## MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

## Tagungsbericht 06/1999

# Mathematische Analyse von FEM für Probleme in der Mechanik

07.02.-13.02.1999

The meeting has been organized by Dietrich Braess (Bochum), Eckard Ramm (Stuttgart) and Christoph Schwab (Zürich). 48 mathematicians and engineers from 12 countries came together attending talks with emphasis on the following topics:

- p- and hp-versions of finite elements
- problems with a small parameter and multigrid methods
- analysis of plate and shell elements
- visco-elasticity, plasticity, and nonlinear effects
- stabilization of mixed methods

## F. ARMERO

## Integration Algorithms for Nonlinear Elastodynamics

We describe in this presentation some recent results in the formulation of integration schemes that possess controllable dissipation in the high-frequency range for general models of finite elastodynamics. More specifically, we present rigorous analyses of the newly developed methods, and consider particular examples that allow a complete comparison with more traditional methods. In particular, the rigorous dissipative character of the schemes becomes clear in these analyses. Different aspects of the numerical implementation of the new schemes in the context of the finite element method are also discussed. Furthermore, we describe the development of these methods in the context of geometrically exact beam and rod theories. Representative numerical simulations are presented illustrating the performance of the newly proposed methods.

## D. BRAESS

## Softening and Stability of EAS Elements

When primal mixed methods are compared with the corresponding classical finite element methods, one observes a softening of the energy functional. It is determined by the  $L_2$ -projection of the gradients (or strains) onto the given finite element space. Instead of specifying the target space of the projection, one may define it by the orthogonal complement. This is in essence the method of enhanced assumed strains. The algebraic relationship was observed by Andelfinger and Ramm and formally proved by Yeo and Lee.

We show now that the inf-sup condition for the mixed method is equivalent to a strengthened Cauchy inequality for the trial spaces in the EAS method. It was often ignored that such a condition has to be satisfied in order to guarantee stability. So it is understood that the first enthusiastic reaction to the EAS method is followed by a more realistic evaluation.

## C. CARSTENSEN

## Averaging Techniques in FEM are Reliable

The standard residual based a posteriori error estimators on finite element computations for second order elliptic equations can be refined: The volume contribution can be replaced by a term which is generically of higher order. Hence, the remaining edge contribution dominates the error estimate, and so do equivalent error indicators. This implies the reliability of the ZZ-estimator on unstructured grids. Numerical examples are provided on Stokes and Lamé problems that indicate the superiority of the averaging techniques.

## R. CODINA

## Stabilization of Incompressibility and Convection Through Orthogonal Sub-Scales in Finite Element Methods

Two apparently different forms of dealing with the numerical instability due to the incompressibility constraint of the Stokes problem are analyzed in this talk. The first of them is the stabilization based on the pressure gradient projection, which consists of adding a certain least-squares form of the difference between the pressure gradient and its  $L^2$ -projection onto the discrete velocity space in the variational equations of the problem. The second is a sub-grid scale method, whose stabilization effect is very similar to that of the Galerkin least-squares method for the Stokes problem. It is shown that the first method can also be recast in the framework of sub-grid scale methods with a particular choice for the space of sub-scales. This leads to a new stabilization procedure, whose applicability to stabilize convection is also studied.

## M. CRISFIELD

# Lower-order Elements for Small and Large Strain Analysis with Incompressibility

The main aim of the current work is large-strain elasto-plastic analysis, for which a multiplicative  $F = F_e F_{\Psi}$  decomposition is adopted. To this end, a range of reasons dictate the use of lower-order elements. The most important of these is contact. Today, many analysts used enhanced assumed strain formulations or, alternatively, the simple Q1 - P0 element. Despite the fact that this element his theoretical limitations (inf-sup), it was shown that very effective non-linear solutions were often obtained. However there are problems and the lecture considered the axi-symmetric resulting problem where severe hour-glassing is found. By re-visiting the original 3-field form of Simo, Taylor et al a form of stabilization was developed.

The second half of the lecture considered linear analysis and turned to the driven cavity flow problem. A wide-range of lower-order elements was considered and tested against a "stiffness test" and Chapelle and Bathe's "numerical Inf-sup test".

#### M. DAUGE

## **Boundary Layers in Thin Structures**

We mainly investigate the behavior of thin elastic solids as the thickness

 $\epsilon \to 0$ .

