MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1999

Mathematical Methods of Geodesy

28.03.99 - 03.04.99

This conference, organized by Willi Freeden (University of Kaiserslautern), Erik W. Grafarend (University of Stuttgart), and Leif Svensson (University of Lund), continued a sequence of Oberwolfach conferences on mathematical methods in geodesy and brought together researchers from mathematics, geodesy, physics, and information technology.

The conference had 35 participants from 6 countries. The program was divided into 7 sessions on different areas. The title of these sessions reflects the variety of themes presented at the conference:

1. *Gravity and Magnetic Field Determination* (M. Bayer, C. Cui, J. Kusche, T. Maier, L. E. Sjöberg, M. Thalhammer)
2. *Numerical Analysis* (D. Potts, R. Reuter, F. Sacerdote, G. Steidl)
3. *Constructive Approximation* (S. Dahlke, O. Glockner, F. J. Narcowich, L. L. Schumaker)
4. *Geodetic Reference Frames, Stochastic Models, Filters, etc.* (A. A. Ardalan, K. Borre, R. Jurisch, B. Richter, H. Römer, V. S. Schwarze)
5. *Geodynamical Problems* (A. M. Abolghasem, S. Beth, E. Groten, D. Wolf)
6. *Inverse Problems* (V. Michel, M. Z. Nashed, E. Schock)
7. *Boundary-Value Problems* (G. Anger, P. Holota)

The topics addressed were given as block of talks followed by a lively discussion and a useful exchange of ideas. One additional afternoon discussion was organized on inverse problems in physical geodesy and the current status of earth's gravitational potential determination. The research summaries posted close to the main lecture hall were extremely helpful to initialize an interdisciplinary exchange of ideas.

All participiants are grateful to the staff of the Oberwolfach institute for making their stay pleasant.

Vortragsauszüge

First Results of an Attempt to Apply a Numerical Method in Earth Deformation Analysis

Amir M. Abolghasem

In order to investigate the deformations of the earth under the application of a force field, different analytical methods have been set forth. Applications of analytical models are restricted by circumstances which may be avoided by numerical techniques. For example, lateral heterogenities have been neglected by perhaps every analytical solution, although they may have profound effects in displacement field of sources such as earthquakes. A numerical model in conjunction with a proper numerical solution technique can overcome this problem.

We have recently started to test the abilities of a numerical technique, i.e. finite element method, in order to compute the earth deformations.

The analysis began with the simplest spherical model, a non gravitational homogeneous isotropic elastic sphere, with the purpose of going step by step towards a self gravitational layered isotropic visco-elastic spherical model. Our solution, although including not more than 1000 tetrahedral elements yet, shows an encouraging agreement with the analytical solutions.

So far deformations of both non-self gravitational and self gravitational isotropic elastic spheres have been solved. On account of the fact that the time-space domain integration is considerably slower and computationally more expensive than the frequency domain analysis, we also investigate the possibility of numerical extraction of normal modes of the sphere. If this method turns out to have good results, it brings the important advantage of fast computation of the displacement field under the application of any force field, as soon as enough natural frequencies are extracted.

Possible further steps:

- Increasing the number of elements.
- Numerical extraction of normal modes.
- Layering of the spherical model.
- Studying the response of a laterally heterogeneous model.
- Possibility of introducing internal discontinuities.
- Possibility of replacing the spherical model by an ellipsoidal model in order to apply geodetic (e.g. GPS) observations as boundary conditions.

The Structure of the Gravitational Field and other Physical Fields: Uncertainties in Inverse Problems

Gottfried Anger

1. Basic physical laws

Newton's law of gravitation (electromagnetic forces)

$$F = \gamma \frac{m_1 m_2}{r^2} \tag{1}$$

is very simple to prove: Let

$$K(x^0, r_1) \subset K(x^0, r_2) \subset \boldsymbol{R^3}$$

be two closed balls, the content of the surface $\partial K(x^0, r)$ is equal to $4\pi r^2$. Further, let

$$v_1 = c_1 \text{ flux of force through } \partial K(x^0, r_1),\ v_2 = c_2 \text{ flux of force through } \partial K(x^0, r_2).$$

From

$$\int\limits_{\partial K(x^0, r_1)} v_1 dS = \int\limits_{\partial K(x^0, r_2)} v_2 dS \tag{2}$$

it follows that

$$4\pi v_1 r_1^2 = 4\pi v_2 r_2^2 \text{ or } v_2 = v_1 \frac{r_1^2}{r_2^2} \sim \frac{1}{r^2} = \frac{1}{|x^0 - y|^2}.$$

The potential energy of two point masses ($\gamma m_1 m_2 = 1$) is given by the line integral along a path $\mathcal{C}$ not containing x^0

$$G_L(x, y) = \int\limits_x^z (K_1 dx_1 + K_2 dx_2 + K_3 dx_3) = \frac{1}{|z - x^0|} - \frac{1}{|x - x^0|}. \tag{3}$$

For these considerations the physical meaning of gravitation is not necessary.

2. Newtonian potential

Let

$$G_L(x, y) = \frac{1}{4\pi} \frac{1}{|x - y|},$$

be the Newtonian kernel. The Newtonian potential is defined by

$$G_L\mu(x) = \int G_L(x, y) d\mu(y). \tag{4}$$

Special measures are $d\mu(y) = \rho(y)dy$, dy volume element, and $d\mu(y) = \sigma(y)dS(y)$, $dS(y)$ surface element. From Green's formula (1828) it follows that

$$\int G_L(x, y)(-\Delta\varphi(y))dy = \varphi(x) \tag{5}$$

for sufficiently smooth test functions φ. If $x \notin \operatorname{supp}\varphi$ from (5) it follows that

$$\int G_L(x, y)\rho(y)dy = 0,\ \rho(y) = -\Delta\varphi(y). \tag{6}$$

Such densities are called non-radiating densites or ghosts or phantoms or artifacts (no real world solutions). Similar formulas hold for other physical fields and for *nuclear magnetic resonance tomography* [3].

Further ghosts (phantoms) arise in discretizing the Riemann integral

$$Af(x) = \int\limits_a^b K(x, y) f(y) dy \approx \sum_{j=1}^N c_{jk} f(y_j) = A_N f(x_k) = g(x_k). \tag{7}$$

If one adds a function φ vanishing at the points y_j we get

$$A_N f = A_N(f + \varphi). \tag{8}$$

The functiuon $f + \varphi$ is also a solution (ghost) of the discretized equation. Such relations also hold in $\boldsymbol{R^n}$. Without additional conditions on f, for instance, $|f < M$, one cannot determine a uniquely determined solution of the discretized equation [3], [4], [5].

If one does not know the physical field on the masses (charges) - right-hand side of a differential equation - then the masses are not uniquely determined [1].

A Main Problem in Geodesy (Geophysics)

The gradient of the gravitational field can be measured along the orbit of a satellite or on the surface of the Earth. One has to determine the downward continuation of this field. Similar problems hold in geophysics relative to the magnetic field (see these Proceedings).

3. Basic Results on Inverse Source Problems
Let $\Omega \subset \boldsymbol{R}^3$ be a bounded open set, $\partial\Omega$ its boundary, $\bar{\Omega} = \Omega \cup \partial\Omega = K$,
supp μ the smallest closed set supporting the measure μ. We introduce for positive μ on $\bar{\Omega}$

$$\mathcal{B}(\mu) = \{\nu : \nu \geq 0, \operatorname{supp}\nu \subset \bar{\Omega}, G_L\nu(x) = G_L\mu(x), x \in \bar{\Omega}\} \tag{9}$$

This set is a convex and weakly compact set of the dual space $C^*(\bar{\Omega})$ of $C(\bar{\Omega})$. Of special interest are the extremal elements of $\mathcal{B}(\mu)$ [1].

4. Inverse Problems
In inverse theory one has to solve equations of the first kind

$$Af = g,\ f \in X,\ g \in Y,\ X, Y \text{ Banach spaces or metric spaces.} \tag{10}$$

If A and f are known, the calculation of Af is called a **direct problem**, if A and g are known the determination of f is called an **inverse problem of the first kind**, if A and f are to be determined and g is known, the **inverse problem is called an inverse problem of the second kind** [2], [4], [5]. Most results in analysis are results to equations of the second kind

$$Af + \lambda f = g, \tag{11}$$

which completely differ from equations of the first kind, if A has good mathematical properties (compact operator).

5. Well-Posed Problems
<u>Definition.</u> Following J. Hadamard (1923) the equation $Af = g$ describing the corresponding inverse problem is called **well-posed**, if

1. For every $g \in Y$ there exists at least one f satisfying the equation $Af = g$ (*existence*).
2. The element f satsfying $Af = g$ is uniquely determined (*uniqueness*).
3. The solution f depends continuously on g (*stability*).

If one of these three definitions is not fullfilled, the problem is called *improperly posed* or *not well-posed* or *ill-posed* [1], [3], [5].

If the inverse $f = A^{-1}g$ is discontinuous, one has the **regularize** the equation $Af = g$, i.e to replace A by a family of operators A_α for which the inverses A_α^{-1} are continuous and to consider the limit of $A_\alpha^{-1}g$ for $\alpha \to 0$ [3], [5]. Following A. G. Yagola (1999) not every ill-posed problem can be regularized.

In medical imaging there are very important inverse problems, such as, computed tomography, impedance computed tomography and magnetic resonance tomography. Relative to these inverse problems **ghosts** exist [3], [4].

6. Medical Diagnosis
The followers of Hippocrates (460 - 377 B.C.) *contend that the history of individual illnesses can be precisely studied only by carefully and precisely registering all symptoms: according to them, the illness as such is beyond our reach.* This result holds for every (complex) system on the Earth, since **no mathematical systems theory** is possible for such complex systems [3], [6].

7. Final Remarks
Problems in technology are reversible, in biology all problems are **irreversible** and no mathematical approach exists for biological problems. In mechanics the sum of the internal forces is zero, a similar fact holds in electrodynamics. Therefore far reaching results exist in these areas. In medicine similar results are not possible. The book of *Parker* [9] is correct from the mathematical point of view. But his method can be applied only to problems, which can uniquely be solved. But in geophysics almost mathematical problems have infinitely many solutions, which is a consequence of very weak measuring data [1], [2], [3], [4]. Following *During* [8] and *Anger* [3] a large part of sciences has to be reorganized in favour of practical experience - **praxis cum theoia**.

