MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 32/1999

Kommutative Algebra und algebraische Geometrie

08.08.-14.08.1999

The conference was organized by J. Herzog (Essen), J. Lipman (West Lafayette) and U. Storch (Bochum). 49 participants from Western and Eastern Europe, the United States, Canada, India and Japan attended the conference. The 31 lectures dealt with most research areas of Commutative Algebra and Algebraic Geometry. New developments in the resolution of singularities and in the resolution of morphisms (principalization, monomialization, toroidalization, factorization by blow-ups and blow-downs) played a central role at the conference. Some talks on Commutative Algebra included homological methods and connections to combinatorics, while other talks took a more geometric point of view. On Wednesday morning and early afternoon, most participants took the opportunity to observe the total eclipse of the sun, another highlight of the conference.

Monday, 9.8.

Luchezar Avramov

FINITE GENERATION OF HOCHSCHILD HOMOLOGY ALGEBRAS

This is an account of joint work with S. Iyengar.

Let k be a commutative noetherian ring and S a flat finiteley generated $k-$Algebra. The Hochschild homology $HH_*(S/k)$ is then a graded, (skew-)commutative $k-$algebra, each $HH_i(S/k)$ is a finitely generated $S-$module, and there is a canonical homomorphism of graded $S-$algebras

$$\omega_{S/k} : \bigwedge^* \Omega_{S/k} \longrightarrow HH_*(S/k)$$

that extends the natural identification $S = HH_0(S/k)$ and $\Omega_{S/k} = HH_1(S/k)$. The classical Hochschild-Kostant-Rosenberg Theorem asserts that $\omega_{S/k}$ is bijective and the $S-$module $\Omega_{S/k}$ is projective if the $k-$algebra S is smooth. The following result is a strong converse:

Theorem. If the $S-$Algebra $HH_*(S/k)$ is finitely generated, then the $k-$algebra S is smooth.

The proof easily reduces to the local case, where it breaks down into two distinct arguments. If the residual characteristic of S is positive, then the statement can be easily deduced from an earlier result of Avramov-Vigué-Poirrier, the Buenos Ayres Cyclic Homology Group, and Rodicio, characterizing smoothness in terms of vanishing of some Hochschild homology groups. The case of characteristic 0 requires an extensive use of minimal DG algebra models, and uses the solution of a conjecture of Quillen by Avramov and Halperin.

Bernd Ulrich

A GENERALIZED PRINCIPAL IDEAL THEOREM AND THE HEIGHT OF IDEALS OF MINORS

This is a report on joint work with David Eisenbud and Craig Huneke.

The Generalized Principal Ideal Theorem by Eisenbud-Evans and Bruns says that if N is a finitely generated module over a Noetherian local ring $(R, \mathbf{m})$ and x is an element of N, then the order ideal $N^*(x) = \{f(x) : f \in \mathrm{Hom}_R(N, R)\}$ has height bounded by the rank of N provided that $x \in \mathbf{m}N$. We investigate the case where $x \notin \mathbf{m}N$. We prove that the conclusion of the Generalized Principal Ideal Theorem remains true as long as there exists a homomorphism from R to a local domain $(S, \mathbf{n})$ so that $x \otimes 1 \in \mathbf{n}(N \otimes_R S)/S-$torsion. We also characterize the modules N in which $\mathrm{ht}\, N^*(x) \leq \mathrm{rank}\, N$ for every $x \in N$.
Refining formulas by Bruns we give bounds for the height of determinantal ideals that take into account the embedding codimension of the ambient ring. For instance we show: Let φ be an $n \times m$ matrix of rank r defined over a Noetherian local ring R of embedding

codimension e, let t be an integer with $I_t(\varphi) \neq R$, and set $\epsilon = \max\{\operatorname{ecodim} R_{\mathbf{q}} : \mathbf{q} \notin V(I_t(\varphi))\}$; if $\operatorname{coker}\varphi$ is not a direct sum of a free module and a torsion module then

$$\operatorname{ht} I_t(\varphi) \leq (r-t)(\max\{m, n+\epsilon\} - t + 1) + n + e\,.$$

The case $t = r$ is due to Faltings.

Craig Huneke

LOCALIZATION OF TIGHT CLOSURE

We discuss some new results concerning the localization of tight closure. Let R be a reduced excellent Noetherian ring of positive characteristic p. We let $q = p^e$ be a varying power of p. The tight closure of an ideal I is

$$I^* = \{x \in R : \exists c \text{ such that } cx^q \in I^{[q]}\, \forall q >> 0\}$$

where c is not in any minimal prime and $I^{[q]} = \langle i^q : i \in I \rangle$. The basic question is does

$$(I^*)_W = (I_W)^*$$

for every multiplicatively closed set $W \subseteq R$. The containment $\subseteq$ always holds. Define a set of primes

$$T(x) = \{Q \in \operatorname{Spec}(R) : Q \text{ is minimal over } I^{(q)} : cx^q\, \forall q >> 0\}$$

where c is a *test element*, i.e. an element in $\bigcup_{I \subset R} I : I^*$, not in a minimal prime. Test elements are known to exist if R is $F-$finite, essentially of finite type over a field or complete local, e.g. Our first result is

Theorem $(I^*)_W = (I_W)^*\, \forall W \iff$

(1) $\mid T(x) \mid < \infty\, \forall x \in R$

(2) If $Q \in T(x)\, \exists N$ such that $Q^{Nq} \subseteq (I^{(q)} : cx^q)Q\, \forall q >> 0$.

Let $Q \in T(x)$. We can define a length function, $\lambda(R_Q/(I^{(q)} : cx^q)Q) = \lambda(q)$. A fundamental question is how does $\lambda(q)$ grow? We conjecture that

$$\lim \lambda(q)/q^{\dim R_Q}$$

exists and a positive answer to this question is closely related, possibly equivalent, to a positive answer to the localization of tight closure.

Shiro Goto

GOOD IDEALS IN GORENSTEIN LOCAL RINGS

My talk is a joint work with S. Jai and K. Watanabe. Let $(A, \mathbf{m})$ be a Gorenstein local ring of $\dim A = d$. Let I be an $\mathbf{m}-$primary ideal in A and assume that $I \supseteq J$ a parameter ideal in A as a reduction. Then we say that I is good if $G(I)$ is a Gorenstein ring and $a(G(I)) = 1 - d$. This is a very strong condition and my purpose is to study good ideals, following the problems: (1) Find a better characterization of good ideals. (2) Estimate $X_A := \{I : I \text{ a good ideal in } A\}$, e.g., clarify how many good ideals are contained in a given Gorenstein local ring. The main results are as follows.

Theorem 1. $\sharp X_A = \infty$ if $d \geq 3$ and $A \supseteq$ a field.

Theorem 2. Let $A = k[[X, Y, Z]]$ and $J = (X^a, Y^b, Z^c)\,(a, b, c \geq 1)$ .Then $X_J = \{I \in X_A : I \supseteq J \text{ as a reduction } \} = \emptyset$ if and only if one of the conditions is satisfied (a) $\{a, b, c\} \ni 1$, or (b) $2 \not| abc$, or (c) $(a, b, c) = (2, 2, \text{odd})$ (or its permutation).

