Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 21/1999

Quadratische und Hermitesche Formen

16.05.-22.05.1999

An der Tagung, welche unter der Leitung von A. Pfister (Mainz), W. Scharlau (Münster) und J.P. Tignol (Louvain-la-Neuve) stattfand, nahmen über vierzig Mathematiker aus dreizehn verschiedenen Ländern teil.
Im Mittelpunkt stand die klassische Theorie der quadratischen Formen über Körpern bzw. Ringen und damit zusammenhängende Gebiete, wie zum Beispiel die Theorie der Algebren mit Involution. Dabei wurden auch Folgerungen aus der Milnor-Vermutung dargelegt, und neuere Resultate über die Rolle der quadratischen Formen (speziell von Pfisterformen) in der Galoiskohomologie vorgestellt. Es gab aber auch Vorträge über Wittgruppen von Schemata und über Wittgruppen von triangulierten Kategorien, sowie über die topologisch motivierten L-Gruppen. Auch Ergebnisse aus der relativ neuen unverzweigten Kohomologie wurden erörtert.
Neben den Hauptvorträgen wurden, wie bei Tagungen am Forschungsinstitut in Oberwolfach üblich, auch viele neuere Entwicklungen und Probleme aus der Theorie der quadratischen und hermiteschen Formen in kleineren Gruppen diskutiert. Es gab also nicht nur das "offizielle" Vortragsprogramm sondern auch eine ganze Reihe von spontan organisierten Vorträgen.

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Vortragsauszüge

J.K. Arason

Some consequences of Voevodsky's results for the $I^n(K)$

This talk is about some applications of the recent far reaching results of Voevodsky together with Orlov and Vishik. These applications were gotten last year in collaboration with R. Elman.
Let K be a field, $\operatorname{char} K \neq 2$. Let ω be a k-fold Pfister form over K. For every $n \geq 1$ we then have that $I^{n+k}(K) \cap W(K)\omega = I^n(K)\omega$. Denoting by $I_\omega(K)$ the annihilator of ω we also have that $I^n(K) \cap I_\omega(K) = I^{n-1}(K)I_\omega(K)$. Using this for $\omega =< 1,1 >^k$ it follows, for a formally real K,

that if $\varphi \in W(K)$ has total signature divisible by 2^n then $\varphi \in I^n(K) + I_{tor}(K)$. It also follows that $I^n(K) \cap I_{tor}(K) = I^{n-1}(K) I_{tor}(K)$.
It is also proved that for $n \geq 2$ the additive group $I^n(K)$ is generated by the class $[\rho]$ of n-fold Pfister forms subject only to the relations

$$[\varphi] = 0 \quad \text{if} \quad \varphi = 0 \quad \text{in} \quad W(K)$$

and

$$[\ll a, b, c \gg \otimes \omega] + [\ll a, b \gg \otimes \omega] = [\ll ac, b \gg \otimes \omega] + [\ll a, c \gg \otimes \omega]$$

for any $a, b \in K^*$ and any $n-2$-fold Pfister form ω.

P. Balmer

A 12-term localization exact sequence

Theorem 1 (Main theorem) *Consider an exact sequence of triangulated categories with duality :*

$$0 \longrightarrow J \xrightarrow{j} K \xrightarrow{q} L \longrightarrow 0$$

such that $\frac{1}{2} \in K$ and such that K and L are noetherian. Then there exists a periodic exact sequence of Witt groups :

$$\begin{array}{ccccccccc}
W^3(L) & \xrightarrow{\partial^3} & W(J) & \xrightarrow{W(j)} & W(K) & \xrightarrow{W(q)} & W(L) & \xrightarrow{\partial^0} & W^1(J) \\
{\scriptstyle W^3(q)}\uparrow & & & & & & & & \downarrow{\scriptstyle W^1(j)} \\
W^3(K) & & & & & & & & W^1(K) \\
{\scriptstyle W^3(j)}\uparrow & & & & & & & & \downarrow{\scriptstyle W^1(q)} \\
W^3(J) & \xleftarrow[\partial^2]{} & W^2(L) & \xleftarrow[W^2(q)]{} & W^2(K) & \xleftarrow[W^2(j)]{} & W^2(J) & \xleftarrow[\partial^1]{} & W^1(L)
\end{array}$$