References:
[1] G. Anger (1990), *Inverse problems in differential equations*, Plenum Publishing, New York, London.
[2] G. Anger, R. Gorenflo, H. Jochmann, H. Moritz and W. Webers (eds.) (1993), *Inverse problems:*

Principles and applications in geophysics, technology and medicine, Akademie- Verlag, Berlin.
[3] G. Anger (1997), *Zur Leistungsfähigkeit der Theorie in den Naturwissenschaften, der Technik und der Medizin - praxis cum theoria (On the power of theory in natural sciences, technology and medicine).* Mitteilungen der Gesellschaft für Angewandte Mathematik und Mechanik **20**. 19 - 36.
[4] G. Anger (1998), *Basic principles of inverse problems in natural sciences and medicine.* Zeitschrift für Angewandte Mathematik und Mechanik **78**, S189 - S192.
[5] G. Anger (1998), *Inverse problems: Regularization of ill-posed problems.* Zeitschrift für Angewandte Mathematik und Mechanik **78**, S235 - S237.
[6] G. Anger (1999), *Zum Verhältnis Mensch - Natur. Grundlegende Prinzipien der messenden Physik: Science is patchwork* (manuscript).
[7] G. Anger and H. Moritz (1999), *Inverse problems and uncertainties in science and medicine* (to be published).
[8] W. L. Duren (1994), *The most urgent problem for mathematics professors.* Notices American Math. Soc. **41**, no. 6, 582 - 586.
[9] R. L. Parker (1994), *Geophysical inverse theory.* Princeton University Press.

World Geodetic Datum 2000

Alireza Ardalan, Erik W. Grafarend

Based on the current best estimates of fundamental geodetic parameters $\{W_0,\ GM,\ J_2,\ \Omega\}$ the *form parameters* of the *Somigliana-Pizetti level ellipsoid*, namely the semi-major axis a *and* semi-minor axis b (or equivalently the linear eccentricity $\varepsilon := \sqrt{a^2 - b^2}$) are computed. There are *six* parameters namely the four fundamental geodetic parameters $\{W_0,\ GM,\ J_2,\ \Omega\}$ and the two form parameters $\{a,\ b\}$ *or* $\{a,\ \varepsilon\}$, which determine the ellipsoidal reference gravity field of *Somigliana-Pizetti type* constraint to two nonlinear condition equations. Their iterative solution leads to best estimates $a = (6\,378\,136.572 \pm 0.053)m$, $b = (6\,356\,751.920 \pm 0.052)m$, $\varepsilon = (521\,853.580 \pm 0.013)m$ for the tide-free geoid of reference *and* $a = (6\,378\,136.602 \pm 0.053)$m, $b = (6\,356\,751.860 \pm 0.052)m$, $\varepsilon = (521\,854.674 \pm 0.015)m$ for the zero-frequency tide geoid of reference. The best estimates of the form parameters of a *Somigliana-Pizetti* level ellipsoid, $\{a,\ b\}$, differ significantly by $-0.398m$, $-0.454m$, respectively, from the data of the Geodetic Reference System 1980.

A 2$^{\text{nd}}$ Generation Wavelet Approach to Approximating Vector Fields and Its' Applications to Geomagnetic Satellite Data

Michael Bayer

In the geosciences the model of Fourier series' (i.e. the expansion into *scalar* spherical harmonics) is commonly used. It has proved a useful and comprehensive tool whenever scalar data on a spherical geometry such as an idealized earth have to be approximated. An important example is the modelling of the geomagnetic field by a geomagnetic potential in terms of spherical harmonics, which has been used since Gauß, who obtained a model up to degree and order four. Quite similar to the methods of geomagnetism are those used for the determination of the geopotential in physical geodesy. There it is important to know about so-called rotationally invariant operators applied to the potential, e.g. how the potential behaves in different altitudes, how its (normal) derivatives may be computed etc. The effect of these operators can be displayed very easily in a scheme named after the famous geodesist Meissl. In particular, the Meissl scheme allows a very comprehensive interpretation when we work with scalar spherical harmonics as basis functions. Since in geomagnetism we also deal with normal and tangential derivatives of a scalar potential modelled by spherical harmonics, there is a bridge between geomagnetism and the Meissl scheme.

However, the disadvantages of this technique are obvious, as there are global support and serious oscillations of higher degrees. Moreover, many available measurements are not of scalar but of vectorial type (as the geomagnetic measurements of the satellite CHAMP). Although there exists a vectorial analogue to scalar spherical harmonics, namely *vector* spherical harmonics, these functions exhibit the same drawbacks as their scalar counterparts, but in addition become singular at the poles as a consequence of their dependence on polar coordinates.

Therefore, we propose basis functions that overcome these difficulties and still allow an interpretation by the extended Meissl scheme, which includes also tangential (i.e. vectorial) components. Further, we use these functions in a multiscale setting, i.e. we define spherical vector wavelets, such that a representation adapted to the number and local structure of the data at hand becomes possible.

As in the one-dimensional case it is possible to define wavelets that can be obtained by dilation and rotation of one single mother wavelet (see (M.Bayer, S.Beth, W.Freeden (1998))). These 1^{st} generation wavelets rely heavily on Fourier techniques as a theoretical tool. An application to geomagnetic data can be found in (Maier, T., Bayer, M.(1998)).

Here we introduce further so-called 2^{nd} generation spherical vector wavelets following the ideas in Sweldens, W. (1995). They are not dilated and rotated copies of one function but rely on a more general multiresolution of the underlying space and a biorthogonal basis construction.

Both approaches can be incorporated into the extended Meissl scheme. As one consequence, it is possible to decompose geomagnetic satellite data of vectorial type (as delivered by MAGSAT or CHAMP) into its' physically relevant parts and to approximate the underlying scalar quantities.

References:

[1] Bayer, M., Beth, S., Freeden, W. (1998): Geophysical Field Modelling by Multiresolution Analysis. *Acta Geod. Geoph. Hung., Vol. 33(2-4), pp. 289-319, 1998*

[2] Maier, T., Bayer, M. (1998): Multiscale Analysis of the Geomagnetic Field. *Progress in Geodetic Science (at GW98), W. Freeden (ed.), Kaiserslautern, 1998.* Shaker Verlag

[3] Sweldens, W. (1995): The Lifting Scheme: A New Philosophy in Biorthogonal Wavelet Constructions. *Wavelet Applications in Signal and Image Processing III, Proc. SPIE 2569, A. F. Laine and M. Unser (eds.), pp. 68-79, 1995*

A Method for Solving the Incompressible Navier–Stokes Equations with Spherical Vector Wavelets

Stefan Beth

The treatment of the incompressible Navier–Stokes equations includes a variety of different techniques like finite element methods, spectral decomposition etc.. In particular, Temam and his co-workers proposed a Nonlinear Galerkin Scheme, which separates the solution and the differential equation in a low frequency and high frequency part. Taking a sufficient large bandwidth for the low frequency part, the terms of the Navier–Stokes equations involving only high frequencies can be simplified, such that one observes two systems of differential equations, which are coupled by the nonlinear term in the Navier–Stokes equations. The idea behind this procedure is seen from numerical point of view: Although the high frequencies have no effect on the momentary solution they influence it over a longer time period. Thus they can not be neglected completely, but every simplification saves computational time.

The Nonlinear Galerkin Scheme also applies to the spherical case, where the corresponding basis functions are the vector spherical harmonics of kind 3. Unfortunately, they are hard to handle, such that as alternative spherical vector wavelets were chosen, which showed first success in applications concerning geophysical field modelling.

The low frequency part can be represented by a single bandlimited mother kernel, while the high frequency part may consist of wavelet basis functions of different scale. The loss of orthogonality has to be paid with the solution of few linear systems, which can be done in advance. Their number depends on the chosen time discretization. The advantage of this approach lies in the simple structure of the new basis functions, as they are generated by only few mother functions, which in addition are localizing in space and frequency. A special choice of wavelets basis functions even allows to install a pyramid scheme in order to examine the structure of the high frequency part of the solution.

Unsolved Problems Connected With Kalman Filtering of GPS Observations

Kai Borre

Real-time applications of the Global Positioning System (GPS) for positioning rely on linear filter theory. Although the Kalman filter is nearly forty years old it appears that some issues are still unsolved.

We present the basics for processing GPS data and at the same time point to our problems. Per definition the ionospheric delay is a positive scalar but filtering often estimates it as a negative number. How to introduce a constraint that changes the model?

Sometimes correlation hurts you, sometimes it helps you. We mention examples of both cases. When modeling the covariance matrices we expect stationary observations and autocorrelation functions of simple form. However practice tells us something different and it would be nice to be able to subtract and model unwanted effects.

Is there a way to recover the residuals after a filtering process—without storing all observations?

Suppose that you have no a priori knowledge about the variance of an observation. How can you tell if the initial observation is to be accepted or rejected? After a few updates the median or mean value help you. But what to do initially?

On the Gradient Tensors of First and Second Order in the Three-Dimensional Space

Chunfang Cui

A set of general expressions of the components of the gradient-tensor of second order with respect to an arbitrary orthogonal curvilinear co-ordinate system has been derived directly using the co-variant derivatives of a co-variant tensor of first rank and using the orthogonality condition.

These expressions should be called the "Borg-formulas" because they have already been, in a rather different form, given by Borg over 30 years ago [1] (It seems, however, the Borg-formulas remain still unknown in the field of Geodesy).

Applying to the gravity potential, then the gradient-tensor of first order describes the gravitational force and that of second order describes the spatial variation of the force (gravity gradiometer tensor). For the research of the gravitation field of the earth we give two sets of special expressions of the gravity gradiometer tensor with respect

1. to a 3-dimensional elliptical co-ordinate system and
2. to the Gaussian orbital co-ordinate system.

The first set can be used for a series expansion of the geopotential which gives, as known, a better fit then the series expansion over the spherical functions [2].

By the second set, the components of the gravity gradiometer tensor are related directly with the orbital variables of a satellite. They provide observation equations for the satellite gravity gradiometer measurement.

