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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 05/1999

Applied and Computational Convexity

January 31 – February 6, 1999

The conference, organized by David Avis (McGill University, Montréal), Peter Gritzmann (TU Munich) and Victor Klee (University of Washington, Seattle), was attended by 32 participants, who gave a total of 25 lectures ranging from 30 to 60 minutes.

The meeting focussed on exciting new developments in the area of Applied and Computational Convexity. The roots of this field lie jointly in geometry, in mathematical programming and in computer science. Typically, the problems are algorithmic in nature, the underlying structures are geometric with special emphasis on convexity, and the questions are usually motivated by practical applications in mathematical programming, computer science, and other less obviously mathematical areas of science.

According to the concept of the conference, the participants (some of whom work in industry) belonged to various different areas of mathematics, computer science and other fields including convexity theory, combinatorics, mathematical programming, numerical analysis, and image processing.

The talks dealt with various topics of the wide spectrum of subjects covered by Applied and Computational Convexity. Some lectures were devoted to geometric aspects of combinatorics and combinatorial optimization. Various lattice point problems were studied, partly from the point of view of integer programming, partly with a view towards questions in the geometry of numbers. Other talks dealt with linear, semi-definite and convex optimization problems, and with geometric aspects of nonlinear optimization. Yet another group of talks focussed on the computation and optimization of geometric functionals, on related layout problems for graphs and on the complex mathematics of visualization and animation of dynamic constructions of mechanical structures and other objects from discrete geometry.

Some of the lectures focussed mainly on theoretical aspects of the field while others presented algorithms for tasks relevant for practical applications, partly including experimental studies. In particular, there was a an evening session devoted entirely to software demonstrations.

In another evening session, some basic open problems were presented and discussed.

The conference showed that even though the participants belonged to different fields that have quite different tool-boxes, approaches and ideas for solving their problems, there is a deep and close connection which is centered around the basic concept of convexity.

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Hans Achatz

Reconstructing a Simple Polytope from its Graph

Let P be a simple polytope with dimension d and $G(P)$ its edge graph. It has been shown that $G(P)$ already determines the complete face-lattice of P. However, the constructive approach used in [Kalai, 88] requires the computation of all orderings in $|vert(P)|$ which is computationally prohibitive for polytopes of even very small sizes. In this talk we propose an algorithm which is still exponential but does work with reasonable computing time for non-trivial simple polytopes.

(Joint work with Peter Kleinschmidt)

Imre Bárány

Test sets in integer programming

Test sets in integer programming provide a way of telling if a feasible solution $z \in \mathbb{Z}^n$ is optimal or not by checking, for every h in the test set, whether $z+h$ is feasible and whether it gives a better value of the objective function. The test set of Scarf, the neighbours of the origin, are associated with a generic $m \times n$ matrix A. The neighbours are a special case of maximal lattice point free convex bodies that form a simplicial complex $\mathcal{K}(\mathcal{A})$ which is, again, associated with the matrix A. In the talk several properties of the neighbours and of the complex $\mathcal{K}(\mathcal{A})$ will be discussed. For instance, the body of the complex turns out to be homeomorphic to $\mathbb{R}^{m-1}$. Moreover, matrices with the same complex $\mathcal{K}$ form a nice polyhedral set.

Anders Björner

Two variations in g-minor

I talk about two minor variations on the theme of the "g-theorem" for polytopal spheres and its possible generalization to all (triangulated) spheres (the "g-conjecture").

First, consider the Scarf complex of a generic $(n+1) \times n$ real matrix (a certain triangulation of $\mathbb{R}^n$) and let f_i be the number of orbits of $(i+1)$-faces under the action of $\mathbb{Z}^n$. H. Scarf asked in 1995 whether it is true that the numbers $f_0, \ldots, f_{\lfloor (n-3)/2 \rfloor}$ determine the numbers $f_0, \ldots, f_{n-1}$. We show how a positive answer (with linear relations between these numbers) is obtained using, among other things, the g-theorem for the boundary of a certain embedded ball.

