MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Group Actions and Curvature

15.08. - 21.08.99

Organizers: U. Abresch (Bochum)
D. Gromoll (Stony Brook)
F. Labourie (Orsay)
W. T. Meyer (Münster)

This meeting brought together a number of young mathematicians with specialists working on various aspects of this field coming primarily from France, Germany, Russia, and USA. The organizers were U. Abresch (Bochum), D. Gromoll (Stony Brook), F. Labourie (Orsay), and W. T. Meyer (Münster).

Recently, nonnegatively curved spaces have again become the focus of intense research leading to some breakthrough results; they were a main topic of this conference. The presentation of this work was well complemented by talks about results on negative curvature and metric invariants in symplectic geometry. Essential links between these topics show up in the study of fundamental groups, holonomy, the geodesic flow, and special geometries, including in particular isoparametric submanifolds, nilmanifolds and quaternionic Kähler manifolds. Analytic aspects spanned index theory, geometric extremal value problems and spectral problems. The 28 lectures started many stimulating and informal work sessions. One substantial new result was established and at least two papers were completed in the course of the conference.

1. Schedule

Mon	Tue	Wed	Thu	Fri
Rong	Grove	Goldman	Hamenstädt	Heintze
Tuschmann	Ziller	Wilking	Benoist	Peyerimhoff
Walschap	Wilhelm	Leuzinger	Weingart	Ghanaat
Bazaikin	Schüth		Cortés	Eberlein
Taimanov	Bérard Bergery		Semmelmann	
Schroeder	Schwachhöfer		Goette	
	DiScala	Schlenker	Katz	

2. Abstracts

Biquotients with integrable geodesic flow

Y. Bazaikin (Novosibirsk)

The main result presenting in this talk is the following

Theorem: *Let $M = K\backslash G/H$ be a biquotient, where H, K are Lie subgroups of the Lie group G, and with a fixed biinvariant metric on G. Let k, h, g be the corresponding Lie algebras of the Lie groups K, H, G. Consider two chains of Lie algebras*

$$h = h_0 \subset h_1 \subset \cdots \subset h_l = g \quad \text{and} \quad k = k_0 \subset k_1 \subset \cdots \subset k_m = g.$$

Let $r_1 = \mathrm{rank}(\{h_i\}_i; V)$, $r_2 = \mathrm{rank}(\{k_j\}_j; V)$, $r_3 = \mathrm{rank}\ g$, where $V = (h+k)^\perp \subset g$. Then the geodesic flow on M has at least $r_1 + r_2 - r_3$ almost everywhere functional *independent Thimm integrals.*

This theorem uses the following definitions:

1) Let g be a Lie algebra, and $h \subset g$ a Lie subalgebra. Let $g = h \oplus p$ and, for $X \in g$, put $N_g(X) = \mathrm{Center}(\ker(\mathrm{ad}(X)))$. Then put

$$\mathrm{rank}((h,g);V) := \max_{X \in V} \dim(\mathrm{pr}_p(N_g(X))),$$

where $V \subset g$ is a vector subspace.

2) If we have a chain $h_0 \subset h_1 \subset \cdots \subset h_n \subset g$, then put

$$\mathrm{rank}(\{h_i\}_i; V) := \sum_{i=0}^{n-1} \mathrm{rank}((h_i, h_{i+1}); \mathrm{pr}_{h_{i+1}}(V)),$$

where $V \subset g$ is a vector subspace.

This definition of a rank of a chain is quite constructive, and using the theorem one can immediately obtain the integrability of the geodesic flow on positively curved Eschenburg examples and positive curved 13-dimensional spaces introduced by the author.

Symplectic actions of compact groups

Y. Benoist (Orsay)

For any symplectic action of a compact connected group on a compact connected symplectic manifold, we show that the intersection of the Weyl chamber with the image of the moment map is a closed convex polyhedron. This extends Atiyah-Guillemin-Sternberg-Kirwan's Convexity Theorem to non hamiltonian actions.
As a consequence, we describe those symplectic actions of a torus which are coisotropic (or multiplicity free), i.e., which have at least one coisotropic orbit.

Non-irreducible pseudo-Riemannian manifolds

L. Bérard Bergery (Nancy)

The holonomy representation of an indecomposable (i.e., not locally a product) pseudo-Riemannian manifold is not necessarily irreducible. The talk described two other types, according to the existence or non-existence of a direct sum decomposition in two isotropic invariant subspaces. Then all pseudo-Riemannian symmetric manifolds of the 3^{rd} type are described as vector bundles over the cotangent bundle of affine symmetric spaces. Also indecomposable manifolds with parallel Ricci are not necessarily Einstein. In the 3^{rd} type, the Ricci operator may satisfy $\mathrm{Ric}^2 = 0$, $\mathrm{Ric} \neq 0$. 2^{nd} type appear for complex manifolds with a complex Einstein non degenerate symmetric 2-form (work with C. Boubel). Various examples of non-irreducible and non symmetric holonomies are presented (work with A.

