Oberwolfach Preprints

OWP 2009 - 18

Mathieu Dutour Sikirić, Achill Schürmann, and Frank Vallentin

The Contact Polytope of the Leech Lattice

Mathematisches Forschungsinstitut Oberwolfach gGmbH
Oberwolfach Preprints (OWP) ISSN 1864-7596

Oberwolfach Preprints (OWP)

Starting in 2007, the MFO publishes a preprint series which mainly contains research results related to a longer stay in Oberwolfach. In particular, this concerns the Research in Pairs-Programme (RiP) and the Oberwolfach-Leibniz-Fellows (OWLF), but this can also include an Oberwolfach Lecture, for example.

A preprint can have a size from 1 - 200 pages, and the MFO will publish it on its website as well as by hard copy. Every RiP group or Oberwolfach-Leibniz-Fellow may receive on request 30 free hard copies (DIN A4, black and white copy) by surface mail.

Of course, the full copy right is left to the authors. The MFO only needs the right to publish it on its website *www.mfo.de* as a documentation of the research work done at the MFO, which you are accepting by sending us your file.

In case of interest, please send a **pdf file** of your preprint by email to *rip@mfo.de* or *owlf@mfo.de*, respectively. The file should be sent to the MFO within 12 months after your stay as RiP or OWLF at the MFO.

There are no requirements for the format of the preprint, except that the introduction should contain a short appreciation and that the paper size (respectively format) should be DIN A4, "letter" or "article".

On the front page of the hard copies, which contains the logo of the MFO, title and authors, we shall add a running number (20XX - XX).

We cordially invite the researchers within the RiP or OWLF programme to make use of this offer and would like to thank you in advance for your cooperation.

Imprint:

Mathematisches Forschungsinstitut Oberwolfach gGmbH (MFO)
Schwarzwaldstrasse 9-11
77709 Oberwolfach-Walke
Germany

Tel +49 7834 979 50
Fax +49 7834 979 55
Email admin@mfo.de
URL www.mfo.de

The Oberwolfach Preprints (OWP, ISSN 1864-7596) are published by the MFO.

THE CONTACT POLYTOPE OF THE LEECH LATTICE

MATHIEU DUTOUR SIKIRIĆ, ACHILL SCHÜRMANN, AND FRANK VALLENTIN

Abstract. The contact polytope of a lattice is the convex hull of its shortest vectors. In this paper we classify the facets of the contact polytope of the Leech lattice up to symmetry. There are $1,197,362,269,604,214,277,200$ many facets in 232 orbits.

1. Introduction

An n-dimensional *lattice* L is a discrete subgroup of the n-dimensional Euclidean space $\mathbb{R}^n$ of the form $L = \{\sum_{i=1}^n \alpha_i b_i : \alpha_1, \ldots, \alpha_n \in \mathbb{Z}\}$ where $b_1, \ldots, b_n$ is a basis of $\mathbb{R}^n$. By $\lambda(L)$ we denote the Euclidean length of non-zero shortest vectors of L and we denote by $\operatorname{Min} L$ the set of *shortest vectors*.

Every lattice comes with two important polytopes: The *contact polytope* of L is the convex hull of its shortest vectors

$$\mathrm{C}(L) = \operatorname{conv}\{v : v \in \operatorname{Min} L\},$$

and the *Voronoi cell* of L is the region of points which are closer to the origin than to other lattice points

$$\mathrm{V}(L) = \left\{x \in \mathbb{R}^n : x \cdot v \leq \frac{1}{2} v \cdot v \text{ for all } v \in L\right\}.$$

Maybe one of the most remarkable lattices is the 24-dimensional Leech lattice Λ_{24}. It has 196560 shortest vectors which is the highest possible number in dimension 24. Its *orthogonal group*, i.e. the group of orthogonal transformations preserving the lattice is the Conway group Co_0. It has $2^{22} \cdot 3^9 \cdot 5^4 \cdot 7^2 \cdot 11 \cdot 13 \cdot 23 = 8,315,553,613,086,720,000$ elements and it is connected to many sporadic simple groups. We refer to the book [4] by Conway and Sloane for an extensive treatment of the Leech lattice.

Borcherds, Conway, Parker, Queen, Sloane [4, Chapter 23, Chapter 25] determine the vertices of the Voronoi cell of the Leech lattice. The Voronoi cell tiles the space $\mathbb{R}^n$ by translations; this gives the *Voronoi cell tiling* of $\mathbb{R}^n$. So, in the context of the Voronoi cell it is natural to consider orbits under the *isometry group* (the group generated by the orthogonal group of the Leech lattice together with lattice translations) acting on the Voronoi cell tiling. We denote the isometry group of the Leech lattice by Co_∞. There are 307 orbits of vertices in the Voronoi cell tiling under the action of Co_∞.

In this paper we determine the facets and their incidence relations of the contact polytope of the Leech lattice. We get the following result.

Date: June 8, 2009.
1991 *Mathematics Subject Classification.* 10E30, 52A43, 94B40.
Key words and phrases. Leech lattice, contact polytope, Conway groups, Voronoi cell.

Theorem 1. *There are* 232 *orbits of facets of* $\mathrm{C}(\Lambda_{24})$ *under* Co_0*.*

The contact polytope and the Voronoi cell are related. To see this relation, we consider

$$\mathrm{C}(L)^* = \left\{ x \in \mathbb{R}^n : x \cdot v \leq \frac{1}{2} v \cdot v \text{ for all } v \in \mathrm{Min}\, L \right\},$$

which is the standard polar polytope scaled by a factor of $\frac{1}{2}\lambda(L)^2$. The faces of $\mathrm{C}(L)$ and of $\mathrm{C}(L)^*$ are in bijection. The bijection reverses the inclusion relation: k-dimensional faces of $\mathrm{C}(L)$ correspond to $(n-k)$-dimensional faces of $\mathrm{C}(L)^*$. In particular, vertices of $\mathrm{C}(L)^*$ correspond to facets of $\mathrm{C}(L)$. For these notions we refer to the standard literature on polytope theory, e.g. the book by Ziegler [15].

We chose the scaling in the definition of $\mathrm{C}(L)^*$ so that it contains $\mathrm{V}(L)$. In the case of the Leech lattice some vertices of $\mathrm{V}(\Lambda_{24})$ and $\mathrm{C}(\Lambda_{24})^*$ are shared. As a side remark: One has equality $\mathrm{C}(L)^* = \mathrm{V}(L)$ if and only if L is a root lattice, see Rajan, Shende [13].

Theorem 2. 164 *orbits of vertices of* $\mathrm{C}(\Lambda_{24})^*$ *are also orbits of vertices of* $\mathrm{V}(\Lambda_{24})$*. They are listed in Table 1. The additional* 68 *orbits of vertices are listed in Table 2.*

We classify the shared vertices in Section 2 and give them in Table 1. In Section 3 we classify the additional vertices of $\mathrm{C}(\Lambda_{24})^*$ which are not vertices of $\mathrm{V}(\Lambda_{24})$. We conclude the paper by Section 4 where we briefly explain our computational techniques.

The data presented here is also electronically available from [20].

2. Shared vertices

In this section we explain the notation used in Table 1 which contains the 164 orbits of shared vertices mentioned in Theorem 2.

The vertices of the Voronoi cell of a lattice are centers of *empty spheres*, i.e. spheres $S(x, \|x\|)$ with center x and radius $\|x\|$ which contain lattice points on their boundary but not in their interior. The convex hull of lattice points on the boundary of such an empty sphere is called the *Delone cell* of the vertex x.

The Delone cells of the Leech lattice are classified by Borcherds, Conway, Parker, Queen, Sloane [4, Chapter 23, Chapter 25] up to the action of the isometry group Co_∞. For this classification they use Coxeter-Dynkin diagrams.

A *Coxeter-Dynkin diagram* with vertex-set $\{1, \ldots, N\}$ is a symmetric $N \times N$ matrix $(m_{ij})_{1 \leq i,j \leq N}$ with ones on the diagonal and $m_{ij} \geq 2$ if $i \neq j$ and $m_{ij} \in \mathbb{N} \cup \{\infty\}$.

A Coxeter-Dynkin diagram is called *simply laced* if $m_{ij} = 2$, 3 or ∞. The *Cartan matrix* of a Coxeter-Dynkin diagram $(m_{ij})_{1 \leq i,j \leq N}$ is the matrix $M = (-\cos \frac{\pi}{m_{ij}})_{1 \leq i,j \leq N}$. A Coxeter-Dynkin diagram is called *spherical* if its Cartan matrix is positive definite and *affine* if its Cartan matrix is positive semidefinite. A Coxeter-Dynkin diagram is called *decomposable* if we can partition its vertex-set into $S_1 \cup S_2$ with $m_{ij} = 2$ if $i \in S_1$ and $j \in S_2$. It is called *indecomposable* otherwise. A Coxeter-Dynkin diagram D admits a unique decomposition into indecomposable Coxeter-Dynkin diagrams $D_1, \ldots, D_r$, which we write as $D = D_1 D_2 \ldots D_r$. The classification of spherical and affine Coxeter-Dynkin diagrams is presented, for example, in Humphreys [9, Section 2.4, 4.7]. Here the famous $A-D-E$ diagrams show up, explained e.g. by Hazewinkel, Hesselink, Siersma, Veldkamp [8]. The spherical, simply laced, indecomposable Coxeter-Dynkin diagrams are a_n for $n \geq 1$, d_n for

$n \geq 4$ and e_n for $6 \leq n \leq 8$. Each diagram corresponds to an indecomposable affine diagram: A_n, D_n and E_n. All these diagrams are pictured e.g. in [4, Figure 23.1].

In the Leech lattice, a Coxeter-Dynkin diagram $(m_{ij})_{1\leq i,j\leq N}$ can be associated with a Delone cell with vertex-set $\{v_1, \dots, v_N\}$ by

$$m_{ij} = \begin{cases} 1 & \text{if} \quad \|v_i - v_j\|^2 = 0, \\ 2 & \text{if} \quad \|v_i - v_j\|^2 = 4, \\ 3 & \text{if} \quad \|v_i - v_j\|^2 = 6, \\ \infty & \text{if} \quad \|v_i - v_j\|^2 = 8. \end{cases}$$

As can been seen in Table 1, different Delone cells may have the same Coxeter-Dynkin diagram.

In Table 1 the rows are sorted first by the squared length $\|v\|^2$ (third column) of the vertex v. Second they are sorted by the size of the stabilizer of v within the orthogonal group of the Leech lattice (fifth column), and then by the number of incident facets of $\mathrm{C}(\Lambda_{24})^*$ (fourth column).

In the second column we give the Coxeter-Dynkin diagrams of the associated Delone cell of v. Note that the diagrams are affine if and only if the squared length of v equals 2, the maximum among shared vertices. In all other cases they are spherical. Furthermore, in the spherical cases the number of incident facets is always equal to the minimum possible number of 24. These observations follow from [4, Chapter 23, Chapter 25].