References:

[1] Borg, S. F., 1963: "Matrix-Tensor Methods in Continuum Mechanics", A. Van Nostraud Comp., Princeton
[2] Heiskanen, W. A. and Moritz, H., 1967: "Physical Geodesy",W. H. Freeman, San Francisco

Adaptive Wavelet Schemes for Elliptic Operator Equations: Analysis and Recent Results

Stefan Dahlke

We are concerned with the numerical treatment of operator equations $Au = F$, where A is a boundedly invertible linear operator. Especially, we are interested in adaptive numerical schemes based on wavelets.

It is well–known that the order of convergence that can be achieved by such adaptive methods depends on the regularity of the exact solution u in the specific scale $B^s_\tau(L_\tau(\Omega)),\ 1/\tau = s/d + 1/p$, of Besov spaces. Therefore we first present some regularity results for some classical model problems. It turns out that the Besov regularity is indeed high enough to justify the use of adaptive schemes. Then we discuss the practical realization of adaptive algorithms. We derive reliable and efficient a posteriori error estimators based on stable multiscale bases. These error estimators lead to adaptive space refinement techniques which are guaranteed to converge in a wide range of cases, including operators of negative order. We also present some numerical experiments, especially for the Poisson equation in an L–shaped domain.

Multiscale Modelling of Geopotentials by Harmonic Wavelets Using Tree Algorithms and Fast Summation Methods

Oliver Glockner

Nowadays, measurements of the earth's gravitational potential are acquired at different altitudes (e.g. terrestrial, air-borne, space-borne) and are of different type (e.g. satellite altimetry, satellite-to-satellite tracking, satellite gravity gradiometry). Since a huge amount of data is available one has to think about the development of powerful numerical algorithms for processing and interpreting the data. In this context it is very helpful to assume the class of approximating functions to constitute a (Sobolev-like) Hilbert space consisting of functions that are harmonic in the space outside an internal (Bjerhammar) sphere Ω_R and therefore in the outer space of the earth's surface Σ, too. In the framework of the Sobolev space $\mathcal{H}$ each observable can be considered as a bounded linear functional of the object function, i.e. the earth's gravitational potential.

Obviously approximate formulae have to be formulated in dependence of the required spatial resolution, since increasing space localization demands increasing data material. A multiscale technique like harmonic wavelet approximation automatically adapts the basis functions from level to level to the required resolution in space domain (zooming in effect). We are led to the discretization of $\mathcal{H}$-convolutions. It turns out that it is possible to obtain coefficients which provide us with an approximation of such convolutions, from coefficients of the finest level, by recursion (pyramid scheme) without going back to the original signal. Such pyramid schemata can be formulated as an exact bandlimited variant and as non-bandlimited variant. The recursion steps as well as the computations of the approximations lead to linear combinations in terms of the kernel functions. In order to increase the performance of the pyramid scheme, especially in the seriously space localizing non-bandlimited case fast summation methods are efficiently applicable. Due to the space localizing character of the kernel functions we propose to explicitly calculate the most influencing part, i.e. the contribution of the vicinity of the target point (near-field), whereas to use a fast approximation for the remaining part (far-field).

Geodetic Problems in Geodynamic Applications

Erwin Groten

Repeat GPS measurements have been used together wity repeat levelling data over an intervall of about 4 years at a large viaduct close to Istanbul in order to investigate the stability of the bridge system. Results of [mm]-accuracy have been obtained. This example has been used to explain, in general, the geodetic problem inherent in modern geodynamics where globally, regionally and locally deformation at the earth's surface are being investigated. One fundamental, partially still unresolved, question is the Datum problem which is particularly influential in case of relative observations as usually applied in geodesy. Related problems of observations (associated precise and specific reference system) as well as of data processing (different types of adjustments in terms of free, classical etc. LQ-techniques) are considered. Also data combination when referred to celestial (GPS etc.) and terrestial reference frames, taken over longer intervals of time, is crucial in geodynamics and still deserve further consideration in view of increased accuracy of modern geodetic techniques.

Application of Variational Methods to Geodetic Boundary Value Problem

Petr Holota

The purpose of this paper is to discuss the use of variational methods in the solution of the fundamtental geodetic boundary value problem associated with the determination of the external gravity field and figure of the Earth. For illustration the famous Stokes problem was approached first and at the same time some apriori estimates were obtained for the disturbing potential and the total horizontal component of its gradient. Then the non-spherical case represented by an oblique derivative boundary value problem was treated. The paper contains a detailed discussion, related to the construction of the bilinear form connected with the problem under consideration and subsequently gives an interpretation of the method in terms of function bases. In paricular the elements of the matrix of the respective Galerkin system were computed and also the accuracy of their approximate representation was estimated.

Reference:

P. Holota: Variational Methods in Geoid Determination and Function Bases, Phys. Chem. Earth (A), Vol. 24, No. 1 pp. 3-14, 1999

A Geometric Analysis of the Linear Gauß-Markov-Model via Plücker-Coordinates

Ronald Jurisch, Georg Kampmann

From a mathematical point of view the problem is the construction of orthogonal projection onto a subspace of $\mathbb{R}^n$. The projections can be constructed, if we choose a base in the subspace. And so there arises the question, what is the "best" base? In our sense, the best "base" should have the following properties:

1. The geometry of the subspace and its orthogonal complement in $\mathbb{R}^n$ should be classified at the same time.

2. The construction of the projection should be able in a simple and obviously way.

In the literature there are various possibilities to choose a base in a special way, for instance the Q-R-decomposition (orthonormal base). But they don't fulfill both properties at the same time. The answer can be found in the field of algebraic geometry. There are studied so-called Plücker-coordinates. These are homogeneous coordinates, which represent the subspace and also the orthogonal complement in an unique way. A geometric analysis for these subspaces can be done in an excellent way. The projections can be constructed as rational function of the Plücker-coordinates.

Regional Gravity Field Recovery from future SST/SGG–Missions using Multi–Scale Approximation

Jürgen Kusche

Description of the problem

The task of the envisaged SST/SGG–missions like CHAMP, SAGE, GRACE, and GOCE is the computation of a global gravitational field with high resolution and precision and – if possible – with repetition in time. Global approaches are aimed at the recovery of spherical harmonic coefficients. The spherical harmonics provide a natural decomposition of the field, but they do not possess any localizing properties in space. Therefore it is not possible to focuse the analysis of satellite mission data to regions of special interest, i.e. for the study of time-dependent phenomena or of the polar regions. On the other hand, regional approaches have been developed to recover gravity anomalies, point masses or spherical spline coeffients. They provide good space-localization, but lack in scale decomposition properties. This means, phenomena on different length scales are not clearly separable.

Approach under investigation
To overcome these difficulties, wavelet and filtering techniques have been proposed recently. Earlier approaches deal with multiple layers of point masses, buried at different depth. In the approach under investigation, wavelet and filtered copies of base functions are combined to form a multi-scale version of the classical least–squares approximation in a Hilbert space. SST and SGG data are considered as a function in time (time–wise approach), and the original observation equations are solved. Special emphasis is laid on the topic of regularization, which is always a crucial point in regional approaches. It is expected that different regularization parameters on different scales may be intruduced with benefit.

Open questions and special topics
Some open questions are related to the following topics (among many other unsolved problems):

- How to "design" a base function system suitable to different SST– and/or SGG–missions
- Interrelation between the various regularizing steps in the analysis: Presmoothing of the data, projection onto subspace, Tykhonov–regularization.
- Choice of multiple regularization parameters
- How to extend the technique on large areas and for global application

Applications and linked projects
At *Geodätische Woche 98*, first results from a simulation experiment on regional gravity field recovery were presented. A two-scale approximation scheme was constructed suitable to a low–low SST gravity field mission and a simulation run using GRACE mission parameters was performed – orbit and data simulation, noise generation, recovery of approximation coefficients and computation of mean gravity anomalies.

It is planned to test the chosen strategy also for polar gravity field determination from non–polar satellite gradiometry. The author is involved in *Workpackage 5: Polar Gap Problem* of the ESA GOCE study *From Eötvös to mGal*, which is currently under preparation by eight european research institutions and managed by the TU Graz. Finally it is worth to mention that scientific work at the approach under consideration is funded by the Deutsche Forschungsgemeinschaft.

References
[1] Kusche, J.: *Regional adaptive Schwerefeldmodellierung für SST–Analysen*, in Freeden, W.: Progress in Geodetic Science at GW 98, Aachen 1998
[2] Kusche, J., Ilk, K.H., Rudolph, S.: *Two–step data analysis for future satellite gravity field solutions: A simulation study*, presented at the 2^nd Joint Meeting of IGC/IGeC at Trieste, 7.–12.9.98, submitted to Bolletino di Geofisica Teoretica ed Applicata
[3]Kusche, J., Ilk, K.H., Rudolph, S.: *Impact of terrestrial data on future satellite gravity field solutions*, poster presented at IGGOS, IAG section II Symposium at Munich, 5.–9.10.98, to appear in IGGOS proceedings
[4] Kusche, J., Rudolph, S.: *Global Gravity Field Recovery from GPS/SST using Regional Focusing Techniques*, in preparation

An Application of Spherical Vectorial Wavelets to the Extraction of Poloidal and Toroidal Vector Fields From MAGSAT Data

Thorsten Maier

The standard technique of geomagnetic field modelling is known as *Gauss-Representation*, i.e. the spherical harmonic expansion of a scalar geomagnetic potential. The expansion coefficients are chosen in a way, that the gradient of the potential fits - in the sense of the $\mathcal{L}^2$-metric - the given vectorial data as good as possible. To guarantee the existence of such a geomagnetic potential, one assumes the corresponding magnetic field to be curl-free which, in connection with Maxwell's equations, means that no electric current densities must be present at the place where the measurements are taken. For Earth-bound or low-atmosphere surveys this is valid, but satellite missions, like MAGSAT or the upcoming CHAMP, acquire their data in the ionosphere where significant electric current densities can be found.

Therefore, the magnetic field, as measured by satellites, cannot be considered to be a gradient field anymore but also contains magnetic contributions from currents on the satellite's track.

From a theoretical point of view, this problem can be resolved by using the so-called *Mie-representation*, i.e. by splitting the magnetic field into *poloidal* and *toroidal* parts. The poloidal fields can be shown to be due to purely tangential current densities, while the toroidal field is created by radial current densities crossing the satellite's orbit. Those radial currents, usually referred to as *field-aligned currents*, and the corresponding magnetic effects are more and more subject of recent geophysical research.