Second, a project with F. Lutz for computerized searches among the triangulations of a manifold is presented. An important motivation for starting this project was an idea for systematically searching for counterexamples to the g-conjecture for 5- and 6-dimensional spheres. No such counterexamples were found, but the program (based on bistellar flips) turned out to be usefull for finding small (and in some cases provably minimal) triangulations of some manifolds. For instance a 16-vertex triangulation of the Poincaré homotopy 3-sphere was found, and it was shown that non-PL d-spheres on $d+3$ vertices exist, for all $d \geq 5$.

Jürgen Bokowski

Not every closed triangulated orientable 2-manifold without boundary can be embedded geometrically in $\mathbb{R}^3$.

B. Grünbaum has formulated the conjecture that every closed triangulated orientable 2-manifold without boundary can be embedded geometrically in $\mathbb{R}^3$, i.e., with flat triangles and without selfintersections. We disprove this conjecture by providing such a closed triangulated orientable 2-manifold of genus 6 which cannot be embedded geometrically in $\mathbb{R}^3$. We use our new fast algorithm for generating oriented matroids with prescribed properties. This algorithm is interesting in its own right as a useful tool for investigations in which the oriented matroid information plays a key role.

(Joint work with António Guedes de Oliveira)

David Bremner

Inner diagonals of convex polytopes

An *inner diagonal* of a polytope P is a segment that joins two vertices of P and that lies, except for its ends, in P's relative interior. A tantalizing conjecture due to von Stengel claims that among simple d-polytopes with $2d$ facets, the maximum number of inner diagonals is achieved by a d-cube. In this talk I will present a characterization of the maximum and minimum number of inner diagonals achievable in 3 dimensions for fixed numbers of vertices or facets. I will also present partial results in higher dimensions, including an interesting relationship to Kalai's new proof (based on rigidity of graphs) of the Lower Bound Theorem.

(Joint work with Victor Klee)

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Andreas Brieden

Largest simplices in bodies: Applications and efficient approximation

With focus on the case of variable j and n, we are concerned with the problem of computing the largest j-dimensional simplex (j-measure) contained in an n-dimensional body K, a problem that is relevant for some important applications. Since even the decision problem related to finding a largest n-dimensional simplex in an n-dimensional $\mathcal{V}$-polytope is $\mathbb{NP}$-hard, the task is that of polynomial-time approximation. We embed the problem in the *Algorithmic Theory of Convex Bodies* developed by *Grötschel*, *Lovász* and *Schrijver* and derive bounds for the accuracy in (oracle-) polynomial-time approximation of largest simplices in bodies.

(Joint work with Peter Gritzmann and Victor Klee)

Komei Fukuda

On the existence of a short admissible pivot sequence for feasibility and linear optimization problems

Recently Fukuda, Lüthi and Namiki have proved the existence of a short admissible pivot sequence from an arbitrary basis to the unique optimal basis, under the assumption that the LP problem is fully nondegenerate. Due to the strong degeneracy assumption, this result cannot be applied to the feasibility problem.

In this paper, for the feasibility problem, we prove the existence of a short admissible pivot sequence from an arbitrary basis to a feasible basis. Regarding the general LP problem, the existence of a short admissible pivot sequence from an arbitrary basis to an optimal basis is proved without any nondegeneracy assumptions. Our constructive proofs are based on techniques that are used in strongly-polynomial basis identification schemes of interior point methods.

(Joint work with Tamás Terlaky)

Martin Henk

On the computation of densest lattice packings of 3-polytopes

The lattice packing problem is the task to find a packing lattice Λ of a given convex body K such that the space is covered as good as possible by the lattice packing $\Lambda + K$. For dimensions $d \geq 4$ such "densest" lattice packings are only known for space fillers, and for the unit ball $\mathbb{B}^d$ the problem is solved for $d \leq 8$. In the planar case several techniques exist to solve the problem, whereas in 3-space densest lattice packings are only known for a few convex bodies (e.g. tetrahedron, octahedron).