Ikemakhen).

A new construction of homogeneous quaternionic manifolds and related geometric structures

V. Cortés (Bonn)

Habilitationsschrift, to appear in Memoirs of the AMS

Let $V = \mathbb{R}^{p,q}$ be the pseudo-Euclidean vector space of signature (p,q), $p \geq 3$ and W a module over the even Clifford algebra $Cl^0(V)$. A homogeneous quaternionic manifold (M,Q) is constructed for any $spin(V)$-equivariant linear map

$$\Pi : \Lambda^2 W \to V.$$

If the skew symmetric vector valued bilinear form Π is nondegenerate then (M,Q) is endowed with a canonical pseudo-Riemannian metric g such that (M,Q,g) is a homogeneous quaternionic pseudo-Kähler manifold.
If the metric is positive definite, i.e., a Riemannian metric, then the quaternionic Kähler manifold (M,Q,g) admits a transitive solvable group of automorphisms. In this special case ($p = 3$) we recover all the known homogeneous quaternionic Kähler manifolds of negative scalar curvature (Alekseevsky spaces) in a unified and direct way. If $p > 3$ then M does not admit any transitive action of a solvable Lie group and we obtain new families of quaternionic pseudo-Kähler manifolds. For $q = 0$ the noncompact quaternionic manifold (M,Q) can be endowed with a Riemannian metric h such that (M,Q,h) is a homogeneous quaternionic Hermitian manifold, which does not admit any transitive solvable group of isometries if $p > 3$.
Finally, the construction has a mirror in the category of supermanifolds. In fact, for any $spin(V)$-equivariant linear map $\Pi : S^2W \to V$ a homogeneous quaternionic supermanifold (M,Q) is associated and, moreover, a homogeneous quaternionic pseudo-Kähler supermanifold (M,Q,q) if Π is nondegenerate.

Homogeneous hyperbolic submanifolds and transitivity of Lorentzian holonomy

A. J. DiScala (Cordoba)

joint work with C. Olmos

We characterize geometrically isometry subgroups of hyperbolic space. As a consequence of this we obtain a direct and conceptual proof of classification results of M. Berger (a question explicitely posed by Bérard-Bergery and Ikemakhen in 1993). Namely, the holonomy group of an irreducible Lorentzian manifold is $SO_0(N,1)$. In particular, irreducible Lorentzian locally symmetric spaces must have constant curvature. Moreover, we obtain the following general result:

Let M be a locally indecomposable Lorentzian manifold. Then its restricted holonomy group either acts transitively on hyperbolic space or transitively on a horosphere.

Another application of our results is that a minimal (extrinsically) homogeneous submanifold of hyperbolic space must be totally geodesic. We also prove the same result for the Euclidean space, which has the following corollary (using the Calabi Rigidity Theorem and the fact that complex immersions are minimal):

A complex isometric immersion from a complex homogeneous space into $\mathbb{C}^N$ must be totally geodesic.

In other words, such isometric immersions can not exist unless the immersed manifold is an affine space.

Geometry of 2-step nilpotent Lie groups

P. Eberlein (Chapel Hill)

We consider an interesting class of compact 2-step nilmanifolds $\Gamma \backslash N$ constructed from representations $\rho : G \to GL(U)$, where $\ker \rho$ is finite, G is a compact, connected Lie group and U is a finite dimensional real vector space. Let $\langle \cdot, \cdot \rangle$ be a $\rho(G)$-invariant inner product on U. This implies $d\rho(g) \subset so(U, \langle \cdot, \cdot \rangle)$. Define $n = U \oplus g$, orthogonal direct sum, where g is equipped with an inner product for which $\mathrm{ad}(g)$ is a family of skew symmetric transformations. Define a 2-step nilpotent structure on n by requiring that $g \subset$ center of n and $\langle [X,Y], Z \rangle_g := \langle Z(X), Y \rangle_U$ for $X, Y \in U$ and $Z \in g$.

Theorem: *The corresponding 2-step, simply connected nilpotent Lie group N with left invariant metric $\langle \cdot, \cdot \rangle$ admits lattices Γ (i.e., there exists a discrete subgroup Γ of N such that $\Gamma \backslash N$ is compact).*

The differential geometric properties of such spaces $\Gamma \backslash N$ are interesting and can be investigated algebraically using the properties of the representation $\rho : G \to GL(U)$, in particular the weight space decomposition of $V = U^{\mathbb{C}}$ in the case that G is semisimple. Understanding the geometry of these examples is important since if Γ^* is a lattice in any simply connected 2-step nilpotent group N^*, then there exists a left invariant inner product $\langle \cdot, \cdot \rangle^*$ on N^* and a Riemannian submersion $q : \Gamma \backslash N \to \Gamma^* \backslash N^*$ such that Γ is a lattice in a suitable group representation example N as above. Moreover the fibers of q are flat, totally geodesic tori. Interesting partial results have already been obtained about the density of closed geodesics in compact 2-step nilmanifolds $\Gamma\backslash N$, where N is a group representation example.