Explanations: The *Witt group* of $(K, \#)$ is then simply the quotient

$$W(K) = W(K, \#) = \frac{\text{isometry classes of symmetric spaces}}{\text{those spaces possessing a lagrangian}}.$$

W^n means that the duality $\#$ is replaced by $T^n \circ \#$, where T is the translation functor of K. In other words, the *higher and lower* Witt groups are simply Witt groups for the *shifted* duality : $W^n(K) = W(K, T^n \circ \#, (-1)^{\frac{n(n+1)}{2}} \cdot \varpi)$. It is easy to see that $W^n \simeq W^{n+4}$ for all $n \in \mathbb{Z}$.

Theorem 2 *Let X be a scheme such that $\frac{1}{2} \in \mathcal{O}_X$. Consider $D^b_{lf}(X)$ to be the derived category of bounded complexes of locally free $\mathcal{O}_X$-modules of finite type (perfect complexes). This is equipped with an obvious duality. The natural homomorphism :*

$$W_{usual}(X) \longrightarrow W\big(D^b_{lf}(X)\big)$$

is an isomorphism. In other words, the usual Witt group can be computed using triangulated categories.

Application to the Gersten complex:

Theorem 3 *Let* $K = D^0 \supset D^1 \supset \cdots \supset D^n \supset D^{n+1} = 0$ *be a filtration of triangulated categories with duality such that every possible quotients are noetherian and such that* $\frac{1}{2} \in K$. *Then there is a converging spectral sequence :*

$$E_1^{p,q} = \begin{cases} W^{p+q}(D^p/D^{p+1}) & \textit{if} \quad 0 \le p \le n \\ 0 & \textit{otherwise} \end{cases} \Longrightarrow W^{p+q}(K)\,.$$

Example 4 *Taking a regular separated scheme* X *of dimension* n *and filtrating* $D^b_{lf}(X)$ *by the codimension of the support of the homology, we obtain the Gersten complex as the* $q = 0$ *line in* E_1. *Very easy considerations give in particular the following new results :*

Theorem 5 *Let* X *be a separated regular integral scheme of dimension* ≤ 4, *such that* $\frac{1}{2} \in \mathcal{O}_X$ *then there exists an exact sequence :*

$$\sum_{x \in X^{(3)}} W(k(x)) \longrightarrow \sum_{x \in X^{(4)}} W(k(x)) \longrightarrow W(X) \longrightarrow W(K) \longrightarrow \sum_{x \in X^{(1)}} W(k(x))$$

where K *is the field of fractions of* X. *In particular, Purity holds for such schemes when dimension is lower or equal to 3.*

The above theorem is a lazy use of the spectral sequence which seems to lead to even more interesting results around the Gersten conjecture.
Application of the localization exact sequence to localization of regular schemes is also of some interest.

E. Becker

Complexity of Hilbert‘s 17th problem

This is a report on the work of J. Schmid (Dortmund/Konstanz).

Theorem 1 *Let* k *be a formally real field with finite Pythagors number* $P_2(k) = P(k)$. *Then there exists a n-fold exponential bound*

$$N = 2^{2^{\cdot^{\cdot^{\cdot^{2^{Cnd^4}}}}}} \Bigg\} n\text{–}\textit{steps}$$

where C *is a constant,* $C \le \max(6, P(k))$ *so that every polynomial*

$$f \in k[X_1, \dots, X_n] \cap \sum k(X_1, \dots, X_n)^2$$

of degree d *has a representation of the type*

$$g^2 f = \sum_{i=1}^{r} h_i^2 \qquad \textit{where} \quad g, h_i \in k[X_1, \dots, X_n]$$

and $r, \deg g, \deg h_i \leq N$.

A corresponding result for the complexity of the real Nullstellensatz is also stated. The proof uses ideas of Daykin (1961) added by quadratic form theory. At present, on the basis of publications, this is the best known bound. Lombardi/Roy have announced a 5-fold exponential bound.