Theorem 3. Suppose $d = 2$. Then the following conditions are equivalent. (1) $I \in X_A$. (2) $I^\ell \in X_A \, \forall \ell > 0$. (3) $I^2 = JI$, $I^\ell \in X_A \, \exists \ell > 0$. (4) $H^1_{\mathbf{m}}(G) = (0)$, $I^\ell \in X_A \, \exists \ell > 0$. (5) $\ell_A(A/I^{n+1})$ is a polynomial for $\forall n \geq 0$ and $I^\ell \in X_A$ for some $\ell > 0$. (6) $\ell_A(A/I^n) = n^2 \ell_A(A/I) \, \forall n \geq 0$. (7) $I^n = J^n : I \, , \forall n \in \mathbb{Z}$. (8) $R(I) \in M$ and $K_R \cong R_+$. (9) R' is a Gorenstein ring and $K_{R'} \cong R'$.

Theorem 4 Suppose A is a two-dimensional rational singularity. Then $I \in X_A$ if and only if I is invertible on minimal resolution $X \longrightarrow \operatorname{Spec} A$. If $d = 1$, we have a one-to-one correspondance between the set of good ideals and certain over rings of A.

Eero Hyry

ON COEFFICIENT IDEALS

Let A be a regular local ring, and let $I \subset A$ be an ideal of positive height. If $J \subset I$ is a minimal reduction, then the coefficient ideal of I relative to J is by definiton the largest ideal α such that $I\alpha = J\alpha$. The importance of this notion, introduced by Aberbach and Huneke, stems from the role it plays in Briançon-Skoda type theorems. We are interested to investigate the coefficient ideal in the situation there the Rees algebra $R_A(I)$ is Cohen--Macauley. We can then show that the coefficient ideal is independent of the minimal reduction J for certain importance classes of ideals. We also consider the connection between adjoint and coefficient ideals which turns out to be related with rational singularities of $R_A(I)$.

Holger Brenner

Affineness of complements of hypersurfaces

We consider the following property of an affine scheme $X = \operatorname{Spec} A$: the complement of every hypersurface is affine. For example this property holds if A is locally factorial (regular) or, due to a theorem of Nagata, if it is two dimensional excellent and normal. If X is normal and affine, we construct a residue class group of the divisor class group $\operatorname{Cl} X$, called the affine class group $\operatorname{ACl} X$. It measures the deviation of this property in a similar way as $\operatorname{Cl} X$ measures the deviation of factoriality. The vanishing of the affine class groups characterizes the property that the complement of every hypersurface is affine.

We show that the affine class group does not change if we replace X by $X \times \mathbf{A} = \operatorname{Spec} A[T]$. So in particular every surface in the affine line over a normal affine surface has an affine complement and thus the intersection of two surfaces has no isolated points (this is not true over a non-normal surface). As a partial generalization of this, we further show for a two dimensional normal affine variety X and a smooth affine curve C over an algebraically closed field that every hypersurface in the product $X \times C$ has an affine complement.

We compute the ACl of hyperbolas, monoidrings and determinantal rings, showing that in these examples the affine class group is just the divisor class group modulo torsion. The computation of ACl of the homogenous coordinate rings A of a ruled surface Y yields $\operatorname{ACl} A = \mathbf{Z}$, suggesting a relation between the affine class group of the cone and the numerical class group of a smooth projective variety.

Martin Kreuzer

ZERO-DIMENSIONAL SCHEMES AND HILBERT-KUNZ FUNCTION

The Hilbert-Kunz function of a 1-dimensional standard graded algebra R over an algebraic extension field of $\mathbb{F}_p$ is known to be of the form $HK_R(i) = \operatorname{mult}(R) \cdot p^i + \varphi(i)$, where $\varphi : \mathbb{N} \longrightarrow \mathbb{N}$ is a periodic function. Using the theory of zero-dimensional subschemes of $\mathbf{P}^n$ we showed how one can effectively compute a number N such that this formula holds for $i \geq N$ and a bound for the periodic length of φ. As an application we saw that the Hilbert-Kunz function of the homogenous coordinate ring of a finite set of $s\,\mathbb{F}_p-$rational points is $HK_R(i) = sp^i - sti$ and does not depend on the geometry of the points.

Tuesday, 9.9.

Orlando Villamayor

A SIMPLIFIED PROOF OF RESOLUTION OF SINGULARITIES

Let X be an irreducible variety over a field k of characteristic zero, let $U \subseteq X$ be the open set of regular points of X, and $\operatorname{Sing}(X) = X - U$. The theorem of desingularization of Hironaka proves that there is a proper birational morphism $X' \xrightarrow{\pi} X$ with the following three properties: a) X' is regular, b) π induces an isomorphism $\pi^{-1}(U) \xrightarrow{\pi} U$, c) $\pi^{-1}(\operatorname{Sing} X)$ is a union of regular hypersurfaces having only normal crossings.

In this talk I present a substantial simplification of the proof of this theorem. In fact this simplified proof avoids Hironaka's notion of normal flatness and his use of Hilbert-Samuel functions as invariants involved in the proof. It is shown that desingularization is a corollary of principalization of ideals included in smooth varieties. For this we apply an algorithm of principalization. The outcome is a desingularization which is also equivariant.

This is joint work with Santiago Encinas.

Herwig Hauser

RESOLUTION PROBLEMS IN POSITIVE CHARACTERISTIC

Given a surface $f(x,y,z) = x^\sigma + y^r z^s g(y,z)$ with $r + s + \operatorname{ord} g \geq \operatorname{ord} f$ where y^r and z^s are exceptional components induced by prior blowups it is natural and common practice to associate to f the resolution invariant (σ, e) where coordinates have to be chosen so that $e = \operatorname{ord} f$ is maximal. Induction on this pair (taken lexicographically) forms the basis of most resolution algorithms in characteristic zero. However, this fails in positive characteristic, where the invariant may increase. Take $f(x,y,z) = x^3 + y^4z^3 + y^2z^6$ in characteristic 3, blow up three times a point, first the origin, then the origin of the $z-$chart followed by the origin of the $Y-$chart. After the second blowup we have strict transform $f'' = x^3 + y^4z^4(y+z)$ with $r = s = 4$, $\sigma = 3$, $e = 1$. After the next blow up, and looking at the "mid point" between the origins of the two charts, we get $f''' = x^3 + z^6(-1+y^2-y^3+y^5)$ with $r = 0$, $s = 6$, $\sigma = 3$ but $e = 2$ not yet realized in the considered coordinates. The change $x \longmapsto x + z^2$ realizes $e = 2$ and shows that the pair (e, σ) has increased in the last blowup.

This phenomen, first observed by Moh (though never exhibited in a true resolution process) has not been overcome yet. We can show, however, that whenever $\bar{r} + \bar{s} \geq 0$ this increase cannot occur ($\bar{r}, \bar{s}$ the remainders of r and s mod σ). The results can be extended to any dimension.

Santiago Encinas

ON CONSTRUCTIVE DESINGULARIZATION OF NON-EMBEDDED

SCHEMES

Let X be a closed subscheme of a regular variety W over a field of characteristic zero. To the embedding $X \subset W$ one may apply algorithms of embedded desingularization. These algorithms have nice properties: compatibility with open immersions, equivariance, and more ...

The scheme X can be embedded in different varieties, say W and W'. If $\dim W = \dim W'$, it follows from the construction of the algorithms that we find the same desingularizations for both embeddings $X \subset W$ and $X \subset W'$. But if $\dim W \neq \dim W'$ then some algorithms will deal to different desingularizations for $X \subset W$ and $X \subset W'$.

Nethertheless, the compatibility with open immersions implies that we can construct desingularization for non-embedded schemes.

Kalle Karu

TOWARDS TOROIDALIZATION

We consider proper morphisms between complex nonsingular algebraic varieties, and study the problem of resolution of singularities of such morphisms.

Definition An embedding $U_X \subset X$ is *toroidal* if it is locally (analytically, étale, formally) isomorphic to a torus embedding. Similarly a morphism is toroidal if it is locally isomorphic to a toric morphism.

