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Modeling communication and movement: from cells to animals and humans

Raluca Eftimie

Communication forms the basis of biological interactions. While the use of a single communication mechanism (for example visual communication) by a species is quite well understood, in nature the majority of species communicate via multiple mechanisms. Here, I review some mathematical results on the unexpected behaviors that can be observed in biological aggregations where individuals interact with each other via multiple communication mechanisms.

1 Communication in biological aggregations

Communication forms the basis of any type of biological interaction: for cells, bacteria, animals and even humans to interact, they first need to communicate with each other. Most generally speaking, we define *communication* to be the process of exchanging information between members of the same species or of different species. In animals and humans, this information exchange occurs via visual, auditory, olfactory, and tactile signals. In cells, communication occurs via signaling pathways that involve signaling proteins and other chemicals. Depending on the type of signals used, communication can be local (for example via short-range tactile signals) or nonlocal (for example via long-range sound or visual signals).

When only one method of communication is used (for example one type of chemical signal), we speak of *single communication* or *single signaling pathway*. The use and effects of signaling mechanisms in cells, bacteria, and animals has been studied intensively over the past fifty years and it is currently quite well understood. If, however, multiple communication mechanisms are used, the interplay between them affects the interactions of an individual (possibly a cell or bacterium) with its neighbors – and it is not fully understood how. Solving this problem has implications to both ecology (to understand how aggregations of bacteria, insects, or animals emerge and persist) and cell biology (to understand, for example, how cells aggregate to form tissues during morphogenesis, or how cancerous aggregations of cells evolve and form solid tumours).

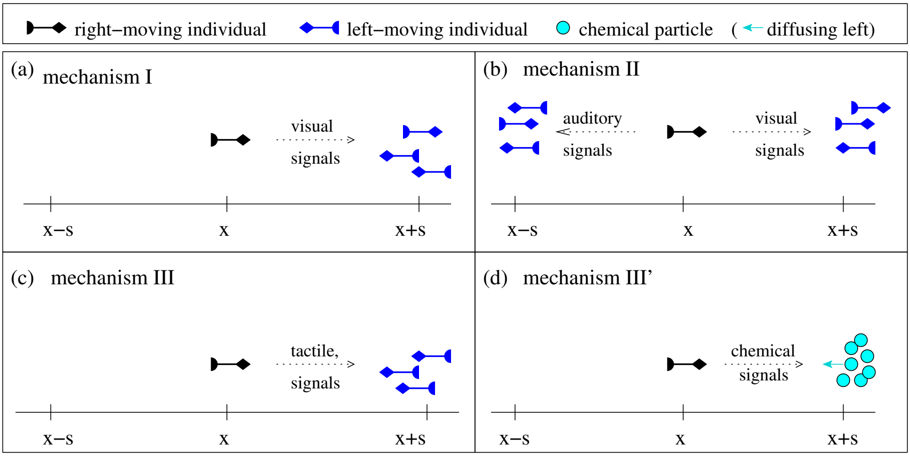

Figure 1: Examples of possible inter-individual interactions via visual, auditory, and tactile communication signals. Shown here are different ways a reference individual positioned at a point in space x can perceive its neighbors: (a) perception, via visual signals, of the neighbors positioned ahead, at $x + s$; (b) perception, via visual and auditory signals, of neighbors positioned ahead (at $x+s$) and behind (at $x-s$); (c) perception, via tactile signals, of only those neighbors positioned ahead at $x + s$ and moving towards the reference individual (This can be observed in *Myxobacteria* organisms, a kind of bacteria that typically travel in swarms.) (d) In *Dictyostelium Discoideum* (a species of amoeba more widely known as slime mold), a reference individual (that is, a cell) can perceive a chemical gradient of cAMP (produced by neighboring cells) that moves towards it.

2 Mathematical approaches

Mathematical models have been used for almost a century to formulate hypotheses regarding the biological mechanisms behind various cell-cell and animal-animal interactions. They are currently used to investigate (among other things) the effect of cell or animal communication mechanisms on the formation, movement, and spatial structure of cell or animal aggregations [4, 2]. This involves methods of mathematical analysis and computer simulations of the solutions of the model.

Mathematical models that describe the transport and movement of cells and animals through a domain can incorporate basic aspects of communication, such as the directionality of communication signals emitted and perceived by neighbors (which can give information about the number of these neighbors and their movement direction). The general form of such models of so-called "hyperbolic" and "kinetic" type that describe movement in one spatial dimension (that is, they apply to domains much longer than wide) is the following (see also [4]):

$$\frac{\partial u^+}{\partial t} + \frac{\partial}{\partial x}(\Gamma(u^+,u^-)u^+) = -\lambda^+(u^+,u^-)u^+ + \lambda^-(u^+,u^-)u^-, \tag{1a}$$

$$\frac{\partial u^-}{\partial t} - \frac{\partial}{\partial x}(\Gamma(u^+,u^-)u^+) = \lambda^+(u^+,u^-)u^+ - \lambda^-(u^+,u^-)u^-. \tag{1b}$$

The functions u^+ and u^- describe the density of right-moving and left-moving cells or animals, respectively. They depend on the position x and the time t; their *partial derivatives* with respect to time, $\frac{\partial u^+}{\partial t}$ and $\frac{\partial u^-}{\partial t}$, describe their change in time. These cells or animals move with velocity Γ, which can depend on the interactions with right- and left-moving neighbors, that is, on u^+ and u^-. There are usually three types of social interactions incorporated into these mathematical models: repulsion from neighbors at close distances, attraction towards neighbors at large distances, and alignment with neighbors at intermediate distances. Thus, individuals can speed up to approach other neighbors further away, or can slow down to avoid colliding with neighbors close by.

