MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30/1999

Geometric and Multibody Mechanics: Nonlinear Dynamics and Control

25.07.-31.07.1999

Organizers

Marsden, Jerrold *(Control and Dynamical Systems, California Institute of Technology, Pasadena)*; **Pfeiffer, Friedrich** *(Engineering, University of Technology, Munich)*; **Scheurle, Jürgen** *(Mathematics, University of Technology, Munich)*.

Topics Covered

The last two decades have seen much progress on the mathematical structures underlying mechanics, such as multibody, robotic and nonholonomic systems, and its applications to engineering problems. The progress has been in the foundational geometric structures, stability and bifurcation theory, computation and visualization. All of these issues are interacting in a very healthy way to produce a rich research output.

This meeting had the aim of allowing workers in the area to exchange ideas and to report on recent advances. The meeting had a healthy mix of senior and established researchers, postdoctoral and graduate students.

The conference focussed on systems typified by multibody mechanics, but was not be limited to this. Elastic and fluid systems as well as relevant theoretical aspects of geometric mechanics were also treated. The specific themes of the conference were as follows:

- **GEOMETRIC AND ANALYTIC FOUNDATIONS.** Including systems with symmetry, bifurcation of mechanical systems with symmetry, nonholonomic systems and multibody systems of engineering interest. Variational principles and their applications, geometric phases.

- **CONTROL THEORY.** Locomotion generation, stabilization of mechanical systems, optimal control, tracking. Gyroscopic systems, including navigation systems.
- **ENERGY-MOMENTUM METHOD.** The energy momentum method in the presence of symmetries and singularities, extensions to nonholonomic systems, applications to specific mechanical systems.
- **MECHANICAL INTEGRATORS.** Symplectic-momentum and energy-momentum integrators, integration of constrained systems, stiff systems, long time integrations. Adaptation to control systems.

Structure of the Meeting

Our philosophy, consistent with the general approach advocated by Oberwolfach was to select about 20–25 senior people and to invite each of them to give a lecture. We also encouraged them to suggest students, postdoctoral fellows and junior faculty to the meeting. In their lecture, they were to also include whoever in their group they felt was appropriate. In addition, there was a poster session for these accompanying persons, which we felt worked out very well. It gave, for example, students a chance to share and discuss their ideas with the entire conference without using up a lecture slot. A number of opportunities in this regard would have been missed had there been only unstructured discussion time. The two together seem to be quite effective.

Abstracts (talks and posters)

Dynamics and Control of Jet Engine Flow

Bjorn Birnir

The basic attractor of nonlinear partial differential equations (PDEs) is the part of the global attractor that attracts a prevalent set in phase space. We give a qualitative description of the basic attractor of a model for axial compression systems and show that it consists of three types of components, uniform (design) flow, surge and stall. The basic attractor can contain more than one stall component. The existence of these components is proven and their stability explored. Numerical results are presented, showing the shape and evolution of stall cells over large parameter regions. Then notions of basic controllablity and basic control are defined for nonlinear PDEs. A quadratic optimal control of the linearized viscous Moore-Greitzer equation is presented and it is confirmed that stall is uncontrollable in this model. A basic control is constructed for the nonlinear viscous Moore-Greitzer equation which can control surge and stall. Some extensions of this construction are discussed. Numerical simulations of the basic control are presented and its performance is compared to the performance of a backstepping control constructed by Banazuk et al. It is shown that the viscous Moore-Greitzer equations with throttle control is not basically controllable, but under certain conditions, adding air injection or bleeding control will make the equations basically controllable.

Symmetry and Reduction in Implicit Generalized Hamiltonian Systems

Guido Blankenstein

In the classical symplectic, or Poisson, reduction theory one considers a symplectic, or Poisson, manifold and a symmetry Lie group G acting on it by symplectic (Poisson) maps. Furthermore, one assumes the existence of an equivariant momentum map J. Then it can be proved that the Hamiltonian system on the original manifold restricts to a Hamiltonian system on the reduced manifold $J^{-1}(\mu)/G_\mu$.

In this work, we generalize the theory to Dirac structures. A Dirac structure D on a manifold $\mathcal{X}$ is a $2n$-dimensional ($n = dim\,\mathcal{X}$) vector subbundle of $T\mathcal{X} \oplus \mathcal{T}^\star\mathcal{X}$, such that $D = D^\perp$, where $D^\perp$ is the orthogonal of D with respect to some bilinear symmetric form on $T\mathcal{X} \oplus \mathcal{T}^\star\mathcal{X}$. Dirac structures are the underlying geometric structures for which we define implicit generalized Hamiltonian systems. Implicit generalized Hamiltonian systems are DAE's, which generalize the classically known Hamiltonian systems. For instance, a mechanical system with (nonholonomic) constraints is an example of an implicit generalized Hamiltonian system. The classical reduction theory can be generalized to these systems, resulting basically in the same reduction scheme.

Optimization of Motion of a Multilink Walking Robot

Nikolai N. Bolotnik

A multilink walking robot consisting of conservatively connected rigid bodies is considered. The bodies are connected by two-degree-of-freedom joints so that the adjacent bodies can rotate with respect to each other about two perpendicular axes. Each rotation is controlled by a geared electric drive. To the terminal links of the chain feet with vacuum cups are attached to enable the robot to move over a surface arbitrarily inclined with respect to the horizontal. In particular, such a surface can be a wall or ceiling. The robot of this design with five links was constructed and tested at the Institute for Problems in Mechanics of the Russian Academy of Sciences. A number of problems of optimal motion planning for this robot have been solved. We have developed an algorithm for planning the time-optimal motion of the robot travelling along a prescribed track. By track we understand here a set of positions on the supporting surface through which the robot must consecutively pass with its feet. Also, we constructed optimal periodic gaits of this robot providing the maximum average speed when moving in a prescribed direction.

Optimal distribution of torques at the joints of a two-body leg of a walking machine were found to provide the maximum friction force between the foot of the leg and the supporting rough surface. This friction force is the propelling force of the robot.

We also have investigated the behaviors of the maximum friction force depending on the position of the foot on the supporting surface and the ratio of the links of the leg. The results of this analysis can be used when designing the leg and planning motions of legged mobile robots.

Visualization of Optimal Trajectories: Robotics, Vehicle Dynamics, Interplanetary Space Probe etc.