This happens with naturally thin structures as plates.

In the situation where the plates are made of a homogeneous isotropic material, we can distinguish membrane and bending loads and describe the behavior of the displacement field submitted to such loadings by a two-scale asymptotics.

Our asymptotic can be derived and yields also a two-scale asymptotic for the other characteristic tensors in the plate, such as strain and stress tensor.

We present numerical experiments showing the boundary layer part of the asymptotics in strain components near the thin part of the boundary.

We moreover give hints about the three-scale asymptotics appearing in thin shells, in the clamped elliptic case.

## L. FRANCA

## Adjoint-residual-free Bubbles for the Advective-Diffusive Equation

We consider conforming Petrov-Galerkin formulations for the advective and advective-diffusive equations. For the linear hyperbolic equation, the continuous formulation is set up using different spaces and the discretization follows with different "bubble" enrichments for the test and trial spaces. Boundary conditions for residual-free bubbles are modified to accommodate with the first order equation case, and regular bubbles are used to enrich the other space. Using piecewise linears with these enrichments, the final formulations are shown to be equivalent to the SUPG method, provided the data is assumed to be piecewise constant. Generalization to include diffusion is also presented.

## B. GUO

## Computation of Elasticity Problems on Non-Smooth Domains in $\mathbb{R}^3$

Based on the analysis of singularities of the solution in neighborhoods of edges, vertices and vertex-edges, mesh and polynomial degree are precisely designed to achieve the optimal rate of convergence. Theoretical and numerical results show that an exponential rate of convergence can be achieved if geometric meshes and bilinear-linear distributions of element degree are used, which is called the h-p version of the finite element method in  $\mathbb{R}^3$ .

Regularity results and approximation results can be generalized to elliptic problems with piecewise analytic data in  $\mathbb{R}^3$  such as polyhedral domains.

## P. HANSBO, R. STENBERG

## On the Use of Nitsche's Method in Computational Mechanics

In this joint presentation we propose the use of a classical method of Nitsche for the purpose of mortaring non-matching meshes. It is shown that Nitsche's method has optimal convergence rates in energy-like and  $L_2$ -norms and is stable provided we add a mesh-dependent penalty-like term with a moderately sized, easily computable, penalty parameter.

The primary use of this approach is for domain decomposition problems, but we also discuss other applications such as contact problems, directional boundary conditions for vector-valued problems, and the discretization of elliptic problems using a discontinuous ansatz.

## U. HANSKÖTTER

## 3D Finite Shell Elements on the Basis of Multi-Director Kinematics

Damage and failure modes in structural components are mostly accompanied by high stress concentrations characterized by very complex through-thickness distributions. This is particularly the case if the shell consists of strongly dissimilar material layers or if the damage develops layer-wisely. Geometrical discontinuities demand precisely working finite elements as well. Therefore shell elements should be able

- to simulate finite rotations and large deformations including transverse strains in the kinematic model,
- to deliver numerically stable results even by distorted meshes while abolishing locking-effects,
- to provide flexibility concerning the discretization in the reference surface and in thickness direction to predict stress concentrations with accurate performance.

For the analysis multi-director shell elements have been developed which are based on higher order polynomial displacement fields in thickness direction. Finite rotations are accomplished by using updated formulation. For the simulation of complex stress distributions multi-layer kinematics has been used. Here one possibility is to transform the multi-director shell element into a 3D multi-director element in the way they can be used for a 3D discretization of the entire shell structure.

#### A. HUERTA

## Adaptivity Based on Error Estimation

Finite Elements and meshless methods may be used together in order to profit from the advantages of both of them. A natural adaptive strategy is to enrich the finite element discretization properly adding meshless particles. In fact, this is a particular case of a hierarchical approach because the degrees of freedom associated with the particles do not alter the value of the interpolant at the finite element nodes.