In the last column we give the MOG (Miracle Octad Generator) coordinates of representatives of each orbit which one has to multiply with α (sixth column). The MOG coordinates form a standard coordinate system for the Leech lattice. They are explained in [4, Chapter 11].

There are 307 orbits of vertices in the Voronoi cell tiling under the action of the isometry group Co_∞ of the Leech lattice. Our computation shows that there are 5297 orbits of vertices of the single Voronoi cell $\mathrm{V}(\Lambda_{24})$ under the action of the smaller, finite orthogonal group of the Leech lattice; 164 of them are shared with $\mathrm{C}(\Lambda_{24})^*$.

3. Additional vertices

There are 68 additional orbits of vertices of $\mathrm{C}(\Lambda_{24})^*$ which are not vertices of the Voronoi cell of the Leech lattice. These additional vertices are characterized by the fact that the distance to a closest lattice point is strictly less than the distance $\|v\|$ to the origin.

Table 2 describes these 68 orbits. Like in Table 1 the rows are sorted (in this order) by the squared length $\|v\|^2$ (third column), the size of the stabilizer of v within the orthogonal group of the Leech lattice (fifth column), and then by the number of incident facets (fourth column).

In the second column we give names for diagrams. The first row corresponds to an exceptional vertex which we explain below. The other 67 rows correspond to graphs which we define later in Section 3.2.

3.1. The exceptional vertex.

The first orbit of vertices is exceptional: Its squared norm $8/3 = 2.666\dots$ is substantially bigger than the squared norm of all other vertices which lie in the interval $[1.92, 2.25]$. Its incidence number of 552 as well as the size of its stabilizer, which is the Conway group Co_3, are also substantially larger than the values for the other vertices. This orbit of vertices is a scaled copy of the vectors of Λ_{24}, having Euclidean norm $\sqrt{6}$.

In the contact polytope $\mathrm{C}(\Lambda_{24})$ this exceptional vertex corresponds to a facet. Since it has maximum norm among all vertices the corresponding facet is closest to the origin and has the largest possible circumsphere among all other facets of $\mathrm{C}(\Lambda_{24})$. This solves a conjecture of Ballinger, Blekherman, Cohn, Giansiracusa, Kelly, Schürmann [1, Section 3.7]. We note that a similar calculation as the one presented here, solves the corresponding conjecture about the contact polytope of the 23-dimensional lattice O_{23}, the shorter Leech lattice, which has 4600 vertices.

The 23-dimensional point configuration, given by the 552 shortest vectors of the Leech lattice defining facets incident to the exceptional vertex, appears in several different contexts: It is universally optimal (Cohn, Kumar [5]), it defines 276 equiangular lines (Lemmens, Seidel [10]), and it defines an extreme Delone cell (Deza, Laurent [6, Chapter 16.3]). Moreover, it contains a wealth of remarkable substructures (see Cohn et. al. [1]), e.g. the highly-symmetric point configurations discussed in the next section, but also others, e.g. the one defined by the McLaughlin graph.

3.2. **The other vertices.** To the remaining 67 orbits of vertices we associate a diagram as follows. Let v be one of these vertices and let $w_1, \dots, w_N$ be shortest vectors of the Leech lattice defining facets incident to v. Only the two inner products 1 and 2 occur between distinct vectors w_i and w_j. So we can define a graph with vertex-set $\{1, \dots, N\}$ and edge-set $\{\{i,j\} : w_i \cdot w_j = 1\}$; the other inner product 2 defines non-edges.

Here again the graphs decompose into connected components where several of these occurring components are highly-symmetric and have been studied in other contexts. We discuss them below, the graphs a_n, d_n and e_n are already described in the previous section, and the remaining ones are in Figure 1.

The *Higman-Sims graph* HS_{100} is the unique strongly regular graph with parameters $(100, 22, 0, 6)$. See Brouwer, Cohen, Neumaier [3, Chapter 13.1].

The *Hoffman-Singleton graph* HS_{50} is the unique strongly regular graph with parameters $(50, 7, 0, 1)$. See [3, Chapter 13.1].

For the *Johnson graph* $J(7,4)$ see [3, Chapter 9.1].

A (k,g)*-cage* is a regular graph of valency k, girth g, which attains the minimum possible number of vertices. The $(5,6)$-cage (incidence graph of a projective plane $\mathrm{PG}(2,4)$) and the $(3,8)$-cage (*Tutte-Coxeter graph*) are unique. See [3, Chapter 6.9] and Tutte [14].

The *Coxeter graph* Cox is the unique distance regular graph with intersection array $\{3,2,2,1;1,1,1,2\}$. See [3, Chapter 12.3].

In Figure 1 we list the remaining graphs. The vertices of these graphs only have degree one (white circles), degree two (sitting on edges, which are not depicted, but see below), or degree three (black circles). We have three kinds of trees: $T^a_b c$ having $a+b+c+4$ vertices, $T^a_b c^d_e$ having $a+b+c+d+e+6$ vertices, and $T^a_b c^d e^f_g$ having $a+b+c+d+e+f+g+8$ vertices; we have 12 other graphs $G_{n,m}$ with n vertices and m edges. In Figure 1 the numbers on the edges show how many vertices of degree 2 sit on them, but in the following four cases we did not put these numbers: The graph $G_{24,30}$ has one vertex of degree 2 on every edge, $G_{25,30}$ is the Petersen graph which has one vertex of degree 2 on every edge, $G_{22,22}$ has three vertices of degree 2 on every edge, and the graph $G_{24,27}$ is the complete bipartite graph $K_{3,3}$ which has two vertices of degree 2 on every edge.

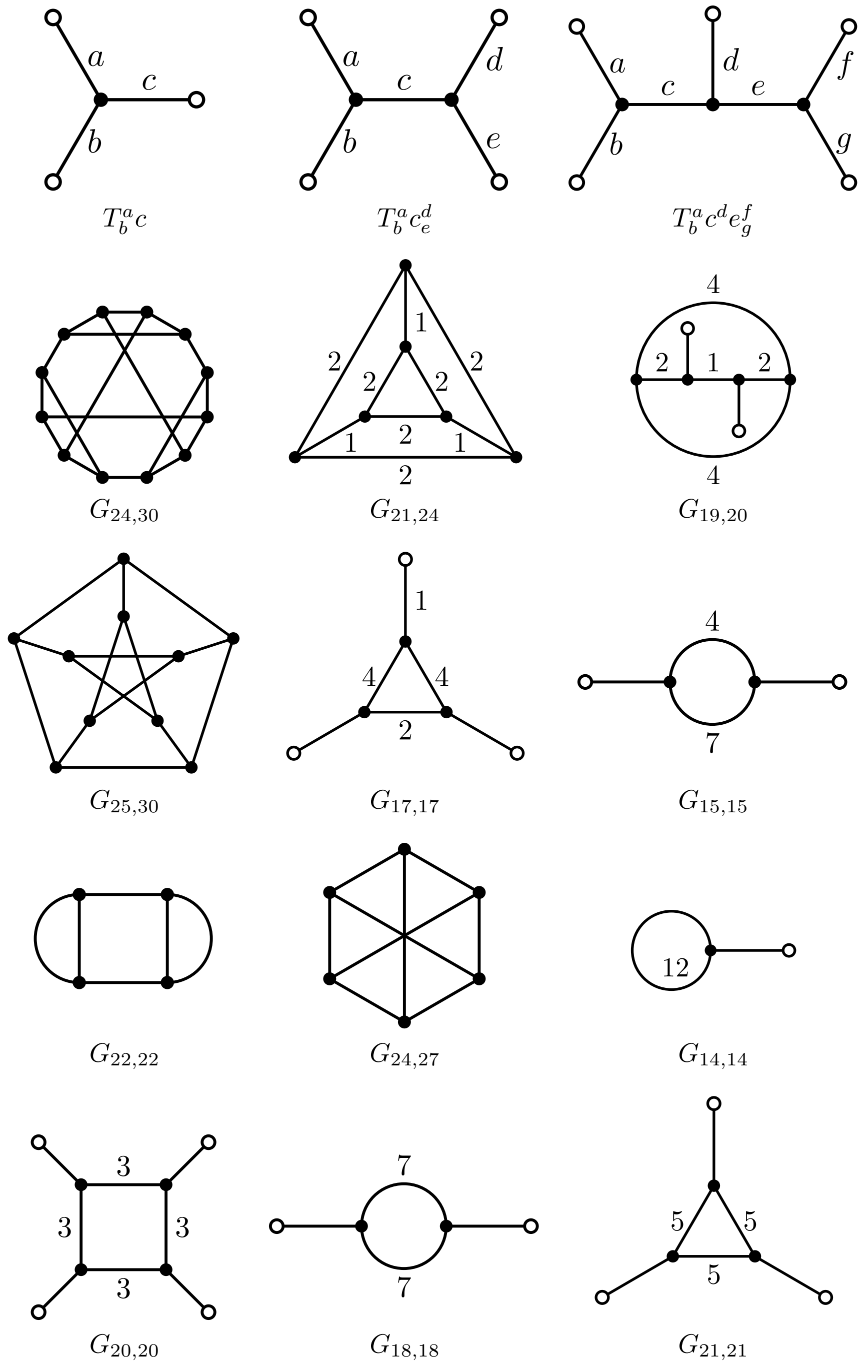

FIGURE 1. DIAGRAMS

4. Computational techniques

Computing the vertices of $C(\Lambda_{24})^*$ from its facets is called a *polyhedral representation conversion problem*. A direct application of standard programs like Fukuda's `cdd` [18] or Avis' `lrs` [16] for this conversion is not feasible due to the extremely large number of vertices.

In order to exploit the symmetries of $C(\Lambda_{24})^*$, we use the *adjacency decomposition method* which is surveyed in Bremner, Dutour Sikirić, Schürmann [2]. An implementation by the first author is available from [17].

The adjacency decomposition method computes a complete list of inequivalent vertex representatives. First one computes an initial vertex by solving a linear program and inserts it into the list of orbit representatives. From any such representative, we compute the list of adjacent vertices, and if they give a new orbit, we insert it into the list of representatives. After finitely many steps all orbits have been treated. Computing adjacent vertices is a lower-dimensional representation conversion problem. So this method can be applied recursively.

For $C(\Lambda_{24})^*$ we had to come up with two case-specific insights:

From [1] it is known that the exceptional vertex of Section 3.1 is indeed a vertex of $C(\Lambda_{24})^*$. We used it as starting vertex of the adjacency decomposition method.