Roland Bulirsch

Films are presented showing the solutions of various ordinary and partial differential equations.

- the flow of the current in a transistor (3-dimensional solution of a system of elliptic PDEs of order 6)
- optimal paths of a robot hand (solution of a 2-point boundary value problem of a system of ODEs of order 6)
- car on a road with wet and icy spots and many other "dirty" situations (initial value problem for a system of ODEs of order 56)
- optimal design and optimal flight path of a space probe to the asteroid 4386 Lüst (multipoint boundary value problem for a system of ODEs of order 92)
- the evolution of the sun: life cycle and its end as a red giant (system of PDEs of parabolic type of order 7)

A Series Expansion Describing the Evolution of Mechanical Control Systems

Francesco Bullo

This talk presents a series expansion that describes the evolution of a mechanical system starting at rest and subject to a time-varying external force. Mechanical systems are described via affine connections as second order systems on a configuration manifold. The goal is to exploit this structure and reduce the dimensionality of the problem. Instead of a series on the full $2n$ dimensional space, the evolution is described as a flow on the configuration space (n dimensional). The treatment relies on some Chronological Calculus tool, see the seminal Agračev and Gamkrelidze ("The exponential representation of flows and chronological calculus", Math. USSR Sbornik, 1978, 35-6, pages 727-785). This expansion generalizes previous results, provides a rigorous mean of analyzing locomotion gaits in robotics, and lays the foundation for the design of motion control algorithms for a large class of mechanical systems.

Bounded Control of Mechanical Systems with Unknown Parameters

Igor M. Cenanievski

The lecture is devoted to designing a control for Lagrangian mechanical systems under uncertainty. We seek the desired control in the form of linear feedback with piecewise constant gains. A linear feedback control (PD-controller) is usually applied for stabilizing mechanical systems, that is, for attaining the terminal state in infinite time. This control law is very simple and does not depend on the parameters of the system. However, the control forces generated by a PD-controller turn out to be unbounded and can violate constraints if they are present. On the other hand, the control forces become too small when the system approaches the terminal state. This implies infinite time of motion. To preserve the advantages of the PD-controller, to meet constraints, and to use control possibilities to full extent we design an algorithm for varying the gains which enables us to steer a mechanical system with unknown parameters to a terminal state in a finite time. The gains tend to infinity; however, the control is bounded and meets the constraints imposed. To illustrate the work of the proposed algorithm a computer simulation of various mechanical systems, including underactuated ones, controlled by this algorithm was performed.

Geometric and Multibody Mechanics: Nonlinear Dynamics and Control

Felix L. Chernousko

Linear and nonlinear mechanical systems subject to unknown but bounded disturbances and bounded control forces are considered. First, various systems with one degree of freedom are examined under different sets of assumptions concerning the bounds imposed on the control force, its rate of change, and disturbances acting on the system. Time-optimal control laws are obtained in an explicit form for these systems. The results are the generalization of the well-known optimal control law for the simplest system with one degree of freedom. The obtained results are used in the frames of the decomposition approach for the design of control for nonlinear Lagrangian systems with many degrees of freedom. For such systems subject to bounded disturbances, the control is also obtained in an explicit form. This feedback control is based on the decoupling of different degrees of freedom (which is achieved under certain assumptions) and on the application of optimal control for each subsystem with one degree of freedom. Sufficient controllability conditions and upper estimates on the time of control are presented.

An Approximation of Dynamical Systems in Banach Spaces with an Application to Shells Dynamics.

Ivica Djurdjevic

In this talk I intend to present a natural approach to an approximation of flows in Banach spaces. In particular, the concept of an Almost-Poisson-mapping (for Hamiltonian structure of elasticity introduced by Ge, Kruse & Marsden in 1996) will be made rigorous. It will be shown that within the class of semilinear problems the above mentioned concept constitutes a special case of the general approach, where no Poisson-structure is needed. Finally, an application to shell dynamics within the framework of linear elastodynamics will be presented. It will turn out that for any shape of the middle surface a dynamical version of Koiters model is asymptotically correct when the thickness parameter tends to zero.

Control of Configuration Variables in Mechanical Systems for Applications to Multibody Systems

Marco Favretti

In this talk we consider a mechanical system where the time evolution of some of the d.o.f. (i.e. configuration variables) is prescribed by a given function u. We suppose that this is realized by suitable active constraints. The dynamic equations for this system are derived in a coordinate-free form. If some conditions on the kinetic energy metric are fulfilled, the case of u discontinuous can be considered. This allows to introduce a notion of 'hyper-impulsive' motion where configurations can suffer first order discontinuities. Applications to control theory are discussed.

References:

1. F. Cardin and M. Favretti, 'Hyper-impulsive Motion on Manifolds', Dynamics of Continuous, Discrete and Impulsive Systems, 4 (1998) 1-21.
2. M. Favretti, 'On the use of configuration variables as control variables in mechanical systems', Proc. 36-th CDC Conference, San Diego, Dec '97, 4210-4215.

Dynamically Controlled Generalized Cell Mapping

Julia Fischer, Rabbijah Guder, Edwin Kreuzer

The introduction of cell to cell mapping techniques has made the study of the global behavior of technical systems feasible. They provide an easy method of finding (all) the different solutions (attractors) belonging to initial conditions everywhere in the phase space.

Moreover, domains of attraction, invariant densities, global measures and other performance characteristics are provided. We discuss recent developments for the generalized cell mapping (gcm). The mathematical description of an autonomous gcm leads to a finite discrete stationary Markov chain where the transition probability matrix completely determines system's behavior. For higher dimensional systems the computational effort is enormous, therefore, adaptive refinement schemes are introduced to compute both the attractors and the domains of attraction with the associated boundaries. The transition probabilities are computed by a discretization of the Frobenius-Perron operator; the algorithm is controlled by a deterministic a priori estimate. Furthermore, perturbations of deterministic models are introduced. There are perturbed systems where the Markov operator (corresponding to the Frobenius-Perron operator in the unperturbed case) is known. Then gcm can be employed to calculate the invariant density of the perturbed system. With examples the efficiency of the approach is demonstrated and the results are visualized. The two posters titled "Generalized Cell Mapping on Different Cell Grids " and "Stability in Dynamical Systems with Perturbations" are related to the lecture.

