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(c) Over Z', the family induces the simultaneous resolution of X, and modulo the action of the Weyl group induces the semiuniversal deformation of X.

Theorem 2: [Kapranov-Vasserot 1998] *The twisted group algebra S "is" indeed the desingularization of X, $D^b(mod\, S) \cong D^b(Coh\, \widetilde{X})$, where $\widetilde{X}$ is the (geometric) desingularization of X.*

Theorem 3: [Buchweitz; Crawley-Boevey] *The quantizations of negative weight of X are of finite representation type with respect to reflexive modules.*

This last result supports H. Knörrers conjecture that finite representation type of maximal Cohen-Macaulay modules should be open in flat families. A more detailed account of the above results will be given in the DMV-Seminar "Quantizations of Kleinian Singularities", Oberwolfach 23.05.–30.05.99.

BERICHTERSTATTER: C. LOSSEN (KAISERSLAUTERN)

Christoph Lossen (Kaiserslautern)

Castelnuovo function, zero-dimensional schemes and singular plane curves

Joint work with Gert-Martin Greuel (Kaiserslautern) and Eugenii Shustin (Tel Aviv)

We study the geometry of families $V = V_d^{irr}(S_1, \ldots, S_r)$ of irreducible complex plane curves of degree d with prescribed (analytic or topological) singularity types $S_1, \ldots, S_r$. The questions about non-emptiness, smoothness, irreducibility and dimension are basic in the geometry of such families. Except for the case of nodal curves, no complete answers are known and one can hardly expect them. Our goal, however, has been to obtain *asymptotically proper* sufficient conditions for V to be non-empty, or *T-smooth* (=smooth of the expected dimension), or irreducible. In 1996, we obtained the first asymptotically proper (general) sufficient condition for the non-emptiness of V (in the case of topological singularity types):

$$\sum_{i=1}^{r} \mu(S_i) < \frac{1}{46} \cdot (d+2)^2$$

(+2 conditions bounding the 5 "worst" singularities) whereas the classically known necessary condition is $\sum_{i=1}^{r} \mu(S_i) \leq (d-1)^2$, that is, the sufficient and necessary conditions differ asymptotically only in the occuring absolute constants. In this talk, we present how the theory of the *Castelnuovo function* associated to a 0-dimensional scheme $X \subset \mathbb{P}^2$ (with corresponding ideal sheaf $\mathcal{J}_{X/\mathbb{P}^2} \subset \mathcal{O}_{\mathbb{P}^2}$),

$$\mathcal{C}_X : \mathbb{Z}_{\geq 0} \longrightarrow \mathbb{Z}_{\geq 0}, \quad d \longmapsto h^1\left(\mathcal{J}_{X/\mathbb{P}^2}(d-1)\right) - h^1\left(\mathcal{J}_{X/\mathbb{P}^2}(d)\right),$$

can be used to obtain a qualitatively new sufficient criterion for T-smoothness:

Theorem. *$V_d^{irr}(S_1, \ldots, S_r)$ is T-smooth (or empty) if $\sum_{i=1}^{r} \gamma(S_i) < d^2 + 6d + 8$ (where $\gamma(S) \leq (\tau(S)+1)^2$ is a new singularity invariant, $\tau(S)$ the Tjurina number).*

We expect this condition to be asymptotically proper in the following sense:

Conjecture: *There exists an absolute constant $A > 0$ such that for any type S there are infinitely many pairs $(r, d) \in \mathbb{N}^2$ such that $r \cdot \gamma(S) \leq A \cdot d^2$ but $V_d^{irr}(r \cdot S)$ is empty or not smooth or has not the expected dimension.*

Ragnar-Olaf Buchweitz (Toronto)

Noncommutative geometry of Kleinian singularities

Let $G \subset \mathrm{SL}(2, \mathbb{C})$ be a finite group, $X = \mathbb{C}^2/G$ the associated Kleinian singularity. Let $Z = Z(G) \subset \mathbb{C}G$ be the centre of the group algebra of G and $Z' \subset Z$ the orthogonal complement of

$$e = \frac{1}{|G|} \sum_{g \in G} g \in Z\,.$$

The complex vectorspace Z' is of dimension $\mu(X) = \tau(X)$, and the natural basis for the simultaneous resolution of the semiuniversal deformation of X.

P. Kronheimer, in his study of hyperkähler structures, described the simultaneous resolution through representations of the quiver associated to the Coxeter-Dynkin diagram of X. Recently, W. Crawley-Boevey and M. Holland (Duke 1998) realized that Kronheimer's construction can be carried much further: let $S = \mathbb{C}[u, v] * G$ be the twisted group algebra associated to G, which according to V. Jones (Notices 1997) should be thought of as the noncommutative desingularization of X.

Theorem 1: [Crawley-Boevey; Holland] *(a) There exists an explicitly constructed quantization (= flat associative deformation) of S over Z.*
(b) This family induces all quantizations of X of negative weight.

distributions on M with singularities, i.e., any modules of vector fields. We discuss also the transversality property which justifies our computations.

Klaus Altmann (Berlin)

Deforming Stanley-Reisner rings

Joint work with Jan Christophersen (Oslo)

Every simplicial complex X on the set $\{1,\ldots,n\}$ gives rise to the so-called *Stanley-Reisner ring* $A(X)$. The corresponding variety $\mathbb{P}(X) := \operatorname{Proj} A(X)$ is a union of planes which looks as X itself. In the talk we have addressed to a question raised by Sorin Popescu at a Oberwolfach meeting last year. Motivated by the search of Calabi-Yau-manifolds arising as smoothings of singular varieties, we are interested in the deformation theory of $\mathbb{P}(X)$.