Curvature, eigenvalues and nilmanifolds

P. Ghanaat (Karlsruhe)
joint work with B. Colbois and E. A. Ruh

For closed n-dimensional Riemannian manifolds M with almost positive Ricci curvature, the Laplacian on one-forms is known to admit at most n small eigenvalues. With strong curvature assumptions we show that if there are n small eigenvalues, then M is diffeomorphic to a nilmanifold, and the metric is almost left invariant. Our result sharpens a recent theorem of Petersen and Sprouse. For the proof, we consider an L^2-orthonormal system of eigenforms $\omega^1, \ldots, \omega^n$ corresponding to the small eigenvalues. Bochner's formula implies that the covariant derivatives $\nabla\omega^k$ are small in the L^2-norm. Our main step consists in obtaining smallness of $\nabla\omega^k$ in the L^∞-norm. This is based on a general deGiorgi-Nash-Moser-estimate on the Laplacian and volume comparison arguments. The mean value theorem then quickly reduces our problem to a comparison theorem for almost Lie groups obtained earlier.

Comparison theorems for scalar curvature, extremal metrics, and rigidity

S. Goette (Orsay)
joint work with U. Semmelmann

Inspired by results of Llarull, Min-Oo, and others, Gromov asked the following question: Which compact connected manifolds M carry Riemannian metrics that cannot be enlarged without making the scalar curvature κ smaller somewhere? More generally, he proposed to also investigate area-nonincreasing maps from some other manifold to M.
Generalizing Llarull's and Min-Oo's approach, we established the following

Theorem: *Let (M, g) be a compact, connected Kähler manifold with* $\mathrm{Ric} \geq 0$ *and canonical bundle $K \to M$. Let $(N, \bar{g})$ be compact, connected, orientable with scalar curvature $\bar{\kappa}$. If $f : N \to M$ is area-nonincreasing with $w_2(N) = f^* w_2(M)$ and $(\hat{A}(N) f^* \mathrm{ch}(K^{1/2}))[N] \neq 0$, then either $\bar{\kappa} = \kappa \circ f$ everywhere, or $\bar{\kappa} < \kappa \circ f$ somewhere on N.*
If M is biholomorphic to $\mathbb{C}P^n$ or the quadric $\mathbb{Q}^n$, we may replace "$\mathrm{ch}(K^{1/2})$" by "vol_M".
If moreover $\mathrm{Ric} > 0$, $\dim_{\mathbb{C}} M \geq 2$ *and $\bar{\kappa} \geq \kappa \circ f$, then N is isometric to $M \times F$, F is Ricci flat, and f is the product projection.*

Note that we may take $N = M$ and $f = \mathrm{id}$ if $\mathrm{Td}(M) \neq 0$ (for example, if $\mathrm{Ric} > 0$). We prove similar theorems for quaternionic Kähler manifolds, and also for manifolds with a nonnegative curvature operator on $\Lambda^2 M$ (but with a slightly weaker rigidity statement). We also give an upper bound for $\min \kappa$ for deformed metrics on algebraic varieties.

Hyperbolic geometry and flat Lorentz 3-manifolds

W. Goldman (College Park)

In 1977 Milnor asked whether a discrete group of affine transformations of $\mathbb{R}^n$ which acts properly is virtually polycyclic. He pointed out that this question is equivalent to whether a free group of rank 2 admits such an action. He proposed starting with a Schottky group in $SO(2,1)$ and adding translational parts. In 1983 Margulis showed such examples do occur. (Fried and Goldman reduced the general question in $\mathbb{R}^3$ to this special construction.) In his 1990 doctoral thesis, Drunn constructed explicit fundamental polyhedra (called crooked planes) and showed that the classical construction of Schottky groups can be carried out in this context. This talk surveyed the geometry of these quotients and some speculations on their classification.

Fundamental groups in positive curvature

K. Grove (College Park)

The so called Chern conjecture (1965) proposed that any abelian subgroup of the fundamental group of a positively curved manifold is cyclic. The following counterexamples were recently discovered (the first by K. Shankar):

Theorem 1 (Shankar): *The Aloff-Wallach space* $M_{1,1} = SU(3)/S^1_{1,1} = SU(3)SO(3)/U(2)$ *(Wilking) as well as the Eschenburg space* $N_{1,1} = M_1 = \begin{pmatrix} z & & \\ & z & \\ & & z \end{pmatrix} \backslash SU(3) / \begin{pmatrix} 1 & & \\ & 1 & \\ & & \bar{z}^3 \end{pmatrix}$ *admit free isometric actions of* $SO(3)$*. In particular,* $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \subset SO(3)$ *occurs as the fundamental group of two positively curved manifolds.*

Theorem 2 (Grove–Ziller): $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ *acts freely and isometrically on the Eschenburg space* $M_p = \begin{pmatrix} z & & \\ & z & \\ & & z^p \end{pmatrix} \backslash SU(3) / \begin{pmatrix} 1 & & \\ & 1 & \\ & & \bar{z}^{p+2} \end{pmatrix}$ *when* p *and* q *are odd and* $(p+1, q) = 1$.