Based on Minkowski's work on critical lattices of 3-dimensional convex bodies we develop a practical algorithm for computing the density of a densest lattice packing of an arbitrary 3-polytope. Using this algorithm we calculate the densities of a densest lattice packing of the regular dodecahedron, icosahedron and of some of the Archimedean solids.

(Joint work with Ulrich Betke)

Fred Holt

Polytopes meeting the conjectured Hirsch bound

For a general linear system consisting of n linear inequalities on d variables, the set of possible solutions is a d-dimensional polyhedron with n facets. If this figure is bounded, we call it a d-polytope with n facets.

In optimizing a linear objective function over this d-polytope, the simplex method moves from vertex to vertex along edges, and so the maximum edge diameter of d-polytopes with n facets provides a bound on the performance of the simplex method under the best selection of edges to follow.

In 1957 W.M. Hirsch conjectured that every d-polytope with n facets has edge diameter at most $n-d$. Recently we developed techniques to construct polytopes which meet this bound $(n-d)$ for some pairs (d,n) with $d<8$ and for all pairs (d,n) with $d \geq 8$.

(Joint work with Vic Klee (U. of Washington) and Kerstin Fritzsche (TU Berlin))

Michael Joswig

POLYMAKE

There are many tools available which allow for the treatment of polytopes on a computer. Among these are Avis' LRS, Fukuda's CDD, Loebel's and Christof's PORTA, The Geometry Center's GEOMVIEW, and more. POLYMAKE is a software package which combines the features of these (and other) programs and goes beyond.

The overall concept of POLYMAKE is as follows. Each polytope is represented as a file. This polytope file is divided into several sections reflecting various properties of the polytope. The user defines a polytope by producing a file containing at least one section. Typically, the polytope is defined in terms of points or inequalities. Then the user can ask about properties of this polytope. Properties which can be deduced from the specification are computed by applying a suitable sequence of rules. The final result is displayed.

Josef Stoer

The complexity of high-order predictor-corrector methods for solving sufficient complementarity problems

Recently, the author and M. Wechs [2] described a class of infeasible-interior-point methods for solving linear complementarity problems (LCP) $Px+Qy=q, (x,y) \geq 0, x^Ty=0$ $(P,Q: n \times n$-matrices), that are sufficient in the sense of Cottle et al. [1]. It was shown that these methods converge superlinearly with an arbitrarily high-order even for degenerate problems with more than one solution or without strictly complementary solution. The complexity of these methods is investigated and it is shown that all these methods, if started appropriately, need at most $0((1+k)^2 n |\log \epsilon|)$ predictor-corrector steps to find an ϵ-solution, and only $0((1+k)\sqrt{n}|\log \epsilon|)$ steps, if the problem has strictly feasible points. Here, k is the sufficiency parameter of the complementarity problem (Väliaho [3]).

Literature: [1] P.W. Cottle, I.-S. Vang, V. Venkateswaran: Sufficient matrices and the linear complementarity problem. Lin. Algebra Appl. 114/115, 231-249 (1989)

[2] J. Stoer, M. Wechs, S. Mizuno: High-order infeasible-interior-point methods for solving sufficient complementarity problems. Math. of Oper. Res. 23, 832-862 (1998)

[3] H. Väliaho: P-matrices are just sufficient. Linear Alg. Appl. 239, 103-108 (1996)

Günter Ziegler

Neighborly Cubical Polytopes

Neighborly cubical polytopes exist: for every $n \geq d \geq 2r+2$, there is a cubical d-polytope C_d^n whose r-skeleton is combinatorially equivalent to that of an n-dimensional cube. We construct the polytopes C_d^n as projections of (suitably deformed) n-dimensional cubes.