The basic information is encoded in the modules $T^1_{A(X)}$ and $T^2_{A(X)}$ which can be calculated using the fine $\mathbb{Z}^n$-grading of $A(X)$. This leads to the following applications: if X is a triangulation of a 2-dimensional manifold, then besides concrete formulas for $T^1_{\mathbb{P}(X)}$, $T^2_{\mathbb{P}(X)}$, we obtain that the localization maps are isomorphisms on the T^2-level. Moreover, one obtains examples of such X for which $\mathbb{P}(X)$ is not smoothable.

Victor Goryunov (Liverpool)

Cyclically-invariant functions and related unitary reflection groups

Finite groups generated by Euclidean reflections became a very common object in various problems of singularity theory since their importance in the classification of critical points of functions was demonstrated by Arnold and Brieskorn. We show that a number of finite groups generated by *unitary* reflections are also naturally related to singularities of functions, namely to those invariant under a unitary reflection of finite order. To establish this, one has to consider function germs on a manifold with boundary and lift them to a cyclic covering of the manifold ramified over the boundary. The construction provides a new notion of roots for the groups under consideration and skew-Hermitean versions of these groups.

Helmut A. Hamm (Münster)

Cohomology of fibres and atypical values

Let $f : \mathbb{C}^n \to \mathbb{C}$ be a polynomial map. It is well-known that even if f is a submersion it may happen that f does not define a topological (or differentiable) fibre bundle (cf. the example of Broughton). One can understand this phenomenon by passing to some compactification. More generally, we may consider a holomorphic map $g : Z \to V$, where Z is a complex space and V an open subset of $\mathbb{C}$, which can be extended to a proper map $\overline{g} : \overline{Z} \to V$. Let us suppose that $Z_\infty := \overline{Z} \setminus Z$ is a Cartier divisor. Let us call $t_0 \in V$ *typical* if there is a neighbourhood W of t_0 such that $\overline{g}|_{\overline{g}^{-1}(W)}$ defines a topological fibre bundle, otherwise *atypical*. If t_0 is typical it is *cohomologically typical*, i.e., for every k: $R^k g_* \mathbb{Z}_Z|_W$ is locally constant, $(R^k g_* \mathbb{Z}_Z)_t \cong H^k(g^{-1}(t), \mathbb{Z}_Z)$, $t \in W$. The latter notion can be extended obviously to sheaf complexes $\mathbf{F}$, e.g., the intersection homology complex. It is convenient to restrict to m-perverse sheaf complexes, e.g., $\mathbb{Z}_Z$ if Z is smooth of pure dimension m or locally a set-theoretic complete intersection of dimension m. Furthermore let us suppose that the cohomology of $\Phi_{\overline{g}} R j_* \mathbf{F}$ is concentrated on a finite set, where $j : Z \hookrightarrow \overline{Z}$. Then (after shrinking V if necessary) the conditions required for a cohomologically typical value are fulfilled for most k, and if the Euler characteristic $\chi_t = \sum_k (-1)^k \mathrm{rk}\, \mathbb{H}^k(g^{-1}(t), \mathbf{F})$ is constant we have a cohomologically typical value. The last result can be viewed as the cohomological counterpart of a result of Parusinski which generalizes itself earlier results of Hà/Lê and Siersma/Tibăr.

1) There exists $C_1 \times C_2 \to S$ étale if and only if $S \cong (C_1' \times C_2')/G$, G acting freely on $C_1' \times C_2'$.
2) Higher dimensions and quotients of polydisks (Yau, Jost, Mok).
3) S birationally equivalent $(C_1 \times C_2)/G$.
4) Existence of a submersion $f : S \to B$, B a curve (Kotschick).
5) Existence of any fibration $f : S \to B$.

In the above, we assume that $b_1\big(B \setminus \{\text{critical values}\}\big) \geq 3$ and that the genus g of the fibres is at least 2. In the examples 3) and 5), the role of $\pi_1(S)$ is taken by the pair $\pi_1^\infty(U) \to \pi_1(U)$, where U is the Zariski open preimage $f^{-1}\big(B \setminus \{\text{critical values}\}\big)$. If $S \setminus U = \bigcup_i \Delta_i$, Δ_i the connected components, then $\pi_1^\infty(U)$ equals the disjoint union of the groups $\pi_1(T_i \setminus \Delta_i)$, T_i being a tubular neighbourhood of Δ_i. For Theorem 5), the conditions are as follows: if $D = \Delta_i = \sum D_i$, and γ_i is a small loop around D_i, then $\pi_1(T_i \setminus D_i) \to \mathbb{Z} \subset \mathbb{F}_{b_1}$ and γ_i has non-zero image — in case f is not as in 4), we require as basic conditions that there exists an exact sequence $1 \to \pi_g \to \pi_1(U) \to \mathbb{F}_{b_1} \to 1$ with a certain cohomological property of non-vanishing of a cup product arising from the spectral sequence for group cohomology (plus the condition $e(U) = 2(g-1)(b_1-1)$).

This research motivated the following conjecture that we hope to prove soon (a generalization of a theorem of Mumford):

Conjecture: *If K is nef on D, i.e., $K \cdot D_i \geq 0$ for all i, then all $\gamma_i \in \pi_1(T \setminus D)$ are non-trivial.*

Emmanuel Ferrand (Grenoble)

Global existence of singularities in contact topology

Let M be a manifold and consider the contact manifolds $ST^*M \xrightarrow{\pi} M$ and $PT^*M \xrightarrow{\pi} M$. Given a legendrian submanifold $\ell \hookrightarrow PT^*M$, $\pi(\ell)$ is a singular hypersurface in M. For example, when $M = \mathbb{R}^2$, $\pi(\ell)$ is a planar curve with cusps and transversal intersections (provided that ℓ is generic). There exists a similar description for $\ell \hookrightarrow ST^*M$.

I have reported several occurences of the following situation: to remove the singularities of $\pi(\ell)$ by a deformation of ℓ, this deformation should break the legendrian knot type of ℓ, i.e., the removal of singularities is impossible by a contact isotopy. (The word "singularity" may have different meanings depending on the examples below.)