Conjecture (Toroidalization). Given a proper surjective morphism $f : X \longrightarrow Y$ there exist compositions of smooth blowups $X' \longrightarrow X$ and $Y' \longrightarrow Y$ such that the induced map $f' : X' \longrightarrow Y'$ is toroidal.

We show that in case Y is a surface the conjecture is reduced to the problem of resolving the singularities of a differential $2-$form with logarithmic poles. For the case when X is also a surface we get the (well-known) result that Conjecture is true. The problem of resolving $2-$forms in dimension ≥ 3 is still open.

Dale Cutkosky

LOCAL AND GLOBAL MONOMIALIZATION OF MORPHISMS

Suppose that $R \subset S$ are regular local rings, essentially of finite type over a field k of char 0, and that V is a valuation ring which dominates S.

We show that if the quotient field of S is finite over the quotient field of R, then $R \longrightarrow S$ can be monomialized. That is, $\exists$ sequences of monoidal transforms $R \longrightarrow R'$ and $S \longrightarrow S'$ such that V dominates S' and $R' \longrightarrow S'$ is a monomial mapping.

As a corollary, we prove a theorem on "simultaneous resolution of singularities". We also prove that a birational map can be locally factored along a valuation. Suppose that

$R \longrightarrow S$ is birational. We show that $\exists$ a local ring T dominated by V such that $R \longrightarrow T$ and $S \longrightarrow T$ are products of monoidal tranforms.

It is of interest to construct a global monomialization of a proper morphism. Suppose that $\Phi : X \longrightarrow S$ is a proper morphism from a nonsingular $3-$fold to a non-singular surface, over an algebraically closed field k of char 0. We construct sequences of monoidal transforms $X_1 \longrightarrow X$ and $S_1 \longrightarrow S$ such that $X_1 \longrightarrow S_1$ is locally a monomial mapping.

Jarosław Włodarczyk

FACTORIZATION THEOREM AND MORSE THEORY IN ALHEBRAIC GEOMETRY

We discuss a Morse-like theory which serves as a basic in a proof of the following factorization theorem

"**Weak Factorization Theorem**". Any birational map $f : X \longrightarrow Y$ between two complete smooth varieties X and Y over a field K of charakteristik 0 can be factored as

$$X = X_0 \cdots \xrightarrow{f_1} X_1 \cdots \xrightarrow{f_n} X_n = Y$$

where each X_i is smooth and f_i is either a blow-up or a blow-down at a smooth center.

The main notion used in the proof is a notion of a birational cobordism.

Definition Let $\varphi : X ... \to Y$ be a proper birational map. A birational cobordism $B = B(X, Y)$ is a variety B such that

a) $K^\times$ acts effectively on B

b) the sets $B_+ = \{x \in B : \lim_{t\to\infty} tx \text{ does not exist}\}$ and

$B_- = \{x \in B : \lim_{t\to 0} tx \text{ does not exist}\}$

are open and

c) there exist geometric quotients (orbit spaces) $B_+/K^\times \cong Y$ and $B_-/K^\times \cong X$

d) The natural embeddings induce the commutative diagramm

$$\begin{tikzcd} & B_+/K^\times \arrow[r, "\cong"] & Y \\ B_+ \cap B_-/K^\times \arrow[ur, "\subset"] \arrow[dr, "\subset"'] & & \\ & B_-/K^\times \arrow[r, "\cong"] & X \arrow[uu, "\varphi"'] \end{tikzcd}$$

This notion is analogous to the notion of the usual cobordism between differentiable varieties in Morse Theory. In the topological setting integrating of the gradient field of the Morse function gives an action of $1-$parameter group of diffeomophism which is isomorphic to the additive group of real number $(\mathbb{R}, +)$ or multiplicative group of real positive numbers $(\mathbb{R}_{>0}, .)$. The critical points of Morse function are the fixed points of this action. Passing through the critical points changes the homotopy type. In the algebraic situation passing through the fixed component induce birational transformation. Looking at the tangent spaces we obtain a local description of these transformations.

Kenji Matsuki

TORIFICATION AND FACTORIZATION OF BIRATIONAL MAPS

We discuss the joint work with D. Abramovich, K. Karu and J.Włodarczyk which gives an affirmative answer to the so-called Weak Factorization Conjecture of birational maps:

Main Theorem. Let $\Phi : X_1 \dots \longrightarrow X_2$ be a birational map between complete nonsingular varieties over K, where K is an algebraically closed field of char $(K) = 0$.

Then Φ is a composite of blowups and/or blowdowns with smooth centers. (The composite or factorization is called "weak" as we do NOT require it consists of blow ups immediatley followed by blowdowns only.)

Moreover,

1) if X_1 and X_2 are both projective, then we may choose a factorization so that all the intermediate varieties are also projective,

2) if Φ induces an isomorphism over an open set U, i.e., $\Phi : X_1 \supset U \overset{\sim}{\longrightarrow} U \subset X_2$, then we may choose a factorization with all the centers outside of U.

The key idea of the proof is that via the theory of Birational Cobordism by Morelli and Włodarczyk we reduce the factorization problem of general birational maps to that of toroidal ones.

Wednesday, 11.8

Ragnar-Olaf Buchweitz

HOCHSCHILD COHOMOLOGY AND SEMIREGULARITY

(joint work with Hubert Flenner (Bochum))

Let X be a complex Kähler manifold, $Z \subseteq X$ an analytic subspace of codimension q and $[Z] \in H^q(X, \Omega_X^q)$ its fundamental class. The vectorspace $H^1(X, \Theta_X)$, Θ_X the tangent sheaf, parametrizes first order deformations of X and contraction against $[Z]$ defines a map $\lrcorner[Z] : H^1(X, \Theta_X) \longrightarrow H^{q+1}(X, \Omega_X^{q-1})$ whose vanishing decides whether the unique horizontal extension of $[Z]$ along a Kodaira-Spencer class $\tau \in H^1(X, \Theta_X)$ remains of Hodge-type (q, q). This is Griffiths' transversality theorem. On the other hand, there is a map $H^1(X, \Theta_X) \longrightarrow T^2_{Z/X}$ whose vanishing decides whether Z can be deformed along τ. S. Bloch asked in 1972 to construct a map $T^2_{Z/X} \longrightarrow H^{q+1}(X, \Omega_X^{q-1})$ that compares these obstructions. Following terminology introduced by Severi (1947) such a potential map is called a "semiregularity map" and Z is semiregular if this map is injective. We construct such a general semiregularity map using the Atiyah-Chern character of Z in X. We indicate applications to the embedded deformation theory of Z, generalizing results by Ran, Kawamata; Artamkin-Mukai. We finally comment on the construction in the singular case, using the theory of the cotangent complex and show how to extend the construction to perfect complexes of $\mathcal{O}_X$-modules on an arbitrary complex space X. Defining finally the Hochschild (co-)homology of X à la Quillen, we show that there is a much more general construction that can be summarized thus: The diagramm

$$\begin{tikzcd} HH^\bullet(X) \arrow[rr, "\lrcorner e^{-at\mathcal{F}}"] \arrow[dr, "\lrcorner ch(\mathcal{F})"'] & & \mathrm{Ext}^\bullet_X(\mathcal{F}, \mathcal{F}) \arrow[dl, "tr(e^{-at\mathcal{F}} \cup *)"] \\ & HH_\bullet(X) & \end{tikzcd}$$

commutes for every perfect complex $\mathcal{F}$, where

$$HH^\bullet(X) := \oplus H^\bullet(\mathrm{RHom}_X(\mathbb{S}^\bullet(\mathbb{L}[1]), \mathcal{O}_X))$$

and $HH_\bullet(X) := H^\bullet(X, \mathbb{S}^\bullet(\mathbb{L}[1]))$ with $\mathbb{S}^\bullet$ the derived symmetric powers of the shifted cotangent complex $\mathbb{L}[1]$ of X.