A Direct Solver for Multi-contact Friction Problems Based on a Generalised Linear Complementarity Formulation

Kilian Funk, Friedrich Pfeiffer

This contribution discusses a solving method for multi-contact problems including Coulomb friction in multi-body systems. Due to couplings between different contacts state transitions from slipping to sticking or from separation to contact in one contact can induce state transitions in other contacts. A systematic approach is made to solve multi-contact friction problems including linear dependencies between different contacts. The method can either be applied to planar problems or three dimensional problems with a polygonal friction cone. A combined friction law for normal and tangential directions is established using the contact accelerations. A linear equation system of the dimension $2n \times 3n$ for planar problems can be derived which correlates the contact accelerations with contact forces (n denotes the number of friction contacts). The introduction of suitable complementarity conditions results in a linear complementarity problem (LCP) in nonstandard formulation. For LCPs in standard formulation the *Lemke*-algorithm is a direct pivoting solver. The main feature of the algorithm is the base exchange step. Using finite state machines for the base exchange step the algorithm can be applied to a much more general set of LCP fomulations. Introducing a finite state machine for each subproblem (in the above case: for each contact) the above described complementarity conditions can be modeled as states and transition conditions.

Numerical Computation of Lyapunov Exponents in Discontinuous Maps Implicitly Defined

Ugo Galvanetto

In the field of stick-slip systems the introduction of low-dimensional implicitly defined maps has often proven very useful in the understanding of the system dynamics. The successive iterates of the maps are computed by means of the integration, in continuous time, of a discontinuous, structurally variable system. The explicit equation of these maps are not known and in general no expression for their Jacobian matrix is therefore available. The dynamics of a three block mechanical system can be characterised by the values of two variables, the relative displacements d21, d31. The values of these variables are constant when both blocks are simultaneously sticking on the driving belt. The dynamics of the system generates a two-dimensional map of the variables d21 and d31. This map is piece-wise continuous and the aim of this note is to discuss some numerical techniques to compute its Lyapunov exponents, basins of attraction, and bifurcation paths. The methods presented in this work can be applied to wider classes of maps since there is no apparent reason to restrict our considerations to maps generated by stick-slip mechanical systems.

On the Structure of Differential Inclusions in Multibody Mechanics

Christoph Glocker

Non-smooth dynamics requires the formulation of the equations of motion as differential inclusions, i.e. as differential equations with set-valued right-hand side. Due to their general character, these set-valued maps have to be further specified to make them suitable for applications. The structure of set-valued force laws that are met in multibody dynamics is thus investigated and discussed with respect to a representation which is suitable for expressing second order ordinary differential inclusions. By writing down the Newton-Euler equations as an equality of measures one may even cover impulsive behavior besides the non-smooth impact-free motion which is also included. However, the measure equation itself is not a complete description of the dynamic problem.It has to be completed by certain force laws, which are here taken asthe ge neralized gradient mappings of (non-smooth, non-convex) displacement potentials and dissipation functions. Unilateral and bilateral constraints on displacement or velocity level, as well as more general set-valued force laws such as dry friction and pre-stressed springs may be treated in this way.

α-Models: Geometric Turbulence Closures

Darryl D. Holm

We discuss analytical, numerical and experimental results for a regularization of the Navier-Stokes equations given by $\partial_t v + u \cdot \nabla v + \nabla u \cdot v + \nabla p = \nu \triangle v + f$, where $u = g \star v$ is the incompressible ($\nabla \cdot u = 0$) mean, or filtered fluid velocity in three dimensions. The NS-α model results when g is chosen to be Helmholtz smoothing by $v = (1 - \alpha^2 \triangle) u$. The NS-$\alpha$ system has global existence and uniqueness of strong solutions. It has a global attractor with finite fractal dimension. The key idea in the proofs of these claims is that the coefficient ∇u in the vortex stretching term in the vorticity equation $\partial_t q + u \cdot \nabla q = q \cdot \nabla u + \nu \triangle q + curl\, f$ for $q = curl\, v$ is bounded in L_2, since the kinetic energy for the NS-α model is the H_1 norm. Note that α is a length scale.

Comparisons with turbulence data for flows in channels and pipes at high Reynolds numbers confirm the physical interpretation of u as the Eulerian mean turbulent fluid velocity. Direct numerical simulations show that the forward cascade of kinetic energy slows and the wavenumber spectrum changes from $k^{-5/3}$ for $k\,\alpha < 1$ to k^{-3} for $k\,\alpha > 1$. The result is a significant increase in computational speed, by a factor of $(\alpha/l_{k_0})^{4/3}$, where l_{k_0} is the Kolmogorov dissipation length.

Waves in Lattices of Coupled Nonlinear Oscillators

Klaus Kirchgässner (This is joint work with Gerard Iooss from Nice.)

In a 1d lattice of nonlinear oscillators, coupled to their nearest neighbors, all travelling waves of moderate amplitude can be found as solutions of a finite dimensional reversible dynamical system. The coupling constant and the inverse wave-speed form the parameter-space. The groundstate consists of a one-parameter family of periodic waves. It is realized in a certain parameter region containing all cases of light coupling. Beyond the border of this region the complexity of wave-forms increases via a succession of bifurcations. An appropriate formulation of this problem is given, the necessary proof of reduction is indicated, together with a normal-form analysis leading to the result on the groundstate and the classification of the first bifurcation. In particular, the existence of socalled nanopterons is shown, i.e. of spatially localized wave-forms with an exponentially small periodic tail at infinity (phonons). Moreover, an application of this analysis to multiple scale problems, where the smooth and slowly variable part of the solution of an elliptic equation is discretized and the fast varying part treated continously, is analysed and error-bounds are given.

Travelling Waves in Nonlinear Chains

Thomas Kriecherbauer

We study the existence of travelling waves in infinite nonlinear chains with nearest neighbor interaction, described by the system

$$\ddot{x}_n(t) = F(x_{n-1}(t) - x_n(t)) - F(x_n(t) - x_{n+1}(t)), \quad n \in \mathbb{Z}, \tag{1}$$

where F describes the (nonlinear) force law and x_n denotes the position of the n-th particle. More precisely, we construct solutions of (1) of the type

$$x_n(t) = nd + \chi(kn - \gamma t), \tag{2}$$

where $\chi : \mathbb{T}^\nu \to \mathbb{R}$ is a function on the ν-dimensional torus and $k, \gamma \in \mathbb{R}^\nu$ denote the frequency vectors. For the construction we expand the function χ in a Fourier series and use a Lyapunov-Schmidt reduction argument to obtain families of solutions of type (2). In the multi-phase case, $\nu \geq 2$, we encounter a small divisor problem which we overcome by a Nash-Moser iteration scheme.