A. Ducros

Rational points over the function field of a real curve

Let R be a real closed field, let C be a smooth, projective, integral R-curve and let K denote its function field. In my thesis, I studied the following problem: let X be a proper, smooth, integral K-variety. If $\prod_P X(K_P) \neq \emptyset$ (here P is running through the set of closed points of C), does X have a K-rational point (if not X is said to be a *counter-example to the Hasse principle*) ? The answer is positive if X is a principal homogeneous space under a connected linear group (the general case is due to Scheiderer); for conic bundles over $\mathbb{P}^1_K$, the Hasse principle doesn't hold, but *if* R *is archimedean*, then the obstruction to the existence of a K-point (provided $X(K_P)$ is non-empty for any P) can be exactly described, in a cohomological as well as in a semi-algebraic geometrical way. But it fails if R is non-archimedean, and I'm now trying to understand this situation; more precisely, I hope that, using some suitable (non-discrete) Krull-valuation on K, the problem can be reduced to a similar question with an archimedean ground field. I gave a positive answer to a (far more elementary) related problem: if X is any proper, integral, smooth variety over a real closed field R without R-point, does it exist a model of X over the ring of archimedean elements of R, whose special fibre has no rational point ? I proved a more general result about the special fibre of a scheme over a valuation ring, using Zariski's description of the valuation spectrum as an inverse limit of proper models.

J. Hurrelbrink

Splitting patterns of quadratic forms

Let k be a field, $\operatorname{char} k \neq 2$. Let q denote an anisotropic quadratic form over k of even dimension.
A quadratic form q is of height 1 if and only if it is similar to a Pfister form over k. In small dimensions, we have compiled the list of splitting patterns of quadratic forms of height 2. We discussed the conjecture on the structure of quadratic forms height 2.

O.T. Izhboldin

Quadratic Form with Maximal Splitting

Let ϕ be an anisotropic quadratic form over a field F. The *first Witt index of* ϕ is defined as $i_1(\phi) := i_W(\phi_{F(\phi)})$ where i_W is the (usual) Witt index. Let us write $\dim \phi$ in the form $\dim \phi = 2^n + m$, where $1 \leq m \leq 2^n$. In [1] Hoffmann proved that $i_1(\phi) \leq m$. We say that ϕ has maximal splitting if $i_1(\phi) = m$. All Pfister neighbors and all forms of dimension $2^n + 1$ have maximal splitting. These examples present exhaustive list of forms with maximal splitting of dimension ≤ 16 except for the case $\dim \phi = 10$. The cases where $\dim \phi = 10$ or $\dim \phi > 16$ are much more complicated. We discuss the following conjectures

Conjecture 1 *Let ϕ be an anisotropic 10-dimensional. Then ϕ has maximal splitting only in the following cases: (i) ϕ is a Pfister neighbor; (ii) ϕ is divisible by a binary form.*

Conjecture 2 *Let $n \geq 4$ be an integer and ϕ be an anisotropic form over F such that $2^{n-1} + 2^{n-3} < \dim \phi \leq 2^n$. Then ϕ has maximal splitting if and only if* $\ker(H^n(F) \to H^n(F(\phi))) \neq 0$.

Applying [2] and [3], we prove these conjectures for all fields F of characteristic zero satisfying condition $\sqrt{-1} \in F$.

[1] Hoffmann, D. W. *Isotropy of quadratic forms over the function field of a quadric.* Math. Z. **220** (1995), 461–476.

[2] Vishik, A. *Integral motives of quadrics.* Max Planck Institut Für Mathematik, Bonn, preprint MPI-1998-13, 1–82.

[3] Orlov, D., Vishik, A., Voevodsky, V. *Motivic cohomology of Pfister quadrics and Milnor's conjecture on quadratic forms*, preprint.

P.Jaworski

On the strong Hasse principle for fields of quotients of power series rings in two variables

Let k be an algebraically closed field of characteristic different from 2 and $K = k((x, y))$ the field of quotients of the ring of formal power series over k.
In my talk I sketched the proof of the Strong Hasse Principle for this field, i.e. of the following property:
a quadratic form over K which is isotropic over completions of K with respect to every discrete valuation, is isotropic over K.
The crucial point of the proof is to show that:
For every quadratic extension K_d of K there exists a regular scheme $\mathcal{Z}$ such that:
- *K_d is the function field of $\mathcal{Z}$,*
- *every 2-fold Pfister form which belongs to the kernels of second residue homomorphisms induced by all prime divisors of $\mathcal{Z}$ is hyperbolic.*

B.Kahn

Unramified cohomology of quadrics: the methods

This talk gave the methods behind the results explained in Sujatha's talk. They involve etale motivic cohomology of a quadric: the coniveau (or Bloch-Ogus) spectral sequence and a "weight" spectral sequence. For some results we use Voevodskys motivic Steenrod operations.