There remains the question of how to numerically obtain the Mie-representation of a given set of vectorial data. [3] introduced a method based on the spherical harmonic analysis of scalar functions which are closely related to the poloidal and toroidal vector fields. This technique, however, involves the evaluation of spherical harmonics which, due to the polynomial character of the harmonics, is numerically disadvantageous. We will present here so-called spherical vectorial wavelets (e.g. [1]) which, completely circumventing the computation of spherical harmonics, enable us to directly model a given vectorial data set and immediately yield a decomposition into the poloidal as well as the toroidal field contributions (see e.g. [2]). Imbedded into a vectorial multi-resolution background, the wavelets show - in contrast to the spherical harmonics - strong localization properties in the space domain and therefore additionally give us the possibility of local reconstructions as well as efficient data compression.

References

[1] Bayer, M., Beth, S., Freeden, W.: (1998) 'Geophysical Field Modelling by Multiresolution Analysis', *Acta Geod. Geoph., Hung., Vol. 33(2-4), pp. 289-319*

[2] Maier, T.: (1998) 'Multiscale Analysis of the Geomagnetic Field', Diploma Thesis, Geomathematics Group, University of Kaiserslautern

[3] Olsen, N. :(1997) 'Ionospheric F Region Currents at Middle and low Latitudes Estimated From Magsat Data', *J. Geoph. Res. 102,* 4563-4576

Theoretical and Numerical Aspects of a Multiscale Method for the Gravimetry Problem

Volker Michel

Let $\overline{B_{int}} := \{x \in \mathbb{R} | \, |x| \leq \beta\}$ be the inner space and $\overline{B^R_{ext}} := \{x \in \mathbb{R} | \, \beta \leq |x| \leq R\}$ be a bounded outer space of a sphere with radius β. On $\mathcal{L}^2\left(\overline{B_{int}}\right)$ and $\mathcal{L}^2\left(\overline{B^R_{ext}}\right)$ we use the inner harmonics $\left\{H^{int}_{n,j}(\beta;\cdot)\right\}_{\substack{n \in \mathbb{N} \\ j=1,\ldots,2n+1}}$ and the outer harmonics $\left\{H^{ext}_{-n-1,j}(\beta;\cdot)\right\}_{\substack{n \in \mathbb{N} \\ j=1,\ldots,2n+1}}$, respectively, as bases for the corresponding subsets of harmonic functions.

The central theme of the talk is the class of Fredholm integral equations of first kind

$$TF := \int_{\overline{B_{int}}} k(x,\cdot) F(x) \, dx = P \tag{12}$$

with harmonic kernel, i.e.

$$k(x,y) = \sum_{n=0}^{\infty} k^{\wedge}(n) \sum_{j=1}^{2n+1} H^{int}_{n,j}(\beta;x) H^{ext}_{-n-1,j}(\beta;y), \; x \in \overline{B_{int}}, \; y \in \overline{B^R_{ext}}. \tag{13}$$

In particular, we are interested in the case $k(x,y) = 1/|x-y|$, where TF is Newton's gravitational potential of a density distribution F. Thus the inversion of T offers a possibility to determine the earth's density distribution from given gravitational data. Unfortunately, this inverse problem is ill-posed for several reasons:

- If P is non-harmonic, no solution exists. Hence, errors is measurements can cause an unsolvable problem. An exact criterion for the solvability is given in the talk.
- The solution is not unique. More precisely, only the harmonic part of the solution can be uniquely reconstructed. The elements of the $\mathcal{L}^2\left(\overline{B_{int}}\right)$-orthogonal space of $Harm\left(\overline{B_{int}}\right)$, the so–called anharmonic functions, form the null space of T. The dilemma is, that for every function in $Harm\left(\overline{B_{int}}\right)$

there exists an infinite–dimensional set of different density distributions, which cause *exactly* the same potential. Only one among these functions is the real solution. Every other function suffers from the ghost phenomenon, that is also known in computer tomography. Moreover, it has been proved in [2] that a pure harmonic reconstruction of a radially symmetric density distribution results in a constant value for the whole earth.

- Finally, if T is restricted to $Harm\left(\overline{B_{int}}\right)$, it is invertible. However, the inverse operator is not continuous. Hence, the solution is not stable, such that small errors in gravitational measurements can cause a completely different solution of (12).

The talk discusses the concept introduced in [1] and [2] for the solution of the integral equation. It consists of two parts: At first, the harmonic solution is reconstructed from gravitational data. Then an appropriate anharmonic part is determined from non-gravitational a priori informations.

Harmonic Solution: For the determination of the harmonic density bandlimited and non-bandlimited kernels are constructed, such that scaling functions and their corresponding scale spaces yield a multiresolution: The sets form a continuously increasing sequence of subsets of $Harm\left(\overline{B_{int}}\right)$ such that the union of all scale spaces is dense in $Harm\left(\overline{B_{int}}\right)$.

Wavelets and detail spaces allow transfers between different scales. This method enables a reconstruction of the harmonic projection of the solution with different space and momentum localization. At low scales a determination of the pure boundaries of the continents is possible. The higher the scale is the more local is the added information, such that finally the Amazonas area, Ayer's Rock and a series of small islands can be detected in the density anomalies of the earth's surface.

The harmonic concept disposes the problems of ill-posedness. The solution is not only unique, as only harmonic functions are considered, but a regularization, developed for arbitrary kernels (13), also enables a stable reconstruction of an arbitrarily good (unique) approximation to the harmonic part of the whole solution.

Anharmonic Solution: As $Anharm\left(\overline{B_{int}}\right)$ is the null space of T, non-gravitational data, generally represented by linear and continuous functionals $\mathcal{F}^n$, have to be used. For this purpose a categorization of the ghosts is given, such that a series of different polynomial basis systems for $Anharm\left(\overline{B_{int}}\right)$ can be derived. One of theses bases is the fundament for the construction of a new Hilbert space of continuous anharmonic functions. The reproducing kernel of the Hilbert space and the functionals $\mathcal{F}^n$ are used to define spline spaces and spline bases, such that an anharmonic spline satisfying the a priori conditions can be determined. Of course, this spline has the usual best approximating properties in its corresponding topology.

The anharmonic method also enables a well-posed reconstruction of an approximation to one component of the solution. Bandlimited and non-bandlimited kernels are available.

One of the fundamental results of [2] is that numerical methods and solution theories, that only use harmonic functions for the determination of the earth's density distribution, are worthless.

References:

[1] Michel, V. (1998) *A Wavelet Based Method for the Gravimetry Problem.* In: Progress in Geodetic Science at GW 98, (W. Freeden ed.), Shaker, 283-298.

[2] Michel, V. (1999) A Multiscale Method for the Gravimetry Problem: Theoretical and Numerical Aspects of Harmonic and Anharmonic Modelling. Doctoral Thesis (submitted), University of Kaiserslautern, Geomathematics Group.

Scattered-Data Quadrature Formulas for Spheres

Francis J. Narcowich

Often we wish to approximate integrals of the form

$$\int_{\mathbb{S}^q} f(p)d\mu(p),$$

given only values of $f \in C(\mathbb{S}^q)$ on a set of scattered points $\mathcal{C} = \{\xi \in \mathbb{S}^q\}$ on the q-sphere $\mathbb{S}^q$. To do this, we seek a quadrature formula of the form

$$Q[f] = \sum_{\xi \in \mathcal{C}} w_\xi f(\xi),$$

where w_ξ is a weight corresponding to the point ξ. Let $\mathbf{H}_L$ denote the spherical harmonics of degree L or less. We want Q to satisfy these properties:

P1. Q is exact for $\mathbf{H}_L$.

P2. The weights w_ξ are nonnegative.

P3. The number of points required from the scattered sites in $\mathcal{C}$ is comparable to the dimension of $\mathbf{H}_L$.

If Q satisfies **P1**, then, by standard arguments, the error we make in using Q is

$$\left| \int_{\mathbb{S}^q} f(p) d\mu(p) - Q[f] \right| \leq \left(\mu(\mathbb{S}^q) + \sum_{\xi \in \mathcal{C}} |w_\xi| \right) \operatorname{dist}_{L^\infty}(f, \mathbf{H}_L) \,.$$

If in addition Q satisfies **P2**, so that $\sum_{\xi \in \mathcal{C}} |w_\xi| = \sum w_\xi = Q[1] = \mu(\mathbb{S}^q)$, then

$$\left| \int_{\mathbb{S}^q} f(p) d\mu(p) - Q[f] \right| \leq 2\mu(\mathbb{S}^q) \operatorname{dist}_{L^\infty}(f, \mathbf{H}_L) \,.$$

The point is that if some of the weights *are* negative, then $\sum_{\xi \in \mathcal{C}} |w_\xi| > Q[1] = \mu(\mathbb{S}^q)$, and the sum $\sum |w_\xi|$ must be controlled separately to ensure stability. Finally, counting equations and parameters, the number of weights is at least the dimension of $\mathbf{H}_L$. Thus **P3** amounts to requiring a nearly optimal number of weights.

Recent work dealing with quadrature formulas for spheres has been done by several reseachers: Driscoll & Healy [Adv. in Appl. Math., **15** (1994), 202-250]; Jetter, Stöckler & Ward [pp. 237-245 in "Computational Mathematics," (Chen, Li, C. Micchelli, Y. Xu, eds.), Marcel Decker, New York, 1998]; Petrushev [SIAM J. Math. Anal., **30** (1998), 155-189]; and Potts, Steidl & Tasche [Math. Comp., **67** (1998), 1577-1590].

The quadrature formulas developed in the papers listed above either put restrictions on $\mathcal{C}$, so that the sites are not truly scattered, or use weights that are of uncertain sign. In this talk, we will discuss quadrature formulas that satisfy **P1-P3**. These formulas were recently developed by Mhaskar, Narcowich & Ward ["Quadrature Formulas on Spheres Using Scattered Data," Center for Approximation Theory Report # 393, Department of Mathematics, Texas A & M University, 1998].