Magnetostrictive Actuation: Models for Control

Perinkulam S. Krishnaprasad

In this talk, we discuss joint work with R. Venkataraman, on low-dimensional, energy-based models for ferromagnetic hysteresis and for magnetostrictive hysteresis. This work is based on a set of reasonable hypotheses to account for magnetization losses, first presented in the work of Jiles and Atherton (J. Magn. Matl. 61 : 48-66, 1986). In contrast with the Preisach formalism for hysteresis, our models are based on local memory. Viewed as state equations for an actuator, our models employ average magnetization M in the material and applied uniform external field H as states and $n = \dot{H}$ as the input. We have used these models for Terfenol-D (a giant magnetostrictive alloy). We have obtained results on the qualitative behaviors of our models, including a result on convergence to a major hysteresis loop. We employ the notion of a Caratheodory solution to accomodate discontinuous changes in $\frac{dM}{dH}$ during reversals.

Some Results on the Hamiltonian Dynamics of Ideal Liquid Bridges

Hans-Peter Kruse

The dynamics of an incompressible, inviscid liquid bridge moving under the influence of surface tension, is studied. It turns out that this problem has a Hamiltonian structure, i.e. there exist a) a phase space $\mathcal{P}$, b) a class $\mathcal{F}$ of admissible functions $F : \mathcal{P} \to \mathbb{R}$, c) a Poisson bracket $\mathcal{F} \times \mathcal{F} \to \{\mathcal{P} \to \mathbb{R}\}$, d) a Hamiltonian $H \in \mathcal{F}$, such that the equations of motion can be written as $\dot{F} = \{F, H\}$ for all $F \in \mathcal{F}$.

The Hamiltonian structure of the problem can be used to study stability and bifurcation behaviour of rigidly rotating fluid cylinders connecting two plates, which represent solutions to the equations of motion of the liquid bridge for any value of the angular velocity $\omega \in \mathbb{R}$. As an example of the kind of result we get we cite the following theorem: Rotating cylinders with base radius d and height h are stable with respect to volume-preserving pertubations with fixed contact lines if

$$\frac{\pi^2 \, d\tau}{h^2} > \max \left(\frac{\tau}{4d} + \frac{\omega^2 d^2}{4}, \omega^2 d^2 \right).$$

Let ω_0 solve

$$\frac{h^2}{4\pi^2 d^2} + \frac{\omega_0^2 h^2 d}{4\tau\pi^2} = 1\,,$$

and let ω_1 solve

$$\omega_1^2 = \frac{\pi^2 \tau}{dh^2}.$$

If $\omega_1 < \omega_0$, then at $\omega = \omega_1$, a branch of $\mathbb{Z}_2$-symmetric, non-cylindrical rotating liquid bridges emanates from the family of rigidly rotating cylinders.

Simulation Approaches to Multibody Dynamics

Ralf Kübler, Subir Saha, Werner Schiehlen

The first part of the lecture is devoted to theoretical aspects of simulator coupling for multibody systems. The modular description is introduced on the mathematical model description level which is the basis for modular simulation. Each subsystem is set up by a general state-space formulation. Time discretization of the subsystems including input and output variables is described. The global system is then formed by interconnections between the inputs and the outputs of these subsystems. On this basis an analysis of zero-stability of the modular numerical integration is presented. It is shown that convergence is only guaranteed if algebraic loops do not exist between the subsystems. Two methods of simulator coupling are proposed, an iterative scheme and the introduction of filters. The theoretical results are illustrated by examples from multibody system dynamics and the numerical results are used to compare the efficiency of both methods, showing the preference of the iterative scheme.

The second part of the lecture deals with recursive simulation of multibody systems. The focus is how to obtain an efficient forward dynamics algorithm for the simulation of multibody systems with large degrees-of-freedom (n), e.g., a free-flying space robot where $n = 12$. It is shown how the Gaussian Elimination technique can be used to obtain an order $'n'$ forward dynamics algorithm for serial-chain systems. It is highlighted that the extension of the algorithm to closed-loop systems is possible if it is accompanied with a suitable velocity constraint relation for the loops that allow one to compute the dependent speeds recursively.

Nonsmooth Dynamical Systems

Tassilo Kuepper

The incorporation of effects like dry friction, state dependent switches or impacts in systems modelled by differential equations leads to Dynamical Systems involving nonsmooth components. Examples widely used for investigations include the friction oscillator with one or several masses, electrical circuits or neural networks. This lecture is concerned with the dynamics of such systems, in particular with respect to bifurcations. While experiments and simulations show the standard scenario of bifurcations including transitions from steady states to periodic orbits and chaotic motions basic techniques of bifurcation theory cannot be applied in a straiightforward way due to the lack of smoothness. We show that the following concepts can be extended to nonsmooth systems:

1. The existence of Lyapunov exponents is established as well as results implying the stability of periodic orbits.
2. The onset of periodic orbits in planar systems is studied with geometrical methods.

Controlled Lagrangians and Mechanical System Stabilization

Naomi Ehrich Leonard

In this talk I describe joint work with A. M. Bloch and J. E. Marsden on a constructive approach to the derivation of stabilizing control laws for Lagrangian mechanical systems with symmetry. Central to our "method of controlled Lagrangians" is the choice of a feedback control law from a family of control laws that produce Lagrangian *closed-loop* dynamics. By making structured modifications to the Lagrangian for the uncontrolled system, we define the closed-loop (controlled) Lagrangian, and the associated Euler-Lagrangian equations give the closed-loop dynamics and the control law. This method has the advantage of making the stabilization problem a matter of energy shaping, and energy methods can be used to produce a Lyapunov function and to find control gains that provide closed-loop stability. Our approach involves both kinetic shaping to stabilize otherwise unstable dynamics and symmetry-breaking potential shaping to provide stability in directions that were originally symmetry directions. Control forces that emulate dissipation are added for asymptotic stability. Our method can be demonstrated for stabilizing balance systems such as an inverted pendulum on a cart as well as for stabilizing steady motions of systems with gyroscopic forces such as a satellite or an underwater vehicle with internal rotors.

