

Tagungsbericht 15/1999

## Geometric and Topological Combinatorics

11.04.–17.04.1999

This workshop was organized by Anders Björner (KTH Stockholm), Gil Kalai (Hebrew University, Jerusalem), and Günter M. Ziegler (TU Berlin).

Its main goal was to present and advance the interplay of geometric, topological and combinatorial tools and ideas. Such interplay can be extremely fruitful, as illustrated by some recurring themes through the lectures. The workshop featured 8 survey style lectures as well as 23 shorter scientific talks.

*Exciting new developments:* Triggered by questions stemming from Vassiliev's work on knot invariants, the study of graph complexes has in the last few years become a very active area of research, with completely unexpected results. In this area, Robin Forman's "Discrete Morse Theory" has found its (first) main domain of applications: thus demonstrating the power of the Discrete Morse Theory as well as the fact that deep interesting geometric-topological structure is "hidden" in some graph complexes. At this workshop, R. Forman presented a survey lecture — and contributions presented by Chari, Jonsson, Shareshian and Wachs related to it.

*Substantial progress:* The Generalized Baues Conjecture disproof was presented in Oberwolfach 1995. Since then positive answers could be reached for quite a number of (geometrically and combinatorially) interesting special cases — see the lectures by Santos (survey), Athanasiadis and Rambau.

Related progress could be achieved very recently in the study of extremal properties of triangulations of subdivisions. The work in this area relates explicit geometric constructions to aspects of complexity (such as NP-completeness), as in the lectures of De Loera, Richter-Gebert and Joswig. There is corresponding progress also on the arithmetic side (studying  $f$ -vectors and flag vectors), as reported at this meeting by Brenti, Bayer and Ehrenborg.

*Unexpected connections:* By coincidence (nearly), Eric Babson lectured on “a space of tetrahedra” followed by Ulli Kortenkamp’s talk on “Dynamic Geometry.” The lectures and subsequent discussions featured most interesting, different points of view, and interest from quite different background was directed at the same types of structures: Very degenerate point-line configurations appear in the construction of smooth compactifications of configuration spaces as “singularities that must be resolved,” but they also occur in any dynamic geometry framework as “singular cases” that must systematically and consistently be taken care of.

In summary — this was an extremely fruitful and lively and successful meeting: and we are grateful for the privilege to enjoy the Oberwolfach Institute’s hospitality for this event.

# Vortragsauszüge

MONDAY:

**Robin Forman:**

**“Opening Lecture: A user’s guide to Discrete Morse Theory”**

Since its introduction in 1925, Morse theory has been a fundamental tool in the investigation of the topology of smooth manifolds. The main idea is that there is an intimate relationship between the topology of a manifold and the critical points of a smooth function on the manifold. In this lecture we will show that this idea can be adapted to provide an interesting new tool for the investigation of the topology of simplicial complexes (and other combinatorial spaces).

**Richard Ehrenborg:**

**“Inequalities for the  $cd$ -index”**

We prove an inequality involving the  $cd$ -indices of a convex polytope  $P$ , a face  $F$  of the polytope and the link  $P/F$ . As a consequence we settle a conjecture of Stanley that the  $cd$ -index of  $d$ -dimensional polytopes is minimized on the  $d$ -dimensional simplex. Moreover, we show how this gives quadratic inequalities on the flag  $f$ -vector of polytopes. Lastly, we present an upper bound theorem for the  $cd$ -index of polytopes.

**Christos Athanasiadis:**

**“Monotone paths on polytopes”**

Let  $P$  be a  $d$ -dimensional convex polytope and  $f$  a generic linear functional on  $P$ . The set of  $f$ -monotone paths on  $P$  is the vertex set of a graph, with adjacency defined by the ‘polygon moves’ along the 2-faces of  $P$ . Reiner has conjectured that this graph is always  $(d - 1)$ -connected. We disprove this conjecture for general  $d$ -dimensional polytopes but prove its validity for polytopes of dimension 3 and simple polytopes of arbitrary dimension  $d$ . We also give a positive answer to the Baues problem for monotone paths and cellular strings in the context of shellable CW-spheres, generalizing results of Billera-Kapranov-Sturmfels and Björner.

(Joint work with Paul Edelman and Victor Reiner.)