A priori error estimates are needed to predict the behavior of this kind of hybrid methods and to design a proper adaptive strategy. In this work a priori error estimates for the hybrid meshless-finite element interpolations are presented. The error estimate depends on the finite element characteristic size h, the order of the finite element interpolation p, the dilation parameter  $\rho$ , and the order of consistency m of the hybrid meshless-finite element method. It is found that the solution is not convergent if only the dilation parameter  $\rho$  goes to zero. The element size h is restrictive and sets up an error threshold that is always exceeded when the dilation parameter is reduced. This result suggests an adaptive strategy consisting on maintaining the finite element interpolation where h does not affect the presented accuracy and suppressing it elsewhere. The a priori estimate reads: let  $u \in C^{m+1}(\bar{\Omega})$ ,  $\Omega \subset \mathbb{R}^n$  be bounded. Then

where 
$$q = m - p$$
.  $||u - (u^h + u^R)||_{\infty} \le h^{p+1}(c_1 h^q + c_2 \rho)$ 

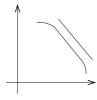
## T. HUGHES

#### Large Eddy simulation and the variational multiscale method

I had an idea how to merge these concept to advantage. I stayed up all night and worked it up before I presented it. It seemed to work out and now it needs some testing and critical evaluation.

The basic idea is this: Push the eddy viscosity into a wave number range  $[\overline{k}, k']$  rather than the traditional range  $[0, \overline{k}]$ . The variational multiscale method accommodates this in a natural way. The results are remarkably superior to the traditional approach.

1) All exact solutions of the Navier-Stokes equations satisfy the model; 2) No accuracy degradation at low wave numbers;



3) correct energy spectra (eg. the -5/3 law); 4) proper kinetic energy decay; 5) backscatter without instability; 6) vanishing Reynold stresses at the mall.

Looking promising!

## B. MIARA

## Asymptotic analysis for geometrically nonlinearly elastic shells

How a shell behaves when its thickness goes to zero?

That is what we are investigating using the **asymptotic formal expansion** of the displacement field; with respect to the thickness, in the case of nonlinearly Saint-Venant-Kirchhoff, elastic material. To be performed, the analysis needs two specific **requirements**.

- 1) no special comparability conditions on the applied forces,
- 2) by linearization we should recover the linear classical models.

We show that when the middle surface of the shell and the associated kinematic boundary conditions do not allow **inextensional displacements** the shell behaves like a **membrane** (the energy defends only on the two-dimensional change of metric tensor) otherwise the model is **flexural**.

To get those limiting models some assumptions on the applied forces have to be made in terms of powers of the thickness of the shell.

The limiting ?-electric shell model where mechanical and electrical displacements are coupled is ? as another example of the power of this approach.

## A. MATACHE

## A Generalized p-FEM for Homogenization Problems

A new finite element method for elliptic problems with locally periodic microstructure of length  $\epsilon > 0$  is developed and analyzed. The idea of this approach is to approximate a Bochner integral by a finite sum truncating a generalized Poisson summation formula.

It is shown that the method converges as  $\epsilon \to 0$ , if the solution of this homogenized problem with optimal order in  $\epsilon$  and exponentially in the number of degrees of freedom independent in  $\epsilon$ .

The computational work of the method is bounded independently of  $\epsilon$ . Numerical experiments demonstrate the feasibility and confirm the theoretical results.

## J. PITKÄRANTA

## New FEM theory in old frame or Old FEM theory in new frame

Suppose we are given a finite element space  $U_h$  and a man-made energy product  $\mathcal{A}_h(\cdot,\cdot)$ . We want to solve

$$u_h \in \mathcal{U}_h : \mathcal{A}_h(u_h, v) = \mathcal{A}(u, v) \quad \forall v \in \mathcal{U}_h^0$$
,

where  $\mathcal{A}(\cdot,\cdot)$  is the energy product given by nature, kinematic constraints may be imposed on  $u_h$ , and homogeneous constraints are improved on  $\mathcal{U}_h^0$ . Let

$$z_h \in \mathcal{U}_h^0: \ \mathcal{A}_h(z_h, v) = (\mathcal{A} - \mathcal{A}_h)(u, v) \quad \forall v \in \mathcal{U}_h^0$$

and let  $\tilde{u}_h = u_h - z_h$ . Then  $\tilde{u}_h$  is the best approximation of u in  $||| \cdot |||_h = \sqrt{\mathcal{A}_h(\cdot, \cdot)}$ , and the error splitting  $u - u_h = (u - \tilde{u}_h) - z_h$  is orthogonal:

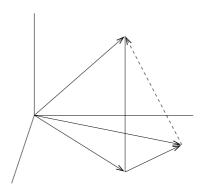
## The task of an engineer:

Design  $\mathcal{U}_h$ ,  $\mathcal{A}_h$ !