Theorem 3 (Grove–Shankar): $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ *acts freely and isometrically on the Aloff-Wallach space* $M_{k,l} = SU(3) / \begin{pmatrix} z^k & & \\ & z^l & \\ & & \bar{z}^{k+l} \end{pmatrix}$ *if and only if* $3 \nmid kl(k+l)$.

In addition, $\mathbb{Z}_6 \oplus \mathbb{Z}_6$ and $\mathbb{Z}_3 \oplus Z_{3q}$, $3 \nmid q$ act freely and isometrically on the Aloff-Wallach-Wilking space.

The main geometric input is the following

Lemma: *If a compact Lie group* $G \subset \text{Iso}(M)$ *(including finite groups) acts freely on* M *with* $\sec M > 0$*, then* $G \cap \text{torus}$, $\text{torus} \subset \text{Iso}(M)$ *is either cyclic or a circle.*

The algebraic input is contained in the following result of Borel (1961):

There is a $\mathbb{Z}_p \oplus \mathbb{Z}_p$ *in a connected compact Lie group* G *and* not *in a* torus $\subset G$ *iff* $\pi_1(G)$ *does not have* p*-torsion.*

Marked length spectrum and volume

U. Hamenstädt (Bonn)

The marked length spectrum of a closed Riemannian manifold M of negative sectional curvature is the function which assigns to a conjugacy class in the fundamental group $\pi_1(M)$ of M the length of the unique closed geodesic in M representing the class. We discuss the following

Theorem: *The marked length spectrum determines the volume of* M*.*

Together with a deep result of Besson, Courtois and Gallot this implies that a negatively curved locally symmetric metric is determined by the marked length spectrum.
The strategy of proof consists in studying the cross ratio on the ideal boundary of the universal covering of M as an integrated form of the natural symplectic structure on the space of geodesics.

Isoparametric submanifolds

E. Heintze (Augsburg)
joint work with X. Liu and C. Olmos

A general definition for isoparametric submanifolds in arbitrary Riemannian spaces is given and it is shown that this behaves nicely with respect to certain Riemannian submersions. In particular one can lift isoparametric submanifolds with flat sections in normal homogeneous spaces to a Hilbert space and exploit Terng's theory of isoparametric submanifolds there. As an application one gets a Chevalley-type restriction theorem which partially generalizes the basic isomorphism $R(G) \to R(T)^W$ in representation theory of compact Lie groups, since the action of G on itself by conjugation gives rise to an isoparametric foliation with flat sections.

Minimal volume with distance bounded below

N. Katz (Bonn)

The talk concerned determining the infimal volume of all Riemannian metrics (or those in a conformal class) on a smooth, compact manifold M with connected boundary V, whose distance function restricted to V is bounded below by a given distance function $\rho : V \times V \to [0, \infty)$. When ρ arises from a Riemannian metric h on V, the infimal volume is Gromov's filling volume. A sufficient condition for minimal volume over a conformal class was stated and new examples were presented.
A new modulus of curves was introduced, adapted to the problem of minimal volume over a conformal class. This gives the minimal volume as the modulus of the set of all rectifiable curves with endpoints in the boundary. This new modulus was shown to be a conformal invariant in that it reproduced the distortion of a ρ-preserving, C^1-diffeomorphism in terms of measures of curves. The filling volume of round spheres (bounding hemispheres with ρ given by the extrinsic distance on the equator) was used to give an upper bound for the first eigenvalue of the Laplacian in terms of the convexity radius and an upper bound on sectional curvature.

Critical exponents of discrete groups and L^2-spectrum

E. Leuzinger (Karlsruhe)

Let Γ be a discrete, torsionfree subgroup of a noncompact semisimple Lie group G. The *critical exponent* of Γ is

$$\delta(\Gamma) := \limsup_{R\to\infty} \frac{|B_R(x_0)\ \cap\ \Gamma \cdot x_0|}{R}$$

(where x_0 is a base point of $X = G/K$). The bottom of the L^2-spectrum of $M = \Gamma\backslash G/K$ is

$$\lambda_0(M) := \inf_{f\in C_0^\infty(M)} \frac{\int_M |\mathrm{grad} f|^2}{\int_M |f|^2}.$$

Let $\rho := \frac{1}{2}\sum_{\alpha\in\Sigma^+} m_\alpha \alpha$ be the halfsum of the positive roots counted with multiplicity and let $\rho_{\min} := \min\{\rho(H) | H \in \overline{a^+}, \|H\| = 1\}$.
Theorem:

a) $\lambda_0(M) = \|\rho\|^2$ *if* $\delta(\Gamma) \in [\ 0, \rho_{\min}\ ]$.

b) $\|\rho\|^2 - (\delta(\Gamma) - \rho_{\min})^2 \leq \lambda_0(M) \leq \|\rho\|^2$ *if* $\delta(\Gamma) \in [\ \rho_{min}, \|\rho\|\ ]$.

c) $\max\{0; \|\rho\|^2 - (\delta(\Gamma) - \rho_{\min})^2\} \leq \lambda_0(M) \leq \|\rho\|^2 - (\delta(\Gamma) - \|\rho\|)^2$ *if* $\delta(\Gamma) \in [\ \|\rho\|, 2\|\rho\|\ ]$.