**Karanbir Sarkaria:**

**“Minimal triangulations of mappings”**

There has been much interest lately in “small triangulations” of *spaces*. In joint work with K.V. MADAHAR we have looked at similar problems for *mappings*. One of these is the following: given a homotopy class  $F$  of maps  $X \rightarrow Y$ , and a triangulation  $L$  of  $Y$ , find least number  $\lambda_L(F)$  such that there is a triangulation  $K$  of  $X$ , having  $\lambda_L(F)$  vertices, and admitting a simplicial map  $K \rightarrow L$  contained in  $F$ . This problem seems tractable when  $Y$  is an  $n$ -sphere,  $L$  its minimal triangulation  $S_{n+2}^n$ , and  $X$  is an orientable  $n$ -manifold. Then  $F$  is characterized by the degree  $d$  of any of its members, and what we want is a simplicial

$n$ -sphere  $K$  with least number of vertices, which admits a degree  $d$  simplicial map  $K \rightarrow L$ . Note that now  $d$  is just the number of  $n$ -simplices contained in the pre-image of any of the  $n$ -simplices of  $L$ .

For  $X = S^2$  the solution of this problem is  $\lambda(0) = \lambda(\pm 1) = 4$ ,  $\lambda(\pm 2) = 7$  and  $\lambda(d) = 2 + 2|d|$  for all other degrees  $d$ .

For  $X = M^2$ , any orientable 2-manifold, the problem is necessarily hard, since the computation of  $\lambda(0)$  is equivalent to the map colour theorem; and  $\lambda(d)$  for  $|d|$  small is probably of the same order of difficulty. However,

for any genus  $g$  we show that  $\lambda(d) = 2 + 2|d| - 2g$  for all degrees  $|d| \geq C_g$ ,

where  $C_g$  is a constant depending just on  $g$ . The above result for  $S^2$  — the case  $g = 0$  — shows that the minimal value for  $g = 0$  is  $C_0 = 3$ ; for the torus — the case  $g = 1$  — we know  $C_1 = 6$ ; for genus 2 surface only that  $C_2 \leq 10$ : computing  $C_g$  for all  $g$  seems an attractive problem. For higher dimensions we do not have complete solution, and we look more at the somewhat easier problem of triangulating some well known maps — Seifert fibrations of  $S^3$  — up to homeomorphisms. For the most basic of these, the Hopf fibration, we got a 12-vertex triangulation  $S^3_{12} \xrightarrow{n} S^2_4$ . This sheds light on Kühnel's  $CP^2_9$ :  $S^3_{12}$  is the boundary of a 17-vertex 4-ball  $D^4_{17}$ ; and performing the simplicial identifications prescribed by  $h$ , one gets the 9-vertex complex projective plane.

**Isabella Novik:**

**“Geometric embeddings”**

We will outline the proof of several necessary conditions for a simplicial complex to be geometrically embeddable in a Euclidean space of given dimension.

**Frank Lutz:**

**“Simplicial and combinatorial manifolds”**

A simplicial manifold is combinatorial if the link of every vertex is a  $PL$ -sphere. We present a heuristic, based on bistellar flips that locally modify the triangulation of a manifold, which we used to obtain vertex-minimal combinatorial triangulations of the following manifolds:

$S^2 \times S^2$	with	11 vertices,
$S^3 \times S^2$	with	12 vertices,
$S^3 \times S^3$	with	13 vertices,
$\mathbb{R}P^4$	with	16 vertices.

We also found a 16-vertex triangulation of the Poincaré homology 3-sphere. This triangulation is the starting point for a simple construction of non- $PL$   $d$ -spheres with  $d + 13$  vertices for  $d \geq 5$ . Moreover, we show that every  $d$ -manifold,  $d \geq 5$ , that can be triangulated with  $n$  vertices, has a non- $PL$  triangulation with  $n + 12$  vertices.

(Joint work with Anders Björner, Stockholm.)

TUESDAY:

**Alexander Schrijver:**

**“Spectrally characterizing linklessly embeddable graphs”**

We show that linkless embeddability of a graph  $G$  is characterized by  $\mu(G) \leq 4$ , where  $\mu$  is the graph parameter introduced by Colin de Verdière, based on spectral properties of matroids associated with  $G$ . The proof uses a Borsuk theorem for antipodal links.

(Joint work with L. Lovász.)