Hubert Flenner

A GENERALIZATION OF A MULTIPLICITY THEOREM OF BÖGER

Let $(A, \mathbf{m})$ be a complete local ring and $J \subseteq I \subseteq \mathbf{m}$ ideals. The Ideal J is called a reduction of I if $JI^n = I^{n+1}$ for $n >> 0$. If I, J are $\mathbf{m}-$primary and A is equidimensional then by a classical result of Rees J is a reduction of I iff $e(I, A) = e(J, A)$. This result was generalized by E. Böger to ideals with $\ell(J) = \operatorname{ht} J$: if $e(J_{\mathbf{p}}, A_{\mathbf{p}}) = e(I_{\mathbf{p}}, A_{\mathbf{p}})$ for all minimal primes of I and if $\sqrt{I} = \sqrt{J}$ then J is a reduction of I. Let now $J \subseteq I \subseteq \mathbf{m}$ be arbitrary ideals in the equidimensional ring A. Then one can assign to I and J a generalized multiplicity $j(I, A)$ resp. $j(J, A)$, which was introduced by Achilles-Manaresi and which generalizes the intersection multiplicity of Fulton for distinguished components resp. the Stückrad-Vogel multiplicities of intersections in projective space. The main result is:

J is a reduction of I if and only if $j(J_{\mathbf{p}}, A_{\mathbf{p}}) = j(I_{\mathbf{p}}, A_{\mathbf{p}})$ for all primes $\mathbf{p}$ of A. (joint work with M. Manaresi, Bologna)

Paul Roberts

THE POSITIVITY OF INTERSECTION MULTIPLICITIES AND SYMBOLIC POWERS OF PRIME IDEALS

Let R be a regular local ring. If $\mathbf{p}$ and $\mathbf{q}$ are prine ideals of R such that $\mathbf{p} + \mathbf{q}$ is primary to the maximal ideal of R and $\dim(R/\mathbf{p}) + \dim(R/\mathbf{q}) = \dim(R)$, then Serre's positivity conjecture states that

$$\chi(R/\mathbf{p}, R/\mathbf{q}) = \sum (-1)^i \text{length}\,(\operatorname{Tor}_i(R/\mathbf{p}, R/\mathbf{q})) > 0\,.$$

We derive several conditions which are equivalent to this conjecture. These conditions are based on the construction of certain bigraded ideals in a bigraded ring constructed by O. Gabber in his proof of Serre's nonnegativity conjecture, and they depend on properties of Hilbert polynomials of bigraded modules. These equivalences imply that the hypotheses of the Serre positivity conjecture for prime ideals $\mathbf{p}$ and $\mathbf{q}$ imply that $\mathbf{p}^{(n)} \cap \mathbf{q} \subseteq \mathbf{m}^{n+1}$, where $\mathbf{p}^{(n)}$ is the n^{th} symbolic power of $\mathbf{p}$ and $\mathbf{m}$ is the maximal ideal of R.

Thursday, 12.8

Sorin Popescu

SYZYGIES OF UNIMODULAR LAWRENCE TORIC IDEALS

The talk is a report on joint work with Dave Bayer and Bernd Sturmfels.
We are interested in the defining ideals of toric subvarieties of $\mathbf{P}^1_k \times \mathbf{P}^1_k \times ... \times \mathbf{P}^1_k$; these are binomial ideals in $2n$ variables of the form

$$J_L = \langle x^a y^b - x^b y^a : a - b \in L \rangle \subset k[x_1, ..., x_n, y_1, ..., y_n]$$

where $(x_i; y_i)$ are the coordinates of the i^{th} factor $\mathbf{P}^1_k$ and $L \subset \mathbb{Z}^n$ is a sublattice. Binomial ideals of the form J_L are called Lawrence toric ideals because of the analogy with the Lawrence lifting construction in polytope theory.

From a different point of view, Lawrence ideals describe also the diagonal embedding of a complete toric variety $X \overset{\triangle}{\hookrightarrow} X \times X$ via Audin-Cox's homogenous coordinate ring of the toric variety $X \times X$.

In the case L is a unimodular lattice, we describe completely the minimal free resolution of the $((\mathbb{Z}^n/L)-$graded) Lawrence toric ideals J_L. A consequence of our description is that the minimal free resolution of J_L is universal, that is it is stable with respect to any term order $<$. This generalizes the fact that the minimal generators of J_L are a universal Gröbner basis, extending this property from generators to all higher syzygies of J_L.

In case L is the graphic or cographic lattice of an oriented graph, we may express properties of the underlying unoriented graph in terms of the Mumford-Castelnuovo regularity of the syzygies varieties of J_L. For instance planarity and the chromatic number may be expressed in this terms.

Irena Peeva

TORIC HILBERT SCHEMES

This talk is on introducing the toric Hilbert scheme which parametrizes all ideals with the same Hilbert fuction as a given toric ideal. I compare the properties of toric Hilbert schemes to those of classical Hilbert schemes. I present a description of the toric Hilbert scheme of a codimension 2 toric variety: the scheme is 2-dimensional, smooth, and has exactly one component which is the closure of the orbit of the toric ideal under the torus action. The talk is on joint results with V. Gasharov and M. Stillman.

Takayuki Hibi

COMPRESSED POLYTOPES

A *compressed* polytope is an integral convex polytope any of those reverse lexicographic triangulations is unimodular. An integral convex polytope $P \subset \mathbb{R}^n$ is called $(0,1)-$*perfect* if, for some $m \geq 1$, there exist
(i) $m \times n\, \mathbb{Z}-$matrix $A = (a_{ij})_{1\leq i\leq m,\, 1\leq j\leq n}$
(ii) $\mathbb{Z}-$vector $b = [b_1, ..., b_m]$
(iii) $\{0,1\}-$vector $c = [c_1, ..., c_n]$
such that

$$P = \left\{ y = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \in \mathbb{R}^n : \begin{array}{c} {}^t b - {}^t c \leq Ay \leq {}^t b \\ \text{(componentwise)} \\ \\ 0 \leq y_j \leq 1 \ (1 \leq \forall j \leq n) \end{array} \right\}.$$

For example, the d$^{\text{th}}$ hypersimplex $\triangle(n; d)$ in $\mathbb{R}^n$ is $(0,1)-$perfect. Also, the order polytope of a finite partially ordered set is $(0,1)-$perfect. It can be proved that any reverse lexicographic initial ideal of the toric ideal of a $(0,1)-$perfect polytope is generated by squarefree monomials. Hence

Theorem. All $(0,1)-$perfect polytopes are compressed.

Note, in addition, that if an $m \times n\, \mathbb{Z}-$matrix A is totally unimodular, then, for any $\mathbb{Z}-$vector $b \in \mathbb{Z}^m$ and for any $0,1-$vector $c \in \mathbb{Z}^m$, the convex polytope consisting of all $y = {}^t[y_1, ..., y_n] \in \mathbb{R}^n$ such that ${}^t b - {}^t c \leq Ay \leq {}^t b$ and $0 \leq y_j \leq 1\, (1 \leq \forall j \leq n)$ is integral. In particular, such a polytope is compressed.