Examples: 1. An analogue of a theorem of Chekanov about the necessity of critical points of the restriction of a function to $\pi(\ell)$ (a generalization of Morse theory).

2. A theorem (joint work with P.E. Pushkar, also proved by M. Entov) about the impossibility to remove fold curves on $\pi(\ell)$ by a contact isotopy of ℓ, for some ℓ for which it is possible to do so by a contact regular homotopy (deforming ℓ through legendrian immersions).

3. (A conjecture by Arnold) Consider a fibre of $\pi : PT^*S^2 \to S^2$. Any legendrian ℓ contact-isotopic to this fibre, in general position with respect to π, must have at least 3 cusps.

Maxim Kazarian (Moscow)

Giambelli-type formulas for distributions on the tangent bundle

Joint work with B. Shapiro (Stockholm)

Let us consider a generic n-dimensional subbundle $\mathcal{V}$ of the tangent bundle TM on a given manifold M. Given $\mathcal{V}$ one can define different degeneracy loci $\Sigma_{\mathbf{r}}(\mathcal{V})$, $\mathbf{r} = (r_1 \leq \cdots \leq r_k)$, on M consisting of all points $x \in M$ for which the dimension of the subspace $\mathcal{V}^j(X) \subset TM(x)$ spanned by all length $\leq j$ commutators of vector fields tangent to $\mathcal{V}$ at x is less than or equal to r_j. We calculate the cohomology classes dual to $\Sigma_{\mathbf{r}}(\mathcal{V})$ using determinantal formulas due to W. Fulton and the expression for the Chern classes of the associated bundle of free Lie algebras in terms of the Chern classes of $\mathcal{V}$. These results are also extended to the case of

Theorem: *Two birationally equivalent Calabi-Yau varieties have the same Hodge numbers.*

We extended these notions to Gorenstein varieties X and effective Cartier divisors D on X such that $\operatorname{supp} D$ contains the singular locus of X, or less restrictive, the locus of log canonical singularities of X. In particular, on the level of Hodge polynomials and Euler characteristics, our invariants admit as special case the stringy E-functions and stringy Euler numbers of Batyrev for log terminal varieties.

Mina Teicher (Ramat Gan)

Braid monodromy type and diffeomorphism type

I defined a *braid monodromy factorization* (BMF) related to an algebraic curve, the *braid monodromy type* (BMT) of an algebraic curve and a virtual BMF. Then I stated the relations between the BMT of a branch curve of a generic projection of an algebraic surface, the diffeomorphism type (Diff) of the related surface, the deformation type (Def) of the surface and other known invariants of an algebraic surface, e.g., the fundamental group of the complement π_1.

$$\begin{array}{ccccccc} \text{Def} & \underset{?}{\overset{\Longrightarrow}{\Longleftarrow}} & \text{BMT} & \underset{?}{\overset{\Longrightarrow}{\Longleftarrow}} & \text{Diff} & \overset{\Longrightarrow}{\not\Longleftarrow} & \text{Homeo} \\ & & \Downarrow & & & & \\ & & \pi_1 & & & & \end{array}$$

I stated the question concerning the relation between BMF and virtual BMF. Certainly there are virtual BMF which are not BMF. How can we identify them?

Mihai Tibăr (Lille)

Singularities of meromorphic functions

Joint work with Dirk Siersma (Utrecht)

We consider a meromorphic function $f : Z \dashrightarrow \mathbb{C}$ on a connected compact complex manifold. The scope is to define and study vanishing cycles. Let

$$X = \left\{(x,t) \in (Z \setminus \mathrm{Pol}(f)) \times \mathbb{C} \,\middle|\, f(x) - t = 0\right\} \xrightarrow{\pi} \mathbb{C}.$$

Fibres of π are fibres of f. We call $H_*(X, F)$ the global vanishing cohomology of f. There is a finite number of atypical fibres of π and we have denoted $F :=$ the general fibre. We prove a direct sum splitting of $H_*(X, F)$ at atypical values and a global Picard-Lefschetz phenomenon. We define singularities of f along the poles $\mathrm{Pol}(f)$ with respect to a weak stratification ("partial Thom stratification") and prove the bouquet struture: $X/F \simeq \bigvee S^{\dim Z}$ in case of isolated singularities. Some related results are proven. Our results extend some local ones on holomorphic functions and some global ones for polynomial functions.

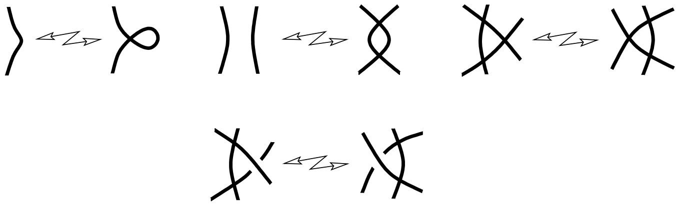
Fabrizio Catanese (Göttingen)

Fundamental groups of open sets of surfaces and fibrations of surfaces

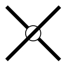
A prototype theorem which generalizes in many possible ways is:

Theorem: *An algebraic surface S is isomorphic to a product of two curves of genera ≥ 2 ($S \cong C_1 \times C_2$, $g_i = g(C_i) \geq 2$) if and only if $\pi_1(S) \cong \pi_{g_1} \times \pi_{g_2}$, $e(S) = 4(g_1-1)(g_2-1)$, where π_g denotes the fundamental group of a curve of genus g and e denotes the Euler number.*

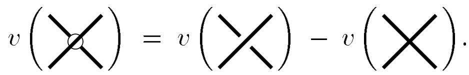
The generalizations go in several directions:



We explain the reasons for a consideration of these objects, using several different descriptions and motivations. The natural theory of finite type invariants is based on a new type of singular crossings, the so-called

- *"semi-virtual" crossings:* 

Given an invariant v of virtual knots, one can extend it to singular virtual knots by putting



An invariant is called *of finite type of degree* $\leq n$ if it vanishes on any singular virtual knot with more than n (semi-virtual) singularities. We show that the virtual isotopy implies isotopy (i.e., knots embed into virtual knots), and that an invariant of virtual knots of degree $\leq n$ is a Vassiliev invariant of degree $\leq n$ of classical knots. We conjecture that any Vassiliev invariant of degree $\leq n$ extends to long virtual knots as an invariant of degree $\leq n$. The conjecture is proven for small values of n.