For checking isomorphy and for computing stabilizers we used the following standard strategy: we characterize a vertex of $C(\Lambda_{24})^*$ by the set of its incident facets and we represent the symmetry group Co_0 as a permutation group acting on the 196560 shortest vectors of the Leech lattice. Then, we use the backtracking algorithm by Leon [11, 12] implemented in [19]. This worked reasonably fast for all the cases except for the two orbits of vertices having the same Coxeter-Dynkin diagram a_1^{25}. The stabilizer of the corresponding Delone cell under the isometry group Co_∞ is the Mathieu group M_{24}. Under the action of M_{24} the 25 vertices of the Delone cell split into two orbits of size 1 and 24. Hence, these two orbits correspond to two distinct orbits of vertices of $C(\Lambda_{24})^*$, one having stabilizer M_{24} and the other having stabilizer M_{23}. The backtracking algorithm of GAP could not decide in reasonable time whether or not two vertices with the same Coxeter-Dynkin diagram a_1^{25} are in the same orbit. So we used the third method of Section 3.5 of [7] to resolve this problem.

Acknowledgements

We started this research during the Junior Trimester Program (February 2008–April 2008) on "Computational Mathematics' at the Hausdorff Institute of Mathematics (HIM) in Bonn. Then, part of this research was done at the Mathematisches Forschungsinstitut Oberwolfach during a stay within the Research in Pairs Programme from May 3, 2009 to May 16, 2009. We thank both institutes for their hospitality and support. The work of the first author has been supported by the Croatian Ministry of Science, Education and Sport under contract 098-0982705-2707. The second and the third author were supported by the Deutsche Forschungsgemeinschaft (DFG) under grant SCHU 1503/4-2.

References

[1] B. Ballinger, G. Blekherman, H. Cohn, N. Giansiracusa, E. Kelly, A. Schürmann, *Experimental study of energy-minimizing point configurations on spheres*, to appear in Experiment. Math.

[2] D. Bremner, M. Dutour Sikirić, A. Schürmann, *Polyhedral representation conversion up to symmetries*, *Proceedings of the 2006 CRM workshop on polyhedral computation*, AMS/CRM Lecture Notes, **48** (2009), 45–71,
[3] A.E. Brouwer, A.M. Cohen, A. Neumaier, *Distance-regular graphs*, Springer, 1989.
[4] J.H. Conway, N.J.A. Sloane, *Sphere packings, lattices and groups (third edition)*, Springer, 1999.
[5] H. Cohn, A. Kumar, *Universally optimal distribution of points on spheres*, J. Amer. Math. Soc. **20** (2007), 99–148.
[6] M. Laurent, M. Deza, *Geometry of cuts and metrics*, Springer, 1997.
[7] M. Dutour Sikirić, A. Schürmann, F. Vallentin, *Complexity and algorithms for computing Voronoi cells of lattices*, Math. Comp. **267** (2009), 1713–1731.
[8] M. Hazewinkel, W. Hesselink, D. Siersma, F.D. Veldkamp, *The ubiquity of Coxeter-Dynkin diagrams (an introduction to the $A-D-E$ problem)*, Nieuw Arch. Wisk. **25** (1977), 257–307.
[9] J.E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge University Press, 1990.
[10] P.W. Lemmens, J.J. Seidel, *Equiangular lines*, J. Algebra **24** (1973), 494–512.
[11] J.S. Leon, *Permutation group algorithms based on partitions, I: Theory and algorithms*, J. Symbolic Comput. **12** (1991), 533–583.
[12] J.S. Leon, *Partitions, refinements, and permutation group computation*, in *Groups and Computation II*, DIMACS Ser. Discrete Math. Theoret. Comput. Sci. **28** (1997), 123–158.
[13] D.S. Rajan, A.M. Shende, *A characterization of root lattices*, Discrete Math. **161** (1996), 309–314.
[14] W.T. Tutte, *A family of cubical graphs*, Proc. Cambridge Philos. Soc. **43** (1947), 459–474.
[15] G.M. Ziegler, *Lectures on polytopes*, Springer, 1995.

Software

[16] D. Avis, *A C-implementation of the reverse search vertex enumeration algorithm*, School of Computer Science, McGill University, Montreal, Canada 1993, `http://www-cgrl.cs.mcgill.ca/~avis/C/lrs.html`
[17] M. Dutour Sikirić, *Polyhedral package*, `http://www.liga.ens.fr/~dutour`
[18] K. Fukuda, *cdd+ reference manual*, Institute for operations research, Swiss Federal Institute of Technology, Zürich, Switzerland, 1995, `http://www.ifor.math.ethz.ch/~fukuda/cdd_home/cdd.html`
[19] The GAP Group, *GAP — Groups, Algorithms, and Programming*, Version 4.4.6, 2005, `http://www.gap-system.org`

Web pages

[20] M. Dutour Sikirić, *Contact polytopes of lattices*, `http://www.liga.ens.fr/~dutour`

M. Dutour Sikirić, Rudjer Bosković Institute, Bijenicka 54, 10000 Zagreb, Croatia
E-mail address: `mdsikir@irb.hr`

A. Schürmann, Mathematics Department, University of Magdeburg, 39106 Magdeburg, Germany
E-mail address: `achill@math.uni-magdeburg.de`

F. Vallentin, Delft Institute of Applied Mathematics, Technical University of Delft, P.O. Box 5031, 2600 GA Delft, The Netherlands
E-mail address: `f.vallentin@tudelft.nl`

	name	$\|v\|^2$	N	g	α	M				O				G			
1.	A_1^{24}	2	48	20891566080	$\frac{1}{2}$	0	2	0	2	0	-2	2	0	-2	0	0	2
						-2	0	-2	0	2	0	0	2	-2	0	2	4
2.	D_4^6	2	30	5760	$\frac{1}{6}$	1	5	1	7	1	-9	7	-5	-5	1	1	5
						-7	1	-3	3	5	1	3	5	-7	3	5	9
3.	$A_5^4D_4$	2	29	864	$\frac{1}{6}$	4	6	0	6	2	-10	2	-6	-2	8	0	-2
						-4	0	-6	2	2	4	6	4	-2	2	6	10
4.	E_6^4	2	28	36	$\frac{1}{12}$	2	8	6	8	2	-18	16	-12	-14	2	0	8
						-16	2	-8	6	10	4	6	8	-12	8	6	18
5.	$A_7^2D_5^2$	2	28	32	$\frac{1}{8}$	7	7	-1	7	3	-11	3	-7	-3	11	1	-5
						-9	-1	-9	3	3	3	9	5	-3	3	7	13
6.	D_6^4	2	28	24	$\frac{1}{10}$	2	8	2	8	0	-14	14	-8	-10	4	0	6
						-12	2	-8	6	8	2	6	8	-12	6	8	14
7.	$A_9^2D_6$	2	27	20	$\frac{1}{10}$	0	8	4	8	0	-16	12	-8	-10	6	2	6
						-14	2	-6	6	10	2	6	6	-10	6	6	14
8.	$A_{17}E_7$	2	26	12	$\frac{1}{18}$	0	12	4	12	-2	-26	22	-18	-16	6	2	8
						-26	4	-8	6	16	2	14	8	-20	14	10	32
9.	$A_{11}D_7E_6$	2	27	8	$\frac{1}{12}$	1	11	5	9	-1	-19	13	-11	-11	3	1	9
						-17	1	-7	5	11	3	9	7	-11	9	5	19
10.	$A_{11}D_7E_6$	2	27	6	$\frac{1}{12}$	-1	9	5	7	-1	-17	15	-13	-9	5	1	7
						-17	3	-5	3	11	3	9	5	-13	9	7	21
11.	$A_{17}E_7$	2	26	6	$\frac{1}{18}$	2	16	8	14	0	-28	20	-16	-18	8	2	12
						-24	4	-10	10	16	4	12	12	-16	14	10	28
12.	$D_{10}E_7^2$	2	27	4	$\frac{1}{18}$	16	18	-2	16	2	-24	6	-16	-6	30	0	-8
						-16	-4	-16	8	8	8	16	12	-8	8	18	30
13.	D_8^3	2	27	4	$\frac{1}{14}$	2	10	4	12	0	-20	16	-12	-14	6	2	10
						-18	2	-8	8	12	2	10	12	-18	8	10	20
14.	$A_{15}D_9$	2	26	4	$\frac{1}{16}$	-1	11	5	11	-1	-23	19	-17	-15	7	3	7
						-21	3	-5	5	15	3	11	9	-17	13	9	29
15.	$D_{10}E_7^2$	2	27	2	$\frac{1}{18}$	4	14	6	14	0	-26	22	-14	-18	10	2	12
						-24	4	-12	10	16	4	12	14	-20	12	12	26
16.	$D_{10}E_7^2$	2	27	2	$\frac{1}{18}$	16	16	-2	16	2	-26	8	-18	-6	28	2	-10
						-18	-4	-18	6	6	6	18	12	-6	6	18	28
17.	E_8^3	2	27	2	$\frac{1}{30}$	22	30	0	30	8	-44	10	-28	-8	42	4	-20
						-28	-6	-32	12	12	8	28	14	-12	12	32	50
18.	E_8^3	2	27	2	$\frac{1}{30}$	31	27	-3	27	3	-43	15	-25	-7	41	-1	-11
						-31	-5	-31	17	11	13	31	19	-11	9	31	49
19.	E_8^3	2	27	2	$\frac{1}{30}$	3	23	3	13	-1	-39	47	-29	-29	11	3	9
						-41	5	-17	11	25	5	21	19	-33	21	19	51
20.	$D_{16}E_8$	2	26	2	$\frac{1}{30}$	6	24	8	20	0	-44	38	-24	-30	14	2	20
						-40	6	-20	16	26	8	20	24	-32	24	18	44

