matrix

associated to $\mathcal{C}$. The vector $q_{\rm ch}$ is governed by the dynamics[2]

$$\mathcal{J}_{\rm ch}\dot{q}_{\rm ch} = f_{\rm ch}(q_{\rm ch}) + u_{\rm ch}, \tag{3}$$

where $\mathcal{J}_{\rm ch} = \mathcal{J}_{\rm ch}^\top > 0$, $-f_{\rm ch}$ is a monotone mapping associated to the pipes and valves' viscous friction, and each $u_{{\rm ch},i}$ is an independent input (associated to properly placed pumps $w_{{\rm E},i}$). Moreover, (3) is shifted passive with passive output $q_{\rm ch}$ and storage function $\mathcal{S}_{\rm ch}(q_{\rm ch}) = \frac{1}{2}(q_{\rm ch} - \bar{q}_{\rm ch})^\top \mathcal{J}_{\rm ch}(q_{\rm ch} - \bar{q}_{\rm ch})$, then the inequality $\dot{\mathcal{S}}_{\rm sh} \leq (u_{\rm ch} - \bar{u}_{\rm ch})^\top (q_{\rm ch} - \bar{q}_{\rm ch})$ holds for all time and for any equilibrium pair $(\bar{u}_{\rm ch}, \bar{q}_{\rm ch})$ of (3).

Proposition 2 (Thermal layer)**.** Let

$$\mathcal{B} = \mathcal{B}_0 {\rm diag}({\rm sign}(q_{\rm E})),\ \mathcal{T} = \frac{1}{2}\left(\mathcal{B} + |\mathcal{B}|\right),\ \mathcal{S} = \frac{1}{2}\left|\mathcal{B} - |\mathcal{B}|\right|.$$

Then, the thermal dynamics of the DH system is described by

$$\begin{bmatrix} \langle V_{\rm E}\rangle & 0 \\ 0 & 0_{n_{\rm N}\times n_{\rm N}} \end{bmatrix} \begin{bmatrix} \dot{T}_{\rm E} \\ 0_{n_{\rm N}} \end{bmatrix} = \underbrace{\begin{bmatrix} -\langle |q_{\rm E}|\rangle & \langle |q_{\rm E}|\rangle \mathcal{S}^\top \\ \mathcal{T}\langle |q_{\rm E}|\rangle & -\langle \mathcal{T}|q_{\rm E}|\rangle \end{bmatrix}}_{=:\mathcal{A}(q_{\rm E})} \begin{bmatrix} T_{\rm E} \\ T_{\rm N} \end{bmatrix} + \begin{bmatrix} B_{\rm pr}P_{\rm pr} - B_{\rm c}P_{\rm c} \\ 0_{n_{\rm N}} \end{bmatrix}, \tag{4}$$

where $B_{\rm pr}$ and $B_{\rm c}$ are suitable constant matrices. Assume that $P_{\rm c}$ is constant and that $q_{\rm E}$ is at equilibrium. Then $\mathcal{A}(q_{\rm E})$ is a Kirchhoff Convection Matrix ($\Rightarrow \mathcal{A}(q_{\rm E}) \leq 0$) and (4) is shifted passive with passive output $T_{\rm pr} := B_{\rm pr}^\top T_{\rm E}$ and storage function $\mathcal{S}_{\rm th}(T_{\rm E}) = \frac{1}{2}(T_{\rm E} - \bar{T}_{\rm E})^\top \langle V_{\rm E}\rangle (T_{\rm E} - \bar{T}_{\rm E})$. Then, $\dot{\mathcal{S}}_{\rm th} \leq (P_{\rm pr} - \bar{P}_{\rm pr})^\top (T_{\rm pr} - \bar{T}_{\rm pr})$ holds for all time and for any equilibrium pair $(\bar{P}_{\rm pr}, \bar{T}_{\rm E})$ of (4).[3]

4. Discussion

Invoking conservation laws and graph theoretic tools we have derived a thermo-hydraulic model of a multi-producer DH system and established that it is shifted passive under certain conditions; this can be useful in the design of *decentralized* passivity-based controllers with closed-loop stability guarantees (see [4]). Our current research is aimed at designing decentralized or distributed control strategies for a coordinated and *fair* distribution of the producer's available heat (see [6]).

References

[1] M. Lund, S. Werner, R. Wiltshire, et al., *4th Generation District Heating. Integrating smart thermal grids into future sustainable energy systems.*, Energy **68** (2014), 1–11.

[2] H. Wang, H. Wang, Z. Haijian, et al., *Optimization modeling for smart operation of multi-source district heating with distributed variablespeed pumps*, Energy **138** (2017), 1247–1262.

[3] J. Machado, M. Cucuzzella and J. Scherpen, *Modeling and passivity properties of district heating systems*, arXiv preprint, 2011.05419, 2021.