This generalizes a classical theorem of Elstrodt, Patterson, Sullivan, Corlette.

Isoperimetric and ergodic properties of horospheres in symmetric spaces of higher rank

N. Peyerimhoff (Bochum)
partially joint work with L. Karp

Motivated by results about spherical means in Riemannian manifolds without conjugate points (by Günther, Farnhammer, Knieper, Eskin/McMullen and others) we prove the following result:

Theorem 1 (Karp, Peyerimhoff): *Let M be a compact locally symmetric space of noncompact type and arbitray rank, H_v a horosphere (perpendicular to $v \in SX$) in the Riemannian unviersal covering $\pi : X \to M$, and $\{K_n\}$ an exhaustion of H_v by increasing compact sets with smooth boundary satisfying the isoperimetric condition*

$$(*) \qquad \lim_{n\to\infty} \frac{\mathrm{area}(\partial K_n)}{\mathrm{vol}(K_n)} = 0.$$

Then we have, for all $f \in C(M)$,

$$\lim_{n\to\infty} \frac{1}{\mathrm{vol}(K_n)} \int_{K_n} f \circ \pi = \frac{1}{\mathrm{vol}(M)} \int_M f.$$

In the rank one case, all horospheres are nilpotent groups and already intrinsic geodesic balls satisfy condition $(*)$ of the theorem. In this case, Theorem 1 can also be deduced by the result of Bowen/Marcus concerning unique ergodicity for horocycle foliations. In higher rank there are only particular horospheres satisfying $(*)$ which we refer to as "good" horospheres (there is one good horosphere associated to each Weyl chamber). These good horospheres are homogeneous manifolds with zero Cheeger isoperimetric constant, even though they seem to have exponential volume growth of intrinsic geodesic balls. The precise statement is as follows (with $\rho := \frac{1}{2}\sum_{\alpha\in\Lambda^+} m_\alpha \alpha$, $m_\alpha = \dim\, g_\alpha$):

Theorem 2: *Let X be a symmetric space of noncompact type and higher rank, $g = k \oplus p$ the Cartan decomposition at $p \in X$ and $T_pX \cong p$. In each closed spherical Weyl chamber*

$W \subset a \cap S_pX$ there is precisely one vector $v \in W$, namely $\rho = \|\rho\|\langle v, \cdot\rangle$, such that the corresponding horosphere H_v is good. Moreover, for any vector $v \in W$ we have the following estimate for the Cheeger isoperimetric constant of the corresponding horosphere:

$$h(H_v) \geq 2 \max_{\substack{w \perp v \\ w \in a \cap S_pX}} |\rho(w)|.$$

Collapsed manifolds with positive pinched curvature

X. Rong (New Brunswick)

The main results presented in this talk are the following:

Theorem A (Rong, 95-96): *Let M be a δ-pinched n-manifold. Then:*

(A1) Either $|\pi_1(M)| \leq \omega(n, \delta)$ or $|\pi_1(M)/\mathbb{Z}_q| \leq \omega(n)$.

(A2) $\pi_q(M)$ has at most $c(n, \delta)$ many isomorphic classes for $q \geq 2n - 3$, or $q \neq 3(\mathrm{mod}4)$ and $q \neq n - 1$.

Theorem B (Fang–Rong, Petrunin–Tuschmann, 98): *Let M be a simply connected n-manifold with δ-pinched curvature. If $\pi_2(M)$ is finite, then* $\mathrm{inj}(M) \geq \epsilon(n, \delta) > 0$.

Theorem C (Petrunin–Rong–Tuschmann, 98): *Let M be a compact T^k-manifold. If M admits a collapse along the T^k-orbits with curvature $\Lambda \leq K_{g_i} \leq 1$, then there is a noncompact Alexandrov space, $Y = Y(M, T^k, g_i)$, with curvature $\geq \Lambda$ (in the Alexandrov comparison sense).*

Theorem D (Rong, 97): *A maximally collapsed n-manifold with δ-pinched curvature has a finite covering of order $\leq \frac{n+1}{2}$ which is diffeomorphic to a lens space, $S^n/\mathbb{Z}_q$, such that $c(n, \delta)^{-1} \leq q$ $\mathrm{vol}(M) \leq c(n, \delta)$.*

Roughly, a collapse along the T^k-orbits is a generalization of collapsing Berger's sphere.