This construction solves a problem of Billera, Babson & Chan. The special case $d=2$ was first established by Goldfarb and Murty. In the special case $n=d+1$, the polytope C_d^{d+1} exists, and is combinatorially unique for even d, by a classification by Blind & Blind. However, a 4-polytope with the graph of the 5-cube need not necessarily be cubical.

(Joint work with Michael Joswig, TU Berlin) and then going to the polar polytope. Instead, we carefully select the shape of a 4-sided face to ensure that an equilibrium on the interior vertices can be extended to the whole graph. In all remaining cases, the polytope must contain a 5-sided face. So far, we have not been able to overcome the technical difficulties that are associated with extending our approach to 5-sided faces.

The question whether the bound C can be improved to a polynomial is open.

Uriel G. Rothblum

Matrix Scaling: Existence and Computation

A scaling of a matrix A is a matrix of the form XAY where X and Y are nonnegative diagonal matrices with positive diagonal elements. Scaling problems concern the identification of scalings of given matrices which have prescribed properties. Results about existence, characterization and computation of a variety of scaling problems will be discussed, with emphasis on complexity analysis of the computation of approximate solutions.

Miklós Simonovits

Computation of Diameter and related questions

A convex body is given in a high dimensional space (Euclidean (ℓ_2^n) or ℓ_p^n) and we wish to get estimates on the basic geometric characteristics of K, like volume, diameter, width, radius of circumscribed ball, etc. K is given by a weak separation oracle. Since the results of Elekes, Bárány and Füredi we know that this is a difficult problem if we restrict ourselves to deterministic algorithms. On the other hand, as Dyer, Frieze and Kannan proved, the volume can be well approximated by randomized algorithms.

Right now the fastest volume approximation uses $O^*(n^5)$ oracle calls (weak separation oracle, Kannan, Lovász, Simonovits) and – as a byproduct – it brings K into nearly isotropic position.

The situation is completely different with the diameter. The diameter cannot be approximated with $o(\sqrt{n/\log n})$ relative error in oracle-polynomial time.

For analogous ℓ_p–results see the FOCS 98 paper of A. Brieden, P. Gritzmann, R. Kannan, V. Klee, L. Lovász and M. Simonovits, and its full version to be published.

Moreover, the final result as well as interesting intermediate results are saved in the polytope file. Thus, asking for a previously computed property does not need a recomputation. It is asserted that the data of the sections within one file is consistent.

POLYMAKE comes with a C++ template class library which can be used to extend and customize the system.

Among POLYMAKE's applications so far are the following: construction of neighborly cubical polytopes, search for and examination of polytopes whose vertex-edge graph diameter attains the known Hirsch bound, investigations about the flag vectors of 4-polytopes.

POLYMAKE is free software. You can download the most recent version from the Internet at URL

`http://www.math.tu-berlin.de/diskregeom/polymake/`

(Joint work with Ewgenij Gawrilow (TU Berlin))

Volker Kaibel

Simple 0/1-Polytopes

Special classes of 0/1-polytopes (i.e., polytopes with 0/1 vertex coordinates) have been extensively studied within the field of polyhedral combinatorics. On the other hand, only a few results are known on the structure of 0/1-polytopes in general. For investigations of general polytopes it is well-known that the class of simple polytopes (i.e., d-dimensional polytopes, where every vertex lies in precisely d facets) is very important. In particular, several extremal questions (as the one for the maximal number of vertices for a given number of facets or the one for the worst-case running time of the simplex-algorithm) reduce to the corresponding ones for simple polytopes. Thus, the question arises if simplicity plays a similar role for the investigation of 0/1-polytopes. In this talk, we show, however, that simple 0/1-polytopes are precisely those 0/1-polytopes that are Cartesian products of 0/1-simplices. In particular, they definitely do not play as an important role in investigating 0/1-polytopes as they do for general polytopes. Some other consequences are that the graphs of simple 0/1-polytopes have cutset expansion one (i.e., every cut in these graphs has at least as many edges as the cardinality of the smaller of the two shores is) and that every polar of a simple 0/1-polytope can be realized as a 0/1-polytope again.