Variational Inequalities, Bounded Variation Regularization and Inverse Source Problems

M. Zuhair Nashed

We first provide several examples of nonlinear ill-posed problems with smooth and nonsmooth operators, and sketch an overview of various approaches to the regularization-approximation of such problems.

The main part of the talk (which is based on joint work with Otmar Scherzer) is to discuss results for bounded variation solutions of nondifferentiable ill-posed problems. A general method is described for obtaining stable approximate solutions for a class of minimization problems for which approximate minimizers can be characterized as solutions of variational inequalities. The functional to be minimized is not assumed to be differentiable, and the minimizers need not satisfy a variational inequality. An application to inverse source problems is considered in detail; convergence and stability are established as a special realization of the theory developed for the general method.

We also consider least-squares regularization methods for ill-posed problems $Af = g$, where A is an operator from a real Banach space into a real Hilbert space using nondifferentiable penalty functionals (as in the case of bounded variation regularization). We show that our results provide a framework for a rigorous analysis of numerical methods based on appropriate Euler-Lagrange equations. This justifies many of the numerical implementation schemes of bounded variation regularization that have been recently proposed in the literature (see [1] and [2] for details of the results and precise formulation of the general setting). Reference [3] is not related to our talk but it deals with regularization of ill-posed variational inequalities for inverse-monotone nonlinear operators.

References:

[1] M. Zuhair Nashed and Otmar Scherzer, Stable approximation of nondifferentiable optimization problems with variational inequalitites, *Contemporary Mathematics*, vol. 204 (1997), 155-170, American Mathematical Society, Providence, RI.

[2] M. Z. Nashed and O. Scherzer, Least squares and bounded variation regularization with nondifferentiable functionals, *Numer. Funct. Anal. Optimiz.*, 19 (1998), 873-901.

[3] F. Liu and M. Z. Nashed, Regularization of nonlinear ill-posed variational inequalities and convergence rates, *Set-Valued Analysis*, 6 (1998)

Preconditioners for Ill–conditioned Toeplitz Systems Constructed from Positive Kernels

Daniel Potts

We are concerned with the iterative solution of "mildly" ill–conditioned Toeplitz systems

$$\boldsymbol{A}_N \boldsymbol{x} = \boldsymbol{b}\,,$$

where $\boldsymbol{A}_N \in \mathbb{C}^{N,N}$ are positive definite Hermitian Toeplitz matrices arising from a continuous non–negative generating function f which has only a finite number of zeros. Often these systems are obtained by discretization of a continuous problem (partial differential equation, integral equation with weakly singular kernel) and the dimension N is related to the grid parameter of the discretization.

Iterative solution methods for Toeplitz systems, in particular the conjugate gradient method (CG–method), have attained much attention during the last years. The reason for this is that the essential computational effort per iteration step, namely the multiplication of a vector with the Toeplitz matrix $\boldsymbol{A}_N$, can be reduced to $\mathcal{O}(N \log N)$ arithmetical operations by fast Fourier transforms (FFT). However, the number of iteration steps depends on the distribution of the eigenvalues of $\boldsymbol{A}_N$. If we allow the generating function f to have zeros, then the related Toeplitz matrices are asymptotically ill–conditioned for $N \to \infty$ and the CG–method converges very slow. Therefore, the really task consists in the construction of suitable preconditioners $\boldsymbol{M}_N$ of $\boldsymbol{A}_N$.

In literature three kinds of preconditioners were mainly exploited, namely band Toeplitz preconditioners, preconditioners based on multigrid methods and preconditioners arising from a matrix algebra $\mathcal{A}_{O_N} := \{\bar{\boldsymbol{O}}'_N(\operatorname{diag}\boldsymbol{d})\boldsymbol{O}_N : \boldsymbol{d} \in \mathbb{C}^N\}$, where $\boldsymbol{O}_N$ denotes a unitary matrix.

For band Toeplitz preconditioners it was proved that the corresponding PCG–methods converge in a number of iteration steps independent of N [1, 2]. Moreover, it is possible to construct superlinear preconditioners. However, there is the significant constraint that the cost per iteration of the proposed procedure should be upper-bounded by $\mathcal{O}(N \log N)$. This implies some conditions on the growth of the bandwidth of the band Toeplitz preconditioners.

The above constraint is trivially fulfilled if we chose preconditioners from matrix algebras, where the unitary matrix $\boldsymbol{O}_N$ has to allow an efficient multiplication with a vector in $\mathcal{O}(N \log N)$ arithmetical operations. Up to now, the only preconditioners of the matrix algebra class which ensure the desired convergence of the corresponding PCG–method are the preconditioners proposed in [3, 4]. Unfortunately, the construction of these preconditioners requires the explicit knowledge of the generating function f.

In this paper, we combine our approach in [4] with the approximation of f by its convolution with a reproducing kernel K_N. The kernel approach was originally given in [5] for positive generating functions. The advantage of the kernel approach is that it does not require the explicit knowledge of the generating function. We restrict our attention to positive kernels. This ensures that our preconditioners are positive definite. Suppose that f has only zeros of even order $\leq 2s$. Then we prove that the condition

$$\int_{-\pi}^{\pi} t^{2k} K_N(t)\,\mathrm{d}t \leq C N^{-2k} \quad (k = 1, \ldots, s)$$

on the kernels K_N is necessary and sufficient to ensure that the eigenvalues of $\boldsymbol{M}_N^{-1}\boldsymbol{A}_N$ are contained in some interval $[a,b]$ $(0 < a \leq b < \infty)$ except for a fixed number (independent of N) of eigenvalues falling into $[b,\infty)$ such that the number of PCG–steps to achieve a fixed precision is independent of N. Typical kernels fulfulling the above property for arbitrary $N \in \boldsymbol{N}$ are R. Chan's B–spline kernels and generalized Jackson kernels.

References:
[1] F. Di Benedetto, G. Fiorentino, and S. Serra. C.G. preconditioning for Toeplitz matrices. *Comp. Math. Appl.*, 25:35 – 45, 1993.
[2] R. H. Chan. Toeplitz preconditioners for Toeplitz systems with nonnegative generating functions. IMA *J. Numer. Anal.*, 11:333 – 345, 1991.
[3] D. Potts and G. Steidl. Preconditioners for ill–conditioned Toeplitz matrices. *BIT*, in print.
[4] T. Huckle. Iterative methods for ill–conditioned Toeplitz matrices. Preprint 1997.
[5] R. H. Chan and M.-C. Yeung. Circulant preconditioners constructed from kernels. SIAM *J. Numer. Anal.*, 29:1093 – 1103, 1992.

Joint work with Gabriele Steidl, University of Mannheim.

Accuracy vs Speed

Richard Reuter

Solving challenging problems in the area of geosciences very often leads to computer programs which are very time consuming, where a considerable amount of time is spent in the evaluation of intrinsic functions like sin, cos, exp, and so on. In order to speedup those programs all parts have to be investigated for performance improvements. This leads to the question if the standard intrinsic functions library can be replaced by an alternative one which performs faster. But what is the price to pay for that? How much is lost in accuracy? In my presentation the accuracy and speed of the double precision (64 bits) functions of the standard intrinsics library “libm“ of the IBM AIX compiler family (xlf, xlc, ...) for the RS/6000 workstations is compared to corresponding functions in the alternative libraries “lmass“ and “lmassv“. “lmass“ is ment for scalar arguments, whereas “lmassv“ accepts vector arguments. MASS is an abbreviation for “Mathematical Acceleration SubSystem“. It turns out that most of the functions in “libm“ return the correctly rounded results and that the functions of “lmass/lmassv“ are considerably faster (up to a factor of 9 for the exp-function on a RS/6000 Mod. 590 workstation) and they are loosing in general at most 1 bit in accuracy. The technique of investigating the accuracy is based on the ulp-concept in [1].

References:
[1] R. C. Agarwal, J. W. Cooley, F. G. Gustavson, J. B. Shearer, G. Slishman, B. Tuckerman: New scalar and vector elementary functions for the IBM System/370, IBM J. Res. Develop. Vol. 30 No. 2, 1986

Synthetic Modelling of Earth Rotation

Burghard Richter

In the synthetic approach of describing the rotation of the earth, we do not try to solve the dynamical Euler/Liouville differential equation, but we start directly from the rotation matrix which connects an earth-fixed reference system with a space-fixed one and which can be modelled in such a way that the resulting rotation vector behaves as expected. The diurnal rotation matrix about the celestial ephemeris pole through the Greenwich sidereal time is extended to one side by the polar motion matrix and to the other side by the precession and the nutation matrices. These matrices, which have only long-periodic variations, describe the direction of the celestial ephemeris pole with respect to both reference systems.

The rotation vector can be obtained in both systems by differentiating the rotation matrix with respect to time. By multiplying it with the earth's tensor of inertia and adding the earth's angular momentum with respect to the earth-fixed system, which both are fairly well known in the earth-fixed system, one gets the angular momentum with respect to the space-fixed system. Its time derivative in the space-fixed system is eventually the torque, which can be compared with the actual torque exerted upon the earth by the moon and the sun. By variation of the diurnal rotation, the polar motion, the precession and nutation matrices, of the inertia tensor, the angular momentum of the earth with respect to the earth-fixed system and of the external torque, one can investigate the interdependences of these factors.

Assuming a rigid and axially symmetric earth, simple examples are presented for a rotation model with a free regular polar motion, for a rotation model with a forced regular precession, and for a combination of these two models.

Conformally and Weyl Invariant Field Theorie

Hartmann Römer

Die Vorstellung der Umskalierung des Raum-Zeitmaßstabes lässt sich in zweifacher Weise formalisieren: erstens als Weyltransformationen der Metrik oder als konforme Transformationen auf der (pseudo) riemannschen Raum-Zeitmannigfaltigkeit. In dem (auf Englisch gehaltenen) Vortrag wurden folgende Themen behandelt:

I. Konforme Transformationen und Weyl-Transformationen

1. Diffeomorphismen
2. Weyl-Transformationen, Isometrien, konforme Transformationen
3. (Konforme) Killingvektorfelder
4. Konforme Gruppe des flachen Raumes für Dimension $D > 2$ und $D = 2$
5. Konforme Kompaktifizierung

II. Konform- und Weyl-invariante Feldtheorien

1. $D = 4$: Teilchenphysik
2. $D = 2$: Statische Mechanik thermodynamischer Systeme am kritischen Punkt in zwei Dimensionen

III. Gravitationstheorie und Weyl-Invarianz

1. Weyl-Tensor
2. "'Dilatonen"
3. Stringtheorie

The Use of Slepian Functions for Local Geodetic Computations

Fausto Sacerdote

The procedure used by D. Slepian to maximize the energy of band-limited functions in a bounded interval has been generalized to a spherical surface by A. Albertella et al., in order to analyze a geodetic boundary-value problem with data lacking on polar caps.