**Art Duval:**

**“Eigenvalues of combinatorial Laplacians”**

We show that the eigenvalues of the combinatorial Laplacian of a shifted simplicial complex are non-negative integers, and can be expressed in terms of higher-dimensional degree sequences of the simplicial complex. We further conjecture that, for an *arbitrary* simplicial complex, this equality becomes a majorization bound for the eigenvalues, and offer some evidence and partial results in this direction.

The theorem and conjecture are the higher-dimensional analogues of a corresponding theorem and conjecture, due to Grone and Grone/Merris, respectively, for graphs.

**Eric Babson:**

**“Configurations of tetrahedra”**

A smooth complete configuration space for tetrahedra is constructed related to the combinatorics of hypersimplices. A similar space is conjectured to be smooth for the higher dimensional simplices. These spaces generalize the Semple triangle space.

**Ulli Kortenkamp:**

**“Dynamic geometry”**

The talk gives an introduction into the research field of Dynamic Geometry. After a general introduction into this new area we focus on solved problems, open problems and some topics that open a new area of research.

Dynamic Geometry covers the aspects of geometry that arise from the introduction of dynamic parameters into geometric constructions. One typical application is interactive geometry software, where a geometric construction is parameterized by points with some degree of freedom (e.g. free points or points on a line). The user is able to move the points with the mouse, and the software should update the construction accordingly. This is easy as long as the construction is completely determined (which is the case when only lines and (intersection-)points are involved), but a completely new problem class arises when ambiguous operations like “intersection of a line and a circle” or “angular bisector” are allowed.

For the theoretical part of Dynamic Geometry it is not necessary to work with interactive 2D geometry. One can reduce the problem to the level of straight line programs over the complex numbers with one dynamic parameter. The fundamental operations of these

programs are addition, subtraction, multiplication and (square) roots. This setup allows a much more general approach to solutions. It furthermore enables us to apply the currently known techniques to several other applications like parametric CAD, computational kinematics, true virtual reality and others.

We will show that a consequent consideration of underlying Riemann surfaces resolves the above mentioned (seemingly) ambiguous situations and gives solutions to most of the current problems in dynamic geometry. Our approach involves a variety of mathematics, starting from Complex Projective Geometry via Cayley-Klein geometries to Analytic Function Theory. Finally, the talk will try to isolate the interesting open (basic and non-basic) problems in that context, which are of rather topological nature and open new areas of research.

**Manoj Chari:**

**“Generalized shellings, discrete Morse functions, and some applications”**

There is a great deal of interest in topological combinatorics in the problem of determining the topology of certain complexes based on graph properties. The discrete Morse theory of Forman has proved to be a very useful tool. In this talk, we define “generalized shellings” for simplicial complexes. We show that constructing generalized shellings for simplicial complexes is equivalent to constructing discrete Morse functions. We demonstrate the usefulness of this perspective for two classes of graph complexes.

**Peter Mani-Levitska:**

**“Convex polytopes and smooth manifolds”**

If a compact topological manifold  $M$  allows a triangulation  $(C, f)$  such that, for every  $x$  in  $C$ , the link of  $x$  in  $C$  is isomorphic to the boundary complex of some polytope  $P(x)$ , and if the reduced configuration space of  $P(x)$  is always simply connected, then  $M$  has a smooth atlas.

We work with small categories of polytopes, which we try to map into smooth categories. There are a number of open questions, but also one curious result: You will find, in dimensions above 7, some polytopes which arise from the simplex by repeated stellar subdivision, and which have nontrivial configuration spaces.

**Jürgen Richter-Gebert & Ulli Kortenkamp:**

**“Cinderella — Dynamic geometry (Software Presentation)”**

Presentation of the Dynamic Geometry Software System *Cinderella*, which will soon be available from Springer-Verlag. See <http://www.cinderella.de>

**Michael Joswig:**

**“POLYMAKE — Convex polytopes (Software Presentation)”**

Presentation of the Software System POLYMAKE for the analysis of convex polytopes. Version 1.3 is available since February 1999.

See <http://www.math.tu-berlin.de/diskregeom/polymake/>

WEDNESDAY:

**John Shareshian:**

**“Graph complexes”**

A monotone graph property is a collection of graphs on a fixed labelled vertex set  $V$  which is closed under removal of edges and permutation of vertices, that is, a simplicial complex on  $\binom{V}{2}$  which admits the vertices of the symmetric group  $S_V$ . After reviewing results of Kahn, Saks and Sturtevant on the topological structure of these complexes and applications to computational complexity, we will examine recent results on particular classes of monotone graph properties which are of interest in combinatorics, algebra, geometry and knot theory.