Convex surfaces in Lorentzian space forms

J. M. Schlenker (Orsay)

Pogorelov (after other's works) proved that any metric with curvature $K > -1$ is induced on a unique convex surface in $\mathbb{H}^3$. We describe a similar (but slightly different) phenomenon in the de Sitter space S^3_1: any metric with $K < 1$, and closed geodesics of length $L > 2\pi$, is obtained on a unique space-like surface in S^3_1.

For surfaces of higher genus, convex embeddings are not possible, so one turns to equivariant embeddings. Gromov proved that, if Σ is a compact surface with genus at least 2, and g is a Riemannian metric on Σ with curvature $K > -1$, then (Σ, g) has an equivariant isometric embedding into $\mathbb{H}^3$, (φ, ρ), such that the group morphism $\rho : \pi_1(\Sigma) \to \mathrm{Isom}(\mathbb{H}^3)$ fixes a 2-plane.

Theorem (F. Labourie, J. M. Schlenker): *If Σ has genus at least 2 and g is a metric over Σ with $K < -1$ (resp. $K < 0$) then (Σ, g) has a unique equivariant isometric embedding into $\mathbb{H}^3_1$ (resp. $\mathbb{R}^3_1$); and, if $K < 1$ and all closed geodesics of $(\tilde{\Sigma}, g)$ have length above 2π, then (Σ, g) has a unique equivariant isometric embedding into S^3_1, whose representation fixes a point.*

Using the duality between $\mathbb{H}^3$ and S^3_1, the result concerning S^3_1 can be reformulated as describing the third fundamental forms of surfaces in $\mathbb{H}^3$.

Bounded geodesics in manifolds of negative curvature

V. Schroeder (Zürich)

We outline the proof of the following results

Theorem 1: *Let M be a complete noncompact Riemannian manifold with sectional curvature $-b^2 \le K \le -a^2 < 0$, $\mathrm{vol}(M) < \infty$ and $\dim(M) \ge 3$. Then, given a point $x \in M$, there exists a complete bounded geodesic through the point x.*

Theorem 2: *Let M be compact with curvature $K < 0$ and $\dim(M) \ge 3$. Then there exists a proper closed subset W of the unit tangent bundle SM which is invariant under the geodesic flow such that $\pi(W) = M$ where $\pi : SM \to M$ is the footpoint projection.*

The proof combines ideas of K. Burns and M. Pollicott together with some additional topological considerations. For these methods the dimension assumption is necessary.

Continuous families of isospectral left invariant metrics on compact Lie groups

D. Schüth (Bonn)

We present a new class of families of closed Riemannian manifolds which are isospectral for the Laplacian on functions. The "classical" way of constructing such manifolds is by the so-called Sunada method. It produced isospectral quotients of a common Riemannian covering manifold by different discrete subgroups of isometries. In particular, those manifolds were

always locally isometric, and were non-simply connected. The first examples of closed, locally non-isometric isospectral manifolds were given by C. Gordon (1992). They arise as principal torus bundles satisfying certain conditions.
The same method later led to the construction of continuous families of isospectral, locally non-isometric metrics on $S^{m-1} \times T^2$ with $m \geq 5$ (Gordon/Gornet/Schüth/Webb/Wilson, 1997), and $S^{m-1} \times K$, where K is any compact Lie group of rank ≥ 2 (Schüth 1997). These manifolds were locally non-homogeneous.
We apply Gordon's method of principal torus bundles in a new way to obtain continuous families of left invariant metrics on the following compact Lie groups: $SO(m) \times T^2$ and $Spin(m) \times T^2$ for $m \geq 5$, $SU(m) \times T^2$ for $m \geq 3$, $SO(n)$ and $Spin(n)$ for $n \geq 9$, and $SU(n)$ for $n \geq 6$. In particular, we thus obtain:

- the first examples of isospectral manifolds which are simply connected and irreducible;
- the first continuous families of isospectral manifolds which are globally homogeneous;
- the first examples of continuous families of isospectral manifolds of positive Ricci curvature.

In most cases, the norm of the Ricci tensor changes during the deformation. This implies, by using the heat invariants for the Laplacian on 1-forms, that the manifolds are not 1-isospectral.

Homogeneous symplectic manifolds with special holonomy

L. Schwachhöfer (Leipzig)

Let (M, ω, ∇) be a triple of a manifold M with a symplectic form ω and a torsion free connection ∇ such that $\nabla\omega \equiv 0$. Then the scalar curvature is defined by the equation $\mathrm{scal}(p) = \mathrm{tr}\ \underline{\mathrm{Ric}_p}^2$, where $\underline{\mathrm{Ric}_p}$ is the endomorphism of T_pM given by $\omega(\underline{\mathrm{Ric}_p}x, y) = \mathrm{Ric}_p(x, y) = \mathrm{tr}\ R_p(\cdot, x)y$. We prove the following theorem

Theorem: *Let (M, ω, ∇) be as above, and suppose that $\mathrm{Hol}^\nabla \subset Sp(V, \omega)$ is a proper absolutely irreducible subgroup. Then the following are equivalent:*

1) *M is locally homogeneous under the action of the group of local diffeomorphisms preserving ω and ∇;*
2) scal *is constant;*
3) *there is a point $p \in M$ with $(\nabla R)_p \neq 0$ for which $\mathrm{scal} - \mathrm{scal}(p)$ vanishes at p of order ≥ 3.*

Moreover, we obtain a complete classification of homogeneous symplectic manifolds with holonomy Hol. As it turns out, not every possible holonomy group can be realized on a homogeneous space. However for each given proper absolutely irreducible subgroup of $Sp(V, \omega)$, there are finitely many homogeneous spaces (M, ω, ∇) with this holonomy.