A universal invariant of virtual knots of degree n is constructed. This technique helps us to prove that any finite type invariant of classical knots can be obtained as a Gauss diagram invariant (by counting an algebraic number of certain subdiagrams of any Gauss diagram; this conjecture dates back to 1993). Some other applications are discussed.

Wim Veys (Leuven)

Topological and related zeta functions on singular varieties

We review recent invariants introduced by Denef/Loeser and Kontsevich on nonsingular varieties and indicate how to extend them to certain singular varieties.

Let X be a nonsingular n-dimensional algebraic variety and D an effective divisor on X. Inspired by the theory of p-adic Igusa zeta functions, Denef and Loeser introduced the motivic (Igusa) zeta function $\mathcal{Z}(D,s)$, defined using the space of arcs of X. We give here a formula in terms of an embedded resolution $h: Y \to X$ of $\operatorname{supp} D$. Let E_i, $i \in T$, denote the irreducible components of $h^{-1}(\operatorname{supp} D)$ and define the natural numbers N_i, ν_i by $h^*D = \sum_{i\in T} N_i E_i$, $K_Y = h^*K_X + \sum_{i \in T}(\nu_i - 1)E_i$. Set also $E_I^\circ := \left(\bigcap_{i\in I} E_i\right) \setminus \left(\bigcup_{\ell \notin I} E_\ell\right)$ for any $I \subset T$. Then

$$\mathcal{Z}(D,s) = \left[\mathbb{A}^1\right]^{-n} \sum_{I \subset T} \left[E_I^\circ\right] \prod_{i \in I} \frac{[\mathbb{A}^1]-1}{[\mathbb{A}^1]^{\nu_i + sN_i} - 1},$$

where $[\ ]$ is the class of a variety in the Grothendieck ring and $\left[\mathbb{A}^1\right]^{-s}$ should be considered as a variable. One can specialize to the level of Hodge polynomials and further to the level of Euler characteristics, obtaining the previously introduced topological zeta functions.

Kontsevich introduced an invariant $\mathcal{E}(D)$ by integrating on the arc space of X, for which the formula in terms of the resolution h is

$$\mathcal{E}(D) = \left[\mathbb{A}^1\right]^{-n} \sum_{I \subset T} \left[E_I^\circ\right] \prod_{i \in I} \frac{[\mathbb{A}^1]-1}{[\mathbb{A}^1]^{\nu_i + N_i} - 1},$$

living in a certain completion of the Grothendieck ring. An important application of this gadget is the following

- If f is the sum of linear and quadratic terms and satisfies (M) then the linear part vanishes.
- If f is quadratic homogeneous and satisfies (M) then $k \leq \rho(\frac{m}{2}) + 1$, where $\rho(m)$ is the *Radon-Hurwitz number*, defined by: $\rho(m)$ depends only on the highest power of 2 dividing m; $\rho(m) = m$ for $m = 0, 1, 2, 4, 8$; $\rho(16m) = m + 8$.
- If $k \leq \rho(\frac{m}{2}) + 1$, then there is a homogeneous quadratic function $f : \mathbb{R}^m \to \mathbb{R}^k$ satisfying (M).

FABIEN NAPOLITANO (PARIS)

Pseudo-homology of complex hypersurfaces

To a complex hypersurface X in $\mathbb{C}^n$ we associate a sequence of non-commutative groups called *pseudo-homology groups* of X. The first of those is $\mathcal{H}_0(X) = \pi_1(\mathbb{C}^n \setminus X)$. The next groups are defined using some generalization of $\pi_1(\mathbb{C}^n \setminus X)$ for the successive bifurcation diagrams of X. Pseudo-homology groups are somehow analogs of classical homology groups: they are computed as a quotient $\operatorname{Ker}\partial / \operatorname{Im}\partial$ associated to some boundary operator ∂; the notion of pseudo subcomplex is defined similarly to the usual notion of subcomplexes in homology; they are invariant by polynomial polynomially invertible maps $\mathbb{C}^n \to \mathbb{C}^n$; they satisfy a theorem of Lefschetz type; etc.. Pseudo-homology groups are defined for simple singularities. Adjacencies of singularities induce morphisms of pseudo-homology groups.

OLEG VIRO (UPPSALA)

Differential topology more friendly to singularities

Traditional foundations of differential topology keep the subject away from singularities. This is neither natural nor easy: the image of a smooth manifold under a differentiable map is pushed out of the theory, there are no quotient objects. Due to this phobia of singularities, differential topology looks more different from algebraic geometry than it is. More “singularity friendly” foundations of differential topology were worked out in the sixties/seventies, but remain unknown even to most of the specialists. They deserve consideration since they provide a great flexibility and terms in which one can discuss glueing and factorizing of smooth manifolds and smoothenings of the results. Metric spaces are incorporated into differential topology. Singularities of functions become parallel to singularities of spaces.

MICHAEL POLYAK (TEL AVIV)

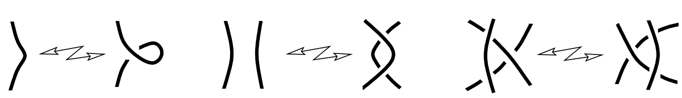
Finite type invariants of classical and virtual knots

Joint work with Mikhail Goussarov (St. Petersburg) and Oleg Viro (Uppsala)

A *virtual knot* is an equivalence class of diagrams with two types of crossings:

- *“real” crossings:* 
- *“virtual” crossings:* 

modulo the Reidemeister type moves:



[AC2] A'Campo, N.: *Generic immersions of curves, knots, monodromy and gordian number.* Publications de IHES, to appear.