Vic Reiner

RESOLUTIONS OF QUADRATIC VERONESE RINGS AND COMBINATORICS

We consider the surjection

$$\mathbb{F}[x_{ij}]_{1\leq i\leq j\leq d} = A \xrightarrow{f} R = \mathbb{F}[t_i t_j]_{1\leq i\leq j\leq d} \subset \mathbb{F}[t_1, ..., t_d]$$

$$X_{ij} \longmapsto t_i t_j$$

and the resolution of R as an $A-$module. An old proposition of Hochster from the 1970's implies that (in this very special case), we have

$$\mathrm{Tor}_i^A(R, \mathbb{F})_\mu \cong \tilde{H}_{i-1}(\triangle_\mu, \mathbb{F}) \ \forall \mu \in \mathbb{N}^d$$

where the subscript μ indicates the $\mu-$multigraded component in the $\mathbb{N}^d-$grading on $A, R, \mathrm{Tor}_\bullet^A(R, \mathbb{F})$, and $\tilde{H}_\bullet$ indicates (reduced) simplicial homology of the simplicial complex $\triangle_\mu$. Here $\triangle_\mu$ is the "bounded-degree graph complex" having vertex set indexed by edges and loops in a complete graph with loops on d vertices, and faces corresponding to edge-subgraphs in which the i^{th} vertex has degree at most i. An alternative computation of this $A, R, \mathrm{Tor}_\bullet^A(R, \mathbb{F})$ comes from work of Jozefiak-Pragacz-Wegman (1981) who resolved $I = \ker(f)$ as the ideal generated by $2 \times 2-$minors of a generic symmetric matrix $(x_{ij})_{1 \leq i,j \leq n}$, $x_{ij} = x_{ji}$, but *only if* char $(\mathbb{F}) = 0$. The resolution is known to depend on char $(\mathbb{F})$, but is not known for char $(\mathbb{F}) > 0$ completely.

We proposed to X. Dang (Ph.D. student, Univ. of Minn.) the following general plan.
(1) Use the char $(\mathbb{F}) = 0$ homology of $\triangle_\mu$ known from JPW to guess homotopy-theoretic facts about $\triangle_\mu$.
(2) Prove these via topological combinatorics.
(3) See what it implies in a characteristic-free way about $A, R, \mathrm{Tor}_\bullet^A(R, \mathbb{F})$.

Here is a typical example of Dang's result:

Theorem. $\triangle_\mu$ is constructible if and only if $\tilde{H}_i(\triangle_\mu, \mathbb{Q}) = 0 \, \forall i$.

Corollary The multidegrees $\mu \in \mathbb{N}^d$ which supports at least one (higher) syzygy in the $A-$resolution of R does not depend on char $(\mathbb{F})$.

Hema Srinivasan

THE ALGEBRAIC FUNDAMENTAL GROUP OF A CURVE SINGULARITY

This talk reports on my joint work with Dale Cutkosky (U. of Missouri).

The topological fundamental group of the knot determined by the germ of an analytically irreducible durve in $\mathbb{C}^2$ has been computed by Brauner in 1928 and also by Kähler and Zariski. These elegant formulas for the generators and relations depend only on the characteristic pairs of a Puiseux series expansion of the curve. We prove an arithmetic analogue of Brauners' theorem valid in all characteristics. Our main theorem is stated in this way.

Theorem. Let $R = k[[x, y]]$, where k is an algebraically closed field of any characteristic. Let $f \in R$ be irreducible and $C = V(f)$. Then there exist positive integers $g, n_1, ..., n_g, m_1, ..., m_g$, $(m_i, n_i) = 1$ such that the prime to p part of the algebraic fundamental group $\Pi_1^{(p)}(\mathrm{Spec}\,(R) - V(f))$ is isomorphic to the prime to p part of the profinite completion of the free group on the symbols $Q_0, Q_1, ..., Q_g, P_1, ..., P_g$ modulo the relations $Q_0 = 1$, $Q_i^{m_i} = P_i^{n_i - n_{i-1}m_i} Q_i^{m_{i-1}m_i}$, $1 \leq i \leq g$ and $P_{i+1} P_i^{y_i} Q_i^{m_{i-1}x_i} = Q_i^{x_i}$, $1 \leq i \leq q - 1$.

$\Pi_1^{(p)}$ is the quotient of Π_1 by the closed normal subgroup generated by its $p-$Sylow subgroups. The integers (m_i, n_i) in the theorem are indeed the characteristic pairs of f. These generators and relations coincide with those in the formula of Brauner.

Stefan Schröer

CONTRACTIBLE CURVES ON NORMAL SURFACES

Let X be a proper normal algebraic surface over a ground field k. Suppose $R \subset X$ is a connected negative definite curve.

Theorem. The curve $R \subset X$ is contractible if and only if there is a Weil divisor A with $A.C = 0$, $C \subset R$, and $A.C > 0$, $C \not\subset R$, satisfying the following properties:

(i) A is Cartier near R.

(ii) $\forall m > 0\, \exists n > 0$, and a numerically trivial Weil divisor N, Cartier near R, such that $nA + N \mid mR = 0$.

The proof rests upon vanishing theorems of Fujita.

Corollary. If $K_X + mR$ is not effective for all $m \geq 0$, then R is contractible.

Suppose D is an arbitrary Weil divisor. Consider the graded ring $R(X, D) := \oplus_{n \geq 0} H^0(X, nD)$ and its homogenous spectrum $P(X, D)$.

Corollary. The scheme $P(X, D)$ is of finite type.

Wolmer Vasconcelos

COHOMOLOGICAL DEGREE AND APPLICATIONS

Let S be a finiteley generated standard graded algebra over a Noetherian local ring $(R, \mathbf{m})$. A cohomological degree $\mathrm{Deg}(\cdot)$ is a numerical function on f.g. $R-$modules satisfying the rules: (i) If $L = H^0_{\mathbf{m}}(M)$, $\mathrm{Deg}\,(M) = \mathrm{Deg}\,(M/L) + \ell(L)$, ℓ = length function; (ii) if $h \in \mathbf{m} \setminus \mathbf{m}^2$ is generic and regular on M, $\mathrm{Deg}\,(M) \geq \mathrm{Deg}\,(M/hM)$; (iii) if M is Cohen-Macauley, $\mathrm{Deg}\,(M) = \deg\,(M)$, the ordinary multiplicity.
Such functions satisfy: (a) $\mathrm{Deg}\,(M) \geq \nu(M)$, ν = min'n gen. function; (b) if R is Artinian, $\mathrm{Deg}\,(S) \geq \mathrm{reg}\,(S)$, the Castelnuovo-Mumford regularity of S.

There is a unique function $\mathrm{Deg}(\cdot)$, $\mathrm{bdeg}\,(\cdot)$, for which equality always holds in (ii) (due to Tor Gunston). It is useful in several calculations of invariants of ideals. It also satisfies

Theorem. For a finiteley generated graded $S-$module $M = \oplus_{n \in \mathbb{Z}} M_n$,

$$\sum_{n \geq 0} \mathrm{bdeg}\,(M_n) t^n = \frac{h(t)}{(1-t)^{\ell}}\,,$$

where $h(t)$ is a polynomial and $\ell = \dim\,(M \otimes R/\mathbf{m})$.

Friday, 13.8

E. Graham Evans

IDEALS ATAINING A GIVEN HILBERT FUNCTION

Let $R = K[X_1, ..., X_n]$. Fix a Hilbert function for an R/I and survey all the ideals that give that Hilbert function. We partial order their free resolutions in the following way:
If

$$... \longrightarrow \oplus_j R[-j]^{\beta_{2j}} \longrightarrow \oplus_j R[-j]^{\beta_{1j}} \longrightarrow R \longrightarrow R/I \longrightarrow 0$$

is one such with graded betti numbers β_{ij} and α is another then $\alpha \geq \beta$ if $\alpha_{ij} \geq \beta_{ij}$ for all i, j.
The theorem of Bigatti-Hulett-Pardue shows there is a unique biggest resolution with the lex seg ideal for that Hilbert function being a representative.