**Paul Edelman:**

**“Counting the interior of a point configuration”**

We prove a formula conjectured by Ahrens, Gordon, and McMahan for the number of interior points for a point configuration in  $\mathbb{R}^d$ . Our method is to show that the formula can be interpreted as a sum of Euler characteristics of certain complexes associated with the point configuration, and then compute the homology of these complexes. This method extends to other examples of convex geometries. We sketch these applications, replicating an earlier result of Gordon, and proving a new result related to ordered sets.

(This is joint work with Victor Reiner.)

**Michelle Wachs:**

**“Bounded degree graph complexes”**

Let  $G$  be a graph, digraph, multigraph or hypergraph. The set of subgraphs of  $G$  for which every node has degree at most  $b$  forms a simplicial complex. Some special cases that have been considered in the recent literature are the matching complex ( $G$  is the complete graph and  $b = 1$ ) and the chessboard complex ( $G$  is a complete bipartite graph and  $b = 1$ ). We describe various techniques for lifting known results on the homotopy and homology of the matching complex and the chessboard complex to the more general bounded degree graph complexes.

(Some of this material is joint work with Dikran Karaguezian and Victor Reiner and with Anders Björner.)

**Jakob Jonsson:**

**“Not 3-connected graphs”**

Using techniques from the discrete Morse theory developed by Robin Forman, we determine the homotopy type of the simplicial complex of not 3-connected graphs on a fixed set of vertices.

**Gil Kalai (chair):**

**“Problem Session”**

THURSDAY:

**Francesco Brenti:**

**“ $P$ -kernels and their applications to geometric combinatorics”**

In the first part of the talk we will survey the theory of  $P$ -kernels introduced by Stanley in [*J. Amer. Math. Soc.* **5** (1992), 805-851], as well as some new developments. This theory includes the toric  $h$ -vector of an Eulerian poset, the Kazhdan-Lusztig polynomials of a Coxeter group (and many of their variations), and the Ehrhart polynomial of a rational polytope.

In the second part of the talk we will apply the theory of  $P$ -kernels to obtain some new results on toric  $h$ -vectors. In particular, we characterize the Eulerian posets such that the toric  $h$ -vectors of its subintervals satisfy certain vanishing properties. The theory of  $P$ -kernels also naturally suggests the definition of an involution on the set of  $P$ -kernels. When applied to Eulerian posets and Coxeter groups, this involution leads to the definition of certain functions in the incidence algebra of  $P$  that are “dual”, in a very precise sense, to the  $R$ -polynomials of a Coxeter group, and to the zeta function of an Eulerian poset. These seem to be new objects, and we will present some of their basic properties.

**Marge Bayer:**

**“The cone of flag vectors of Eulerian partially ordered sets”**

The face lattices of convex polytopes belong to the class of Eulerian posets. In every interval of these ranked posets, the number of elements of even rank equals the number of elements of odd rank. The flag vector of a ranked poset gives the numbers of chains for the various rank sets. This talk discusses the closed convex cone of flag vectors of Eulerian posets. (The linear span is determined by the generalized Dehn-Sommerville equations.) We completely determine the cone for Eulerian posets up through rank 7, and give some general results for both extreme rays and facets of the cone in higher ranks. The approach uses work of Billera and Hetyei on flag vectors of ranked posets, and introduces “half-Eulerian” partially ordered sets. An open question is whether the cone of Eulerian posets equals the cone of doubles of half-Eulerian posets. The latter cone is contained in the former and, by a  $cd$ -index argument, contains the face lattices of convex polytopes. (This is joint work with Gabor Hetyei, and is work in progress.)

**Michael Joswig:**

**“A counterexample to the cubical upper bound conjecture”**

Kalai conjectured that the neighborly cubical polytopes would maximize the  $f$ -vector for a given (power of 2) number of vertices among all cubical spheres. However, by local surgery a counter-example can be constructed from a 4-dimensional cubical polytope with the same graph as the 6-cube.

(This is joint work with Günter M. Ziegler.)



**Masahiro Hachimori:**

**“Decomposition of 3-spheres and 3-balls and knots consisting of few edges”**

In Lickorish’s paper in 1991 it is shown that a triangulated 3-sphere is nonshellable if it has a knot with three edges and complicated enough. We extend this result to nonconstructibility and remove the complexity condition of the knot. We also apply a similar argument to vertex-decomposability, and show that a 3-sphere (or 3-ball) with a knot consisting of at most five edges is not vertex decomposable.