Jean Michel Kantor

Universal counting of lattice points in polytopes

Given a lattice polytope P (with underlying lattice $\mathbb{L}$), the universal counting function $\mathcal{U}_P(\mathbb{L}') = |P \cap \mathbb{L}'|$ is defined on all lattices $\mathbb{L}'$ containing $\mathbb{L}$. Motivated by questions concerning lattice polytopes and the Ehrhart polynomial, we study the equation $\mathcal{U}_P = \mathcal{U}_Q$. In particular, we claim:

Theorem: Assume P, Q are $\mathbb{L}$-polytopes with identical universal counting function. Then, for every primitive $z \in \mathbb{L}^*$,

$$rvol\ P(z) + rvol\ P(-z) = rvol\ Q(z) + rvol\ Q(-z)$$

Corollary: Assume P is an $\mathbb{L}$-polytope. Then, apart from lattice translates, there are only finitely many $\mathbb{L}$-polytopes with the same universal counting functions as P.

Also

Theorem: Suppose P and Q are $\mathbb{L}$-polygons. Then $\mathcal{U}_P = \mathcal{U}_Q$ if and only if the following two conditions are satisfied.

(i) Area (P)=Area (Q)

(ii) There exist $\mathbb{L}$-polygons X, Y such that $P = X + Y$ and $Q + X - Y$ (Minkowski Sums).

The higher dimensional case is still work in progress.

(Joint work with Imre Bárány)

Gyula Károlyi

New results related to the Erdös-Szekeres theorem

A set of points, in general position in d-space, is called *convex* if it is the vertex set of a convex polytope. Let, for $n > d \geq 2$, $f(n,d)$ denote the smallest integer such that any set of at least $f(n,d)$ points, in general position in $\mathbb{E}^d$, contains a convex set of size n. The existence of $f(n,d)$, $n > d \geq 2$, follows from the Erdös-Szekeres theorem. As for lower bounds, we prove that, for every $d \geq 2$, there exists a constant $c = c(d) > 1$ such that, if $n > d$, then

$$f(n,d) = \Omega(c^{\sqrt[d-1]{n}})\ .$$

If we are given $f(k,d)$ points, in general position in $\mathbb{E}^d$, we can separate a convex set of size k. Repeating this process, eventually we obtain a partition

sequence with the same base points. In this talk we prove that a corresponding reachability problem that asks for deciding whether one can move continuously from one instance of a construction sequence to another is a hard problem.

Theorem: Let A and B be two instances of a construction sequence that has only "Join", "Meet", and "Angular Bisector" as primitive operations. It is PSPACE-hard to decide whether A can be continuously transferred to B by moving the base elements in the real projective plane (on a path that avoids singular situations).

Günter Rote

Realizations of 3-polytopes with integral vertices

By a theorem of Ernst Steinitz (1922), every 3-connected planar graph with n vertices can be realized as a convex polytope. Richter-Gebert (1996) has shown that this can be done with integer vertex coordinates between $-C$ and C, where $C = 43^n$ if the graph contains a triangle, and $C = 2^{13n^2}$ for general graphs. We extend the case where a linear number of bits is sufficient from graphs with a triangle to graphs containing a quadrilateral face, with a bound of $C = 2^{12n}$.

Previously, Das and Goodrich (1995) have given an algorithm that works for triangulated 3-polytopes and selects rational vertex-coordinates with polynomially many bits for the numerator and the denominator, i.e., $C = 2^{n^{O(1)}}$. However, their algorithm has the advantage that it takes only a linear number of steps (including arithmetic operations on numbers of size at most C).

The approach of Das and Goodrich is more closely related to the original proof of Steinitz, who builds up the polytope from simpler polytopes by making local changes. (Steinitz's original proof yields a doubly-exponential bound for C.)