The Geometry of Optimal Control for Affine Connection Control Systems

Andrew D. Lewis

The Maximum Principle of Pontryagin is applied to a system, affine in controls, whose drift vector field is the geodesic spray for an affine connection on a manifold Φ and whose control vector fields are vertical lifts to $T\Phi$ of vector fields on Φ. It results that one is interested in the Hamiltonian vector field on $T^*T\Phi$ which is the cotangent lift of Z, the geodesic spray. We describe the geometry of this vector field, giving the relationship between its Hamiltonian structure and the structure associated with the affine connection. The key to this is the "adjoint Jacobi equation" which is a one-form version of the Jacobi equation which describes the variation of geodesics.

Variational Integrators, Newmark and Collision Algorithms

Jerry Marsden, Couro Kane

This lecture was motivated by the study of systems in which complex collision sequences can occur, such as the fragmentation and shattering of solids. In these problems, amongst many others, small fragments with sharp corners are produced which subsequently undergo complicated collisions leading to extreme dynamical sensitivity.

We make use of integration methods that are based on discretizations of variational principles and nonsmooth analysis. The Newmark algorithm is shown to be variational in the sense of Veselov (and hence is symplectic). This algorithm is extended from the smooth case to the nonsmooth context motivated by collision algorithms. We have included friction and forcing into the variational framework via discretizations of the Lagrange d'Alembert principle and Lagrangian product formulas (splitting methods). Several computer simulations illustrate the results.

Antisymmetry, Pseudospectral Methods and Conservative PDSs

Robert McLachlan

"Dual composition", a new method of constructing energy-preserving discretizations of conservative PDSs, is introduced. It extends the summation-by-parts approach to arbitrary differential operators and conserved quantities. Links to pseudospectral, Galerkin and Hamiltonian methods are discussed. For the equation $\dot{u} = D(u)\frac{\delta H}{\delta u}$, where $D(u)$ is a linear differential operator and H is the conserved energy, we first compute $\frac{\delta}{\delta u}$, then project to a domain on which $D(u)$ is skew-adjoint, then apply $D(u)$, then project to the chosen finite-dimensional function space in which the approximation of u lies.

The function spaces and (weighted residual) projections can be chosen compatibly so that the resulting discretization preserves a discrete energy. In some cases (when H or D are simple enough) the method can coincide with standard Galerkin or pseudospectral methods. In these cases it sheds new light on the skew-adjoint structure of Chebyshev spectral differentiation and suggest a generalization of the Fourier antialiasing technique to Chebyshev spectral methods.

Symplectic Reduction by Stages

Gerard Misiolek

This is a follow-up talk to the one given by T. Ratiu and based on the joint work with J. Marsden, M. Perlmutter and T. Ratiu. I present two results that were obtained recently as applications of the symplectic reduction by stages. The first describes the structure of coadjoint orbits of semi-direct products and the second gives an interpretation of the Gelfand-Fuchs cocycle as the curvature of a right-invariant mechanical connection on the principal S^1-bundle $\hat{D}(S^1) \to D(S^1)$.

Drift of Relative Equilibria of Hamiltonian Systems with Symmetry

George Patrick

In a Hamiltonian system with symmetry, relative equilibria at nongeneric momenta have weaker stability than those at generic momenta. For example, in mechanical systems there is in general an absence of orientation stability at zero total angular momentum. This allows interesting dynamics of orientation for coupled rigid bodies and for point vortices on the plane or on the sphere.

I present results of numerical simulations which illustrate this effect, both for coupled bodies and for point vortices on the sphere, and I outline how to find a Hamiltonian system for the dynamics of the orientation.

Symplected Reduction by Stages

Matthew Perlmutter

Results on the problem of symplectic reduction by stages for the case of a symmetry group that is a central extension of a group G, denoted $\hat{G}$, are presented. The theory guarantees that if $\hat{G}$ acts symplectically on (P, Ω), and admits an equivariant momentum map $J_{\hat{G}} : P \to \hat{\mathfrak{G}}^{\star}$, then the 2-step reduced spaces obtained by first reducing by the $\mathbb{R}$-

action and then by the G-action are symplectically diffeomorphic to the reduced space $J_{\hat{G}}^{-1}(v)/\hat{G}_v, v \in \hat{\mathfrak{G}}^\star$.

We specialize to the case $P = T^\star Q$. To carry out the reduction in this case, we develop a non-canonical cotangent bundle reduction theory, since after the first reduction we obtain a cotangent bundle with a magnetic term. If we take $Q = \hat{G}$ we get a systematic description of the coadjoint orbits and, in certain cases, an interpretation of the Kostant-Kirillov symplectic forms as curvatures of appropriate connections.

Symplectic Reduction by Stages

Tudor S. Ratiu

In this talk we will present a general theorem for reduction of symplectic manifolds by stages with special emphasis on semidirect products and central extensions. The applications are the heavy top and the orbits of the Bott-Virasoro group. Reduction by stages for the action of a semidirect product of a Lie group with a representation space is first performed by the normal subgroup and then by the isotropy subgroup of an isotropy subgroup of the contragredient representation. It is shown that the result of these two successive reductions coincides with the reduction of the entire semidirect product. The same technique is utilized to deal with general central extensions and this will be presented in the companion talk by Gerard Misiolek. The poster by Matthew Perlmutter will present the general theory.

Stability of Relative Equilibria

Mark Roberts

A relative equilibrium p_e of a Hamiltonian H that is invariant under a proper action of a Lie group G on a symplectic manifold (M, ω) is said to be A-stable, for any subset A of G, if for every open neighbourhood U of p_e there exists an open neighbourhood V such that if $p(0) \in V$ then $p(t) \in AU$ for all t. Assume M has an equivariant momentum map J and let $\mu_e = J(p_e)$. The normal space to Gp_e at p_e can be identified with $(\mathfrak{G}_{\mu_e}/\mathfrak{G}_{p_e})^\star \oplus N_1$, where N_1 is the symplectic normal space. Let $q : \mathfrak{G}^\star_{\mu_e} \to \mathfrak{G}^\star_{\mu_e}//G_{\mu_e}$ denote the Hausdorff quotient space of the coadjoint action of G_{μ_e} and define Z_{μ_e} to be the "tangent space" to $q^{-1}(q(0))$.