[AC3] A'Campo, N.: *Planar trees, slalom curves and hyperbolic knots.* Preprint.

Antonio Campillo (Valladolid)

Geometry of singular foliations on the projective plane

Joint work with Jorge Olivares, Guanajuato/Mexico

A singular foliation on the projective plane $\mathbb{P}^2$ becomes the same object as a rational map $\phi : \mathbb{P}^2 \dashrightarrow \check{\mathbb{P}}^2$ whose graph $G_\phi \subset \mathbb{P}^2 \times \check{\mathbb{P}}^2$ is contained in $\mathbb{I} = \{(q, L) \,|\, q \in L\}$, the incidence variety. The indeterminacy ideal I of ϕ corresponds to the singular subscheme of the foliation and it defines a zerodimensional scheme which is a locally complete intersection. The support of I is the singular set S and for every $q \in S$ one has local invariants given by $\mu_q = \ell(\mathcal{O}_{\mathbb{P}^2,q}/I_q)$ and $\nu_q = \mathrm{ord}_{\mathfrak{m}_q}(I_q)$. The colength of I, $\ell(I)$, is $r^2 + r + 1$ where the integer r is the degree of the foliation, i.e., the degree of the map ϕ ($r = 0$ if ϕ is not dominant, i.e., if the foliation is a radial one). The map ϕ is given by the linear system of $\phi^*\check{\ell}$ where $\check{\ell}$ is a line in $\check{\mathbb{P}}^2$, i.e., a point q in $\mathbb{P}^2$. The degree $r + 1$ divisor $\phi^*\check{\ell}$ is called the *polar* of the pole q of $\check{\ell}$ with respect to the foliation.

We show how the linear system of polars allows to answer some questions about the singular subscheme of the foliation as the following ones:

- Compute the space $H^0(I(s))$ for every value of $s \geq 0$.
- Decide when the subscheme I determines the foliation.
- Characterize those zerodimensional subschemes which are the singular subscheme of some foliation.

We show the following results on the questions:

Theorem 1: *If I is the singular scheme of a foliation of degree r then one has $h^0(I(s)) = 0$ if $s \leq r$, $h^0(I(r+1+t)) = (t+1)(t+3)$ if $0 \leq t \leq r-1$, and $h^0(I(s)) = \frac{1}{2}(s+1)(s+2) - (r^2+r+1)$ if $s > 2r$.* Moreover, one can explicitly give a basis for the spaces $H^0(I(s))$.

Theorem 2: *If $r \neq 1$ then the singular subscheme of the foliation determines it uniquely.*

Theorem 3: *Let I be a zero-dimensional locally complete intersection subscheme of $\mathbb{P}^2$ of colength $r^2 + r + 1$. Then there exists a foliation of degree r having it as singular subscheme if and only if $h^0(I(r+1)) = 3$, the linear system given by $H^0(I(r+1))$ has no base curves and there exist bases $\{A, B, C\}$ for $H^0(I(r+1))$ and $\{L, M, N\}$ for $H^0(\mathcal{O}_{\mathbb{P}^2}(1))$ such that $LA + MB + NC = 0$.*

Theorem 4: *Let I be a zero-dimensional subscheme of $\mathbb{P}^2$ of colength 7. Then I is the singular scheme of a foliation of degree 2 if and only if one has $h^0(I'(2)) = 0$ for every subscheme I' of I of colength 6 (i.e., if there are no 6 points of I on a conic).*

Elmer G. Rees (Edinburgh)

On a question of Milnor on singularities of maps

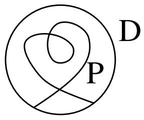
In his book on singularities, J. Milnor considered functions $f : \mathbb{R}^m, 0 \to \mathbb{R}^k, 0$ satisfying

$$(M): \quad df_x \text{ has rank } k \text{ for all small } x \neq 0 \text{ and } df_0 \text{ has rank} < k\,.$$

He proved that if f satisfies (M) then there is a fibration $S_\varepsilon \setminus K \to S^{k-1}$ where S_ε is a small sphere centred at 0 and $K = S_\varepsilon \cap f^{-1}(0)$. He asked if there are "non-trivial" examples of maps f satisfying (M) (other than complex analytic maps $\mathbb{C}^n \to \mathbb{C}$); in particular he asked: for which m, k are there examples? He gave examples based on the quaternions and on Cayley numbers.

In this talk, proofs of the following results are given:

Many knots $L(P)$ of divides D are not the local knot of a plane curve singularity. For instance, the knot of the divide



is a *fibered* knot (i.e., the complement of the knot admits a fibration over the circle) with Milnor number 4 and monodromy matrix

$$\begin{pmatrix} 3 & -1 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & -1 \end{pmatrix}$$

with trace 1. Calling a divide P *connected* if the union of its immersed copies of the interval is a connected subset of D, we have

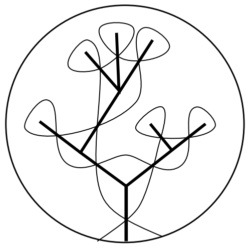
Theorem 2: *The link $L(P)$ of a connected divide P is a fibered link.*

The monodromy diffeomorphism can be constructed directly from the combinatorics of the divide. The proof of Theorem 2 uses a function $F_P : B \to \mathbb{C}$ which is along $D = \mathbb{R}^2 \cap B$ holomorphic, i.e., satisfies the Cauchy-Riemann equations. $F_P : B \to \mathbb{C}$ is constructed from a self indexing Morse function $f_P : D \to \mathbb{R}$ with $\mu = 2\delta_P - r + 1$ singularities and $f_P^{-1}(0) = P$.