Given a subset B of the unit sphere S, the problem of maximizing the ratio

$$\frac{\int_B f^2 d\sigma}{\int_S f^2 d\sigma} = \frac{\sum^L f_i f_j \int_B Y_i Y_j d\sigma}{\sum_L f_i^2}$$

where f is a $L^2(S)$ band-limited function (up to degree L), $\{Y_i\}$ is an orthonormal basis (typically, spherical harmonics, here, for simplicity, labelled with only one index), is reduced to the determination of eigenvalues and eigenvectors of the matrix $A_{ij} = \int_B Y_i Y_j d\sigma$. The functions obtained as combinations of the original basis functions Y_i, using as coefficients the components of the normalized eigenvectors are orthonormal on S, and, in addition, are orthogonal on B, with squared norm equal to the corresponding eigenvalue.

What happens in practice is that, if B is a spherical belt, so that the integration with respect to longitude is over an interval with amplitude 2π, the problem can be solved separately order by order, and, for a given maximum degree L and for arbitrary order, one finds a number of eigenvalues very close to 1 and a number very close to 0, roughly proportional respectively to the amplitude of the belt and of its complement. Only few eigenvalues have intermediate values. Therefore, it is possible to choose a set of functions which span a subspace whose power is essentially 0 on the belt (or outside it) and can be disregarded if only data in the corresponding region are available.

If B is a spherical rectangle with amplitude $2\pi/n$ in longitude, it is possible to extend periodically the function to the whole belt, so that only orders multiple of n are involved.

Consequently, a function defined on a rectangular region can be represented in terms of a number of basis functions that is roughly proportional to the extension of the region. Furthermore, the basis functions are defined in terms of spherical harmonics, so that harmonic extensions and, more generally, geodetic operators, can be easily applied. Yet, apparently there is no simple relation between functions corresponding to different maximum degrees, so that apparently it is not possible to obtain higher resolutions with simple procedures, similar to the ones introduced in multiresolution analysis for wavelets.

Error Estimates for Band-Limited Spherical Regularization Wavelets in an Inverse Problem of Satellite Geodesy

Eberhard Schock, Sergei Pereverzev

We consider the integral equation

$$Af(x) = \frac{1}{4\pi R} \int\limits_{\Omega_R} \frac{d^2}{dr^2} \left(\frac{r^2 - R^2}{|x-y|^3}\right) f(y) d\omega_R(y) = g(x)$$

for computing the gravitational potential f at the surface of the earth Ω_R from a measured function g at satellite amplitude r.

This problem is exponentially ill-posed. In the thesis of F. Schneider (Kaiserslautern) there are develloped numerical algorithms based on band-limited spherical wavelets. We present results on the asymptotic behaviour of the error. We show the connection between analytic properties of the solution, the rate of convergence and the choice of the regularization parameters and the parameter of band-limitation with respect to the error in the measured data.

Splines on Spherical Triangulations

Larry L. Schumaker

This talk presents an overview of the recent development of a theory of spline functions defined on the sphere. Such splines have a variety of applications in CAGD, surface approximation, scattered data fitting, and finite element solution of PDE's.

We begin with a natural way to define spherical barycentric coordinates, and introduce direct analogs of the classical Bernstein-Bézier polynomials defined on triangles (here defined on spherical triangles). We discuss a variety of properties of such SBB-polynomials, including a deCasteljau algorithm, subdivision, degree raising, smoothness conditions between spherical patches, etc. We also present several results from the constructive theory of spherical splines, including a dimension result. Finally, several practical data fitting and interpolation methods are discussed along with numerical results for some test problems.

Coordinate Systems and Observation Frames in Curved Space-Time

Volker S. Schwarze

Modern geodetic space-techniques have to be modeled within a consistent physical framework. The appropriate physical framework is the general relativity.

Starting with the space-time metric up to first post-Newtonian order a set of local charts is presented which is suitable for astronomic and geodetic use. Based on the Gram-Schmidt pseudo-orthonormalization technique it is shown how a four-leg is constructed which is pseudo-orthonormal with respect to some given metric. This is the starting point to give explicit expressions for the coordinate transformations between the charts under use as well as for deriving geodetic observation equations being consistent up to required order with general relativity.

On the Topographic Effects by Gravimetric Geoid and Quasi Geoid Determinations

Lars E. Sjöberg

Stokes integral formula is the basis for gravimetric geoid determination. It requires that 1) there are no topographic masses ($\Rightarrow$ direct topographic effect) and 2) the gravity anomaly be downward continued from the Earth's surface to the sea-level ($\Rightarrow$ effect of downward continuation). Finally, 3) after Stokes Formula has been employed, the topography is restored, yielding the so-called indirect effect on the geoid.

Traditionally geodesists are using very appropriate estimates of the above effects. Based on the assumption of constant topographic density we present surface integrals for the direct and indirect effects as functions of topographic elevation. Comparison with curvent planar approximation reveal errors reaching as much as $0.5m$ in the highest mountains, showing that traditional methods for geoid determination must be improved to reach the goal of the $10cm$ geoid or even better. In this respect we consider also the routines for determining the effect 3) above, where the basic problem is the solution of the Poisson's integral equation for the gravity anomaly without (complete) removal of topographic masses. The improved formulas are confirmed from comparisons with GPS derived geoidal modulations.

References:

[1] Sjöberg, L. E., Nahavandchi, H. (1999): The indirect effect in geoid determination by the Stokes-Helmert method, J. Geodesy, No. 1

[2] Nahavandchi, H. (1998): Division of Geodesy Report Nr. 1041, Royal Institut of Technology, Stockholm (Dissertation)

Fast Fourier Transforms for Nonequispaced Data

Gabriele Steidl

Let $\Pi^d := [-\frac{1}{2}, \frac{1}{2})^d$ and $I_N := \{k \in \boldsymbol{Z}^d : -\frac{N}{2} \leq k < \frac{N}{2}\}$, where the inequalities hold componentwise. For $x_k \in \Pi^d$, $v_j \in N\Pi^d$, and $f_k \in \boldsymbol{C}$, we are interested in the fast and robust computation of the *discrete Fourier transform for nonequispaced data* (NDFT)

$$f(v_j) = \sum_{k \in I_N} f_k \, \mathrm{e}^{-2\pi \mathrm{i} x_k v_j} \qquad (j \in I_M)\,.$$

For arbitrary nodes, the direct evaluation of the above sums takes $\mathcal{O}(N^d M^d)$ arithmetical operations, too much for practical purposes. For equispaced nodes $x_k := \frac{k}{N}$ $(k \in I_N)$ and $v_j := j$ $(j \in I_N)$, the values $f(v_j)$ can be computed by the well-known *fast Fourier transform* (FFT) with only $\mathcal{O}(N^d \log N)$ arithmetical operations. However, the FFT requires sampling on an equally spaced grid, which represents a significant limitation for many applications. For example, most geographical data are sampled at individual observation points or by fast moving measuring devises along lines. Using the ACT method (*a*daptive weight, *c*onjugate gradient acceleration, *T*oeplitz matrices) introduced by Feichtinger et al. for the approximation of functions from scattered data , one has to solve a system of linear equations with a block Toeplitz matrix as system matrix. The entries of this Toeplitz matrix are of the above form and should be computed by efficient NDFT algorithms. Further applications of the NDFT range from frequency analysis of astronomical data to modelling and imaging.

We give a unified approximative approach for the fast computation of NDFT and estimate the approximation error for various window functions.. Further, we prove another advantage of our NDFT algorithm, namely its robustness with respect to roundoff errors, a feature which is well-known from the classical FFT. Numerical tests confirm our theoretical expectations.

Future Satellite Gravity Missions and their Impact on Studying the Earth's Interior

Markus Thalhammer

1. Motivation

As soon as one of the currently planned satellite gravity missions (GRACE, GOCE) will have successfully been realized a gravity field model becomes available that reveals unprecedented features with respect to accuracy, resolution and homogeneity.

Among other topics this will enable the geophysical community to get ahead in the recovery of the internal structure and composition of the earth. Because of the basic nonuniqueness of this problem gravity field data have to be combined with other geophysical information. Especially the progress in seismic tomography which led to global, highly resolved datasets of seismic parameters promises a significant step towards this aim.

For a correspondingly accurate interpretation of the new data one has to develop a theory of the inverse gravity problem that allows to introduce such additional geophysical information in a flexible manner.

2. Inverse gravitational problem (IGP)

The new data will describe the gravitational potential V in the exterior $C\Omega$ of the earth Ω in terms of its spherical harmonics expansion up to a certain degree and order : L : $V^{ext} = \sum_{l,m}^{L} \nu_{lm} Y_{lm}$. The aim of the IGP is to infer the generating mass distribution $\rho \in L_2$, supp $\rho = \Omega \subset \mathbb{R}^3, \rho \geq 0$, which is linearly related to that observable by the Newton-operator P, $V^{ext}(x) = G\int_\Omega \rho(y) l^{-1}(x,y) d\Omega(y) =: P\rho$, as a special solution of the underlying Poisson-differential equation, $\Delta V(x) = -4\pi G\rho(x)$.

It is well known that this integral equation of the first kind is ill posed because of nonuniqueness which can generally be shown by Green's theorem. The Newton operator P leads to an orthogonal decomposition, $L_2(\Omega) = N(P) \oplus N(P)^\perp$, where $N(P)^\perp$ covers the so-called harmonic densities ρ_h and $N(P)$ the anharmonic densities ρ_a. This orthogonal decomposition of ρ with respect to P, $\rho = \rho_h + \rho_a$, leads to the fact that ρ_h is uniquely determined by the given potential coefficients ν_{lm} whereas the anharmonic part ρ_a remains completely hidden to V^{ext} or any measured functional thereof.