[4] J. Machado, M. Cucuzzella, N. Pronk and J. Scherpen, *Adaptive control for flow and volume regulation in multi-producer district heating systems*, arXiv preprint, 2103.06568, 2021.

[2] These results are based on those of [7] treating single producer DH systems.

[3] In [3] we describe a procedure to write (4) as an ODE (imposing some conditions to ensure that (4) has differential index 1). Here we keep the DAE representation to save space. We refer the reader to [5] for a comprehensive port-Hamiltonian modeling of DH systems.

[5] S. Hauschild, N. Marheineke, V. Mehrmann, et al., *Port-hamiltonian modeling of district heating networks*, Progress in Differential-Algebraic Equations II, (2020), 333–355.
[6] A. Vandermeulen, B. van der Heijde, and L. Helsen, *Controlling district heating and cooling networks to unlock exibility: A review*, Energy **151** (2018), 103–115.
[7] C. De Persis and C. Kallesoe, *Pressure regulation in nonlinear hydraulic networks by positive and quantized controls*, IEEE Tr. Cont. Sys. Tech. **19** (2011), 1371–1383.
[8] T. Scholten, C. De Persis and P. Tesi, *Modeling and control of heat networks with storage: The single-producer multiple consumer case*, IEEE Tr. Cont. Sys. Tech. **25** (2015), 414–427.

The port-Hamiltonian framework for poroelastic network models

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(joint work with R. Altmann, V. Mehrmann)

We study an energy-based formulation of equations modeling porous material saturated by $m \in \mathbb{N}$ fluid networks as they appear in geosciences or medical applications. The mathematical description [1, 5] in a Lipschitz domain $\Omega \subseteq \mathbb{R}^d$ with $d \in \{2, 3\}$ is given by the coupled system of partial differential equations

$$\rho\, \partial_{tt} u - \nabla \cdot \big(\sigma(u)\big) + \sum_{i=1}^{m} \nabla(\alpha_i p_i) = \hat{f} \quad \text{in } (0,T] \times \Omega, \tag{1a}$$

$$\partial_t \Big(\alpha_i \nabla \cdot u + \frac{1}{M} p_i\Big) - \nabla \cdot \Big(\frac{\kappa_i}{\nu_i} \nabla p_i\Big) - \sum_{j \neq i} \beta_{ij}(p_i - p_j) = \hat{g}_i \quad \text{in } (0,T] \times \Omega, \tag{1b}$$

which has to be solved for the displacement field $u\colon [0,T] \times \Omega \to \mathbb{R}^d$ of the porous media and the pressure $p_i\colon [0,T] \times \Omega \to \mathbb{R}$ of the ith ($i = 1, \dots, m$) fluid network. Note that these are averaged quantities across (infinitesimal) cubic elements. Within this system, the stress tensor σ models the linear elastic stress-strain constitutive relation

$$\sigma(u) = 2\mu\, \varepsilon(u) + \lambda\, (\nabla \cdot u)\, \mathcal{I}, \qquad \varepsilon(u) = \tfrac{1}{2}\left(\nabla u + (\nabla u)^T\right)$$

with the Lamé coefficients μ and λ and the identity tensor $\mathcal{I}$. Further, α denotes the Biot-Willis fluid-solid coupling coefficient, M the Biot modulus, κ the permeability, ρ the density, and ν the fluid viscosity. The right-hand side $\hat{g}$ represents an injection or production process and $\hat{f}$ denotes the volume-distributed external forces. In several applications the term $\rho \partial_{tt} u$ appearing in (1a) is assumed small enough that it can be neglected, yielding the so-called *quasi-static* formulation.

As sample applications, we first discuss cerebral edema [7], where one distinguishes the arterial blood network, the capillary network, the cerebrospinal fluid network, and the venous blood network, yielding a total of $m = 4$ compartments. A second application stems from powder-bed fusion [6], an advanced additive manufacturing technology. The resulting thermoelasticity model is mathematically equivalent to linear poroelasticity.

Using appropriate function spaces, cf. [2, 3] and performing a first-order reformulation, we can rewrite the weak formulation of (1) as abstract operator differential-algebraic equation. The key observation to obtain a *port-Hamiltonian*

(pH) descriptor system, as introduced in [4], is to include the commonly omitted second-order term $\rho\partial_{tt}u$. Hereby, we mimic the finite-dimensional properties of a pH system in the infinite-dimensional setting, such that a semi-discretization in space via mixed finite elements yields a finite-dimensional pH system. We emphasize that in the pH formulation, the term $\rho\partial_{tt}u$ can be set zero without affecting the pH structure.

References

[1] E. C. Aifantis, *Continuum basis for diffusion in regions with multiple diffusivity*, J. Appl. Phys., **50** (1979), 1334–1338.