The approach of Richter-Gebert exploits the connection between 3-polytopes and *stresses* on the edges of a plane projection of a polytope, which goes back to Maxwell (1864). We fix a triangular face as exterior face, interpret all remaining edges as springs with elasticity 1, and compute the equilibrium of this mechanical system. This amounts to solving a system of linear equations, and leads to vertex coordinates of size at most 42^n. The same procedure was also used by Tutte (1960) for obtaining a nice drawing of a planar graph.

The next step calculates the polytope which projects onto this drawing. We can do this directly by exploiting the geometric significance of the edge weights.

Finally, in the case that the graph contains no triangles, but a quadrilateral face, we depart from the traditional approach of realizing the dual graph ous optimization problem

$$\min f(x) + \mu \sum_{i=1}^{n} x_i(1-x_i)$$
$$\text{s.t. } g(x) \leq 0,\ 0 \leq x_i \leq 1, i = 1, \ldots, n$$

for some $\mu \geq \mu_0$. Can you compute μ?

5. Consider the following quadratic problem

$$\min f(x) = c^T x + \tfrac{1}{2} x^T Q x$$
$$\text{s.t. } x \geq 0$$

where Q is an $n \times n$ symmetric matrix, and $c \in \mathbb{R}^n$. The complexity of finding (or proving existence) of Kuhn-Tucker points for the above quadratic problem is NP-hard.

Consider the related problem

$$\min f(x) = c^T x + \tfrac{1}{2} x^T Q x$$
$$\text{s.t. } 1 \geq x \geq 0$$

where Q is negative definite, i.e., the function $f(x)$ is concave. What is the complexity of the problem of computing a local minimum?

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[2] R. Horst, P.M. Pardalos and V. Thoai, *Introduction to Global Optimization*, Kluwer Academic Publishers (1995).

Jürgen Richter-Gebert

Reachability Problems in Dynamic Geometry

Geometric (ruler and compass) constructions play a central role in geometry. While static aspects of these constructions have been widely explored (which configurations are constructable with a given set of tools), dynamic aspects have been hardly investigated.

Usually a geometric construction consists of a set of freely chosen base objects (say some points in the plane) and a sequence of construction steps. A natural notion of "continuity" in the configuration space of a construction arises when one moves the base objects and traces the paths of the dependent objects. However, intrinsic ambiguities in certain construction steps arise (a line and a circle have two intersections, two lines have two angular bisectors), which produce distinct geometric instances for the same construction

into convex k-sets, and a remaining set of size $< f(k,d)$. However, one can do it better. In fact, it is not too difficult to prove that if $d \geq 3$ and n is large enough, then any set of kn points, in general position in $\mathbb{E}^d$, can be partitioned into n convex subsets of size k. This is not true, however, in the planar case, even if $k = 4$. Answering a problem due to Joe Mitchell, we design a fast algorithm which decides if a given set of $4n$ points in the plane can be partitioned into n convex quadrilaterals.

Finally, we give an account on a recent progress concerning the existence of large empty convex subsets.
(Joint work with János Pach and Géza Tóth)

Leonid Khachiyan

Integer Optimization on convex semi-algebraic sets

Let Y be a convex set in $\mathbb{R}^k$ defined by polynomial inequalities and equations of degree at most $d \geq 2$ with integer coefficients of binary length at most l. We show that if the set of optimal solutions of the integer programming problem $\min\{y_k | y = (y_1, \ldots, y_k) \in Y \cap \mathbb{Z}^k\}$ is not empty then the problem has an optimal solution $y^* \in Y \cap \mathbb{Z}^k$ of binary length $ld^{O(k^4)}$. For fixed k, our bound implies a polynomial-time algorithm for computing an optimal integral solution y^*. In particular, we extend Lenstra's theorem on the polynomial-time solvability of linear integer programming in fixed dimension to semidefinite integer programming. We also give a linear-time algorithm for real semidefinite optimization in fixed dimension for the real number model of computation.
(Joint work with Lorant Porkolab)

James Lawrence

Valuations and Uniform Oriented Matroids

Associated with each n-point oriented matroid is a valuation on the lattice of subcomplexes of the boundary of the n-dimensional cross-polytope. The valuation determines the structure of the oriented matroid. There exists a characterization of the valuations associated with uniform oriented matroids which involves only linear equations, linear inequalities, and integrality. Thus the n-point, rank r, uniform oriented matroids are characterized as the set of points having coordinates in $\mathbb{Z}$ which lie in a certain polytope.