If one restricts to a spherical shape of the boundary, Σ_R, that completely includes Ω, then a three-dimensional orthonormal basis for $\rho \in L_2(\Sigma_R)$ can simply be established: $\rho(r,\vartheta,\lambda) = \sum_{l,m,k} \rho_{lmk} D_{lmk}$ where the basis functions read as $D_{lmk}(r,\vartheta,\lambda) := (r/R)^l P_k(r^2;l) Y_{lm}(\vartheta,\lambda)$ ($P_k(r^2;l)$: Jacobi polynomials). The coefficients of ρ_h ($k=0$) are then given to $\rho_{lm0} = \overline{\rho}(2l+1)\sqrt{(2l+3)/3}\nu_{lm}$ ($\overline{\rho} = \rho_{000}$: mean density of the earth) which then leads to the harmonic series representation

$$\rho_h = \overline{\rho} \sum_{l=0}^{L_{max}} \sum_{-l}^{l} [(2l+1)(2l+3)/3](r/R)^l \nu_{lm} Y_{lm}.$$

Thereof the harmonic densities of currently existing gravity models such as OSUxx, EGM96, etc can simply be calculated, but such a density distribution is purely artificial, i.e. has no immediate physical meaning. If one works on a global basis in this spectral domain the information content of external gravity about the internal mass distribution has been completely exploited. Any information about the anharmonic part ρ_a must arise from other geophysical observables.

3. A solution strategy of the combined IGP in terms of extremal measures

As the pure IGP leaves one with the total ignorance of the nullspace $N(P)$, it would be desirable to find a proper description of this space that allows to introduce any further (geophysical) information and to calculate the resulting change of its structure. The concept of measure seems to be a well-suited tool to approach this goal. For the space $M^+(\overline{\Omega})$ of non-negative RADON-measures μ one has to investigate the set $B(\mu) = \{\eta \in M^+(\overline{\Omega}) : \int h(x)d\eta(x) = \int h(x)d\mu(x), \forall h$ harmonic in $\Omega\}$. $B(\mu)$ is a weakly compact convex set and according to a theorem of Krein & Mil'man each measure $\eta \in B(\mu)$ can be composed as a linear (convex) combination of extremal elements of $B(\mu)$, $ex\, B(\mu)$.

For the most simple case of radialsymmetric density distributions, i.e. only ν_{00} be given, a general construction scheme can be developed, which allows to include any preliminary information about $\rho = \rho(r)$ and to map this knowledge into the spectral domain, i.e. into the coefficients $\rho_{lmk}, k \neq 0$, as well.

4. Sensitivity analysis and preliminary inversion results

From a practical point of view any numerical solution of the IGP has to take into account the quality of the data, i.e. its accuracy and resolution. For the gravity data these informations can be taken from simulation results of the aforementioned satellite missions in terms of the maximum degree L of the expansion together with the accuracies of the series coefficients, $\sigma(\nu_{lm})$.

Therefore a forward computation scheme has been developed that calculates the spherical harmonics expansion, i.e. coefficients $\delta\nu_{lm}$, of a prescribed mass distribution $\delta\rho(r, \vartheta, \lambda)$ inside the earth. A comparison with the accuracy spectrum $\sigma(\nu_{lm})$ then allows to separate the spectrum $\delta\nu_{lm}$ into a part $\delta\nu_{lm}$ the satellite measurements are sensitive to and its complement that cannot be recovered at all.

If applied to single mass anomalies of various extensions, depths and density contrasts these calculations give valuable hints to a proper choice of the discretization in the following inversion procedure.

Additionally it is also possible to separate these anomalous masses into their harmonic and anharmonic parts. This has also been performed for more complicated mass configurations, e.g. various isostasy models and subduction scenarios.

As to the numerical solution of the combined IGP itself a simulation software has been developed that uses a modification of classical linear programming for the inversion step. The current version uses a simplified parametrization in the space domain (volumetric spherical blocks) where any additional geophysical information is simulated as knowledge of the density within such a block together with a certain accuracy measure (variance).

References

[1] Anger, G.: A characterization of the inverse gravimetric source problem through extremal measures. Reviews of Geophysics and Space Physics, vol. 19, no. 2, pp.299-306, 1981.
[2] Schulze, B.; Wildenhain, G.: Methoden der Potentialtheorie für elliptische Differentialgleichungen beliebiger Ordnung. Birkhäuser, Basel, 1977.
[3] Kounchev, O.: Zeros of non-negative sub-biharmonic functions and extremal problems in the inverse source problem for the biharmonic potential. In "Inverse problems: principles and applications in geophysics, technology and medicine", edited by G. Anger et al., Akademie Verlag, Berlin, pp. 243-250, 1993.

Viscoelastic Models of Deformation and Gravity Change

Detlef Wolf, Guoying Li, Zdeněk Martinec and Malte Thoma

Overview

The causal description of long-period deformations of the earth is commonly based on the field theory of *gravito-viscoelastodynamics*. In Section 1.4: *System Theory and Modelling* of the GeoForschungsZentrum Potsdam, one research group applies this theory to study deformations and gravity changes caused by glacial loading. Besides the interpretation of data related to glacial–isostatic adjustment, the research group is also concerned with the derivation of new solutions to the *incremental field equations* of gravito-viscoelastodynamics for more realistic earth models. Problems of current interest are here

- Deriving analytical solutions for initially hydrostatic *compressible* earth models
- Deriving analytical/numerical solutions for *laterally heterogeneous* incompressible earth models

Compressibility
The incremental field equations of gravito-viscoelastodynamics (*e.g.* Wolf, 1991a, 1997) used to describe long-period deformations of the earth are conventionally solved in a simplified form valid on the assumptions of Maxwell viscoelasticity and incompressibility (*e.g.* Wolf, 1991b; Amelung & Wolf, 1994). Whereas the generalization of the solutions for arbitrary types of linear viscoelasticity does not pose serious problems (*e.g.* Wolf, 1994, 1997; Rümpker & Wolf, 1996; Wieczerkowski, 1999), the derivation of solutions for compressible viscoelasticity is an area of current theoretical research.

Previous analytical approaches to compressible viscoelasticity have generally assumed that the earth is composed of a sequence of radially symmetric *homogeneous shells*. Analytical solutions to the incremental field equations for this type of viscoelastic earth models have been given by Wu & Peltier (1982), Wolf (1985), Vermeersen *et al.* (1996), Hanyk *et al.* (1999) and Wieczerkowski (1999). Corresponding solutions for elastic earth models were first derived by Love (1911) and later in more general form by Gilbert & Backus (1968) and Martinec (1984).

The *physical* stability of compressible elastic earth models consisting of homogeneous spherical shells has already been questioned by Love (1911); the studies by Plag & Jüttner (1995), Hanyk *et al.* (1999) and Wieczerkowski (1999) have extended the investigations of stability to viscoelastic earth models. The results of these studies show that, for particular parameter values and deformation wavelengths, the solutions for both elastic and viscoelastic earth models become singular. Physically, these singularities are related to the assumption of shells of homogeneous density: Since the radial distribution of the density in the hydrostatic initial state must be consistent with the assumption of compressibility, a shell of homogeneous density implies that, with the effect of self-compression removed, the density of the material decreases with depth, which, in turn, means that the initial state is inherently unstable.

In order to model the influence of compressibility on viscoelastic perturbations correctly, the following is required:

- Calculation of the radial distributions of density, pressure and gravity for a hydrostatic initial state in consistency with the assumption of compressibility
- Consideration of this hydrostatic initial state when deriving solutions for the compressible incremental state

The problem has first been solved in simplified form for a gravitationally decoupled and compositionally homogeneous plane earth model (Wolf & Kaufmann, 1999). More recently, the analytic solution has been derived for a gravitationally coupled spherical earth model consisting of a compositionally homogeneous viscoelastic mantle and a fluid core (Li & Wolf, 1999). At present, this solution is extended to a spherical earth model consisting of an arbitrary number of compositionally homogeneous viscoelastic shells. The generalized solution is based on the propagator-matrix method and provides analytic expressions for the matrix elements (Thoma, in preparation).

Lateral heterogeneity
The derivation of solutions to the field equations of gravito-viscoelastodynamics for laterally heterogeneous earth models is complicated by the fact that variations of viscosity in the lateral direction may reach two orders of magnitude. If the variations are assumed to be restricted to a factor of, say, five at the most, *first-order perturbation theory* may be used to obtain analytical expressions. This approach has been employed to obtain computational results for a gravitationally decoupled and compositionally homogeneous plane earth model (Kaufmann & Wolf, 1999).

In the general case of arbitrarily large variations of viscosity in the lateral direction, nearly all results obtained so far have been based on commercial *finite-element codes* (*e.g.* Kaufmann *et al.,* 1997). These codes were originally developed for engineering applications and, as such, usually assume gravitational decoupling, plane geometry and a stress-free initial state. The consequence of the last assumption is that finite-element earth models are unstable with respect to surface loading. In order to avoid these instabilities, the finite-element codes must be adapted to the case of a hydrostatic initial state. This modification introduces buoyancy forces into the viscoelastic incremental state, which stabilize the response. The stabilization is usually achieved by *ad hoc* modifications of the boundary conditions, an approach which is strictly justified only under special circumstances.

A more promising approach is the derivation of solutions for laterally heterogeneous earth models which are gravitationally coupled, spherical and in a hydrostatic initial state. In order to test the accuracy of these solutions, the exact solutions for simple types of lateral heterogeneity are required. For this purpose, the *semi-analytical solution* for a simple 2-D spherical earth model consisting of two eccentrically nested spheres has recently been derived (Martinec & Wolf, 1999). As the next step, the development of general codes based on the *spectral finite-difference scheme* and valid for arbitrary 2-D or 3-D spherical earth models is being initiated.