This work looks for smallest ones

1. (– + Charalambous) There may not be a smallest
2. (Rodriguez) There may not be a smallest if we restrict to just betti numbers instead of graded betti numbers
3. (Richert) There may not be a smallest if we rstrict to the R/I which are Gorenstein. [This was a question of Tony Geramita.]
4. (Richert) If I is a sequence then there can be more than one minimal.

Quite generally the minimal elements in this partial order seem difficult to find.

Vesselin Gasharov

HILBERT FUNCTIONS

Let I be a homogenous ideal in a polynomial ring $R = k[X_1, ..., X_k]$, where k is a field. Green gave an upper bound for the Hilbert function $H(R/(h, I), d)$ for all $d \geq 1$, where h is a generic linear form in R. He proved that $H(R/(h, I), d) \leq H(R/(x_n, L), d)$, where L is a lexicographic ideal with the same Hilbert function as I. Herzog and Popescu extended Green's result to generic forms h of arbitrary degree in the case $\operatorname{char} k = 0$. Their proof uses Gröbner bases theory and reduction to the case of strongly stable ideals. I extend the Herzog-Popescu resul to arbitrary characteristic. I also give analogous results for strongly stable ideals in rings with restricted powers of the variables.

Dorin Popescu

BETTI NUMBERS FOR SPECIAL MONOMIAL IDEALS

Let K be a field of characteristic $p > 0$, $R = K[X_1, ..., X_n]$ and I a monomial ideal generated by a minimal set of monomials $G(I)$. For a $u \in G(I)$ set $m(u) = \max\{t : X_t \mid u\}$ and let $m(I) = \max_{u\in G(I)} m(u)$. Eliahou-Kervaire gave minimal free resolutions and nice formulas for Betti numbers for stable ideals. Our purpose is to find such formulas for more complicated monomial ideal, namely products of Frobenius $p-$powers of stable ideals $I = \prod_{t=1}^{s} I_t^{[p^{r_t}]}$, where I_t is stable, $0 \le r_1 \le r_2 \le ... \le r_s$ are integers and $I_t^{[p^{r_t}]}$ is the p^{r_t} Frobenius power of I.

Theorem 1(Ene-Pfister-Popescu). If for every j, $1 \le j < s$, I_j contains $X_{m(I_{j+1})}^{p^{r_{j+1}-r_j}} - 1$ then
i) the Betti numbers

$$\beta_j(R/I) = \sum_{q=1}^{s} \sum_{u\in G(I_q)} \prod_{t>q} \mid G(I_t) \mid \begin{pmatrix} m(u)-1 \\ j-1 \end{pmatrix}, \, j \ge 2,$$

the case $j = 0, 1$ being trivial,

$$\text{ii) } \operatorname{reg}(I) = \max_{1\le q\le s}\{\sum_{t>q}^{s} p^{r_t} d_t + \max_{u\in G(I_q)}[p^{r_t}\deg u + (m(u)-1)(p^{r_q}-1)]\}$$

$d_t = \max_{v\in I_t} \deg v$, the right hand of equality ii) we denote by $\mathrm{pa}(I)$.

Theorem 2 (Herzog-Popescu). Suppose for all $1 \le t < s$ and each $u \in G(I_t)$, $u = x_1^{a_1} ... x_n^{a_n}$ it holds

1) $x_2^{a_1+a_2} x_3^{a_3} ... x_n^{a_n} \in I$

2) $\sum_{i=1}^{e} \alpha_i \le (e-1)(p^{k_{e+1}-k_e} - 1)$ for all e, $2 \le e \le n$

Then $\operatorname{reg}(I) \le \mathrm{pa}(I)$. Moreover if I is a principal $p-$Borel ideal, that is the minimal $p-$Borel ideal containing one monomial this equality becomes equality, i.e. $\operatorname{reg}(I) = \mathrm{pa}(I)$.

Corollary 3 (Herzog-Popescu). The Pardue conjecture holds.

Reinhold Hübl

VALUATIONS AND DERIVATIONS

This talk deals with three problems:

1) Assume $\varphi : (R, \mathbf{m}) \longrightarrow (S, \mathbf{n})$ is an injective local homomorphism of local noetherian domains. Find sufficient conditions for the induced map $\hat{\varphi} : \hat{R} \longrightarrow \hat{S}$ to be injective again.

2) Let R be a complete regular local ring containing a field and with perfect residue class field $k = R/\mathbf{m}$ which it contains, and let $J \subseteq R$ be an ideal. Does there exist an $\ell = \ell(R, J)$ such that, whenever $f \in R$ with $\delta(f) \in J^{n+\ell}$ for all $\delta \in \mathrm{Der}_k(R)$, there exists an $a \in R$ with $\delta(a) = 0$ for all $\delta \in \mathrm{Der}_k(R)$ and with $f - a \in J^n$? Is it possible to bound ℓ in some uniform way?

3) Let k be a field, $\operatorname{char} k = 0$, let $(R, \mathbf{m})/k$ be a reduced local algebra essentially of finite type, and let $(S, \mathbf{n})/k$ be a local algebra, essentially of finite type. If $\epsilon : S \longrightarrow R$ is a surjective local morphism inducing an isomorphism $\epsilon^* : \Omega^1_{S/k} \otimes_S R \longrightarrow \Omega^1_{R/k}$, does this imply that ϵ is bijective?

Using valuations and their interplay with differential forms one can prove the following results

In the situation of 1) assume that R and S are analytically irreducible and that there exists a prime divisor u of S which is positive on $\mathbf{n}$ and which restricts to a prime divisor of R. Then there exist $a, b \in \mathbb{N}$ with $\mathbf{n}^{a\ell+b} \cap R \subseteq \mathbf{m}^{\ell} \, \forall \ell \in \mathbb{N}$. Thus $\hat{\varphi}$ is injective. If S/R is essentially of finite type, such prime divisors exist.

2) The first part has a positive answer, however it is not known whether these constants can be bounded uniformly. There is some evidence that $\ell(R, J) \leq \dim - 1$ might hold, however this is only proved if $\operatorname{char} k = 0$ and in some special cases (This is joint work with S.D. Cutkosky).

3) Assume $R = P/J$ for some smooth local $k-$algebra P/k. The problem can be reduced to either of the following questions:
a) If $f \in R$ with $f^n \in J^{n+1}$ for some $n \in \mathbb{N}$, does this imply, that $f \in \mathbf{m}J$?
b) Let $\mathbf{a}(J) = \{x \in R : \; v(x) \geq v(J) + v(\mathbf{m})$ for all prime divisors of R and $v(x) \geq v(J) + 1$ for all prime divisors with $v(J) > 0\}$. Is it true that $\mathbf{a}(J) \subseteq \mathbf{m}J$?
A positive answer to either a) or b) would give a positive answer to the original problem. Positive answers however are only available in some special cases.

Marc Chardin

CASTELNUOVO-MUMFORD REGULARITY AND LIAISON

joint work (in progress) with Bernd Ulrich.

Let k be a field, $R = k[x_0, ..., x_n]$, $\mathbf{m} = (x_0, ..., x_n)$. Consider a homogenous ideal $I = (f_1, ..., f_t)$ in R, set $d_i := \deg f_i$ and assume for simplicity that $d_1 \geq d_2 \geq ... \geq d_t$.