N.A. Karpenko

On anisotropy of orthogonal involutions

Let F be a field (of any characteristic), D a central division F-algebra of degree a power of a prime p, X the Severi-Brauer variety of D, and n the dimension of X.
For any element α of the Chow group $\mathrm{CH}^n(X \times X)$, the integer $d_1(\alpha)$ is defined by the equality (cf. [1], Example 16.1.4) $(pr_1)_{*(\alpha)} = d_1(\alpha) \cdot [X]$, where $(pr_1)_*$ is the push-forward with respect to the projection pr_1 of the product $X \times X$ onto the first factor. Using the second projection pr_2 instead of pr_1, one defines the integer $d_2(\alpha)$.

Proposition 1 *For any $\alpha \in \mathrm{CH}^n(X \times X)$, there is a congruence*

$$d_1(\alpha) \equiv d_2(\alpha) \pmod{p} .$$

The proof of Proposition **??** is based on the following

Lemma 2 ([2], Prop 2.1.1) *There exists a field extension E/F such that for any $\alpha \in \mathrm{CH}^i(X)$ with $i > 0$ the element $\alpha_E \in \mathrm{CH}^i(X_E)$ is divisible by p.*

Now we assume that $p = 2$, that char $F \neq 2$, and that D is supplied with an orthogonal involution σ.

Theorem 3 *The involution $\sigma_{F(X)}$ of the split $F(X)$-algebra $D_{F(X)}$ is anisotropic.*

Proof Let Y be the involution variety of σ ([3]). We recall that Y is a closed subvariety of X of codimension 1.
Suppose that the involution $\sigma_{F(X)}$ is isotropic. Then there is a rational morphism $f\colon X \to Y$. Let α be the element of $\mathrm{CH}^n(X \times X)$ given by the closure of the graph of f. We have $d_1(\alpha) = 1$. Since f is not dominant, we have $d_2(\alpha) = 0$, a contradiction with Proposition **??**. □

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[2] N. A. Karpenko. *Grothendieck's Chow motives of Severi-Brauer varieties*, St. Petersburg Math. J. **7** (1996), no. 4, 649–661

[3] D. Tao. *A variety associated to an algebra with involution*, J. Algebra **168** (1994), no. 2, 479–520

M. Knebusch

Specialization and generic splitting in the presence of characteristic 2

Let $\lambda : K \longrightarrow L \cup \infty$ be a place, o its valuation ring, k its residue class field. We call quadratic o-module $M = (M, q)$ non degenerate, if M has finite rank, the bilinearform B_q associated with q is nondegenerate on $M/QL(M)$, and every primitiv vector x of $QL(M)$ has value $q(x)$ in o^*. Based on this notion we define "good reduction" (GR) and "fair reduction" (FR) of nondegenerate quadratic forms φ over K with respect to λ.
We say a form φ over K has GR if $\varphi \simeq (E, q)$ with $E \simeq K \otimes_o M$, M nondegenerate over o, and has FR if $M \otimes_o k$ is nondegenerate over k, and in both cases put

$$\lambda_*(\varphi) = \text{the form corresponding to } L \otimes_{o,\lambda} M \,.$$

The general principles of generic splitting theory can be established in a satisfactory way using GR, but FR is needed to lift nondegenerate forms over a field of $\mathrm{char}\, 2$ to a field of $\mathrm{char}\, 0$. Happily generic splitting also works with FR.

D. Lewis

Classification theorems for central simple algebras with involution

The involutions in this paper are algebra anti-automorphisms of period two. Involutions on endomorphism algebras of finite-dimensional vector spaces are adjoint to symmetric or skew-symmetric bilinear forms, or to hermitian forms. Analogues of the classical invariants of quadratic forms (discriminant, Clifford algebra, signature) have been defined for arbitrary central simple algebras with involution. In this paper it is shown that over certain fields these invariants are sufficient to classify involutions up to conjugation. For algebras of low degree a classification is obtained over an arbitrary field.