Theorem 1: If $d^2(H - J_\xi)(p_e)$ is definite on $(Z_{\mu_e} \cap (\mathfrak{G}_{\mu_e}/\mathfrak{G}_{p_e})^\star) \oplus N_1$, then p_e is G-stable. Here ξ is the "drift velocity" of p_e.

Theorem 2: If p_e is G-stable, then (i) p_e is $\mathfrak{C}G_{\mu_e}$-stable for any neighbourhood $\mathfrak{C}$ of 1 in G; (ii) if $G_{\mu_e} = WK$, were W and K are subgroups of G_{μ_e} with K compact, then p_e is $A_{\mathfrak{C}}(W)K$-stable, where $A_{\mathfrak{C}}(W) = \{gWg^{-1} : g \in \mathfrak{C}\}$.

These theorems are proved in joint work with Andrew Lewis and George Patrick using a local normal form for Hamiltonian vector fields obtained with Jeroen Lamb and Claudia Wulff.

Numerical Integration of Constrained Hamiltonian Systems

Werner M. Seiler

We discuss two underlying Hamiltonian equations for constrained systems. We show their relations to projection methods as they are often used for numerical integration and study the stability of the constraint manifold under the corresponding flows. This gives some indications about the behavior of projection methods and explains why momentum projections are so much more effective than position projections. Finally, we briefly indicate the relation to the impetus striction approach for setting up the equations of motion, namely that there the momentum projections are already included in the differential equations.

Geometry of Diffeomorphism Groups for Manifolds with Boundary

Steve Shkoller

Let (M, g) be a $\mathcal{C}^\infty$ compact m-dimensional oriented Riemannian manifold with $\mathcal{C}^\infty$ boundary ∂M, and let $D^s = \{\eta \in H^s(M, \tilde{M}) |\, \eta \text{ bijective}, \eta^{-1} \in H^s(M, \tilde{M}), \eta \text{ leaves } \partial M \text{ invariant}\}$ denote the group of Hilbert class diffeomorphisms of M (here $\tilde{M}$ denotes the double). Let $D^s_\mu = \{\eta \in D^s |\, \eta^\star(\mu) = \mu\}$ where μ is a volume form. We prove that for $s > m/2 + 1$, $D^s_{\mu,D} = \{\eta \in D^s_\mu |\, \eta_{|\partial M} = e\}$ and $D^s_{\mu,N} = \{\eta \in D^s_\mu |\, T\eta_{|\partial M} : H^{s-1/2}(N) \to H^{s-3/2}_n(N)\}$ are $\mathcal{C}^\infty$ subgroups of D^s_μ. We then prove that the solution of the partial differential equation $\partial_t(1 - \alpha^2 \triangle_r)\, u + \nabla u\, (1 - \alpha^2 \triangle_r)\, u - \alpha^2\, \nabla u^t \cdot \triangle_r u = -grad\, p,\ div\, u = 0, u(0) = u_0, \triangle_r = -(d\,\delta + \delta\, d) + 2Ric$ with boundary conditions $u = 0$ on ∂M or $g(u, n) = 0$ and $(\nabla_n u)^t + S_n(u) = 0$ on ∂M are geodesics of the right invariant metric on $D^s_{\mu,D}$ or $D^s_{\mu,N}$ given at the identity by

$$< X, Y >_e = \int_M [g(X(x), Y(x)) + \frac{\alpha^2}{2}\, \bar{g}\, (\mathcal{L}_X\, g, \mathcal{L}_Y\, g)]\, \mu$$

Namely, there exists an interval $I = [-t_0, t_0]$ depending only on $|u_0|_{H^s}$ and a unique geodesic of $< \cdot, \cdot >$ $\dot{\eta} \in \mathcal{C}^\infty(I, TJ^s_\mu)$ for $J^s_\mu = D^s_{\mu,D}$ or $D^s_{\mu,N}$ such that $u(t) = \dot{\eta} \circ \eta(t)^{-1}$ is a solution of the partial differential equation.

Stability Analysis of Relative Equilibria of Tethered Satellite Systems

Alois Steindl, Hans Troger

After shortly explaining the concept of tethered satellite systems and giving some practical applications of this concept, we present two different mathematical models of such systems. The first and simpler one is a finite dimensional model consisting of a rigid massless or massive rod and two point masses. The second more complicated infinite dimensional model consists of two endbodies, which either could be point masses or rigid bodies, which are connected by a massive perfectly flexible and extensible string.

For both systems relative equilibrium positions for a circular motion around the Earth are presented. For the simple finite dimensional model this can be done in great generality. For the infinite dimensional model we restrict to planar configurations. Besides trivial configurations for which the tether is in a straight position there exist also nontrivial relative equilibrium configurations if the position of one satellite relative to the other is controlled. By means of the reduced energy momentum method we asses the stability of all these relative equilibria. In case of the nontrivial equilibrium positions the corresponding calculations must be performed numerically.

Nonholonomic Robot Stabilization by Vision-based Time-varying Controls

Dimitris P. Tsakiris

The problem of stabilizing a nonholonomic system to a desired pose is considered in this presentation, which describes joint work with Claude Samson (INRIA). In particular, the use of time-varying state feedback controls is examined, for which vision data are used to approximate the state of the nonholonomic mobile manipulator, whose docking or parallel parking maneuvers we attempt to automate. The vision data are provided by a camera carried by the arm of the mobile manipulator, which tracks a target of reference, as the robot moves. Two approaches are considered: the first involves continuous homogeneous time-varying controls, where the state information is updated at frame rate from vision. These controls render, in the presence of ideal state information, the closed-loop system homogeneous of degree zero with respect to an appropriate dilation and are robust with respect to additive perturbations of strictly positive degree of homogeneity. However, this may not be sufficient in cases where the state information is inferred from the sensory data by approximations or by using a crudely calibrated sensory apparatus. This introduces errors in the model parameters, which may destabilize the system. Thus, a second approach is explored, namely hybrid time-varying controls involving a combination of open and closed-loop phases, where the state information is updated from vision data only at the beginning of each period of the periodic open-loop controls. Extensive experimental evaluation was performed using a prototype with real-time vision and control capabilities.