TABLE 1. SHARED VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
21.	$D_{16}E_8$	2	26	2	$\frac{1}{30}$	0	24	12	22	0	−50	36	−28	−28	12	2	20
						−42	4	−16	12	26	6	20	18	−30	24	16	44
22.	$D_{16}E_8$	2	26	2	$\frac{1}{30}$	−2	20	10	20	−2	−44	38	−28	−26	10	2	14
						−42	10	−14	10	26	4	22	16	−34	24	18	52
23.	$D_{16}E_8$	2	26	1	$\frac{1}{30}$	27	29	−3	27	3	−43	15	−27	−9	45	1	−13
						−27	−5	−31	15	11	13	27	21	−11	11	31	49
24.	$D_{16}E_8$	2	26	1	$\frac{1}{30}$	25	29	−3	25	3	−41	11	−29	−9	47	1	−15
						−29	−5	−27	13	11	15	25	21	−15	13	31	51
25.	D_{12}^2	2	26	1	$\frac{1}{22}$	4	16	8	18	0	−32	26	−20	−22	10	2	16
						−30	6	−12	10	20	4	14	16	−26	14	14	32
26.	a_1d_{24}	$\frac{17296}{8649}$	25	1	$\frac{1}{93}$	4	76	36	68	0	−152	108	−84	−88	40	8	64
						−128	16	−52	44	80	20	64	60	−92	72	48	140
27.	$a_1a_2d_{22}$	$\frac{13252}{6627}$	25	1	$\frac{1}{141}$	−4	94	46	90	−12	−206	176	−136	−128	58	16	68
						−194	40	−64	52	124	20	100	78	−160	112	82	244
28.	$a_1e_8^3$	$\frac{7440}{3721}$	25	6	$\frac{1}{61}$	−2	50	26	42	−6	−94	78	−62	−58	10	6	38
						−82	6	−34	26	50	10	38	34	−62	54	38	94
29.	$a_1d_{16}e_8$	$\frac{7440}{3721}$	25	1	$\frac{1}{61}$	−4	40	24	40	−8	−88	76	−64	−56	32	12	32
						−80	12	−28	20	56	12	40	32	−64	48	32	108
30.	$a_1a_2d_{14}e_8$	$\frac{5764}{2883}$	25	1	$\frac{1}{93}$	78	88	−8	78	12	−126	32	−90	−28	148	4	−48
						−90	−18	−84	40	36	44	78	66	−48	40	96	156
31.	a_1a_{24}	$\frac{5200}{2601}$	25	2	$\frac{1}{51}$	0	36	16	32	−4	−76	64	−52	−48	20	8	24
						−68	12	−24	20	44	8	36	28	−56	40	28	88
32.	$a_1a_8e_8^2$	$\frac{5200}{2601}$	25	2	$\frac{1}{153}$	−12	108	68	100	−20	−220	188	−156	−148	84	28	84
						−204	28	−76	52	140	36	92	76	−156	116	84	268
33.	$a_1a_8d_{16}$	$\frac{5200}{2601}$	25	1	$\frac{1}{153}$	134	138	−6	150	34	−214	74	−134	−46	222	14	−94
						−134	−30	−166	66	66	58	134	94	−66	66	166	250
34.	$a_1a_2a_6e_8^2$	$\frac{5080}{2541}$	25	2	$\frac{1}{231}$	218	246	−22	190	22	−338	94	−202	−78	322	−10	−82
						−230	−34	−242	138	82	106	218	154	−82	74	214	386
35.	$a_1a_2a_6d_{16}$	$\frac{5080}{2541}$	25	1	$\frac{1}{231}$	2	166	62	158	−14	−334	294	−230	−214	86	46	110
						−310	50	−74	74	206	30	158	106	−262	182	114	418
36.	$a_1d_{10}d_{14}$	$\frac{4416}{2209}$	25	1	$\frac{1}{47}$	6	34	18	38	0	−70	56	−42	−48	24	4	32
						−64	10	−28	22	42	8	32	34	−56	28	30	66
37.	$a_1a_{16}e_8$	$\frac{4112}{2057}$	25	1	$\frac{1}{187}$	−8	128	72	120	−24	−272	232	−192	−176	96	32	96
						−248	40	−88	64	168	40	120	96	−200	144	104	328
38.	$a_1a_3d_{14}e_7$	$\frac{2047}{1024}$	25	1	$\frac{1}{64}$	54	60	−6	54	8	−86	22	−62	−20	102	2	−34
						−62	−12	−58	26	26	30	54	44	−32	28	66	108
39.	$a_1d_{12}^2$	$\frac{4048}{2025}$	25	2	$\frac{1}{45}$	2	34	18	30	2	−74	54	−42	−46	18	2	30
						−62	6	−26	22	38	10	30	30	−46	34	22	66
40.	$a_1^2a_{15}e_8$	$\frac{1921}{961}$	25	1	$\frac{1}{62}$	−4	42	24	40	−8	−90	76	−64	−58	32	12	32
						−82	12	−28	20	56	14	40	32	−66	48	34	110

TABLE 1 (CONTD.). SHARED VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
41.	$a_1a_{10}d_{14}$	$\frac{3716}{1859}$	25	1	$\frac{1}{143}$	122	132	−12	122	20	−194	52	−138	−44	224	8	−80
						−138	−26	−132	56	56	68	122	98	−72	64	148	240
42.	$a_1a_4d_{14}e_6$	$\frac{3628}{1815}$	25	1	$\frac{1}{165}$	49	109	69	129	15	−245	211	−149	−165	55	11	119
						−231	41	−79	73	153	37	73	109	−179	125	109	245
43.	$a_1^2a_9d_{14}$	$\frac{3608}{1805}$	25	1	$\frac{1}{95}$	0	76	36	68	0	−156	112	−88	−92	40	8	64
						−132	16	−52	44	84	20	64	60	−96	72	48	140
44.	$a_1^2a_9d_{14}$	$\frac{3608}{1805}$	25	1	$\frac{1}{95}$	81	89	−7	81	13	−129	33	−93	−29	149	5	−53
						−93	−17	−87	37	37	45	81	65	−47	43	97	159
45.	$a_1a_9e_7e_8$	$\frac{3608}{1805}$	25	1	$\frac{1}{95}$	−6	66	42	62	−14	−138	118	−98	−90	50	18	50
						−126	18	−46	30	86	22	58	46	−98	74	54	166
46.	$a_1a_2a_{14}e_8$	$\frac{3608}{1805}$	25	1	$\frac{1}{95}$	76	96	0	76	20	−140	36	−84	−24	148	−4	−48
						−84	−28	−100	52	48	24	84	44	−28	40	100	160
47.	$a_1a_3a_{21}$	$\frac{1781}{891}$	25	1	$\frac{1}{198}$	−2	132	58	130	−16	−288	248	−198	−184	84	28	94
						−272	50	−92	76	172	30	140	104	−220	156	116	342
48.	$a_1a_5d_5d_{14}$	$\frac{1733}{867}$	25	1	$\frac{1}{102}$	87	95	−9	85	13	−139	35	−99	−31	161	5	−57
						−99	−19	−93	41	41	49	87	69	−51	45	105	171
49.	$a_1^2a_2a_7d_{14}$	$\frac{1727}{864}$	25	1	$\frac{1}{144}$	−7	97	49	93	−9	−209	179	−133	−131	55	13	71
						−203	43	−67	55	127	17	103	81	−163	115	85	247
50.	$a_1^2a_2a_7d_{14}$	$\frac{1727}{864}$	25	1	$\frac{1}{144}$	122	134	−12	122	20	−196	50	−140	−44	226	8	−80
						−140	−26	−132	56	56	68	124	98	−72	64	148	242
51.	$a_1a_2a_7e_7e_8$	$\frac{1727}{864}$	25	1	$\frac{1}{144}$	−1	119	55	107	17	−215	181	−131	−149	65	27	81
						−205	29	−69	65	137	7	65	71	−133	101	83	241
52.	$a_1a_4a_6d_{14}$	$\frac{3428}{1715}$	25	1	$\frac{1}{245}$	210	228	−20	206	32	−334	84	−238	−76	384	12	−136
						−238	−46	−224	96	96	116	210	166	−124	108	252	412
53.	$a_1^2a_2a_{13}e_8$	$\frac{3400}{1701}$	25	1	$\frac{1}{189}$	−10	130	74	122	−26	−274	234	−194	−178	98	34	98
						−250	38	−86	62	170	42	122	94	−202	146	102	334
54.	$a_1^2a_2a_{13}e_8$	$\frac{3400}{1701}$	25	1	$\frac{1}{189}$	181	195	−17	155	17	−283	71	−167	−63	263	−11	−71
						−175	−35	−199	111	71	83	181	119	−71	55	179	319
55.	$a_1a_4a_{12}e_8$	$\frac{3248}{1625}$	25	1	$\frac{1}{325}$	−16	224	128	208	−48	−472	400	−336	−304	168	56	168
						−432	64	−152	104	296	72	208	160	−344	248	176	576
56.	$a_1a_{10}e_6e_8$	$\frac{3232}{1617}$	25	1	$\frac{1}{231}$	−16	160	96	152	−32	−336	288	−240	−216	120	40	120
						−304	48	−112	72	208	56	144	112	−240	176	128	408
57.	$a_1a_{11}d_{13}$	$\frac{3172}{1587}$	25	1	$\frac{1}{69}$	1	55	27	49	1	−113	81	−65	−69	29	5	47
						−95	11	−39	31	59	13	47	45	−71	51	35	101
58.	$a_1a_{11}d_5e_8$	$\frac{3172}{1587}$	25	1	$\frac{1}{69}$	−4	48	28	44	−10	−100	86	−72	−64	36	12	36
						−92	14	−32	22	62	16	44	34	−72	52	38	122
59.	$a_1a_2d_{10}d_{12}$	$\frac{3172}{1587}$	25	1	$\frac{1}{69}$	−4	46	22	42	−6	−98	86	−64	−62	28	4	32
						−98	22	−34	28	58	8	46	42	−82	58	40	118
60.	$a_1^2a_2^2a_{11}e_8$	$\frac{1535}{768}$	25	1	$\frac{1}{96}$	92	100	−8	78	8	−142	36	−84	−32	134	−4	−36
						−90	−18	−102	56	36	42	92	60	−36	28	92	162