J. Minac

Field theory and the cohomology of some Galois groups

Let F be a field whose characteristic is not 2. We write $F^{(2)}$ for the field obtained by adjoining all of the square roots of the elements of F. Now let $F^{\{3\}} = (F^{(2)})^{(2)}$ and $G_F^{(2)} = Gal(F^{(2)}/F)$. Recall that $F^{(3)}$ is the extension of $F^{(2)}$ obtained by adjoining the square roots of the elements $\alpha \in F^{(2)}$ such that $F^{(2)}(\sqrt{\alpha})$ is Galois over $F^{(2)}$. $F^{(3)}$ is a Galois extension of F and we recall that the w-group of F is the Galois group $g_F = Gal(F^{(3)}/F)$. Using the surjectivity part of Merkurjev‘s theorem $k_2(F) \simeq H^2(F)$ we show that $F^{(3)} = F^{\{3\}}$ iff F is a C-field.
Set $J = (F^{(2)})^*/((F^{(2)})^*)^2$, and

$$J_1 = J^{G_F^{(2)}}, \dots, \; (J/J_i)^{G_F^{(2)}} = J_{i+1}/J_i\,, \dots,$$

so that $0 \subset J_1 \subset J_2 \subset \dots \subset J$. In this talk I discussed a result reminiscent of Hilbert‘s theorem 90 which gives a description of the $G_F^{(2)}$-module J_2 depending on the knowledge of g_F. This talk was a report on part of my current joint work with A. Adem, W. Gao and D. Karaguenzian.

J.P. Monnier

Signature Map and Unramified Cohomology

Let X be a variety of dimension d over a real closed field R. The total signature homomorphism $\Lambda : W(X) \to Cont(X(R), \mathbf{Z})$ is defined on the Witt ring of X with values in the set of continuous functions from the real points of X to $\mathbf{Z}$. Parimala and Mahé asked the question whether the exponent of the cokernel of Λ is bounded by 2^{d+1}, for smooth varieties of dimension d over R. We prove that this is true when $d \leq 4$ and more generally in any dimension if the Witt group of X satisfies a purity theorem. Moreover, for smooth surfaces we prove that $4.Cont(X(R), \mathbf{Z}) \subseteq Im\Lambda$ if and only if the graded Witt ring is isomorphic to the graded unramified cohomology ring. For real rational surfaces, we show that the previous condition is fulfilled and we describe completely the image of Λ.

J.F. Morales

Lie algebras and quadratic forms

We show that the Hasse-Witt invariant of the Killing form of a semisimple Lie-algebra L of inner type is essentially given by the Tits invariant of the irreducible representation of L with highest weight one-half of the sum of the positive roots. For Lie algebras of external type the Hasse-Witt invariant of the Killing form depends also on cohomological invariants attached to a two-fold extension of the symmetry group of the Dynkin diagram of L. Specializing L suitably one can derive from these general results known formulae for trace forms of étale algebras, central simple algebras and central simple algebras with involution.

R. Parimala

Hasse principle for classical groups over function fields of curves over p-adic number fields

(Joint work with R. Preeti)
Let k be a number field, Ω a set places, $\Gamma = Gal(\overline{k}/k)$, $\Gamma_v = Gal(\overline{k_v}/k_v)$, $v \in \Omega_k$. Let G be a semisimple simply connected linear algebraic group defined over k. We prove that the kernel of the map

$$H^1(\Gamma, G) \longrightarrow \prod_{v \in \Omega_k} H^1(\Gamma_v, G)$$

is trivial in the following cases:

1. G is of type B_n, C_n, D_n, G_2, F_4.
2. G is of type A_n and is of the form $SU(h)$, h a hermitian form over a divison algebra of prime degree with an involution of second kind.

If $k = k(X)$ is a rational function field in one variable, for G any semisimple simply connected linear algebraic group defined over k, the corresponding Hasse kernel is trivial.