Using the work of Kronheimer and Mrowka on the Thom conjecture concerning the minimal genus of smooth surfaces in $\mathbb{P}^2(\mathbb{C})$ representing d times the generator of $H_2\big(\mathbb{P}^2(\mathbb{C}), \mathbb{Z}\big)$, we have, as for links of plane curve singularities:

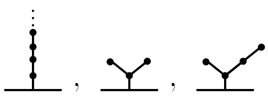
Theorem 3: *The unknotting number of the link $L(P)$ equals the number δ_P of double points of P.*

Let $n \geq 1$ be an integer. There exist infinitely many knots with unknotting number n, but only finitely many types of divides P with $\delta_P = n$. So the knots $L(P)$ with $\delta_P = n$ are a minority among the knots with unknotting number n. Given a rooted tree t there is a slalom divide P_t:



Theorem 4: *The knots of the slalom divides for the trees*

A_n :



are the knots of the singularities A_{2n}, E_6, respectively E_8. For all other rooted trees the knot of the slalom divide is a fibered hyperbolic knot.

The monodromy diffeomorphism T of the hyperbolic knots of Theorem 4 is a pseudo-Anosov diffeomorphism of a surface of genus g with one boundary component. The diffeomorphism T is the product of 2g positive Dehn twists T_i, $1 \leq i \leq 2g$, such that the union of the core cycles of the twists T_i is a spline in the surface F_g. The dual graph of the intersections of the system of core cycles corresponds to the Dynkin diagram. So as long as the Dynkin diagram is not a classical Dynkin diagram, the complement of the knot admits a complete hyperbolic metric and a fibration over the circle. The hyperbolic metric is up to an isometry unique by a celebrated rigidity theorem of Mostow.

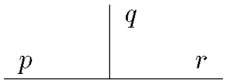
[AC1] A'Campo, N.: *Real deformations and complex topology of plane curve singularities.* Annales de la Faculté des Sciences de Toulouse, to appear.

Jonathan Wahl (Chapel Hill)

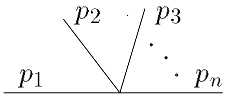
A generalization of Brieskorn homology 3-spheres

Work in progress with Walter Neumann

Let $p, q, r \geq 2$ pairwise relatively prime. The Brieskorn singularity $\{x^p + y^q + z^r = 0\} \subset \mathbb{C}^3$ has link $\Sigma(p, q, r)$ which is an integral homology 3-sphere ("IHS"); setting a coordinate to 0 gives a knot in Σ. We summarize the data of the IHS and the 3 knots by the *"splice diagram"* of Eisenbud/Neumann:



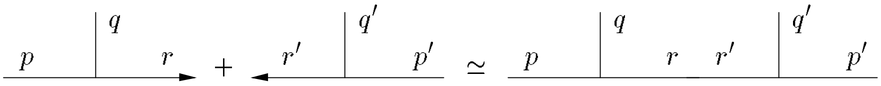
Similarly, one has Brieskorn complete intersections:



We study Gorenstein singularities – or even ICIS's – whose link is an IHS. We have an old

Conjecture: [Neumann/Wahl, 1987] *Given an ICIS whose link is an IHS Σ. Then the Casson invariant of Σ equals $\frac{1}{8}$ times the signature of the Milnor fibre.*

This was proved by us in the late '80's for a number of examples, including Brieskorn complete intersections, and singularities $z^n = f(x, y)$ (f irreducible, n relatively prime to the characteristic pairs). Examples were hard to find; topologically, they arise by "splicing" diagrams as above, e.g., in the simplest case



Which occur as links of singularities?

Theorem 1: *Suppose $r \in \langle p', q' \rangle$ and $r' \in \langle p, q \rangle$, say $r = \alpha q' + \beta p'$, $r' = \alpha' q + \beta' p$. Then*

$$\begin{cases} x^p + y^q = u^\alpha v^\beta \\ u^{p'} + v^{q'} = x^{\alpha'} y^{\beta'} \end{cases}$$

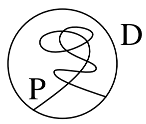
is an ICIS with IHS link as above.

Theorem 2: *For the examples in Theorem 1, the above Conjecture is true.*

We have in fact a general algorithm which associates to any splice diagram with "semi-group condition" (as for r, r' above) a system of equations which ought to be an ICIS whose link is the IHS described by the diagram. In many cases (so far), we have verified the above Conjecture. We conjecture the semi-group conditions are necessary.

Norbert A'Campo (Basel)

Real deformations and complex topology



A *divide* P is a generic system of immersions of copies of $[0, 1]$ in the unit disk $D \subset \mathbb{R}^2$ with $\partial([0, 1]) \subset \partial D$. To a divide P we associate a link in the 3-sphere in $\mathbb{R}^4 = \mathbb{R}^2 + \mathbb{R}^2 = \mathbb{R}^2 + i\mathbb{R}^2$ by

$$L(P) := \left\{(x, u) \in TD = D \times \mathbb{R}^2 \;\middle|\; x \in P,\, u \in T_x P,\, \|x\|^2 + \|u\|^2 = 1\right\}.$$

Links of plane curve singularities are of this type by

Theorem 1: *Let $f : \mathbb{C}^2 \to \mathbb{C}$ have an isolated singularity at 0 such that the unit ball $B \subset \mathbb{C}^2$ is a Milnor ball for the singularity. Assume that the local branches have real parametrizations $p_i : \mathbb{C} \to \mathbb{C}^2$, $i = 1, \dots, r$, i.e., $p_i(\mathbb{R}) \subset \mathbb{R}^2$. Let $\overline{p}_i : \mathbb{R} \to \mathbb{R}^2$ be a generic small deformation such that the system $\bigcup_{i=1}^r \overline{p}_i(\mathbb{R}) \cap D$ is a divide P in D, with δ_P double points, $\mu(f) = 2\delta_P - r + 1$. Then the links $L(P)$ and $\partial B \cap \{f = 0\}$ are equivalent.*

with normal crossings in a complex 2-dimensional manifold Z. Given a $\vec{v} \in (T_0\Delta)^*$ and $\vec{w}$, a tangent vector at the strict preimage in $h^{-1}(0)$, it is possible to find a notion of paths and homotopies in the central fibre of h such that the corresponding homotopy classes form a group $\pi_1(Z_{\vec{v}}, \vec{w})$ which is isomorphic to π_1(Milnor fibre). We call this group the *nearby fundamental group*, and it is possible to define certain iterated integrals along its elements. Let $\bar{\mathcal{J}} \subset \mathbb{Z}\pi_1(Z_{\vec{v}}, \vec{w})$ denote the augmentation ideal in its group ring.