Decomposition of a Finite Rotation

Jens Wittenburg, Lübomir Lilov

Given: The axis $\vec{\chi}$ and the angle χ of a rotation and the axes of three consecutive rotations $1, 2, 3$ (all four axes are fixed in the same reference space). To be determined: The angles χ_1, χ_2, χ_3 of the three rotations such that their resultant is the prescribed rotation $(\vec{\chi}, \chi)$.

The following problem has the same solution. Given is a body in a two-gimbal suspension. In the null position the outer gimbal axis, the axis common to both gimbals and the axis common to the body and to the inner gimbal are aligned with the rotation axes $3, 2$ and 1, respectively. Starting from this null position the angles χ_3, χ_2, χ_1 result in the prescribed rotation $(\vec{\chi}, \chi)$ of the body.

Explicit solutions are obtained from a quaternion and arise from a dyad representation of rotation. The solutions are real for arbitrary $(\vec{\chi}, \chi)$ if and only if axis 2 is orthogonal to both, axis 1 and axis 3. In critical cases χ_1 and χ_3 are indeterminate. Only $\chi_1 + \chi_3$ or $\chi_1 - \chi_3$ is determined. Indeterminacy conditions and the associated solutions for χ_2 and $\chi_1 \pm \chi_3$ are given.

Underwater Vehicle Stabilization

Craig Woolsey

An underwater vehicle may be modeled by a Hamiltonian system by treating the vehicle as a rigid body in a perfect fluid. An ellipsoidal vehicle modeled in this way has an unstable relative equilibrium corresponding to steady translation along the ellipsoids longest axis. Using internal rotors, this equilibrium can be stabilized through feedback which shapes the kinetic energy and preserves the Hamiltonian structure. This closed-loop Hamiltonian then leads to a Lyapunov function for the relative equilibrium via the energy-Casimir method. The Lyapunov function, in turn, indicates an appropriate choice of feedback dissipation to render the equilibrium asymptotically stable.

Authors of this Report:

J. E. Marsden, J. Scheurle

Participants

Anayevski, I.M.: Moscow, chern@ipmnet.ru

Birnir, Bjorn: Santa Barbara, birnir@math.ucsb.edu

Blankenstein, Guido: Enschede, g.blankenstein@math.utwente.nl

Bolotnik, Nikolai N.: Moscow, bolotnik@ipmnet.ru

Bulirsch, Roland: München, bulirsch@mathematik.tu-muenchen.de

Bullo, Francesco: Urbana, bullo@uiuc.edu

Chernousko, Felix L.: Moscow, chern@ipmnet.ru

Djurdjevic, Ivica: Stuttgart, ivica@mathematik.uni-stuttgart.de

Favretti, Marco: Padova, favretti@math.unipd.it

Fischer, Julia: Hamburg, julia.fischer@tu-harburg.de

Funk, Kilian: Garching, funk@lbm.mw.tu-muenchen.de

Galvanetto, Ugo: Padova, galva@caronte.dic.unipd.it

Glocker, Christoph: Garching, glocker@lbm.mw.tu-muenchen.de

Guder, Rabbijah: Hamburg, guder@tu-hamburg.de

Holm, Darryl: Los Alamos, dholm@lanl.gov

Kane, Couro: Pasadena, kane@cds.caltech.edu

Kirchgässner, Klaus: Stuttgart, kirchgaessner@mathematik.uni-stuttgart.de

Kreuzer, Edwin: Hamburg, kreuzer@tu-harburg.de

Kriecherbauer, Thomas: München, tkriech@rz.mathematik.uni-muenchen.de

Krishnaprasad, Perinkulam S.: College Park, krishna@isr.umd.edu

Kruse, Hans-Peter: München, kruse@mathematik.tu-muenchen.de

Kübler, Ralf: Stuttgart, rk@mechb.uni-stuttgart.de

Küpper, Tassilo: Köln, kuepper@mi.uni-koeln.de

Leonard, Naomi Ehrich: Princeton, naomi@balan Lewis, Andrew D.: Kingston, Ontario, andrew@mast.queensu.ca

Marsden, Jerrold E.: Pasadena, marsden@cds.caltech.edu

McLachlan, Robert: Aukland, r.mclachlan@massey.ac.nz

Misiolek, Gerard K.: Notre Dame, gmisiole@hilbert.helios.nd.edu

Patrick, George: Saskatoon, patrick@math.usask.ca

Perlmutter, Matt: Pasadena, perl@cds.caltech.edu

Pfeiffer, Friedrich: München, pfeiffer@lbm.mw.tu-muenchen.de

Ratiu, Tudor S.: Lausanne, Tudor.Ratiu@epfl.ch

Roberts, Mark: Coventry, mark@maths.warwick.ac.uk

Saha, Subir K.: Dehli, saha@mech.iitd.ernet.in

Scheurle, Jürgen: München, scheurle@mathematik.tu-muenchen.de

Schiehlen, Werner: Stuttgart, wos@mechb.uni-stuttgart.de

Schmah, Tanya: Lausanne, Tanya.Schmah@epfl.ch

Seiler, Werner M.: Mannheim, seiler@euler.math.uni-mannheim.de

Shkoller, Steve: UC Davis, shkoller@math.ucdavis.edu

Steindl, Alois: Vienna, Alois.Steindl@tuwien.ac.at

Troger, Hans: Vienna, hans.troger@tuwien.ac.at

Tsakiris, Dimitris: Heraklion, Crete, Dimitris.tsakiris@sophia.inria.fr

Vocke, Tanja: München, vocke@mathematik.tu-muenchen.de

Wittenburg, Jens: Karlsruhe, wittenburg@itm.uni-karlsruhe.de

Woolsey, Craig: Princeton, cwoolsey@princeton.edu

Tagungsteilnehmer

Prof.Dr. Igor M. Ananyevski
Institute for Problems of
Mechanics
Russian Academy of Sciences
Prospekt Vernadskogo 101-1

Moscow 117 526
RUSSIA

Prof.Dr. Bjorn Birnir
Dept. of Mathematics
University of California

Santa Barbara , CA 93106
USA

Guido Blankenstein
Dept. of Mathematical Sciences
Twente University
P.O. Box 217

NL-7500 AE Enschede

Prof.Dr. Nikolai N. Bolotnik
Institute for Problems of
Mechanics
Russian Academy of Sciences
Prospekt Vernadskogo 101-1