## The task of a mathematician:

Estimate the two error terms - from above and from below!

## The Big Picture:



## E. RANK

## The p-Version of the FEM for some Elasto-Plastic Problems

The p-version of the Finite Element Method is considered superior to the classical h-version for many linear problems. Yet it is still an open question, whether high quality of this method with respect to accuracy such efficiency will also be observed for more complex non-linear problems. This lecture rests on investigations of the p-version for the deformation theory of plasticity with special emphasis on a comparison with a benchmark test, which was carried out earlier for the adaptive and non-adaptive h-version by other authors. Finally we present a domain decomposition method, where one part of the structure with elasto-plastic behaviour is analysed by a classical h-approach, whereas the other, linear elastic part is approximated by a p-version.

## B. REDDY

## Enhanced Strains, Stability and Affine Approximations

The variational setting of the method of enhanced assumed strains is reviewed, and its relationship to classical stabilization methods is established, by reducing the EAS problem, through projection, to one involving displacements only.

The notion of the equivalent parallelogram associated with a quadrilateral is introduced; this is the affine figure closest to a given quadrilateral, in a sense made precise. The affine-approximate method of EAS is then defined to be the method obtained by replacing quadrilaterals in a finite element mesh with their equivalent parallelogram. The formulation thus obtained is analysed, and found to converge at the rate 0(h), provided that the distortion of quadrilaterals measured by a scalar distortion parameter, is also 0(h).

## J. SCHÖBERL

## Robust Multigrid Methods for Plate and Shell Problems

This talk deals with the construction and analysis of robust multigrid preconditioners for parameter dependent elasticity problems in primal variables. A stable finite element discretization based on mixed methods is used.

First, we present a two-level method, which fits into the framework of additive Schwarz techniques. It is shown that block Jacobi preconditioners in combination with robust grid transfer operators lead to preconditioners with uniformly bounded condition numbers. Then, new results establishing optimal and robust convergence of the according multigrid method are presented. These results apply to Reissner-Mindlin plate model as well as to nearly incompressibility.

Finally, numerical results verifying the theoretical results are shown. Additional experiments show good numerical behaviors also for shell problems implemented by means of thin 3d elements.

## D. SCHÖTZAU

## Time Discretization of Parabolic Problems by the hp-Version of the DG Finite Element Method

The discontinuous Galerkin Finite Element Method for the time discretization of parabolic problems is analyzed in a hp-version context. We show that this method can achieve exponential rates of convergence for piecewise analytic (in time) solutions if geometrically refined time steps and linearly increasing approximation orders are used. In h-version approaches such singular solution behaviour can be resolved by the use of algebraically graded time steps. Finally, we address the hp-discretization in time and space and explain how exponential rates of convergence in time and space can be obtained by the use of certain spatial mesh design principles.

#### K. SCHWEIZERHOF

# Removal of incompressibility locking for 3D-solid/solid-shell elements with low order interpolation

Incompressibility locking is occurring in FE approximations of continuous problems with low order polynomials, if materials like rubber of plasticity are analysed. This nonphysical phenomenon can be removed by a modification of the deformation gradient for small and large strain formulations. Various methods have been suggested in the literature, however most of them show some deficiencies under certain circumstances - neither solution leads to a robust element for arbitrary situations. The contribution focuses on a efficient solution reduced integration scheme for the large deformation regime which under the condition of an isochoric volumetric split no longer shows volumetric locking. However, with the constant pressure assumption the robustness of the 3-linear element is hot and instabilities may occur. This can be avoided by enhancing the fully integrated deviatoric part with some shore of the part. This is motivated by the observation that the under-integrated volumetric part contains no longer any resistance against the hourglass kinematics, however, the stabilization needs an heuristically chosen parameter.

Solid-shells FE formulations - linear and quadratic - are modified with enhanced strains resp. assumed strains to improve the benching and in-plane behavior, however they still show come locking in the incompressible regime. This can be removed in the same fashion and a stabilization can be applied as for the solid elements. The numerical tests show the improved behavior, in particular of the quadratic elements. A mesh refinement study reveals as after the result that for many problems the difference in the various improvements concerning the enhancements in in-plane direction have less influence on the results than choosing a sufficiently refined mesh.