Table 1 (contd.). Shared vertices

	name	$\|v\|^2$	N	g	α	M				O				G			
61.	$a_1a_2a_8e_6e_8$	$\frac{3040}{1521}$	25	1	$\frac{1}{117}$	20	80	40	88	4	−172	144	−108	−120	44	12	84
						−164	36	−56	60	116	28	60	80	−132	80	68	172
62.	$a_1a_2a_4a_{10}e_8$	$\frac{2968}{1485}$	25	1	$\frac{1}{495}$	472	516	−44	404	44	−736	188	−428	−168	692	−20	−188
						−472	−92	−520	288	188	212	472	308	−188	148	476	832
63.	$a_1a_2a_9d_{13}$	$\frac{1469}{735}$	25	1	$\frac{1}{210}$	−14	140	68	132	−18	−304	262	−194	−190	80	14	106
						−292	62	−98	86	182	22	146	120	−242	170	128	362
64.	$a_1a_2a_9d_5e_8$	$\frac{1469}{735}$	25	1	$\frac{1}{210}$	202	216	−20	170	20	−310	80	−182	−72	296	−8	−80
						−202	−38	−220	120	80	92	202	134	−80	64	200	352
65.	$a_1d_{10}e_7^2$	$\frac{2736}{1369}$	25	2	$\frac{1}{37}$	2	30	14	22	−2	−58	42	−34	−30	14	2	26
						−46	10	−22	22	30	6	26	22	−42	34	26	58
66.	$a_1a_{17}e_7$	$\frac{2736}{1369}$	25	2	$\frac{1}{37}$	0	28	12	28	4	−52	48	−36	−36	12	8	20
						−52	8	−16	16	36	0	20	20	−36	28	20	64
67.	$a_1^2a_4a_{19}$	$\frac{1351}{676}$	25	1	$\frac{1}{52}$	0	36	16	34	−4	−76	66	−52	−48	22	8	24
						−70	12	−24	20	46	8	36	28	−58	40	30	90
68.	$a_1a_6a_{18}$	$\frac{2392}{1197}$	25	1	$\frac{1}{399}$	4	276	124	260	−28	−584	504	−404	−372	172	64	192
						−536	100	−176	148	356	60	260	212	−440	308	228	696
69.	$a_1a_{17}d_7$	$\frac{1151}{576}$	25	2	$\frac{1}{48}$	3	41	21	35	−1	−75	55	−43	−47	19	7	33
						−65	9	−29	25	41	11	33	31	−45	37	25	75
70.	$a_1a_{17}d_7$	$\frac{1151}{576}$	25	1	$\frac{1}{144}$	−1	101	49	95	−13	−209	179	−149	−137	67	19	67
						−191	35	−61	53	131	25	95	77	−155	113	83	251
71.	$a_1^2a_3(d_{10})^2$	$\frac{1151}{576}$	25	1	$\frac{1}{48}$	7	35	19	39	−1	−69	57	−43	−49	23	5	33
						−65	11	−29	23	43	9	33	35	−57	29	31	69
72.	$a_1d_7d_{10}e_7$	$\frac{1151}{576}$	25	1	$\frac{1}{48}$	9	37	17	39	−1	−69	57	−41	−47	25	5	31
						−65	11	−31	27	41	9	31	37	−57	31	31	69
73.	$a_1a_{15}d_9$	$\frac{2176}{1089}$	25	2	$\frac{1}{33}$	30	30	−2	30	6	−46	14	−30	−10	50	2	−18
						−30	−6	−34	14	14	14	30	22	−14	14	34	54
74.	$a_1a_2d_8e_7^2$	$\frac{2164}{1083}$	25	2	$\frac{1}{57}$	54	52	−4	50	4	−82	24	−58	−20	88	4	−32
						−58	−14	−56	20	20	20	54	38	−20	20	56	88
75.	$a_1a_5d_9d_{10}$	$\frac{1013}{507}$	25	1	$\frac{1}{78}$	72	74	−6	64	10	−112	26	−72	−22	122	2	−42
						−72	−16	−78	38	38	34	72	48	−30	30	78	126
76.	$a_1a_3a_4a_{17}$	$\frac{999}{500}$	25	2	$\frac{1}{100}$	2	72	26	66	−10	−146	126	−100	−88	34	14	42
						−138	18	−44	34	92	12	74	48	−112	74	54	178
77.	$a_1a_3a_4d_{10}e_7$	$\frac{999}{500}$	25	1	$\frac{1}{100}$	90	98	−8	82	14	−138	36	−90	−30	164	2	−50
						−90	−24	−90	42	42	46	90	64	−44	48	100	166
78.	$a_1d_6d_8d_{10}$	$\frac{1920}{961}$	25	1	$\frac{1}{31}$	5	23	11	25	−1	−45	37	−27	−31	15	3	21
						−41	7	−19	17	27	5	21	23	−37	19	21	45
79.	$a_1^2d_8^2e_7$	$\frac{1920}{961}$	25	2	$\frac{1}{31}$	6	24	8	22	0	−44	40	−24	−30	16	2	20
						−42	6	−20	16	28	8	20	24	−34	24	20	46
80.	$a_1^2d_8^2e_7$	$\frac{1920}{961}$	25	2	$\frac{1}{31}$	30	28	−2	26	2	−46	12	−30	−12	48	2	−16
						−30	−8	−30	10	12	12	30	20	−12	10	32	48

TABLE 1 (CONTD.). SHARED VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
81.	$a_1a_4a_{13}e_7$	$\frac{1748}{875}$	25	1	$\frac{1}{175}$	3	143	55	131	17	−247	225	−163	−177	53	35	93
						−249	37	−69	73	165	7	93	83	−177	129	95	305
82.	$a_1a_3a_{11}d_{10}$	$\frac{1732}{867}$	25	1	$\frac{1}{51}$	46	48	−4	46	8	−70	20	−46	−16	80	4	−28
						−46	−10	−48	20	20	24	46	34	−24	24	52	84
83.	$a_1a_2a_{15}d_7$	$\frac{1732}{867}$	25	2	$\frac{1}{51}$	−1	37	17	35	−5	−73	63	−53	−49	23	7	23
						−67	11	−17	17	47	9	35	25	−55	41	27	91
84.	$a_1a_9d_8e_7$	$\frac{1688}{845}$	25	1	$\frac{1}{65}$	63	59	−5	55	5	−95	25	−63	−23	99	3	−35
						−63	−17	−65	25	25	23	63	41	−25	21	65	103
85.	$a_1d_8^3$	$\frac{1680}{841}$	25	6	$\frac{1}{29}$	0	24	12	20	0	−48	36	−28	−28	8	4	16
						−40	4	−16	12	24	4	20	16	−28	24	16	44
86.	$a_1a_3d_6d_8e_7$	$\frac{799}{400}$	25	1	$\frac{1}{40}$	5	29	13	31	−3	−57	51	−37	−41	15	3	25
						−51	7	−25	23	33	3	25	29	−51	25	31	57
87.	$a_1a_2a_7d_8e_7$	$\frac{767}{384}$	25	1	$\frac{1}{96}$	5	77	29	65	11	−133	127	−89	−103	35	17	51
						−135	23	−47	43	91	5	43	53	−95	71	57	163
88.	$a_1a_2a_3a_9d_{10}$	$\frac{749}{375}$	25	1	$\frac{1}{150}$	134	146	−8	134	22	−206	52	−142	−46	236	10	−82
						−142	−28	−138	58	58	70	134	100	−68	72	148	246
89.	$a_1a_4a_5d_8e_7$	$\frac{1468}{735}$	25	1	$\frac{1}{105}$	35	67	39	95	13	−159	129	−87	−99	45	1	81
						−145	39	−61	51	91	35	51	67	−105	71	71	159
90.	$a_1a_{12}^2$	$\frac{1456}{729}$	25	4	$\frac{1}{27}$	0	20	8	20	0	−40	36	−28	−24	8	4	12
						−36	8	−12	8	24	4	16	12	−28	20	16	48
91.	$a_1a_3a_5d_6d_{10}$	$\frac{725}{363}$	25	1	$\frac{1}{66}$	60	64	−6	56	8	−92	22	−60	−20	106	4	−36
						−60	−14	−60	28	28	32	60	42	−30	30	66	108
92.	$a_1^2a_3a_5d_8e_7$	$\frac{725}{363}$	25	1	$\frac{1}{66}$	24	46	24	52	8	−100	80	−58	−66	20	0	52
						−92	16	−34	32	62	18	32	46	−70	44	36	100
93.	$a_1^2a_3a_5d_8e_7$	$\frac{725}{363}$	25	1	$\frac{1}{66}$	64	60	−6	56	4	−96	26	−64	−24	102	4	−36
						−64	−18	−64	24	24	24	64	42	−26	22	66	104
94.	$a_1^2a_5a_{11}e_7$	$\frac{721}{361}$	25	1	$\frac{1}{38}$	0	30	12	28	4	−54	48	−36	−38	12	8	20
						−54	8	−16	16	36	2	20	20	−38	28	22	66
95.	$a_1a_7a_{10}e_7$	$\frac{703}{352}$	25	1	$\frac{1}{176}$	3	139	59	127	13	−251	225	−167	−177	61	31	93
						−249	41	−81	77	165	11	93	91	−177	129	103	301
96.	$a_1a_3a_7d_7e_7$	$\frac{675}{338}$	25	1	$\frac{1}{52}$	19	37	19	39	5	−79	61	−45	−53	17	1	41
						−73	11	−27	27	49	13	27	37	−55	35	27	79
97.	$a_1^2a_3^2a_7d_{10}$	$\frac{675}{338}$	25	1	$\frac{1}{52}$	46	50	−4	46	8	−72	18	−48	−16	82	4	−28
						−48	−10	−48	20	20	24	48	34	−24	24	52	86
98.	$a_1a_2d_6d_8^2$	$\frac{1348}{675}$	25	2	$\frac{1}{45}$	−4	28	16	30	−6	−68	56	−40	−38	16	4	20
						−62	22	−22	16	40	8	34	24	−52	34	28	76
99.	$a_1a_3a_5a_6d_{10}$	$\frac{671}{336}$	25	1	$\frac{1}{168}$	154	162	−12	146	22	−234	56	−154	−54	264	10	−90
						−154	−36	−154	66	66	78	154	108	−80	76	168	278
100.	$a_1a_{13}d_5e_6$	$\frac{671}{336}$	25	1	$\frac{1}{168}$	−28	118	78	122	−20	−246	216	−168	−144	66	16	84
						−226	48	−88	36	148	44	108	70	−168	128	98	300