Theorem: *There is a natural $\mathbb{Z}$-MHS on $\mathbb{Z}\pi_1(Z_{\vec{v}}, \vec{w})/\bar{\mathcal{J}}^{s+1}$ for $s \geq 1$, and the local system of $\mathbb{Z}$-MHSs*

$$\left\{\mathbb{Z}\pi_1(Z_{\vec{v}}, \vec{w})/\bar{\mathcal{J}}^{s+1}\right\}_{\vec{v}\in(T_0\Delta)^*}$$

is a nilpotent orbit of MHSs on $(T_0\Delta)^*$ whose monodromy is induced by the geometric monodromy of h.

Finally we sketched ideas about a more intrinsic way to define the nearby homotopy group using logarithmic structures in the sense of Kato, Illusie, Fontaine. We suggested a construction similar to the real blow-up construction of A'Campo. But here the points on a divisor with normal crossings are replaced by algebraic tori instead of real ones. We call this construction the *vector blow-up.*

Victor A. Vassiliev (Moscow)

Order complexes of singular sets and topology of spaces of nonsingular projective varieties

I present a method of computing homology groups of the spaces $N(d,n)$ of all nonsingular hypersurfaces of degree d in $\mathbb{C}P^n$, or, equivalently, of the spaces $\Pi(d,n)\setminus\Sigma$ of all homogeneous (of degree d) polynomials $\mathbb{C}^{n+1} \to \mathbb{C}$ defining such nonsingular hypersurfaces. This problem is the natural "complexification" of the rigid isotopy classification of real algebraic hypersurfaces, and our method can be applied also to this problem. If $d=2$, then $\Pi(d,n)\setminus\Sigma$ is homotopy equivalent to the Lagrangian Grassmannian $U(n+1)/O(n+1)$. For some other pairs (d,n) we have the following formulae for the Poincaré polynomials $P_{d,n}$ of the groups $H^*\big(N(d,n),\mathbb{Q}\big)$:

$$P_{3,2} = (1+t^3)(1+t^5), \quad P_{3,3} = (1+t^3)(1+t^5)(1+t^7), \quad P_{4,2} = (1+t^3)(1+t^5)(1+t^6).$$

Conjecture: *For any n: $H^*\big(N(3,n),\mathbb{Q}\big) \simeq H^*\big(PGL(n+1,\mathbb{C}),\mathbb{Q}\big)$.*

On the other hand, I don't know any explicit realization of the 6-dimensional generators of $H^*\big(N(4,2)\big)$.

The method of calculation is based on the study of the *discriminant variety* Σ consisting of singular polynomials (whose locally finite homology group is related to $H^*\big(\Pi(d,n)\setminus\Sigma\big)$ via the Alexander duality). The main technical tools are the *conical resolutions* of discriminant varieties (which are a far generalization of the inclusion-exclusion formula, and also of *simplicial resolutions* used in algebraic geometry) and the *topological order complex* of singular sets. Our calculations lead to many natural and beautiful problems of algebraic geometry and topology, in particular on the classification of all possible singular sets of hypersurfaces of a given degree, and on homology groups of such classes.

[Vas] V.A. Vassiliev: *How to calculate homology groups of spaces on nonsingular algebraic projective varieties.* Proc. Steklov Math. Inst. (1999).

András Szücs (Budapest)

Elimination of singularities

Our aim is to generalize the following two theorems:

Theorem: [Whitney] *If M^2 is a closed surface and $f : M^2 \to R^2$ is stable smooth, then $\#\Sigma^{1,1} \equiv \chi(M^2)$.*

Theorem: [Eliashberg, Levine] *If $\chi(M^2)$ is even, then f is homotopic to a $\Sigma^{1,1}$-free map.*

Problem: Let M^n, P^{n+k} be smooth manifolds, $f : M^n \to P^{n+k}$ a stable map, P stably parallelizable, η a maximal singularity of f. Is then f cobordant to an η-free map?

We require also that the cobordism has no more complicated singularities than f:

1. *Primary obstructions* (necessary conditions): The Gysin map f_* must annihilate the Thom polynomial associated to η, and also all the higher Thom polynomials. (The latter express the cohomology class in $H^*(M^n)$ dual to the characteristic cycles of the η-stratum.)
2. *Secondary obstructions* (sufficient conditions): The Postnikov invariants of some classifying spaces occuring in [R-Sz]. These always have finite order.

From this we get the following

Corollary: *Let f, M, P, η be as above and suppose that all primary obstructions vanish. Then there exists an integer $L \neq 0$ such that Lf is cobordant to an η-free map by a cobordism having no more complicated singularities than those of f.*

There are examples showing that secondary obstructions do exist and so the map f itself may not be cobordant to an η-free map. The main tool in the proof is the article

[R-Sz] Rimanyi, R.; Szücs, A.: *Pontrjagin-Thom-type construction for maps with singularities.* Topology (1998).

Yosi Yomdin (Rehovot)

Center-focus problem, composition of polynomials, moments, etc.