Moscow 117 526
RUSSIA

Prof.Dr.Dr.h.c. Roland Bulirsch
Zentrum Mathematik
TU München

80290 München

Prof.Dr. Francesco Bullo
Coordinated Science Laboratory
University of Illinois
1308 W. Main Street

Urbana , IL 61801
USA

Prof.Dr. Felix L. Chernousko
Institute for Problems of
Mechanics
Russian Academy of Sciences
Prospekt Vernadskogo 101-1

Moscow 117 526
RUSSIA

Ivica Djurdjevic
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57

70569 Stuttgart

Prof.Dr. Marco Favretti
Dipartimento di Matematica Pura
e Applicata
Universita di Padova
Via Belzoni, 7

I-35131 Padova

Julia Fischer
Technische Universität
Hamburg-Harburg
Arbeitsbereich Meerestechnik II
Eißendorfer Str. 42

21073 Hamburg

Kilian Funk
Lehrstuhl B für Mechanik
Technische Universität München

85747 Garching

Prof.Dr. Ugo Galvanetto
Department of Aeronautics
Imperial College of Science, Tech.
and Medicine, South Kensington
Prince Consort Road

GB-London SW7 2BY

Dr. Christoph Glocker
Institut B für Mechanik
Techn. Universität München

85747 Garching

Prof.Dr. Rabbijah Mazid Guder
Technische Universität
Hamburg-Harburg
Arbeitsbereich Meerestechnik II
Eißendorfer Str. 42

21073 Hamburg

Prof.Dr. Darryl D. Holm
Theoretical Division
Los Alamos National Laboratory
University of California
MS-B284

Los Alamos , NM 87545
USA

Prof.Dr. Couro Kane
Graduate Aeoronautical Laboratories
Mail Code 105-50
Caltech

Pasadena , CA 91125
USA

Prof.Dr. Klaus Kirchgässner
Mathematisches Institut A
Universität Stuttgart
Postfach 801140

70569 Stuttgart

Prof.Dr.-Ing. Edwin Kreuzer
Technische Universität
Hamburg-Harburg
Arbeitsbereich Meerestechnik II
Eißendorfer Str. 42

21073 Hamburg

Prof.Dr. Tassilo Küpper
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Dr. Thomas Kriecherbauer
Mathematisches Institut
Universität München
Theresienstr. 39

80333 München

Prof.Dr. Naomi Ehrich Leonard
Department of Mechanical and
Aerospace Engineering
Princeton University

Princeton , NJ 08544-5263
USA

Prof.Dr. Perinkulam S. Krishnaprasad
Institute for Systems Research
A.V. Williams Building
University of Maryland

College Park , MD 20742
USA

Prof.Dr. Andrew D. Lewis
Department of Mathematics and
Statistics
Queen's University

Kingston, Ontario K7L 3N6
CANADA

Dr. Hans-Peter Kruse
Zentrum Mathematik
TU München
Arcisstr. 21

80333 München

Prof.Dr. Ljubomir Lilov
Faculty of Mathematics & Informatic
Bulgarian Academy of Sciences
Blvd. A. Ivanov, 5

1126 Sofia
BULGARIA

Ralf Kübler
Institut B für Mechanik
Universität Stuttgart

70550 Stuttgart

Prof.Dr. Jerrold E. Marsden
Control and Dynamical Systems
Caltech 116-81

Pasadena , CA 91125
USA

Prof.Dr. Robert McLachlan
Mathematical Sciences Research
Institute
2223 Fulton Street
1000 Centennial Drive

Berkeley , CA 94720
USA

Prof.Dr. Gerard K. Misiolek
Department of Mathematics
University of Notre Dame

Notre Dame , IN 46556-5683
USA

Prof.Dr. George Patrick
Dept. of Mathematics
University of Saskatchewan
106 Wiggins Road

Saskatoon Sask. S7N 5E6
CANADA

Prof.Dr. Matt Perlmutter
CDS 107-81
Caltech

Pasadena , CA 91125
USA

Prof.Dr. Friedrich Pfeiffer
Institut B für Mechanik
Technische Universität

80290 München

Prof.Dr. Tudor S. Ratiu
Departement de Mathematiques
Ecole Polytechnique Federale
de Lausanne

CH-1015 Lausanne

Dr. Mark Roberts
Mathematics Institute
University of Warwick
Gibbet Hill Road

GB-Coventry , CV4 7AL

Dr. Subir K. Saha
c/o Prof. W. Schiehlen
Inst. B für Mechanik
Universität Stuttgart
Pfaffenwaldring 9

70550 Stuttgart

Prof.Dr. Jürgen Scheurle
Zentrum Mathematik
TU München

80290 München

Prof.Dr. Werner Schiehlen
Institut B für Mechanik
Universität Stuttgart

70550 Stuttgart

Tanya Schmah
Departement de Mathematiques
Ecole Polytechnique Federale
de Lausanne

CH-1015 Lausanne

Werner M. Seiler
Lehrstuhl für Mathematik I
Fak. für Mathematik und Informatik
Universität Mannheim
Seminargebäude A 5

68159 Mannheim

Prof.Dr. Steve Shkoller
CNLS, MS B258
Los Alamos National Laboratory

Los Alamos , NM 87545
USA

Prof.Dr. Alois Steindl
Institut für Mechanik der
Technischen Universität Wien
Wiedner Hauptstr. 8 - 10

A-1040 Wien

Prof.Dr. Hans Troger
Institut für Mechanik der
Technischen Universität Wien
Wiedner Hauptstr. 8 - 10

A-1040 Wien

Dr. Dimitris Tsakiris
Inst. of Computer Science (ICS)
Found. or Research and Technology
Vassilika Vutan
P.O. Box 1385

71110 Heraklion, Crete
GREECE

Tanja Vocke
Zentrum Mathematik
TU München
Arcisstr. 21

80333 München

Prof.Dr. Jens Wittenburg
Institut für Technische Mechanik
Fakultät für Maschinenbau
Universität Karlsruhe

76128 Karlsruhe

Craig Woolsey
Department of Mechanical and
Aerospace Engineering
Princeton University

Princeton , NJ 08544-5263
USA