Table 1 (contd.). Shared vertices

	name	$\|v\|^2$	N	g	α	M				O				G			
101.	$a_1a_6a_9d_9$	$\frac{629}{315}$	25	1	$\frac{1}{210}$	196	198	−10	180	30	−300	70	−196	−66	318	6	−110
						−196	−44	−210	90	90	86	196	132	−90	82	210	346
102.	$a_1a_{11}d_7e_6$	$\frac{1248}{625}$	25	2	$\frac{1}{25}$	6	18	10	18	2	−38	30	−22	−26	10	2	18
						−34	6	−14	14	22	6	14	18	−26	18	14	38
103.	$a_1e_6^4$	$\frac{1248}{625}$	25	48	$\frac{1}{25}$	6	22	10	22	2	−38	30	−22	−22	10	6	18
						−34	6	−14	14	22	2	10	18	−22	22	14	38
104.	$a_1a_2a_4^2a_{14}$	$\frac{1208}{605}$	25	2	$\frac{1}{55}$	0	40	16	36	−4	−80	72	−56	−48	20	8	24
						−76	12	−24	20	48	8	40	28	−60	40	32	96
105.	$a_1a_4^2a_9e_7$	$\frac{1208}{605}$	25	2	$\frac{1}{55}$	48	56	−4	44	8	−76	20	−48	−16	92	0	−28
						−48	−16	−48	24	24	24	48	32	−24	28	56	92
106.	$a_1^2a_{11}e_6^2$	$\frac{599}{300}$	25	1	$\frac{1}{60}$	−7	43	27	41	−5	−87	75	−63	−51	27	7	33
						−79	15	−31	15	55	17	39	25	−63	47	35	105
107.	$a_1^2a_7^2d_9$	$\frac{577}{289}$	25	2	$\frac{1}{34}$	30	32	−2	30	6	−48	12	−32	−10	52	2	−18
						−32	−6	−34	14	14	14	32	22	−14	14	34	56
108.	$a_1a_3a_7^2e_7$	$\frac{577}{289}$	25	2	$\frac{1}{34}$	2	26	12	24	4	−48	44	−32	−36	12	6	18
						−48	8	−16	16	32	2	16	18	−34	24	20	58
109.	$a_1a_4a_7a_{13}$	$\frac{559}{280}$	25	1	$\frac{1}{280}$	6	206	82	182	−16	−408	366	−294	−246	102	44	132
						−388	72	−108	92	248	38	182	136	−296	202	160	498
110.	$a_1a_4a_6a_7e_7$	$\frac{559}{280}$	25	1	$\frac{1}{280}$	208	258	−20	278	46	−302	112	−272	−86	446	−16	−148
						−202	−80	−322	148	148	106	208	174	−122	90	316	504
111.	$a_1^2a_4a_5a_7e_7$	$\frac{539}{270}$	25	1	$\frac{1}{180}$	7	143	63	131	17	−255	229	−171	−189	65	35	97
						−253	45	−85	81	169	7	89	95	−181	125	107	305
112.	$a_1^2a_4a_5a_7e_7$	$\frac{539}{270}$	25	1	$\frac{1}{180}$	160	182	−12	138	26	−242	64	−160	−50	306	−2	−90
						−150	−56	−160	78	78	84	160	106	−76	92	180	304
113.	$a_1d_4d_6^2d_8$	$\frac{1056}{529}$	25	2	$\frac{1}{23}$	3	17	7	19	−1	−33	29	−21	−23	9	1	15
						−29	5	−15	13	19	3	15	17	−29	13	17	33
114.	$a_1a_6a_{11}d_7$	$\frac{1048}{525}$	25	1	$\frac{1}{105}$	−4	88	48	72	−4	−160	124	−100	−112	36	20	68
						−144	20	−64	56	96	24	68	64	−100	76	52	160
115.	$a_1a_6e_6^3$	$\frac{1048}{525}$	25	6	$\frac{1}{105}$	−14	74	50	62	−10	−146	134	−110	−94	58	10	62
						−134	26	−62	22	98	38	62	46	−110	86	62	182
116.	$a_1^2a_3d_6^2d_8$	$\frac{511}{256}$	25	1	$\frac{1}{32}$	6	24	10	26	−2	−46	38	−28	−32	14	2	22
						−42	6	−20	18	28	4	22	24	−40	18	22	46
117.	$a_1a_5^2d_8e_6$	$\frac{1012}{507}$	25	2	$\frac{1}{39}$	11	27	15	31	5	−59	49	−35	−39	13	1	29
						−53	11	−21	19	35	11	19	27	−41	27	23	59
118.	$a_1a_2a_{11}d_5e_6$	$\frac{1012}{507}$	25	2	$\frac{1}{39}$	−4	28	16	24	−6	−56	50	−40	−32	16	4	20
						−56	10	−16	10	34	8	28	18	−40	28	22	70
119.	$a_1^3a_5^3e_7$	$\frac{1012}{507}$	25	2	$\frac{1}{39}$	1	31	13	29	5	−55	49	−37	−41	13	9	21
						−55	9	−17	17	37	1	19	21	−39	27	23	67
120.	$a_1a_5d_5d_6d_8$	$\frac{485}{243}$	25	1	$\frac{1}{54}$	9	41	17	43	−3	−77	67	−49	−53	23	3	37
						−71	11	−33	31	45	7	37	41	−67	29	39	77

TABLE 1 (CONTD.). SHARED VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
121.	$a_1a_9a_{11}d_4$	$\frac{479}{240}$	25	1	$\frac{1}{120}$	104	114	−8	116	22	−164	52	−104	−38	184	0	−60
						−104	−26	−128	60	60	40	104	74	−46	46	128	198
122.	$a_1a_9^2d_6$	$\frac{880}{441}$	25	4	$\frac{1}{21}$	2	14	6	14	−2	−30	26	−22	−22	10	6	10
						−26	6	−6	6	18	2	14	10	−26	14	10	38
123.	$a_1a_2^2a_8e_6^2$	$\frac{880}{441}$	25	2	$\frac{1}{63}$	−12	48	32	40	−8	−88	80	−60	−52	24	4	36
						−96	16	−28	16	56	12	44	28	−60	44	36	112
124.	$a_1d_6^4$	$\frac{880}{441}$	25	24	$\frac{1}{21}$	18	18	−2	22	6	−26	10	−22	−6	30	−2	−14
						−14	−2	−26	10	10	6	18	14	−10	10	22	34
125.	$a_1^2a_7a_{11}d_5$	$\frac{431}{216}$	25	1	$\frac{1}{72}$	64	66	−4	70	14	−94	32	−64	−22	110	0	−36
						−58	−16	−82	36	36	26	64	46	−26	26	76	120
126.	$a_1^2a_7a_{11}d_5$	$\frac{431}{216}$	25	1	$\frac{1}{72}$	−1	55	23	51	1	−103	93	−67	−69	25	11	33
						−101	13	−37	33	65	7	49	39	−77	53	43	121
127.	$a_1a_5a_7e_6^2$	$\frac{431}{216}$	25	2	$\frac{1}{72}$	17	51	33	55	11	−107	91	−71	−81	19	−5	47
						−95	17	−39	37	65	23	37	51	−71	51	41	107
128.	$a_1^2a_9^2d_5$	$\frac{391}{196}$	25	2	$\frac{1}{28}$	1	23	11	21	1	−45	33	−25	−29	13	5	19
						−39	7	−15	15	27	5	15	17	−27	19	15	41
129.	$a_1a_3a_5a_{11}d_5$	$\frac{391}{196}$	25	2	$\frac{1}{28}$	−2	20	10	18	−4	−40	36	−30	−24	12	4	14
						−40	6	−12	8	24	6	20	12	−28	20	16	50
130.	$a_1^2d_5d_6^3$	$\frac{391}{196}$	25	3	$\frac{1}{28}$	7	23	7	21	−1	−39	37	−23	−27	13	1	19
						−37	5	−19	17	23	5	19	23	−33	15	21	39
131.	$a_1a_2^2a_4a_{11}d_5$	$\frac{748}{375}$	25	2	$\frac{1}{75}$	−4	56	28	48	−10	−108	98	−80	−64	32	12	36
						−104	14	−32	22	66	16	52	34	−76	52	42	134
132.	$a_1a_2a_9^2d_4$	$\frac{724}{363}$	25	4	$\frac{1}{33}$	27	33	−3	33	5	−45	13	−27	−11	51	1	−15
						−27	−9	−33	15	19	11	27	21	−13	15	37	55
133.	$a_1a_2d_4d_6^3$	$\frac{724}{363}$	25	6	$\frac{1}{33}$	6	24	10	28	0	−48	44	−28	−32	12	0	20
						−40	6	−24	22	26	6	20	24	−40	18	26	48
134.	$a_1a_3a_7^2d_7$	$\frac{720}{361}$	25	4	$\frac{1}{19}$	−1	17	9	13	−1	−29	23	−19	−19	5	3	13
						−27	3	−11	9	17	5	13	11	−17	13	9	29
135.	$a_1a_8^3$	$\frac{720}{361}$	25	12	$\frac{1}{19}$	2	14	2	10	−2	−26	26	−18	−18	6	2	6
						−26	2	−10	6	18	2	14	10	−22	14	10	34
136.	$a_1^3d_4d_6^3$	$\frac{720}{361}$	25	2	$\frac{1}{19}$	2	14	6	16	−2	−28	24	−16	−18	8	0	12
						−22	6	−14	12	14	2	12	14	−24	10	16	28
137.	$a_1a_3a_7a_9d_5$	$\frac{319}{160}$	25	1	$\frac{1}{80}$	81	73	−5	65	15	−115	25	−71	−31	113	1	−45
						−81	−19	−85	35	35	31	81	47	−35	27	75	131
138.	$a_1a_3a_5d_4d_6^2$	$\frac{293}{147}$	25	2	$\frac{1}{42}$	42	38	−6	34	4	−58	14	−36	−10	68	2	−24
						−42	−10	−36	20	20	22	42	24	−18	18	42	66
139.	$a_1a_7^2d_5^2$	$\frac{576}{289}$	25	8	$\frac{1}{17}$	−2	14	6	10	−2	−22	22	−18	−14	6	−2	6
						−26	6	−6	6	14	2	10	10	−18	14	10	30
140.	$a_1^2a_3d_4^2d_6^2$	$\frac{287}{144}$	25	2	$\frac{1}{24}$	3	19	7	21	−1	−35	31	−21	−23	9	1	15
						−29	5	−17	15	19	3	15	17	−29	13	19	35

TABLE 1 (CONTD.). SHARED VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
141.	$a_1a_2a_4^2a_9d_5$	$\frac{269}{135}$	25	2	$\frac{1}{90}$	7	77	33	71	3	−147	103	−79	−91	39	15	65
						−129	21	−45	45	93	15	45	51	−85	57	45	131
142.	$a_1^4a_7^3$	$\frac{255}{128}$	25	6	$\frac{1}{32}$	26	30	0	30	8	−44	10	−32	−8	46	4	−20
						−28	−6	−36	12	12	12	32	18	−12	12	32	54
143.	$a_1^2a_4a_7^2d_5$	$\frac{249}{125}$	25	2	$\frac{1}{50}$	46	44	−6	46	18	−72	16	−48	−14	72	6	−30
						−48	−6	−54	18	18	18	48	30	−18	18	46	84
144.	$a_1a_2a_3a_7^2d_5$	$\frac{484}{243}$	25	4	$\frac{1}{27}$	−2	26	14	20	−2	−40	34	−26	−26	8	6	18
						−40	4	−16	14	26	8	18	14	−22	14	14	40
145.	$a_1^3d_4^4d_6$	$\frac{448}{225}$	25	8	$\frac{1}{15}$	2	12	4	14	0	−22	18	−12	−14	6	2	10
						−18	2	−10	10	12	2	10	12	−18	8	12	22
146.	$a_1a_6^4$	$\frac{448}{225}$	25	24	$\frac{1}{15}$	2	10	6	10	−2	−22	18	−14	−14	2	2	6
						−22	6	−6	2	14	−2	10	6	−18	10	10	26
147.	$a_1^3a_3a_5a_7^2$	$\frac{215}{108}$	25	2	$\frac{1}{36}$	30	32	−2	34	10	−52	12	−36	−10	52	2	−22
						−32	−6	−38	14	14	14	36	22	−14	14	34	60
148.	$a_1^3a_3a_5a_7^2$	$\frac{215}{108}$	25	4	$\frac{1}{36}$	−5	33	21	23	−5	−55	43	−35	−35	11	7	25
						−53	9	−21	17	33	11	25	19	−29	21	17	55
149.	$a_1^2a_3^3a_7^2$	$\frac{199}{100}$	25	4	$\frac{1}{20}$	−2	18	10	14	−2	−30	24	−20	−20	6	4	14
						−30	4	−12	10	18	6	14	10	−16	12	10	30
150.	$a_1^3a_5^3a_7$	$\frac{191}{96}$	25	2	$\frac{1}{48}$	38	46	−4	46	12	−72	18	−48	−12	66	0	−24
						−40	−6	−52	24	16	20	48	30	−16	16	44	82
151.	$a_1a_5^4d_4$	$\frac{336}{169}$	25	48	$\frac{1}{13}$	2	10	2	6	−2	−18	18	−10	−10	6	2	6
						−18	6	−2	2	10	2	10	6	−14	10	6	26
152.	$a_1d_4^6$	$\frac{336}{169}$	25	2160	$\frac{1}{13}$	14	10	−2	10	10	−14	6	−10	−6	14	2	−10
						−14	−6	−18	6	6	6	10	10	−6	6	10	22
153.	$a_1^3a_2a_5^4$	$\frac{292}{147}$	25	16	$\frac{1}{21}$	15	21	−3	21	5	−33	7	−21	−5	27	1	−9
						−15	−3	−21	9	7	11	21	15	−7	9	19	37
154.	$a_1^3a_2d_4^5$	$\frac{292}{147}$	25	120	$\frac{1}{21}$	2	18	6	24	2	−30	24	−18	−20	6	2	14
						−24	2	−12	12	18	4	14	18	−24	10	18	30
155.	$a_1^6a_3d_4^4$	$\frac{127}{64}$	25	24	$\frac{1}{16}$	1	13	5	17	1	−23	19	−13	−15	5	3	11
						−19	1	−11	9	13	3	11	13	−19	7	13	23
156.	$a_1^9d_4^4$	$\frac{240}{121}$	25	48	$\frac{1}{11}$	0	8	4	12	0	−16	12	−8	−10	4	2	8
						−12	2	−8	6	8	2	8	10	−14	4	10	16
157.	$a_1a_4^6$	$\frac{240}{121}$	25	240	$\frac{1}{11}$	12	12	−4	4	0	−16	4	−12	−4	16	0	−4
						−8	−4	−8	4	4	4	8	8	−4	4	12	20
158.	$a_1a_3^8$	$\frac{160}{81}$	25	2688	$\frac{1}{9}$	8	16	0	8	0	−12	0	−4	−4	12	0	0
						−4	0	−8	4	4	8	4	8	−4	4	12	12
159.	$a_1^{21}d_4$	$\frac{96}{49}$	25	5760	$\frac{1}{7}$	2	6	2	8	0	−10	8	−4	−6	0	2	6
						−8	0	−4	2	6	0	2	6	−8	4	8	10
160.	$a_1a_2^{12}$	$\frac{96}{49}$	25	190080	$\frac{1}{7}$	0	8	4	4	−4	−8	8	−4	−4	4	−4	4
						−8	4	−8	4	4	0	4	8	−4	4	4	12