(This is joint work with Günter M. Ziegler.)

**Mario Salvetti:**

**“Cohomology of Coxeter and Artin groups”**

We describe some constructions in combinatorial topology for the spaces  $K(W, 1)$  and  $K(G_W, 1)$ , where  $W$  is a finitely generated Coxeter group and  $G_W$  is the Artin group of type  $W$ . From these, we deduce algebraic complexes which calculate the cohomology of any Artin group and any Coxeter group with coefficients in any module.

**Eva Maria Feichtner:**

**“Rational versus real cohomology algebras of low-dimensional toric varieties”**

The rational cohomology groups of a compact quasi-smooth toric variety are completely determined by the combinatorial data of its defining simplicial fan. We will review results on the cohomology algebras of toric varieties, and present examples which show that the ring structure is not combinatorially determined in general.

In low dimension, the situation turns out to be surprisingly intricate: We show that the real cohomology algebra of a compact toric variety in complex dimension 2 is completely determined by the combinatorial data of its defining fan. We derive this result from special properties of the Hodge decomposition for the cohomology of quasi-smooth, projective toric varieties. For rational coefficients the cohomology algebra is no longer combinatorially determined. Also, our examples show that neither the real nor the complex cohomology algebras of compact quasi-smooth toric varieties are combinatorial invariants in general.

**Mark de Longueville:**

**“Coordinate subspace arrangements”**

Every simplicial complex  $\Delta \subset 2^{[n]}$  on the vertex set  $[n] = \{1, \dots, n\}$  defines a real resp. complex arrangement of coordinate subspaces in  $\mathbb{R}^n$  resp.  $\mathbb{C}^n$  via the correspondence  $\Delta \ni \sigma \mapsto \text{span}\{e_i : i \in \sigma\}$ . The linear structure of the cohomology of the complement of such an arrangement is explicitly given in terms of the combinatorics of  $\Delta$  and its links by the Goresky–MacPherson formula. Here we derive, by combinatorial means, the ring structure on the integral cohomology in terms of data of  $\Delta$ . We provide a non-trivial example of different cohomology rings in the real and complex case. Furthermore, we give an example of a coordinate arrangement that yields non-trivial multiplication of torsion elements.

FRIDAY:

**Francisco Santos:**

**“Recent progress on the Baues problem”**

Given a polytope projection  $\pi : P \rightarrow Q$ , Billera and Sturmfels (1992) introduced the concept of “subdivision of  $Q$  induced by  $\pi$ ” (or  $\pi$ -induced), which essentially is any polyhedral subdivision of  $Q$  whose cells are projections of faces of  $P$ . The poset of all  $\pi$ -induced subdivisions is usually called the “Baues poset” of the projection  $\pi$ . Originally, the generalized Baues conjecture of Billera, Kapranov and Sturmfels asked whether the Baues poset has the homotopy type of a sphere of dimension  $\dim(P) - \dim(Q)$ . This was disproved in the general case but there are some interesting cases open. For example, if  $P$  is a simplex then  $Baues(\pi : P \rightarrow Q)$  is the poset of all polyhedral subdivisions of the point configuration  $\pi(\text{vertices}(P))$ , and no example in which this is not a homotopy sphere is known.

This talk addresses the following two recent developments:

1) There are triangulations of arbitrarily large numbers of points in dimension 4 and with bounded number of geometric bistellar neighbors. This implies that the posets of all subdivisions of these points configurations are very weakly connected and is evidence against the conjecture that they are always connected.

2) The polyhedral Cayley Trick implies that there is a bijection between the Baues poset of mixed subdivisions of a Minkowski sum of polytopes  $P_1, \dots, P_k$  and all subdivisions of a certain polytope (the Cayley embedding) obtained from them. As applications we show:

- since the Lawrence polytope of a family of vectors equals the Cayley embedding of the corresponding segments, there is a bijection between the zonotopal tilings of an arbitrary polytope and the polyhedral subdivisions of the associated Lawrence polytope. This provides a new proof of the Bohne-Dress theorem on zonotopal tilings and has connections to oriented matroid theory.
- since the product of a  $k$ -simplex and a polytope  $Q$  equals the Cayley embedding of  $k + 1$  equal copies of  $Q$ , the Cayley Trick can be applied backwards in order to study subdivisions of this product via mixed subdivisions of  $Q + \dots + Q$ , which has smaller dimension. We apply this to understand subdivisions of products of simplices and to obtain triangulations of cubes with small number of simplices.