[2] R. Altmann, R. Maier, and B. Unger, *Semi-explicit discretization schemes for weakly-coupled elliptic-parabolic problems*, Math. Comp., **90** (2021), 1089–1118.

[3] R. Altmann, V. Mehrmann, and B. Unger, *Port-Hamiltonian formulations of poroelastic network models*, arXiv e-print arXiv:2012.01949 (2020). Accepted to appear in Math. Comput. Model. Dyn. Sys.

[4] C. Beattie, V. Mehrmann, H. Xu, and H. Zwart, *Port-Hamiltonian descriptor systems*, Math. Control Signals Systems, **30** (2018), 1–27.

[5] M. A. Biot, *General theory of three-dimensional consolidation*, J. Appl. Phys. **12** (1941), 155–164.

[6] Z. Luo and Y. F. Zhao, *A survey of finite element analysis of temperature and thermal stress fields in powder bed fusion Additive Manufacturing*, Additive Manufacturing, **21** (2018), 318–332.

[7] B. Tully and Y. Ventikos, *Cerebral water transport using multiple-network poroelastic theory: application to normal pressure hydrocephalus*, J. Fluid Mech. **667** (2011), 188–215.

From Velocity to Manifold Turnpikes

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(joint work with Kathrin Flaßkamp and Sina Ober-Blöbaum)

Recently, the turnpike phenomenon, which refers to a similarity property of solutions of parametric Optimal Control Problems (OCPs), was *rediscovered* and successfully exploited in diverse applications like, e.g., Model Predictive Control [2] or the numerical solution of (infinite-dimensional) optimal control problems, see, e.g., [10, 16]. Moreover, the link between the turnpike property and dissipativity was established, see, e.g., [1, 7, 9] or the recent survey article [11]. However, in contrast to the *classical* works by Dorfman, Samuelson, and Solow (see [12] and the references therein), where so-called growth paths are considered, nowadays most authors are concerned with analyzing systems optimally operated at some steady state or periodic orbit in a finite- or an infinite-dimensional setting, see, e.g., [18] or [15], respectively.

Before proceeding let us briefly recall the concept of optimal operation at steady state in context of the continuous-time minimization problem

$$\min_{L^\infty([0,T],\mathbb{R}^m)} \frac{1}{T}\int_0^T \ell(x(t;x^0,u),u(t))\,\mathrm{d}t \tag{1}$$

subject to the initial value problem

$$\dot{x}(t; x^0, u) = f(x(t; x^0, u), u(t)), \qquad x(0; x^0, u) = x^0 \tag{2}$$

where the vector field $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is continuous and locally Lipschitz w.r.t. its first argument in order to guarantee (local) existence and uniqueness. Moreover, let the stage cost $\ell : \mathbb{R}^n \times \mathbb{R}^m$ be continuous and bounded from below. Of course, one may add state and/or control constraints. Then, assuming $f(x^*, u^*) = 0$, i.e. let x^* be a (controlled) steady state, the system (2) is said to be optimally-operated at steady state if, and only if,

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \ell(x(t; x^0, u), u(t))\, \mathrm{d}t \geq \ell(x^*, u^*)$$

holds for all $x^0 \in \mathbb{R}^n$ and $u \in L^\infty([0, T], \mathbb{R}^m)$. This optimal steady state is a solution to

$$\min_{x\in\mathbb{R}^n, u\in\mathbb{R}^m} \ell(x, u) \quad \text{subject to} \quad f(x, u) = 0,$$

which is significantly easier to solve than the previous OCP. The turnpike phenomenon implies that for varying initial conditions x^0 and varying horizon lenghts, the the optimal solutions are *close* to (x^*, u^*) for most of the time. This knowledge can be exploited in the analysis (and numerical solution) of the OCP (1). The key assumptions to establish turnpike properties of OCPs are (strict) dissipativity and some reachability/controllability property of the underlying dynamics.

Inspired by Vorotnikov's notion of partial stability [17] and the original works dealing with growth paths (again, we refer to [12]), we extended this notion to so-called velocity steady-states in [4] for mechanical systems. Here, the OCP was augmented by a terminal state constraint in order to adequately reflect the control objective of state transition with minimal energy. The resulting OCP yields an optimal rate of travel.

In view of mechanics, such energy-efficient solutions are often intrinsically linked to symmetry-induced motions, e.g., translational or rotational ones, cp. [8] as well as [6] and the references therein. In [5], the work [4] was extended by a dissipativity-based characterization, which motivated the work [13], in which the linear-quadratic case was essentially fully covered. Then, the notions were adopted to the nonlinear setting in [3], where symmetry-related manifolds were characterized as energy-optimal motions – again via dissipativity. To this end, a manifold consisting of basic motions – so-called trims was the set, in which optimal solutions spent most of the time with an initial and a leaving arc for the transient phases at the beginning and the end of the optimisation horizon, respectively.