Table 1 (contd.). Shared vertices

	name	$\|v\|^2$	N	g	α	M				O				G			
161.	a_1^{25}	$\frac{31}{16}$	25	40320	$\frac{1}{8}$	2	8	2	8	0	−12	8	−4	−8	−2	2	8
						−8	0	−6	2	6	0	2	8	−8	4	8	12
162.	a_1^{25}	$\frac{52}{27}$	25	443520	$\frac{1}{9}$	3	9	3	9	−1	−13	9	−3	−9	−1	1	9
						−9	1	−9	1	7	1	3	9	−9	3	9	13
163.	a_1^{25}	$\frac{48}{25}$	25	10200960	$\frac{1}{5}$	1	5	1	5	−1	−7	5	−1	−5	−1	1	5
						−5	1	−5	1	5	1	1	5	−5	1	5	7
164.	a_1^{25}	$\frac{48}{25}$	25	244823040	$\frac{1}{5}$	4	4	4	4	0	−8	4	−4	−4	8	0	−4
						−4	0	−4	0	0	0	4	4	−4	4	4	4

TABLE 1 (CONTD.). SHARED VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
1.	exceptional	$\frac{8}{3}$	552	495766656000	$\frac{1}{3}$	2	2	2	2	−2	−6	2	−2	−2	6	2	2
						−2	2	−2	2	2	2	2	2	−2	2	2	6
2.	$\mathrm{HS}_{100}\ a_1$	$\frac{9}{4}$	101	44352000	$\frac{1}{4}$	1	3	3	3	−1	−7	3	−1	−3	5	1	3
						−5	3	−3	3	3	1	3	3	−3	3	3	7
3.	$\mathrm{HS}_{50}\ a_2$	$\frac{32}{15}$	52	126000	$\frac{1}{15}$	2	10	2	14	−2	−22	18	−14	−10	2	−2	14
						−22	2	−2	2	14	6	14	10	−22	14	6	22
4.	$J(7,4)\ a_3$	$\frac{25}{12}$	38	2520	$\frac{1}{12}$	7	15	3	15	3	−17	3	−13	−3	15	3	−11
						−13	−3	−11	3	5	3	7	5	−5	9	13	19
5.	$(5,6)\ a_1^2$	$\frac{52}{25}$	44	80640	$\frac{1}{5}$	4	6	0	4	0	−8	2	−4	−2	8	0	0
						−4	0	−4	2	2	4	4	4	−2	2	6	8
6.	Cox a_4	$\frac{72}{35}$	32	168	$\frac{1}{35}$	6	30	14	26	−6	−66	38	−34	−26	18	14	14
						−34	14	−22	14	22	14	30	18	−26	26	18	66
7.	$G_{24,30}\ a_5$	$\frac{49}{24}$	29	24	$\frac{1}{24}$	4	20	10	18	−4	−44	28	−24	−18	12	8	10
						−24	10	−16	10	16	8	20	12	−18	18	12	44
8.	$(3,8)\ a_3$	$\frac{100}{49}$	33	1440	$\frac{1}{7}$	7	5	−1	7	3	−9	5	−5	−3	9	1	−5
						−7	−1	−9	3	3	3	7	5	−3	3	7	11
9.	$(3,8)\ a_1a_2$	$\frac{55}{27}$	33	720	$\frac{1}{18}$	15	21	−3	15	−1	−27	7	−15	−5	27	1	−3
						−15	−3	−15	9	7	11	15	15	−7	9	19	31
10.	$G_{21,24}\ a_6$	$\frac{128}{63}$	27	6	$\frac{1}{63}$	10	54	26	50	−10	−114	74	−66	−46	30	18	26
						−62	26	−42	26	42	22	50	34	−46	50	30	114
11.	$G_{19,20}\ a_7$	$\frac{81}{40}$	26	2	$\frac{1}{40}$	34	34	0	34	0	−60	14	−36	−16	62	4	−12
						−36	−2	−40	28	12	16	36	26	−12	20	40	70
12.	$G_{25,30}\ d_4$	$\frac{164}{81}$	29	120	$\frac{1}{9}$	2	8	4	8	0	−14	10	−6	−8	6	2	6
						−12	2	−6	6	8	2	6	6	−8	6	6	14
13.	$G_{25,30}\ a_1a_3$	$\frac{99}{49}$	29	60	$\frac{1}{14}$	12	16	−2	12	0	−20	6	−12	−4	22	0	−4
						−12	−2	−12	8	4	8	12	10	−6	6	14	24
14.	$G_{17,17}\ a_8$	$\frac{200}{99}$	25	1	$\frac{1}{99}$	88	84	0	84	0	−148	36	−88	−40	152	8	−32
						−88	−8	−100	68	32	40	88	64	−32	48	100	172
15.	$T_1^1 4^0 2_0^0$	$\frac{121}{60}$	25	1	$\frac{1}{60}$	54	52	0	50	0	−90	20	−54	−24	92	4	−20
						−54	−6	−60	40	20	24	54	38	−20	28	60	104
16.	$G_{15,15}\ a_9$	$\frac{121}{60}$	24	1	$\frac{1}{60}$	6	42	18	46	−4	−90	72	−58	−52	20	0	44
						−84	14	−28	22	54	16	44	42	−72	48	34	90
17.	$G_{22,22}\ d_5$	$\frac{244}{121}$	27	24	$\frac{1}{11}$	−1	9	3	5	−1	−17	13	−11	−11	3	1	7
						−13	3	−7	9	9	1	7	7	−13	9	7	17
18.	$G_{24,27}\ a_4$	$\frac{272}{135}$	28	36	$\frac{1}{45}$	46	34	−6	46	18	−62	26	−38	−14	62	6	−30
						−38	−6	−54	18	18	18	38	30	−18	18	46	74
19.	$G_{14,14}\ a_{10}$	$\frac{288}{143}$	24	1	$\frac{1}{143}$	126	122	−2	122	6	−210	58	−126	−54	222	10	−50
						−126	−14	−146	94	50	58	126	94	−46	70	146	246
20.	$T_0^3 4_0^1$	$\frac{288}{143}$	24	1	$\frac{1}{143}$	16	100	44	108	−8	−216	172	−136	−124	48	0	104
						−200	32	−68	56	128	40	104	100	−168	116	80	216