They also turn from left to right at rate λ^-, and from right to left at rate λ^+. These turning rates can depend on interactions with neighbors. (Animals may, for example, turn around to approach neighbors positioned behind them.) Generally, interactions with neighbors occur only if individuals can perceive their neighbors. Figure 1 shows examples of possible communication (perception) mechanisms between animals (mechanisms I–III) or cells (mechanism III'). The models in Figure 1 and equations 1a and 1b can be generalized to describe movement in two spatial dimensions (see [5]).

3 Results

Using these mathematical models, my collaborators and I have shown that many of the spatial and spatio-temporal patterns displayed by biological aggregations (such as stationary aggregations, aggregations travelling in a linear manner, or zigzagging aggregations) can be explained by the interplay between the repulsive/alignment/attractive interactions between individuals (cells, bacteria, animals, etc.) and the different biological mechanisms employed to perceive these individuals (via visual, auditory, tactile, or chemical stimuli, or combinations of these stimuli) [4].

Moreover, we have shown that the movement of aggregations formed of individuals that use only one communication mechanism is generally periodic, that is, there is a repeating pattern. In contrast, the movement of biological aggregations formed of individuals that use different communication mechanisms could be chaotic [3] – see, for example, the periodic and chaotic zigzagging behaviors shown in Figure 2(a) and (b), respectively. The use of multiple communication mechanisms by individuals in the same community can also lead to the spatial segregation of these aggregations (see Figure 2(c)-(d)).

In addition, the use of different communication mechanisms by different members of the aggregation can lead to behaviors that cannot be obtained when all individuals communicate with their neighbors in the same way. For example, when all individuals use one communication mechanism (like solely mechanism I or mechanism II in Figure 1), it is possible to obtain no spatial patterns – that is, individuals can be evenly spread over the whole domain. When some individuals in the community communicate via mechanism I while other individuals communicate via mechanism II, however, it is possible to obtain moving aggregations [3].

Once we have determined these aggregation patterns (with the help of mathematical software), two of the most interesting questions are: "What mathematical mechanisms lead to the formation of the patterns, and what mechanisms govern the transitions between different patterns?" Using a mathematical method called "weakly nonlinear analysis", we have been able to show that many of the aggregation patterns do not persist for very long times – they are *unstable* [1]. Moreover, these patterns can co-exist for the same parameter values. This implies that one can observe transitions between the patterns (for example between stationary and moving aggregations) without any change in the parameters that describe individual movement, such as speed and turning rates.

This field is a great opportunity for the use of mathematical modeling and analysis techniques to reproduce and investigate aggregation patterns observed in various species: from flocks of birds, schools of fish, and swarms of insects to various cellular and bacteria aggregations. Moreover, when there are no experimental tools available, these models (and the mathematical tools used to analyze them) can be used to propose hypotheses about the role of cell or animal communication on the formation and structure of biological aggregations.

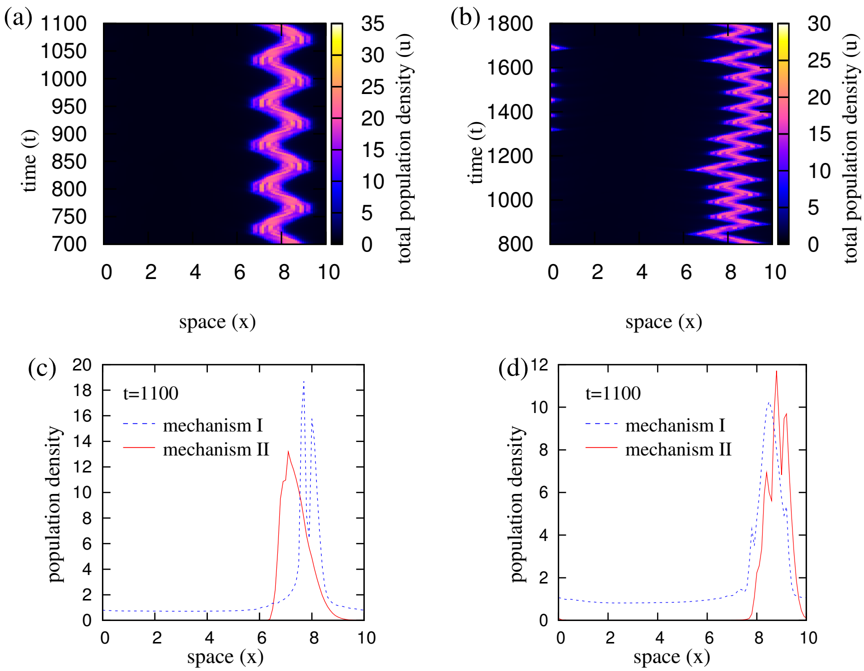

Figure 2: Example of zigzagging patterns and the structure of the aggregations when some individuals in the population can perceive only those neighbors ahead of them (mechanism I in Figure 1(a)) and other individuals can perceive neighbors positioned both behind and ahead of them (mechanism II in Figure 1(b)). Figure 2 shows the total population density $u = u^+ + u^-$ at various points in space $x \in [0, 10]$ and at various points in time t. (a) Periodic zigzags; (b) chaotic zigzags; (c) spatial structure of periodic zigzag aggregations from panel (a) (at $t = 1100$, when the aggregation is moving to the left); (d) spatial structure of chaotic zigzag aggregations from panel (b) (at $t = 1100$, when the aggregation is moving to the right). In Panels (c) and (d), the population that uses communication mechanism II is positioned towards the front end of the aggregation (with respect to its current moving direction).

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Raluca Eftimie *is a senior lecturer in mathematical biology at the University of Dundee.*
reftimie@dundee.ac.uk

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Mathematisches Forschungsinstitut
Oberwolfach gGmbH
Schwarzwaldstr. 9 – 11
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