These notions can be further generalized to consider unbounded sets as turnpike attractors for linear port-Hamiltonian systems

$$\dot{x}(t) = (J - R)Qx(t) + Bu(t)$$

with output $y(t) = B^\top Qx(t)$. Here, the energy balance equation

$$\frac{\mathrm{d}}{\mathrm{d}t} H(x(t)) = u(t)^\top y(t) - \|R^{\frac{1}{2}} Qx(t)\|_2^2$$

is automatically satisfied. The interesting part is that the term $u(t)^\top y(t)$ corresponds to the supplied energy while the dissipation is reflected by the positive semi-definite matrix R. The intriguing aspect is that the energy-based modelling approach, i.e. the port-Hamiltonian framework, automatically provides a dissipation inequality w.r.t. the conservative subspace (or the conservative manifold in the nonlinear setting). Hence, we get the key assumption as an outcome of an adequate modelling approach.

In conclusion, the turnpike property (and/or the respective dissipativity notions) turned out to be an essential tool in mathematical systems theory, which is widely applied in economics, mechanics and many other application areas. Combining it with an energy-based modelling approach like the port-Hamiltonian one, alleviates the verification of the required assumptions.

References

[1] T. Damm, L. Grüne, M. Stieler, K. Worthmann *An exponential turnpike theorem for dissipative discrete time optimal control problems*, SIAM Journal on Control and Optimization 52:3, 1935-1957, 2014.

[2] M. Diehl, R. Amrit, J.B. Rawlings *A Lyapunov function for economic optimizing model predictive control*, IEEE Transactions on Automatic Control 56:3, 703-707, 2010.

[3] T. Faulwasser, K. Flasskamp, S. Ober-Blöbaum, M. Schaller, K. Worthmann: *Manifold Turnpikes, Trims and Symmetries*, arXiv e-print 2104.03039

[4] T. Faulwasser, K. Flasskamp, S. Ober-Blöbaum, and K. Worthmann, *Towards velocity turnpikes in optimal control of mechanical systems*, IFAC-PapersOnLine 52:16, 490-495, 2019.

[5] T. Faulwasser, K. Flasskamp, S. Ober-Blöbaum, and K. Worthmann, *A dissipativity characterization of velocity turnpikes in optimal control problems for mechanical systems*, arXiv e-print 2002.04388

[6] K. Flasskamp, S. Ober-Blöbaum, K. Worthmann, *Symmetry and Motion Primitives in Model Predictive Control*, Mathematics of Control, Signals, and Systems 31:4, 455-485, 2019.

[7] T. Faulwasser, M. Korda, C. Jones, D. Bonvin, *On turnpike and dissipativity properties of continuous-time optimal control problems*, Automatica 81, 297-304, 2017.

[8] E. Frazzoli, M.A. Dahleh, E. Feron, *Maneuver-based motion planning for nonlinear systems with symmetries*, IEEE transactions on robotics 21:6, 1077-1091, 2005.

[9] L. Grüne and M.A. Müller, *On the relation between strict dissipativity and turnpike properties*, Systems & Control Letters 90, 45-53, 2016.

[10] L. Grüne, M. Schaller, A. Schiela, *Exponential sensitivity and turnpike analysis for linear quadratic optimal control of general evolution equations*, Journal of Differential Equations 268:12, 7311-7341, 2020.

[11] T. Faulwasser, L. Grüne, *Turnpike Properties in Optimal Control: An Overview of Discrete-Time and Continuous-Time Results*, arXiv preprint arXiv:2011.13670, 2020.

[12] L.W. McKenzie, *The Dorfman-Samuelson-Solow Turnpike Theorem*, International Economic Review 4:1, 29-43, 1963.

[13] D. Pighin, N. Sakamoto, *The turnpike with lack of observability*, arXiv preprint arXiv:2007.14081, 2020.

[14] M. Schaller, F. Philipp, T. Faulwasser, K. Worthmann, B. Maschke, *Control of port-Hamiltonian systems with minimal energy supply*, arXiv preprint arXiv:2011.10296, 2020.

[15] E. Trélat, C. Zhang, and E. Zuazua, *Steady-state and periodic exponential turnpike property for optimal control problems in Hilbert spaces*, SIAM Journal on Control and Optimization 56:2, 1222-1252, 2018.

[16] E. Trélat and E. Zuazua, *The turnpike property in finite-dimensional nonlinear optimal control*, Journal of Differential Equations 258, 81-114, 2015.
[17] V. Vorotnikov, *Partial stability and control*, Springer Science & Business Media, 2012.
[18] M. Zanon, L. Grüne, and M. Diehl, *Periodic optimal control, dissipativity and MPC*, IEEE Transactions on Automatic Control 62:6, 2943–2949 2016.

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