Let be given a polynomial vectorfield on the plane $\mathbb{R}^2$,

$$(*) \qquad \begin{cases} \dot{x} = -y + F_1(x,y)\,, \\ \dot{y} = \ \ x + F_2(x,y)\,. \end{cases}$$

The classical Poincaré Centre-Focus problem asks for explicit conditions on the coefficients of F_1 and F_2 for $(*)$ to have a centre at the origin, that is, to have all the trajectories near the origin closed. We discuss relations of the problem with some questions in analysis, algebra, and, in particular, in singularity theory. The following example may be illustrative:

Example: Consider an elliptic curve in $\mathbb{C}^2$ given by $z_1^2 + z_2^3 + c = 0$, $c \neq 0$. Let P and Q be two polynomials in $\mathbb{C}[x,y]$. Give conditions on P and Q for the vanishing of all the "moments" $m_{ij} = \int P^i Q^j dP$, $m'_{ij} = \int P^i Q^j dQ$ (i.e., conditions for the forms $P^i Q^j dP$ and $P^i Q^j dQ$ to be exact *for any* i, j).

Rainer Kaenders (Düsseldorf)

On De Rham homotopy theory for plane curve singularities

Given a plane curve singularity $f : (\mathbb{C}^2, 0) \to (\mathbb{C}, 0)$ one can associate to it a mixed Hodge structure on the fundamental group of the Milnor fibre. For the construction of this mixed Hodge structure (MHS) we need a way to represent the fundamental group of the Milnor fibre on the central fibre. To do this, we first consider instead of f its semistable reduction $h : (Z, D^+) \to (\Delta, 0)$ over a disk $\Delta \subset \mathbb{C}$, where the central fibre $h^{-1}(0)$ is a reduced divisor

Vortragsauszüge

Wolfgang Ebeling (Hannover)

Singular moonshine

We consider a normal surface singularity (X, x) with $\mathbb{C}^*$-action which is smoothable and the Milnor fibre compactifies (after resolution of singularities) to a minimal K3 surface. By a result of H. Pinkham, the minimal good resolution of (X, x) looks as follows: There is one central curve E of genus g which is intersected by n rational curves $E_1, \ldots, E_n$. This is equivalent to (X, x) being a *Fuchsian singularity.*

We assume that the embedding dimension of (X, x) is less than or equal to 4. These cases were classified by I. Dolgachev and P. Wagreich. One gets 31 isolated hypersurface singularities and 26 icis in $(\mathbb{C}^4, 0)$. The characteristic polynomial of the classical monodromy operator of (X, x) can be written in the form

$$\phi(\lambda) = \prod_{m|h} (\lambda^m - 1)^{\chi_m} \quad \text{for } \chi_m \in \mathbb{Z} \text{ and for some } h \in \mathbb{N}.$$

For this we use the symbolic notation (*"Frame shape"*) $\pi = \prod_{m|h} m^{\chi_m}$. K. Saito has defined a dual Frame shape

$$\pi^* = \prod_{k|h} k^{-\chi_{h/k}}.$$

Theorem: *The symbol $\pi\pi^*$ is a 24-dimensional self-dual Frame shape which is the Frame shape of an automorphism of the Leech lattice.*

There are 28 self-dual Frame shapes of the automorphism group of the Leech lattice with trace > 1. By our construction we get 24 of these symbols. To a Frame shape one can associate an η-product

$$\eta_\pi(\tau) = \prod_{m|h} \eta(m\tau)^{\chi_m}, \quad \tau \in \mathbb{H},$$

where η is the Dedekind η-function. It turns out that in our cases $\eta_{\pi\pi^*}$ is a modular function for a discrete subgroup Γ of $\mathrm{PSL}(2, \mathbb{R})$ containing $\Gamma_0(h)$. The genus of Γ is zero and $\eta_{\pi\pi^*}$ is a generator of the function field of Γ.

Claus Hertling (Toulouse/Bonn)

Frobenius manifolds and moduli spaces for hypersurface singularities

Theorem: *The set of right equivalence classes of isolated hypersurface singularities in one μ-homotopy class carries a natural complex structure.*

More precisely, it is an analytic geometric quotient for the action of the algebraic group of k-jets ($k \geq \mu + 1$) of coordinate changes on the algebraic variety of k-jets of functions in one μ-homotopy class.

The proof uses work of K. Saito and M. Saito: they proved that the base of a semiuniversal unfolding of an isolated hypersurface singularity can be equipped with the structure of a Frobenius manifold. The metric corresponds to a primitive form of K. Saito, and, by work of M. Saito, to an opposite filtration for Steenbrinks Hodge filtration on the cohomology of the Milnor fibre. M. Saito makes use of the Brieskorn lattice and a result of Malgrange. The multiplication on the tangent bundle was defined by K. Saito with the Kodaira-Spencer map. It is worth to be studied for itself. Manin and I observed that it satisfies the following (most natural integrability) condition:

$$\mathrm{Lie}_{X \circ Y}(\circ) = X \circ \mathrm{Lie}_Y(\circ) + Y \circ \mathrm{Lie}_X(\circ)$$

for any two local vector fields. A manifold with such a multiplication is called an *F-manifold.* It has many nice properties of which I could mention only a few in the talk.

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1999

Singularitäten

02.05. – 08.05.1999

The conference was organized by G.-M. Greuel (Kaiserslautern), J.H.M. Steenbrink (Nijmegen) and V.A. Vassiliev (Moscow) and attended by about 50 participants from Europe, North America, Israel and Japan. Singularity theory showed to be a very vivid subject. Certain breakthroughs have taken place and it was interesting to see how this has developed during the last years. Moreover, singularity theory has opened to neighbouring fields, connecting them in various ways. This was represented during the conference in 24 talks which were devoted, besides singularity theory itself, to mathematical physics, knot theory, representation theory, general and differential topology, dynamical systems, Frobenius manifolds, algebraic geometry, combinatorics and computer algebra. Besides the official talks demonstrations of the computer algebra system SINGULAR and about image processing have taken place, as well as there was a special session about the Gauß-Manin connection.