TABLE 2. ADDITIONAL VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
21.	$T_0^3 4_0^1\ a_{10}$	$\frac{288}{143}$	24	1	$\frac{1}{143}$	128	128	0	120	0	−216	48	−128	−56	216	8	−48
						−128	−16	−144	96	48	56	128	88	−48	64	144	248
22.	$G_{24,27}\ a_2^2$	$\frac{296}{147}$	28	72	$\frac{1}{21}$	−2	14	6	14	−2	−30	26	−18	−18	6	2	10
						−30	6	−6	6	18	2	18	10	−26	18	10	38
23.	$G_{22,22}\ a_1a_4$	$\frac{161}{80}$	27	12	$\frac{1}{40}$	33	43	−5	33	1	−57	17	−37	−11	61	−1	−13
						−37	−5	−37	23	13	21	33	29	−17	15	41	69
24.	$T_0^{13} 4_0^1$	$\frac{13864}{6889}$	24	1	$\frac{1}{83}$	76	76	0	68	0	−124	28	−76	−32	124	4	−28
						−72	−8	−84	56	28	32	76	52	−28	36	84	144
25.	$T_0^{12} 4_0^2$	$\frac{163}{81}$	24	1	$\frac{1}{36}$	4	26	10	26	−2	−54	44	−34	−32	12	0	26
						−50	8	−18	14	32	10	26	26	−42	30	20	54
26.	$T_0^{14} 4_0^0$	$\frac{1159}{576}$	24	1	$\frac{1}{48}$	5	35	15	37	−3	−73	57	−45	−41	17	1	35
						−67	11	−23	19	43	13	35	33	−55	39	27	73
27.	$T_3^6 0\ a_{11}$	$\frac{169}{84}$	24	1	$\frac{1}{84}$	74	72	−2	74	4	−122	34	−74	−32	130	6	−30
						−74	−8	−86	54	30	34	74	56	−28	40	86	144
28.	$T_3^6 0\ a_{11}$	$\frac{169}{84}$	24	1	$\frac{1}{84}$	75	75	1	71	1	−127	27	−75	−33	127	5	−29
						−75	−11	−85	57	29	31	75	51	−29	37	85	145
29.	$T_0^1 4_0^1\ a_1a_{11}$	$\frac{169}{84}$	24	1	$\frac{1}{84}$	76	76	0	70	0	−126	28	−76	−32	126	4	−28
						−74	−10	−86	56	28	34	76	52	−28	36	84	146
30.	$G_{20,20}\ d_6$	$\frac{340}{169}$	26	8	$\frac{1}{13}$	3	11	3	7	1	−19	17	−11	−15	5	1	9
						−17	3	−9	7	11	3	7	11	−13	11	7	19
31.	$T_0^{11} 4_0^3$	$\frac{7984}{3969}$	24	1	$\frac{1}{63}$	56	56	0	52	0	−96	20	−56	−24	96	4	−24
						−56	−8	−64	44	24	24	56	36	−20	28	64	108
32.	$T_5^{15} 0$	$\frac{24784}{12321}$	24	1	$\frac{1}{111}$	98	94	−2	98	6	−162	46	−98	−42	170	10	−42
						−98	−10	−114	70	38	46	98	74	−38	54	114	190
33.	$T_3^{16} 0\ a_1$	$\frac{2059}{1024}$	24	1	$\frac{1}{64}$	57	57	1	53	1	−97	21	−57	−25	97	3	−23
						−57	−9	−65	43	23	23	57	37	−21	29	65	111
34.	$T_3^3 0\ a_2a_{12}$	$\frac{392}{195}$	24	1	$\frac{1}{195}$	174	174	2	166	2	−294	62	−174	−78	294	10	−70
						−174	−26	−198	130	70	70	174	114	−66	86	198	338
35.	$T_0^{10} 4_0^4$	$\frac{197}{98}$	24	1	$\frac{1}{28}$	25	23	−1	25	1	−41	13	−25	−11	43	3	−11
						−25	−3	−29	17	9	11	25	19	−9	13	29	47
36.	$T_2^{18} 0$	$\frac{886}{441}$	24	1	$\frac{1}{42}$	2	28	12	30	−4	−62	52	−42	−36	16	2	26
						−58	10	−18	14	38	10	30	26	−50	34	24	68
37.	$T_0^1 4_0^0\ a_{13}$	$\frac{225}{112}$	24	1	$\frac{1}{56}$	51	51	−1	47	1	−85	19	−51	−21	83	1	−19
						−49	−7	−57	37	19	23	51	35	−19	23	55	97
38.	$G_{18,18}\ d_7$	$\frac{452}{225}$	25	4	$\frac{1}{15}$	−1	13	9	11	−1	−23	19	−15	−15	3	3	9
						−21	1	−9	5	13	3	9	7	−13	13	9	23
39.	$T_0^9 4_0^5$	$\frac{5224}{2601}$	24	1	$\frac{1}{51}$	6	38	18	38	−2	−78	58	−46	−42	22	2	38
						−70	14	−26	22	46	14	38	34	−58	42	30	78
40.	$G_{20,20}\ a_1a_5$	$\frac{488}{243}$	26	4	$\frac{1}{27}$	23	29	−3	23	1	−39	11	−25	−7	41	−1	−9
						−25	−3	−27	15	9	13	23	19	−11	11	27	45

Table 2 (contd.). Additional vertices

	name	$\|v\|^2$	N	g	α	M				O				G			
41.	$T_2^3 0\ a_{15}$	$\frac{289}{144}$	24	1	$\frac{1}{48}$	43	43	1	41	1	−73	15	−43	−19	71	3	−19
						−43	−7	−49	31	17	17	43	27	−17	21	49	83
42.	$T_0^8 4_0^6$	$\frac{289}{144}$	24	1	$\frac{1}{24}$	4	16	8	20	−2	−36	26	−20	−22	14	2	18
						−34	8	−14	12	20	6	18	16	−26	18	16	36
43.	$T_0^2 7_0^2\ d_8$	$\frac{580}{289}$	25	2	$\frac{1}{17}$	16	14	−2	16	2	−24	10	−16	−6	26	2	−10
						−16	−4	−18	6	6	6	16	12	−6	6	18	26
44.	$T_0^7 4_0^7$	$\frac{4432}{2209}$	24	2	$\frac{1}{47}$	4	36	12	28	0	−72	60	−44	−48	16	0	32
						−64	8	−28	20	40	12	32	36	−52	40	24	68
45.	$G_{21,21}\ d_5$	$\frac{339}{169}$	26	6	$\frac{1}{26}$	5	23	11	21	1	−41	29	−21	−25	13	5	19
						−35	7	−15	15	23	5	15	17	−23	19	15	41
46.	$G_{18,18}\ a_1 a_6$	$\frac{351}{175}$	25	2	$\frac{1}{70}$	61	75	−5	61	5	−101	29	−65	−19	105	−1	−25
						−65	−9	−69	39	25	33	61	49	−25	27	69	117
47.	$T_0^1 7_0^1\ d_9$	$\frac{724}{361}$	24	2	$\frac{1}{19}$	2	16	8	12	0	−30	20	−16	−16	10	2	14
						−24	6	−12	12	16	4	14	12	−20	16	12	30
48.	$G_{21,21}\ a_5$	$\frac{728}{363}$	26	6	$\frac{1}{33}$	30	26	−2	34	10	−46	18	−30	−10	46	2	−22
						−26	−6	−38	14	14	14	30	22	−14	14	34	54
49.	$G_{21,21}\ a_2 a_3$	$\frac{385}{192}$	26	6	$\frac{1}{48}$	−2	32	14	30	−6	−70	58	−44	−40	14	2	22
						−70	14	−20	14	44	4	38	24	−56	38	26	86
50.	$T_0^1 7_0^0\ d_{10}$	$\frac{884}{441}$	24	1	$\frac{1}{21}$	3	15	7	17	−1	−31	25	−19	−19	9	1	15
						−29	5	−11	9	19	5	15	15	−25	15	13	31
51.	$T_0^2 7_0^2\ a_1 a_7$	$\frac{485}{242}$	25	1	$\frac{1}{44}$	38	46	−4	38	4	−64	18	−40	−12	66	0	−16
						−40	−6	−44	24	16	20	40	30	−16	16	44	74
52.	$T_1^8 0\ d_{11}$	$\frac{1060}{529}$	24	1	$\frac{1}{23}$	2	16	8	16	0	−36	28	−22	−22	8	0	16
						−32	4	−12	10	20	6	16	16	−26	18	12	34
53.	$T_1^6 0\ a_1 d_{12}$	$\frac{1252}{625}$	24	1	$\frac{1}{25}$	23	23	1	21	1	−37	7	−23	−9	37	1	−9
						−21	−5	−27	17	9	9	23	15	−9	9	25	43
54.	$T_0^1 7_0^1\ a_1 a_8$	$\frac{649}{324}$	24	1	$\frac{1}{108}$	97	111	−9	93	9	−157	45	−97	−31	161	−1	−41
						−97	−17	−109	59	41	49	97	73	−41	39	109	181
55.	$T_1^5 0\ d_{14}$	$\frac{1684}{841}$	24	1	$\frac{1}{29}$	28	28	−2	24	2	−44	10	−26	−10	42	0	−10
						−26	−4	−30	18	10	12	26	18	−10	10	28	50
56.	$T_0^1 7_0^0\ a_1 a_9$	$\frac{1692}{845}$	24	1	$\frac{1}{65}$	11	47	23	51	1	−95	77	−63	−67	25	5	49
						−89	19	−33	27	59	11	39	47	−77	43	39	95
57.	$T_0^1 7_0^0\ a_1 a_9$	$\frac{1692}{845}$	24	1	$\frac{1}{65}$	59	67	−5	55	5	−95	25	−59	−19	97	−1	−25
						−59	−11	−65	35	25	29	59	43	−25	23	65	109
58.	$T_1^8 0\ a_1 a_{10}$	$\frac{1079}{539}$	24	1	$\frac{1}{154}$	138	142	2	120	38	−208	74	−138	−34	240	−10	−82
						−116	−52	−184	82	82	52	138	82	−42	62	162	260
59.	$T_1^8 0\ a_1 a_{10}$	$\frac{1079}{539}$	24	1	$\frac{1}{154}$	−12	120	36	102	−8	−216	206	−144	−148	50	12	76
						−202	16	−84	96	130	4	88	88	−174	116	106	262
60.	$T_0^1 11_0^1\ e_6$	$\frac{2168}{1083}$	25	2	$\frac{1}{57}$	−2	46	18	30	−2	−86	66	−54	−50	22	2	38
						−70	14	−34	38	46	6	38	34	−66	50	38	94

TABLE 2 (CONTD.). ADDITIONAL VERTICES

	name	$\|v\|^2$	N	g	α	M				O				G			
61.	$T_1^{18}0\ a_1$	$\frac{15856}{7921}$	24	1	$\frac{1}{89}$	-1	61	29	55	-3	-129	109	-89	-77	37	9	47
						-121	25	-41	33	81	15	65	47	-105	73	53	151
62.	$T_1^{18}0\ a_1$	$\frac{15856}{7921}$	24	1	$\frac{1}{89}$	78	90	2	70	14	-134	30	-78	-26	134	-6	-46
						-78	-30	-94	50	46	22	78	38	-30	34	94	150
63.	$T_0^1 11_0^1\ a_6$	$\frac{2368}{1183}$	25	2	$\frac{1}{91}$	82	78	-2	90	22	-122	46	-82	-26	130	10	-58
						-74	-18	-106	38	38	38	82	58	-38	38	98	150
64.	$T_0^1 11_0^1\ a_2a_4$	$\frac{2432}{1215}$	25	2	$\frac{1}{135}$	-8	92	44	84	-12	-196	172	-128	-112	44	8	64
						-196	44	-56	44	116	16	104	72	-152	104	80	236
65.	$T_1^6 0\ a_1^2 a_{11}$	$\frac{1351}{675}$	24	1	$\frac{1}{90}$	80	90	-8	80	10	-128	40	-80	-26	136	0	-36
						-80	-14	-92	48	36	40	80	62	-34	34	92	150
66.	$T_1^6 0\ a_1^2 a_{11}$	$\frac{1351}{675}$	24	1	$\frac{1}{90}$	82	94	-6	76	6	-132	34	-82	-26	132	-2	-34
						-80	-16	-92	50	34	40	82	58	-34	30	90	152
67.	$T_1^5 0\ a_1 a_{13}$	$\frac{4048}{2023}$	24	1	$\frac{1}{119}$	106	118	-10	106	14	-170	54	-106	-34	178	2	-50
						-106	-18	-122	62	46	54	106	82	-46	46	122	198
68.	$T_1^5 0\ a_1 a_{13}$	$\frac{4048}{2023}$	24	1	$\frac{1}{119}$	109	123	-9	101	9	-177	45	-109	-35	173	-5	-45
						-105	-21	-121	67	45	53	109	77	-45	39	117	201

Table 2 (contd.). Additional vertices