**Jörg Rambau:**

**“The generalized Baues problem for cyclic polytopes”**

The Generalized Baues Problem asks whether for a projection of polytopes  $P \xrightarrow{\pi} Q$  the partially ordered set of all  $\pi$ -induced subdivisions of  $Q$  has the homotopy type of a sphere of dimension  $\dim(P) - \dim(Q) - 1$ . Because it is known that the answer to the general problem is negative one is interested in special cases like special classes of polytopes. In fact: The answer is affirmative for all projections  $C(n, d') \rightarrow C(n, d)$  between cyclic polytopes given by forgetting the last  $d' - d$  coordinates.

The main ingredients of the proof of this result are presented as well as connections to related questions.

(Joint work with Christos Athanasiadis and Francisco Santos; obtained during the conference “Geometric and Topological Combinatorics 1998, Kotor, Montenegro, Yugoslavia.)

**Jesús De Loera:**

**“Minimal and maximal triangulations of convex polytopes”**

In this talk I will discuss several properties of triangulations of convex polytopes that optimize the number of top dimensional simplices, such as how they behave under coordinate changes, deletion and contraction or computational complexity.

(Some of these results are joint work with A. Below and J. Richter-Gebert.)

**Jürgen Richter-Gebert:**

**“Polytopes in small dimensions — construction methods and results”**

This talk focuses on various constructions for polytopes in dimensions between four and six. Among the topics that will be touched are:

- Finding minimal triangulations of the boundary of a 4-polytope is NP hard.
- One can construct 4-polytopes with “very stiff” 2-faces:  
“For any  $n$ -gon  $G$  with algebraic coordinates one can find a 4-polytope  $P$  that has  $G$  as a 2-face such that in every realization of  $P$  this 2-face has the same shape as  $G$  (up to projective equivalence).”

As an application of the latter we show that

- It is NP-hard to find a coordinatization that maximizes the maximal number of simplices in a triangulation of the boundary.
- One cannot reasonably generalize the notion of an antiprism.

**Laura Anderson:**

**“Polytope bundle theory”**

A polytopal bundle is a combinatorial analog to a topological sphere bundle, in which the base space is replaced by a poset and the fibers are replaced by combinatorial types of convex polytopes. The combinatorial analog to continuity is expressed in terms of polytopal subdivision. We have recently proven that the category of polytopal bundles is equivalent to the category of piecewise-linear sphere bundles. As immediate consequences we get new topological results on various categories of combinatorial types of polytopes.

(Joint work with Nikolai Mnëv.)

**Christian Haase:**

**“Crepan resolutions of toric l.c.i.-singularities”**

The question whether or not a given variety admits crepan (full) resolutions of its singularities is subtle in general. In the case of toric local complete intersections it translates to the question whether or not certain polytopes admit unimodular triangulations. This question can be answered positively by a surprisingly simple induction argument.

(Joint work with D.I. Dais and G.M. Ziegler.)

**Volkmar Welker:**

**“Subspace arrangements and resolutions of modules”**

In commutative algebra one is interested in finite or infinite minimal resolutions of finitely generated modules over quotients of a polynomial ring. The Betti-numbers  $\dim_k \operatorname{Tor}_i^R(M, k)$  associated to this resolution are of particular interest. Combinatorial formulas are available if  $R$  is the polynomial ring and  $M = R/I$  for a monomial or toric ideal and for  $R =$  affine semigroup ring and  $M = k$ . We show that for a monomial ideal there is an alternative formula involving the lattice of lcm's of the generators and if  $R =$  affine semigroup ring modulo a monomial ideal there is an expression using relative homology of order complexes. In both cases the formulas can advantageously be used to derive new and old results in commutative algebra but also allow to build geometric models (i. e., spaces  $X$  such that  $\operatorname{Tor}_i^R(M, k) = H^i(X; K)$ ). In both cases  $X$  can be chosen as the complement of an arrangement of linear subspaces. The subspaces involved are either spans of coordinate basis vectors or thin diagonals defined by some coordinates being equal.

Berichterstatter: G. M. Ziegler

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