The point spectrum of the Dirac operator on noncompact symmetric spaces

U. SEMMELMANN (München)
joint work with S. Goette

In this work we consider the Dirac operator D on a Riemannian symmetric space M of noncompact type. Using representation theory we completely determine the point spectrum of D. We prove the following result:

Theorem: *Let M be a Riemannian symmetric space of noncompact type and let D be the Dirac operator acting on spinors over M. Then the following statements are equivalent:*

i) the point spectrum of D is nonempty;

ii) the point spectrum of D is precisely $\mathrm{spec}_p(D) = \{0\}$; moreover, as a G-module, $\ker(D)$ is irreducible and isomorphic to the discrete series representation with Harish Chandra parameter ρ_t;

iii) the $\hat{A}$-genus of the compact dual of M is non zero;

iv) each irreducible factor of M is isometric to $U(p, q)/U(p) \times U(q)$, with $p + q$ odd.

Integrable geodesic flow with positive topological entropy

I. A. TAIMANOV (Novosibirsk)

We present the following result from our joint paper with A. V. Bolsinov (Moscow): There is a three-dimensional compact real-analytic Riemannian manifold M^3 such that

1) its geodesic flow is (Liouville) integrable by C^∞ first integrals and not integrable in terms of real-analytic first integrals;

2) the Liouville entropy of the geodesic flow vanishes and the topological entropy of this flow is positive;

3) the fundamental group of the manifold, $\pi_1(M^3)$, has exponential growth;

4) the phase space of this flow contains a two-torus which is invariant under the translations along trajectories per unit time and this translation is given by an Anosov (hyperbolic) automorphism of the torus.

Asymptotically flat manifolds

W. Tuschmann (Leipzig)
joint work with A. Petrunin

Let M^m be an asymptotically flat m-manifold which has cone structure at infinity. We show that M has a finite number of ends and classify for simply connected ends all possible cones at infinity (except for dim $M = 4$ where it is not clear if one of the theoretically possible cones, $\mathbb{R} \times \mathbb{R}^+$, can actually arise). This leads to a complete classification of asymptotically flat manifolds with nonnegative sectional curvature: The universal covering of such a manifold is isometric to $\mathbb{R}^{m-2} \times S$, where S is an asymptotically flat surface.
Here a complete noncompact Riemannian manifold M with a marked point p is said to be asymptotically flat if $\limsup_{|px|\to\infty} |K_x| \ |px|^2 = 0$, where $|K_x|$ denotes the maximal absolute value of the sectional curvatures at $x \in M$, and (M, g) has by definition cone structure at infinity if there exists a locally compact metric cone C with vertex o such that for any sequence of numbers $\epsilon_n \to 0$ the pointed Gromov–Hausdorff limit of $((M, \epsilon_n g), p)$ exists and such that this limit is isometric to (C, o).

Transitive holonomy and rigidity in nonnegative curvature

G. Walschap (Norman)
joint work with L. Guijarro

We explore the relationship between the twisting of a vector bundle ξ over a manifold M and the action of holonomy groups of connections on ξ. In particular, if the holonomy group of some connection does not act transitively on the unit sphere bundle of ξ, then for any map of a sphere $f : S^l \to M$ into M, the pullback bundle $f^*\xi$ admits a nowhere-zero section. Thus, if the bundle is twisted enough, then every connection has transitive holonomy.
As a consequence, if such a bundle occurs as the normal bundle $\nu(S)$ of a soul S in an open manifold with nonnegative sectional curvature, then the exponential map $\exp : \nu(S) \to M$ must be a diffeomorphism, the metric projection of M onto S is C^∞, and the ideal boundary

$M(\infty)$ of M consists of a single point.