A. Ranicki

Algebraic transversality

The L-groups $L^n(A)$ $(n \in \mathbb{Z})$ of a ring with involution A are the cobordism groups of A-module chain complexes C with an n-dimensional Poincaré duality $H^{n-*}(C) \cong H_*(C)$. The 0th L-group $L^0(A)$ is just the Witt group of A. 'Algebraic transversality' is an algebraic analogue for chain complexes with Poincaré duality of the geometric transversality of manifolds, generalizing the Higman linearization trick for matrices. Among other things, algebraic transversality can be used to prove a Künneth formula for the L-groups of a Laurent polynomial extension $A[z, z^{-1}]$ (with involution $\overline{z} = z^{-1}$) of the type

$$L^n(A[z, z^{-1}]) \;=\; L^n(A) \oplus L^{n-1}(A)$$

and a Mayer-Vietoris exact sequence for the L-groups of an amalgamated free product $A = A_1 *_B A_2$ of the type

$$\cdots \to L^n(B) \to L^n(A_1) \oplus L^n(A_2) \to L^n(A) \to L^{n-1}(B) \to \dots \,.$$

Results of this kind were first proved geometrically for group rings, using surgery theory.

References:

1. "Exact sequences in the algebraic theory of surgery", Mathematical Notes 26, Princeton (1982)
2. "Algebraic L-theory and topological manifolds", Tracts in Mathematics 102, Cambridge (1992)
3. "On the Novikov conjecture", Proc. 1993 Oberwolfach Conference on the Novikov Conjectures, Rigidity and Index Theorems, Vol. 1, LMS Lecture Notes 226, 272–337, Cambridge (1995)
4. "High-dimensional knot theory", Springer Monograph in Mathematics (1998)

U. Rehmann

Galois cohomological characterization of certain anisotropic orthogonal groups

For orthogonal groups of excellent quadratic forms a natural definition of Galois cohomological invariants in all dimensions can be given, which allows to characterize, modulo Milnor's conjecture, these groups up to isomorphy. These examples contain anisotropic groups of arbitrarily high dimensions.

J.P. Serre

The role of Pfister forms in Galois cohomology

Several examples of Pfister forms associated with $H^1(k, G)$ were given for:

$$G = PGL_2,\ G_2,\ F_4,\ PGL_4,\ \mathrm{Spin}_7,\ \mathrm{Spin}_8 \text{ and } E_8\,.$$

Unpublished results of Rost, and of Rost-Serre-Tignol, were discussed, in connection with the recent study of "essential dimension" by Reichenstein-Youssin (to appear).

R. Sujatha

Unramified cohomology of quadrics : Results

References: B. Kahn, M. Rost, R. Sujatha, Unramified Cohomology of quadrics, American Journal of Math. 120 (1998), 841-891.
B. Kahn, R. Sujatha, Unramified cohomology of quadrics II, III, preprints.

Let F be a field (char $F \neq 2$), q a quadratic form over F, X_q the associated smooth projective quadric, and $F(X_q)$ the function field. We list the results proved on $\ker \eta^i$, $\operatorname{coker} \eta^i$, χ^i $(i \leq 4)$ where these are the maps:

$$\eta^i : H^i(F, \mathbb{Q}/\mathbb{Z}(i-1)) \longrightarrow H^i_{nr}(F(X_q), \mathbb{Q}/\mathbb{Z}(i-1))$$

and

$$\chi^i : I^i(F) \longrightarrow I^i_{nr}(F(X_q)) \,.$$

We prove that these maps are surjections for anisotropic Pfister quadrics. The results are joint work with Kahn-Rost, and some others with B. Kahn, who reported the methods used to prove the results.

M. Szyjewski

Symmetric bilinear forms over schemes, applications of e^0 and perspectives for e^1

For a line bundle L over a scheme X there is exact (co)functor $\hat{} \otimes L$:

$$A\hat{} \otimes L = \mathcal{H}om_{\mathcal{O}_X}(A, L)$$

on the exact category of vector bundles on X. One defines a Witt group $W(X, L)$ of symmetric bilinear forms with values in L as a factor of free abelian group of isomorphism classes of pairs (A, a), where $a : A \to A\hat{} \otimes L$ is an isomorphism such that $a\hat{} \otimes L = a$, modulo metabolic ones. In the particular case of trivial bundle $L = \mathcal{O}_X$ this is the "usual" Witt ring of a scheme X, defined by Knebusch: $W(X, \mathcal{O}_X) = W(X)$. Forgetting a yields the homomorphism