There is a finite set of vectors in the space of valuations such that mutations of the uniform oriented matroids correspond to translations by vectors in this set.

Jesús de Loera

Computing minimal and maximal triangulations of convex polytopes

In this talk we discuss the problem of finding triangulations that minimize or maximize the number of top dimensional simplices. A classical case of study has been the n-cube, but here we discuss the problem for arbitrary convex polytopes.

New results include that fast recognition of stacked polytopes, heuristics for finding minimal triangulations of 3-polytopes, NP-hardness of minimizing over a subset of the simplices, #P-hardness of finding maximal regular triangulations in arbitrary dimension and the structure of minimal triangulations for archimidean solids.

Shmuel Onn

Partitions: Optimization and Structure

We develop a framework for partition problems that provides broad expressive power and efficient solution of a variety of optimization problems. In this talk I will describe some of the outcomes of our work, in particular:

- A hierarchy of polytime algorithms for partition problems with convex objectives, parameterized by the number of criteria and number of parts.
- The asymptotics of the maximum number of separable partitions in Euclidean and VC spaces, and DS-sequence based tight lower and upper bounds on the complexity of the corresponding enumeration problem.
- An algorithm for constructing universal Gröbner bases for vanishing ideals of point configurations, polytime in fixed number of variables, and the structure and asymptotical properties of corner cut polyhedra.

References:

- A polynomial time algorithm for shaped partition problems (with F. Hwang and U. Rothblum), SIAM J. Opt., to appear.
- Separable partitions (with N. Alon), Disc. App. Math. 91:39-51, 1999.
- Cutting Corners (with B. Sturmfels), Adv. App. Math., to appear.

János Pach

Drawing graphs – Does convexity make a difference?

Let G be a planar graph of n vertices, $v_1, \dots, v_n$, and let $\{p_1, \dots, p_n\}$ be a set of n points in the plane. We present an algorithm for constructing in $O(n^2)$ time a planar embedding of G, where vertex v_i is represented by point p_i and each edge is represented by a polygonal curve with $O(n)$ bends.

This bound is asymptotically optimal in the worst case. In fact, if G is a planar graph containing at least m pairwise independent edges and the vertices of G are randomly assigned to points in convex position, then, almost surely, every planar embedding of G mapping vertices to their assigned points and edges to polygonal curves has at least $m/20$ edges represented by curves with at least $m/40^3$ bends. Does this remain true if the vertices of G are randomly assigned to the elements of any fixed set of n points in the plane, not necessarily in convex position?

(Joint work with R. Wenger)

Panos Pardalos

Some open questions in convexity and combinatorial optimization

In this talk we discuss the following problems and related open questions:

1. How difficult is it to check convexity of a function (or a set)? In particular what is the complexity of the problem of checking convexity of a multivariable polynomial of degree at least 4? We conjecture that this is NP-hard.

2. It is well known that many classes of functions that appear in optimization can be expressed as the difference of two convex functions (dc representation). What is the "best" dc representation of a function?

3. Is there an efficient computational procedure to check if a set of spheres centered at selected vertices of a polytope cover the polytope? This problem appears in space covering techniques for global optimization problems.

4. The general integer programming problem

$$\begin{aligned}&\min f(x)\\ &\text{s.t. } g(x) \le 0,\ x \in \{0,1\}^n\end{aligned}$$

can be formulated (under certain conditions) as an equivalent continu-