References:

[1] Amelung, F. & Wolf, D. (1994) Viscoelastic perturbations of the earth: significance of the incremental gravitational force in models of glacial isostasy, *Geophys. J. Int.,* **117,** 864–879.
[2] Gilbert, F. & Backus, G. (1968) Elastic-gravitational vibrations of a radially stratified sphere, in Hermann, G., ed., *Dynamics of Stratified Solids,* American Society of Mechanical Engineers, New York.
[3] Hanyk, L., Matyska, C. & Yuen, D.A. (1999) Secular gravitational instability of a compressible viscoelastic sphere, *Geophys. Res. Lett.,* in press.
[4] Kaufmann, G. & Wolf, D., (1999) Effects of lateral viscosity variations on postglacial rebound: an analytical approach, *Geophys. J. Int.,* in press.
[5] Kaufmann, G., Wu, P. & Wolf, D., (1997) Some effects of lateral heterogeneities in the upper mantle on postglacial land uplift close to continental margins, *Geophys. J. Int.,* **128,** 175–187.
[6] Li, G. & Wolf, D. (1999) Load-induced gravitational–viscoelastic relaxation of a spherical earth with compressional density stratification, *Geophys. J. Int.,* to be submitted.
[7]Love, A.E.H. (1911) *Some Problems of Geodynamics,* Cambridge University Press, Cambridge.
[8]Martinec, Z. (1984) Free oscillations of the earth *Trav. Géophys.,* **591,** 117–236.
[9] Martinec, Z. & Wolf, D. (1999) Gravitational viscoelastic relaxation of eccentrically nested spheres, *Geophys. J. Int.,* in press.
[10] Plag, H.-P. & Jüttner, H.U. (1995) Rayleigh–Taylor instabilities of a self-gravitating earth, *J. Geodyn.,* **20,** 267–288.
[11] Rümpker, G. & Wolf, D. (1996) Viscoelastic relaxation of a Burgers half space: implications for the interpretation of the Fennoscandian uplift, *Geophys. J. Int.,* **124,** 541–555.
[12] Vermeersen, L.L.A., Sabadini, R. & Spada, G. (1996) Compressible rotational deformation, *Geophys. J. Int.,* **126,** 735–761.
[13] Wieczerkowski, K. (1999) Gravito-Viskoelastodynamik für verallgemeinerte Rheologien mit Anwendungen auf den Jupitermond Io und die Erde, *Ph.D. Thesis,* University of Münster.
[14] Wolf, D. (1985) The normal modes of a uniform, compressible Maxwell half-space, *J. Geophys.,* **56,** 100–105.
[15] Wolf, D. (1991a) Viscoelastodynamics of a stratified, compressible planet: incremental field equations and short- and long-time asymptotes, *Geophys. J. Int.,* **104,** 401–417.
[16[Wolf, D. (1991b) Boussinesq's problem of viscoelasticity, *Terra Nova,* **3,** 401–407.
[17] Wolf, D. (1994) Lamé's problem of gravitational viscoelasticity: the isochemical, incompressible planet, *Geophys. J. Int.,* **116,** 321–348.
[18] Wolf, D. (1997) Gravitational viscoelastodynamics for a hydrostatic planet, *Veröff. Deut. Geodät. Komm., Reihe C,* **452,**1–96.
[19] Wolf, D. & Kaufmann, G. (1998) Effects due to compressional and compositional density stratification on load-induced Maxwell–viscoelastic perturbations, *Geophys. J. Int.,* submitted.
[20] Wu, P. & Peltier, W.R. (1982) Viscous gravitational relaxationm, *Geophys. J. R. Astron. Soc.,* **70,** 435–485.

Email-Liste

Name	E-Mail Adress
Abolghasem, Amir M.	amir.abolghasem@gis.uni-stuttgart.de
Anger, Gottfried	-
Ardalan, Alireza	ardalan@gis.uni-stuttgart.de
Bayer, Michael	m_bayer@mathematik.uni-kl.de
Beth, Stefan	beth@mathematik.uni-kl.de
Borre, Kai	borre@kom.auc.dk
Büchler, Bernd	buechler@mathematik.uni-kl.de
Cui, Chunfang	cui@lego.bv.tu-berlin.de
Dahlke, Stephan	dahlke@igpm.rwth-aachen.de
Freeden, Willi	freeden@mathematik.uni-kl.de
Glockner, Oliver	glockner@rhrk.uni-kl.de
Grafarend, Erik W.	grafarend@gis.uni-stuttgart.de
Groten, Erwin	groten@ipg.ipgs.verm.tu-darmstadt.de
Holota, Petr	gope@asu.cas.cz
Jurisch, Ronald	jurisch@vw.hs-anhalt.de
Kusche, Jürgen	kusche@uni-bonn.de
Maier, Thorsten	tmaier@physik.uni-kl.de
Michel, Volker	michel@mathematik.uni-kl.de
Narcowich, Francis J.	fnarc@math.tamu.edu
Nashed, M. Zuhair	nashed@math.udel.edu
Nutz, Helga	nutz@mathematik.uni-kl.de
Potts, Daniel	potts@math.mu-luebeck.de
Reuter, Richard	reuter@fh-aachen.de
Richter, Burghard	richter@dgfi.badw-muenchen.de
Römer, Hartmann	hartmann.roemer@physik.uni-freiburg.de
Sacerdote, Fausto	fausto@dicea.unifi.it
Schock, Eberhard	schock@mathematik.uni-kl.de
Schumaker, Larry L.	s@mars.cas.vanderbilt.edu
Schwarze, Volker S.	schwarze@gis.uni-stuttgart.de
Sjöberg, Lars E.	sjoberg@geomatics.kth.se
Steidl, Gabriele	steidl@math.uni-mannheim.de
Svensson, Leif	leif.svensson@math.lth.se
Thalhammer, Markus	thama@step.iapg.verm.tu-muenchen.de
Witte, Bertold	witte@uni-bonn.de
Wolf, Detlef	dasca@gfz-potsdam.de

Tagungsteilnehmer

Amir M. Abolghasem
Geodätisches Institut
Universität Stuttgart
Geschwister-Scholl-Str. 24D

70174 Stuttgart

Prof.Dr. Gottfried Anger
Rathausstr. 13, Wg. 11/09

10178 Berlin

Alireza A. Ardalan
Geodätisches Institut
Universität Stuttgart
Geschwister-Scholl-Str. 24D

70174 Stuttgart

Michael Bayer
Technomathematik, AG Geomathematik
Universität Kaiserslautern
Kurt-Schumacher-Str. 26

67663 Kaiserslautern

Stefan Beth
Technomathematik, AG Geomathematik
Universität Kaiserslautern
Kurt-Schumacher-Str. 26

67663 Kaiserslautern

Prof.Dr. Kai Borre
Dept. of Math. & Computer Sciences
Inst. for Electronic Systems
University of Aalborg
Fredrik Bajers Vej 7

DK-9220 Aalborg

Bernd Büchler
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße

67663 Kaiserslautern

Dr. Chunfang Cui
Inst. für Geodäsie und Geoinformatik
Straße des 17. Juni 135

10623 Berlin

Dr. Stephan Dahlke
Institut für Geometrie und Praktische Mathematik
RWTH Aachen
Templergraben 55

52062 Aachen

Prof.Dr. Willi Freeden
Fachbereich Mathematik
Universität Kaiserslautern

67653 Kaiserslautern

Oliver Glockner
Technomathematik, AG Geomathematik
Universität Kaiserslautern
Kurt-Schumacher-Str. 26

67663 Kaiserslautern

Prof.Dr.Dr.h.c. Erik W. Grafarend
Geodätisches Institut
Universität Stuttgart
Geschwister-Scholl-Str. 24D

70174 Stuttgart

Prof.Dr. Erwin Groten
Institut für Phys. Geodäsie
TU Darmstadt
Petersenstr. 13

64287 Darmstadt

Dr. Petr. Holota
Inst. of Geodesy, Topography and Cartography
25066 Zdiby 98

Praha - vychod
CZECH REPUBLIC

Dr. Ronald Jurisch
FB Vermessungswesen
TH Anhalt
Bauhausstr. 1

06846 Dessau

Dr. Jürgen Kusche
Theoretische Geodäsie
RFWU Bonn
Nussallee 17

53115 Bonn

Thorsten Maier
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße

67663 Kaiserslautern

Volker Michel
Technomathematik, AG Geomathematik
Universität Kaiserslautern

67653 Kaiserslautern

Prof.Dr. Francis J. Narcovich
Center for Approximation Theory
Texas A & M University

College Station , TX 77843-3368
USA

Prof.Dr. M. Zuhair Nashed
Department of Mathematical Sciences
University of Delaware
501 Ewing Hall

Newark , DE 19716-2553
USA

Helga Nutz
Kurt-Schumacher-Str. 17

67663 Kaiserslautern

Dr. Daniel Potts
Institut für Mathematik
Med. Universität zu Lübeck
Wallstr. 40

23560 Lübeck

Prof.Dr. Richard Reuter
FB Elektrotechnik und
Informationstechnik
Eupener Str. 70

52066 Aachen

Dr. Burghard Richter
DGFI
1. Abt. der Geodäsie
Marstallplatz 8

80539 München

Prof.Dr. Hartmann Römer
Fakultät für Physik
Universität Freiburg
Hermann-Herder-Str. 3

79104 Freiburg

Prof.Dr. Fausto Sacerdote
Universita Degli Studi di Firenze
Dipartimento di Ingegneria Civile
Via S.Marta 3

I-50139 Firenze

Prof.Dr. Eberhard Schock
Fachbereich Mathematik
Universität Kaiserslautern

67653 Kaiserslautern

Prof.Dr. Larry L. Schumaker
Dept. of Mathematics
Vanderbilt University
Stevenson Center 1326

Nashville , TN 37240
USA

Volker S. Schwarze
Staatl. Vermessungsamt
Waiblingen
Fronacker Str. 36

71332 Waiblingen

Prof.Dr. Lars E. Sjöberg
Department of Geodesy
The Royal Institute of Technology

S-10044 Stockholm

Prof.Dr. Gabriele Steidl
Fakultät für Mathematik und
Informatik - Dekanat -
Universität Mannheim
Seminargebäude A 5

68159 Mannheim

Prof.Dr. Leif Svensson
Dept. of Mathematics
University of Lund
Box 118

S-221 00 Lund

Dr. Markus Thalhammer
Institut für Astronomische und
Physikalische Geodäsie
TU München
Arcisstr. 21

80333 München

Prof.Dr. Bertold Witte
Theoretische Geodäsie
RFWU Bonn
Nussallee 17

53115 Bonn

Dr. Detlef Wolf
Geo Forschungszentrum
Telegrafenberg A 17

14473 Potsdam