Vanishing theorems for quaternionic Kähler manifolds

G. Weingart (Bonn)
joint work with U. Semmelmann

The deRham-cohomology of a compact Riemannian manifold $(M, \langle\cdot,\cdot\rangle)$ with holonomy group $\mathrm{Hol} \subset O_n(\mathbb{R})$ decomposes in isotypical subspaces according to

$$H^{\bullet}_{\mathrm{dR}}(M,\mathbb{C}) = \bigoplus_{\substack{\pi \text{ irred. repr.} \\ \text{of Hol}}} \mathrm{Hom}(\pi, \Lambda^{\bullet}\mathbb{C}^n) \otimes \mathcal{H}_\pi$$

where $\mathcal{H}_\pi$ is the kernel of a second order elliptic diff. operator $\Delta_\pi : \Gamma\pi(M) \to \Gamma\pi(M)$ on sections of the abstract vector bundle $\pi(M) := \mathrm{Hol}M \times_{\mathrm{Hol}} \pi$. Similarly the eigenspaces of a twisted Dirac operator $D : \Gamma\$ \otimes \mathcal{R} \to \Gamma\$ \otimes \mathcal{R}$ decomposes if $\mathcal{R}$ is a geometric vectorbundle associated to $\mathrm{Hol}M$ with induced connection and the curvature acts on $\mathcal{R}$ by scalar multiplication. We consider the case of quaternionic Kähler holonomy where there is a large family of geometric vectorbundles meeting this condition.
Comparing the squares of the twisted Dirac operators with twist $\mathcal{R}$ in this family to the operator $\Delta_\pi : \Gamma\pi(M) \to \Gamma\pi(M)$ we get a strong vanishing theorem for the kernels of these twisted Dirac operators on compact quaternionic Kähler manifolds, both of positive or negative scalar curvature. Examples of applications include:
Theorem (originally proved by **Kramer, Semmelmann, Weingart, 97):** *If a quaternionic Kähler manifold is spin and scalar curvature $K > 0$, then the square of the untwisted Dirac operator satisfies $D^2 \geq \frac{K}{4}\frac{n+3}{n+2}$.*
Theorem (for positive scalar curvature originally due to **Salamon):** *If π is an irreducible representation of $\mathrm{Hol} = Sp(1)Sp(n)$ such that*

$$\mathrm{Hol}_{Sp(1)Sp(n)}(\pi, \Lambda^{\bullet}\mathbb{C}^{4n}) \neq \{0\},$$

then $\mathcal{H}_\pi = \{0\}$ unless π is equal to $\Lambda^{a,a}_{\mathrm{top}}E$, $a = 0,\ldots,n$ (if $K > 0$) or π is equal to $\Lambda^{a,a}_{\mathrm{top}}E$, $a = 0,\ldots,n$ or $\mathrm{Sym}^{2n-a-b}H \otimes \Lambda^{a,b}_{\mathrm{top}}E$ (if $K < 0$). In particular, all odd Betti numbers of M, b_{2i+1}, with $2i+1 < n$, vanish. Moreover, $b_{2i+1} \leq b_{2i+3}$ for all $2i+1 < 2n$ (which is not covered by Kramer's theorem).

Examples of manifolds with quasi-positive curvature

F. Wilhelm (Riverside)

In this talk we discussed the following two theorems:

Theorem: *The metric on the Gromoll-Meyer sphere can be perturbed to one that has positive sectional curvature almost everywhere and an isometric $SO(3)$-action.*

Theorem: *The unit tangent bundle of S^4 admits a metric with positive sectional curvature almost everywhere that has the following properties:*

(a) *The connected component of the identity of the isometry group is $SO(4)$ and it contains two copies of S^3 that act freely.*

(b) *The set of points with 0-sectional curvatures is the union of 2 copies of $S^3 \times S^3$ that intersect along a common $S^2 \times S^3$.*

Corollary: *There is a homology $\mathbb{C}P^3$ that is not a cohomology $\mathbb{C}P^3$ that admits a metric with positive sectional curvature almost everywhere.*

This last 2 results are joint work with P. Petersen.

All of these metrics have flat totally geodesic T^2's, so they do not admit perturbations whose sectional curvature are positive to first order.

On fundamental groups of manifolds of nonnegative curvature

B. Wilking (Münster)

We will characterize the fundamental groups of compact manifolds of (almost) nonnegative Ricci (sectional) curvature. Actually it turns out that the known necessary conditions are sufficient as well.

Moreover, we reduce the Milnor problem – are the fundamental groups of open manifolds of nonnegative Ricci curvature finitely generated? – to manifolds with abelian fundamental groups.

Finally, we prove for each positive integer n that there are only finitely many finite simple groups acting effectively on some complete n-manifold of nonnegative Ricci curvature.

Bundles with nonnegative curvature

W. Ziller (Philadelphia)

joint work with K. Grove

A group action of G on M is called cohomogeneity one if $\dim(M/G) = 1$. In the particular case of $M/G =$ interval we prove

Theorem 1: *Every cohomogeneity one manifold with singular orbits of codimension* 2 *carries an invariant metric with nonnegative curvature.*

This is a rich class of manifolds, as exhibited by

Theorem 2: *Every principal* $SO(k)$ *bundle over* S^4 *admits a cohomogeity one action by* $SO(k) \times SO(3)$ *with singular orbits of codimension* 2*.*

In particular one obtains

Corollary 1: *Every vector bundle over* S^4 *admits a complete metric with* $\sec \geq 0$*.*

Corollary 2: 15 *of the* 27 *exotic spheres in dimension* 7 *admit infinitely many metrics with nonnegative curvature.*

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