$$e^0_L : W(X, L) \longrightarrow \hat{H}^0(1, \hat{} \otimes L\,,\, K_0(X))$$

which generalizes the dimension index $e^0 : W(F) \to \mathbb{Z}/2\mathbb{Z}$. In the most important case of trivial bundle L we abbreviate notation to e^0. The map e^0 is a ring homomorphism, which in many cases is surjective. If X is a projective smooth curve over an algebraically closed field, then e^0 is an isomorphism. In many cases the groups

$$E^{(-1)^i}(X, L) = \hat{H}^i(1, \hat{} \otimes L\,,\, K_0(X))$$

may be explicitly computed. Such a computations for even-dimensional quadrics of maximal index, or projective line over a suitable ring together with surjectivity of e^0 yield existence of non-extended Witt classes. An exact sequence of E-groups for a variety, closed subvariety and its open complement (which shows importance of groups E^{-1} and values in a line bundle) yields estimation of orders of E-groups of Grassmann varieties. For grassmannian of planes e^0 is surjective and there exist non-extended Witt classes.
On the level of classifying spaces seems clear how to define in analogous manner a homomorphism e^1 from the kernel of e^0 to suitably subfactor of $K_1(X)$.

K. Szymiczek

Witt equivalence of number fields and p-ranks of class groups

Two fields K and L are *Witt equivalent* when their Witt rings of symmetric bilinear forms are isomorphic. For global fields we have the following local-global principle.
Two global fields are Witt equivalent if and only if their places can be paired so that corresponding completions are Witt equivalent.
See R. Perlis, K. Szymiczek, P. E. Conner and R. Litherland, Matching Witts with global fields. *Contemp. Math.* **155** (1994), 365–387.

The following theorem was presented.
For each prime factor p of n, each infinite class of Witt equivalent number fields of degree n contains a field with arbitrarily large $p-$rank of ideal class group.

The Theorem admits a reformulation in terms of Hilbert class fields:

For each prime factor p of n, each infinite class of Witt equivalent number fields of degree n contains a field with infinite Hilbert $p-$class field tower.

J. van Geel

The u-invariant of function fields of curves over p-adic fields

Let k be a p-adic field, $p \neq 2$, and let $K = k(C)$ be the function field of a smooth projective curve over k. In [3] David Saltman showed that the index of elements in the 2-component of the Brauer group of K is bounded by 4. In [1] this result is used to prove that an anisotropic quadratic form over K has dimension ≤ 22, in other terms the u-invariant $u(K) \leq 22$. Parimala and Suresh (cf. [2]) showed that $u(K) \leq 10$ (the u-invariant is expected to be 8). In order to obtain this better bound they improved Saltman's result by giving more precise information on the biquadratic splitting fields for elements in the 2-component of $Br(K)$ and they extend Saltman's results to $H^3(K, \mathbb{Z}/2\mathbb{Z})$. They obtain the following important result: *If $\alpha_i \in H^3(K, \mathbb{Z}/2\mathbb{Z})$, $1 \leq i \leq n$, then there exist elements $f, g, h_i \in K^*$ such that $\alpha_i = (g) \cup (g) \cup (h_i)$. In particular every element in $H^3(K, \mathbb{Z}/2\mathbb{Z})$ is a symbol.* In the talk we showed how Saltman's result is used to bound the u-invariant of K. We gave some of the ideas behind Saltman's result. (A gap in Saltman's proof was found by Gabber who also corrected it, cf. the review of [3] in the Zentralblatt). And we gave a survey of the results of Parimala and Suresh.

[1] Hoffmann, D., Van Geel, J., *Zeros and Norm Groups of Quadratic forms over Function Fields in One variable over a Local Non-Dyadic Field*, J. Ramanujan Math. Soc. 13 (2), (1998), 85-110.

[2] Parimala, R., Suresh, V., *Isotropy of quadratic forms over function fields of curves over p-adic fields*, to appear IHES.

[3] Saltman, D., *Division Algebras over p-adic curves*, J. Ramanujan Math. Soc. 12 (1), (1997), 25-47. (cf. review Zentralblatt für Math. 902.16021)

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