## B. SZABO

## Quality Assurance in the Numerical Simulation of Mechanical Systems

In order to achieve reliability in the numerical simulation of mechanical systems it is necessary to control both the errors of idealization and the errors of discretization. The errors of discretization are controlled by a hierarchic family of finite element spaces. Analogously, the errors of idealization are controlled by a hierarchic family of mathematical models. For example, in the treatment of beams, plates, and shells the construction of hierarchic models involves the determination of director functions such that the exact solution of the kth model converges to the exact solution of the fully three-dimensional model based on the theory of elasticity. More generally, mathematical models must account for nonlinear phenomena, such as material nonlinearities, geometric nonlinearities, mechanical contact conditions, etc. The results of some recently completed research and some current problems under investigation are discussed and examples presented.

## K.Y. SZE

## Trapezoidal Locking and Solid Shell Finite Element Formulation

The source and remedies for trapezoidal locking in QUAD4 and HEXA8 elements are reviewed. The scaled hybrid stress formulation and the assumed transverse normal strain appear to be the only finite element technologies to circumvent the locking problem. With a new HEXA8 solid-shell element, shear locking is resolved by assumed natural transverse shear strain, trapezoidal locking is resolved by assumed physical transverse normal strain, whereas membrane, Poisson and incompressibility locking are surmounted by hybrid stress formulation which is also effective to enhance the element accuracy. The computation cost of the element is significantly reduced by orthogonalizing the non-constant stress modes w.r.t. the constant ones and enforcing admissible sparsity in the flexibility matrix of the element. Popular numerical benchmark tests are conducted and the element accuracy is promising compared to the state-of-the-art solid-shell elements.

## R. TAYLOR

## Mixed-Enhanced Formulation for Tetrahedral Finite Elements

This work considers the solution of problems in three-dimensional solid mechanics using tetrahedral finite elements. A formulation based on a mixed-enhanced treatment permits direct approximations of displacements, mean stress (pressure) and volumetric deformations. The approximations for displacement and mean stress are conventional  $C^0$  interpolations with nodal parameters at the vertices of the tetrahedral element. The approximation for volumetric deformation is also linear in each element but discontinuous between contiguous elements. The enhanced deformation terms are computed from the gradient of a bubble function. The form adopted permits direct use of any constitutive model which is strain (deformation) driven – including, therefore, elastic and inelastic forms to be easily used. Both small and finite deformation problems are addressed. Solution of sample problems including nearly incompressible situations are given and compared to mixed hexahedral element results. The element is locking free for situations considered.

## C. WIENERS

## Multigrid Methods for Prandtl-Reuss plasticity

We consider the application of multigrid methods to problems in plasticity.

For the discretization in time, we introduce a Runge-Kutta method which is based on an algorithmic formulation with a response function for the nonlinear material behaviour. In the particular case of linear hardening B-stability can be proved for algebraically stable Runge-Kutta schemes. For stiffly accurate methods the scheme can be derived as the limit of a viscoplastic regularization. We tested the method numeri-

cally for a 2D benchmark problem for perfect plasticity with assumed plain strain. We compared the asymptotic behaviour of different time schemes, various discretizations  $(P_1, Q_1, Q, P_0, P_2, Q_2, P_1, Q_2, Q_1)$  and adaptive meshes.

Finally, we present numerical examples for plasticity with linear hardening and exponential hardening on large 3D problems, where we applied a geometric multigrid method together with an algebraic multigrid method for the solution of the linear problems. Here, we can obtain a good parallel efficiency for the resulting algorithm combining the plasticity model with multigrid.

## P. WRIGGERS

#### On Smooth Contact Discretization

Finite deformation contact problems are associated with large sliding in the contact area. Thus, during an analysis a slave node can slide over several master segments within the actual finite element discretization. Standard contact discretization is based on a linear boundary interpolation. Hence the outside normal field is not smooth. This leads often to convergence problems in penalty or Lagrangian multiplier algorithms. To present this a  $C^1$ -continuous interpolation is introduced using Hermitian or Bézier splines. The associated development of the discretization for finite deformations regarding contact of two or more deformable bodies is developed for the case of frictionless and frictional contact. By means of examples the new contact scheme is compared with existing ones using linear interpolation. It is shown that the smooth interpolation is more robust and yields better approximations of the contact forces.

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