Definition If M is a finitely generated $R-$module which is graded,

$$\operatorname{reg}(M) := \min\{\mu : \; H^i_{\mathbf{m}}(M)_{>\mu - i} \, \forall i\}$$

If $X = \operatorname{Proj}(R/I)$ we set $\operatorname{reg}(X) = \operatorname{reg}(R/I^{\text{sat}})$.

We extend the following result, valid in characteristic 0:

Theorem (Butzam,Ein,Lazarsfeld) If $X = \operatorname{Proj}(R/I)$ is smooth, purely of codimension r,

$$\operatorname{reg}(X) \leq d_1 + ... + d_r - r\,.$$

Our two main result are:

Theorem 1. Set $X = \mathrm{Proj}\,(R/I)$ and $r := \mathrm{codim}\, X$. If X is locally a complete intersection outside a finite number of points, and the locus of irrational singularities has at most dimension one, if $\mathrm{char}\, k = 0$ then:

$$\mathrm{reg}\,(X^{\mathrm{top}}) \leq d_1 + ... + d_r - r\,;$$

there X^{top} is the top dimensional component of X.

Theorem 2. If $X := \mathrm{Proj}\,(R/I)$ is locally a complete intersection and $F-$rational, purely of codimension r,

$$\mathrm{reg}\,(R/I) \leq \frac{(\dim X + 2)!}{2}(d_1 + ... + d_r - r)\,.$$

NB $\mathrm{reg}\,(X) \leq \mathrm{reg}\,(R/I)$.

The proof of Theorem 1 relies on Kodaira type vanishing theorem and the following result:

Theorem 3. Let A be a noetherian local ring, essentially of finite type over a field of characteristic 0. Let $y_1, ..., y_r$ be a regular sequence in A, $\mathbf{a} := (y_1, ..., y_r)$ and $L_i := \sum_{i=1}^{r} U_{ij} y_j$ in $A[U_{ij}]_{1\leq i\leq s,\, 1\leq j\leq r}$ (U_{ij} are variables).
If $A/\mathbf{a}$ have rational singularities, so has $A[U_{ij}]/(L_1, ..., L_s) : \mathbf{a}\,.$

This result allows to show that, in the situation of Theorem 1, r "general" elements of I of degrees $d_1, ..., d_r$ links X to a scheme satisfying Kodaira vanishing (for the structure sheaf), from that the liaison sequence gives the conclusion. Theorem 2 is proved by recursion on $\dim X$, using the fact that the intersection of X with its "generic" link satisfies the same type of hypotheses.

Michal Kwieciński

REDUCING FLATNESS TO TORSION-FREENESS

joint work with André Galligo (Nice)

Suppose $f : X \longrightarrow Y$ is a complex algebraic morphism, Y is smooth and X is reduced and of pure dimension. We prove that

f is flat $\Longleftrightarrow$ the canonical map $X \times_Y ... \times_Y X \longrightarrow Y$ (n times) has no vertical (irreducible or embedded) components, $n = \dim Y$.

The proof includes a new construction of a "support hypergerm", which allows us to overcome the lack of generic flatness in almost finitely generated modules and perform flat dimension computations.

The above theorem is a version of a conjecture of Vasconcelos (1997):

A is $R-$flat $\Longleftrightarrow$ $A^{\otimes_R^n}$ is $R-$torsion free ($n = \dim R$) under suitable assumptions on A and R. It generalizes a classical result of Auslander (1961).

Related work on characterizing flatness when Y is not smooth (Kwieciński, 1998) and openness (Kwieciński, Tworzewski, 1996) is also discussed.

Uwe Nagel

LIAISON OF MODULES

Liaison theory for ideals of a Gorenstein ring R is well understood and useful for ideals of low codimension. Thus one might wonder about an analogous theory for modules. The talk addresses this problem. The modules we are using for linkage are called quasi-Gorenstein modules. These are perfect $R-$modules whose canonical module is isomorphic to the module itself (up to a degree shift in the graded case). Such modules exist in abundance. Using these quasi-Gorenstein modules we describe a construction which gives rise to the definition of $m-$linkage of modules. This relation is symmetric. Thus, it generates an equivalence relation which is called $m-$liaison. The corresponding $m-$liaison classes consist of unmixed modules of fixed dimension. The concept of $m-$liaison can be specialized in various ways. Fixing a free $R-$module F one defines $m-$liaison for submodules of F using $m-$liaison. In case $F = R$ it turns out that ideals I, J of R are $m-$linked by an ideal K of R iff K is a Gorenstein ideal and $K : I = J$, $K : J = I$, that means, I and J are Gorenstein linked by K. Yoshino and Isogowa proposed 1998 a different concept of linkage of Cohen-Macauley modules. It is equivalent to $m-$liaison where the linking modules are always free R/I modules, R being a complete intersection.

If M and N are $R-$modules of codimension c which are evenly linked then their corresponding $c-$syzygy modules are stably equivalent. Hence the intermediate local cohomology modules of M and N are isomorphic (up to a degree shift). This gives necessary conditions for modules being $m-$linked. If R is a domain we have the following sufficient condition. If M has codimension one then M is $m-$linked to R/aR for some $a \neq 0$ iff M is perfect of codimension one.

Berichterstatter: Holger Brenner (Bochum)

Name, First	University	e-mail
Avramov, Luchezar	West Lafayette	avramov@math.purdue.edu
Berger, Robert	Saarbrücken	rberger@math.uni-sb.de
Brenner, Holger	Bochum	brenner@cobra.ruhr-uni-bochum.de
Brodmann, Markus	Zürich	brodmann@math.unizh.ch
Bruns, Winfried	Osnabrück	winfried@mathematik.uni-osnabrueck.de
Buchweitz, Ragnar-Olaf	Toronto	ragnar@math.utoronto.ca
Chardin, Marc	Paris	chardin@math.jussieu.fr
Cutkosky, Dale	Columbia	dale@cutkosky.math.missouri-edu
Encinas, Santiago	Valladolid	sencinas@cpd.uva.es
Evans, E. Graham	Illinois	graham@symcom.math.uiuc.edu
Flenner, Hubert	Bochum	Hubert.Flenner@rz.ruhr-uni-bochum.de
Gasharov, Vesselin	Ithaca	vesko@math.cornell.edu
Goto, Shiro	Kawasaki-shi	goto@math.meiji.ac.jp
Hauser, Herwig	Innsbruck	herwig.hauser@uibk.ac.at
Herzog, Jürgen	Essen	juergen.herzog@uni-essen.de
Hibi, Takayuki	Osaka	hibi@math.sci.osaka-u.ac.jp
Hübl, Reinhold	Regensburg	Reinhold.Huebl@mathematik.uni-regensburg.de
Huneke, Craig	Lawrence	huneke@math.ukans.edu
Hyry, Eero	Helsinki	Eero.hyry@helsinki.fi
Jouanolou, Jean-Pierre	Strasbourg	jouanolo@math.u-strsbg.fr
Karu, Kalle	Boston	kllkr@math.bu.edu
Kreuzer, Martin	Regensburg	Martin.Kreuzer@mathematik.uni-regensburg.de
Kunz, Ernst	Regensburg	ernst.kunz@mathematik.uni-regensburg.de
Kwieciński, Michal	Krakow	kwiecins@im.uj.edu.pl
Lehmkuhl, Thomas	Göttingen	lehmkuhl@uni-math.gwdg.de
Lipman, Joseph	West Lafayette	lipman@math.pardue.edu
Matsuki, Kenji	West Lafayette	kmatsuki@math.pardue.edu
Nagel, Uwe	Paderborn	uwen@uni-paderborn.de
Nastold, Hans-Joachim	Münster	elke@math.uni-muenster.de
Patil, Dilip	Bochum	dilip.patil@ruhr-uni-bochum.de
Peeva, Irena	Ithaca	irena@math.cornell.edu
Popescu, Dorin Mihail	Essen	dorin@stoilow.imar.ro
Popescu, Sorin	New York	psorin@math.columbia.edu
Reiner, Victor	Minneapolis	reiner@math.umn.edu
Roberts, Paul C.	Salt Lake City	roberts@math.utah.edu
Römer, Tim	Essen	sm0061@uni-essen.de
Schenzel, Peter	Halle	schenzel@mathematik.uni-halle.de
Schröer, Stefan	Bochum	s.schroeer@ruhr-uni-bochum.de
Srinivasan, Hema	Columbia	hema@math.missouri-edu
Storch, Uwe	Bochum	Uwe.Storch@ruhr-uni-bochum.de
Stückrad, Jürgen	Leipzig	stueckrad@mathematik.uni-leipzig.de
Ulrich, Bernd	East Lansing	ulrich@math.msu.edu
Valla, Guiseppe	Genova	valla@dima.unige.it
Vasconcelos, Wolmer V.	New Brunswick	vasconce@math.rutgers.edu
Vetter, Udo	Oldenburg	vetter@mathematik.uni-oldenburg.de
Villamayor, Orlando	Madrid	villamayor@vam.es
Waldi, Rolf	Regensburg	rolf.waldi@mathematik.uni-regensburg.de
Welker, Volkmar	Berlin	welker@math.tu-berlin.de
Włodarczyk, Jaroslaw	Warszawa	jwlodar@mimuw.edu.pl

Tagungsteilnehmer

Prof.Dr. Luchezar Avramov
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907
USA

Prof.Dr. Robert Berger
Fachbereich 9 - Mathematik
Universität des Saarlandes
Postfach 151150

66041 Saarbrücken

Dr. Holger Brenner
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Markus Brodmann
Mathematisches Institut
Universität Zürich
Winterthurerstr. 190

CH-8057 Zürich

Prof.Dr. Winfried Bruns
Fachbereich Mathematik/Informatik
Universität Osnabrück
Albrechtstr. 28

49076 Osnabrück

Prof.Dr. Ragnar-Olaf Buchweitz
Dept. of Mathematics
University of Toronto
100 Saint George Street

Toronto, Ontario , M5S 3G3
CANADA

Prof.Dr. Marc Chardin
Institut de Mathematiques, T. 46
UMR 9994 du CNRS, 3eme etage,
Universite Pierre et Marie Curie
4 place Jussieu, B.P. 191

F-75252 Paris Cedex 05

Prof.Dr. Dale Cutkosky
Dept. of Mathematics
University of Missouri-Columbia

Columbia , MO 65211-0001
USA

Prof.Dr. Santiago Encinas
Dept. Matematica Aplicada
E.T.S. Arquitectura
Avda. Salamanca s/n

E-47014 Valladolid

Prof.Dr. E. Graham Evans
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall
1409 West Green Street

Urbana , IL 61801
USA

Prof.Dr. Hubert Flenner
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Vesselin Gasharov
Dept. of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Prof.Dr. Shiro Goto
Meiji University
Department of Mathematics
School of Science and Technology
Higashi-mita 1-1-1, Tama-ku

Kawasaki-shi 214
JAPAN

Prof.Dr. Herwig Hauser
Institut für Mathematik
Universität Innsbruck
Technikerstr. 25

A-6020 Innsbruck

Prof.Dr. Jürgen Herzog
FB 6 - Mathematik und Informatik
Universität-GH Essen

45117 Essen

Prof.Dr. Takayuki Hibi
Department of Mathematics
Graduate School of Science
Osaka-University
Toyonaka

Osaka 560-0043
JAPAN

Dr. Reinhold Hübl
Fakultät für Mathematik
Universität Regensburg

93040 Regensburg

Prof.Dr. Craig Huneke
Department of Mathematics
University of Kansas
405 Snow Hall

Lawrence , KS 66045-2142
USA

Prof.Dr. Eero Hyry
National Defnece College
Santahamina

SF-00-860 Helsinki

Prof.Dr. Jean-Pierre Jouanolou
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Dr. Kalle Karu
Dept. of Mathematics
Boston University
111 Cummington Street

Boston , MA 02215
USA

Martin Kreuzer
Fakultät für Mathematik
Universität Regensburg

93040 Regensburg

Prof.Dr. Ernst Kunz
Fakultät für Mathematik
Universität Regensburg

93040 Regensburg

Dr. Michal Kwiecinski
Instytut Matematyki
Uniwersytet Jagiellonski
ul. Reymonta 4

30-059 Krakow
POLAND

Thomas Lehmkuhl
Königsallee 41c

37081 Göttingen

Prof.Dr. Joseph Lipman
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907-1395
USA

Prof.Dr. Kenji Matsuki
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907-1395
USA

Dr. Uwe Nagel
FB 17: Mathematik/Informatik
Universität Paderborn

33095 Paderborn

Prof.Dr. Hans-Joachim Nastold
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Prof.Dr. Dilip Patil
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA
Universitätsstr. 150

44801 Bochum

Dr. Irena Peeva
Dept. of Mathematics
Cornell University
Malott Hall

Ithaca , NY 14853-4201
USA

Prof.Dr. Dorin Mihail Popescu
FB 6 - Mathematik und Informatik
Universität-GH Essen

45117 Essen

Sorin Popescu
Dept. of Mathematics
Columbia University
MC 4417
2990 Broadway

New York , NY 10027
USA

Prof.Dr. Victor Reiner
Department of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.

Minneapolis , MN 55455
USA

Prof.Dr. Paul C. Roberts
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112-0090
USA

Tim Römer
FB 6 - Mathematik und Informatik
Universität-GH Essen

45117 Essen

Dr. Peter Schenzel
Fachbereich Mathematik u.Informatik
Martin-Luther-Universität
Halle-Wittenberg

06099 Halle

Stefan Schröer
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Hema Srinivasan
Dept. of Mathematics
University of Missouri-Columbia

Columbia , MO 65211-0001
USA

Prof.Dr. Uwe Storch
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Jürgen Stückrad
Fakultät für Mathematik/Informatik
Universität Leipzig
Postfach 920

04009 Leipzig

Prof.Dr. Bernd Ulrich
Department of Mathematics
Michigan State University

East Lansing , MI 48824-1027
USA

Prof.Dr. Giuseppe Valla
Dipartimento di Matematica
Universita di Genova
Via L. B. Alberti, 4

I-16132 Genova

Prof.Dr. Wolmer V. Vasconcelos
Dept. of Mathematics
Rutgers University
Busch Campus, Hill Center

New Brunswick , NJ 08903
USA

Prof.Dr. Udo Vetter
Fachbereich Mathematik
Universität Oldenburg
Postfach 2503

26111 Oldenburg

Prof.Dr. Orlando Villamayor
Departamento de Matematicas, C-XV
Universidad Autonoma de Madrid
Ciudad Universitaria de Cantoblanco

E-28049 Madrid

Prof.Dr. Rolf Waldi
Fakultät für Mathematik
Universität Regensburg

93040 Regensburg

Prof.Dr. Volkmar Welker
Fachbereich Mathematik
Universität Marburg

35032 Marburg

Prof.Dr. Jaroslaw Wlodarczyk
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2

00-913 Warszawa
POLAND