MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 38/1999

L^2-invariants and K-theory

19.09.-25.09.1999

The aim of this conference was to bring together mathematicians from such different fields as Topology, Non-Commutative Geometry, Spectral Theory and Algebra to provoke an interchange of methods, results and ideas. The topic of the conference is related to all these areas. In particular Methods from L^2-theory are used in all the above mentioned fields to prove results within that field. Therefore apart from talks presenting recent research results there was a series of lectures on the Baum-Connes Conjecture, the Atiyah Conjecture and the Isomorphism Conjecture in algebraic K- and L-theory. These talks explained the corresponding conjectures, presented the current state of the art concerning these conjectures and discussed relations between them. The organizers hope that the conference initiated new cooperation and thank the institute for its hospitality.

Schedule

Monday

9:15 - 9:45	Wolfgang Lück	Introduction
10:00 - 11:00	Thomas Schick	Approximating L^2-signatures
11:15 - 12:15	Michel Rumin	What do we hear on forms at the infinity of some nilpotent Lie groups?
16:00 - 17:00	Peter Teichner	Knot concordance and L^2-signatures
17:15 - 18:15	Mike Davis	L^2-homology of Coxeter groups

Tuesday

9:30 - 10:30	Alain Valette	On the Baum-Connes Conjecture: Introduction
11:00 - 12:00	Pierre Julg	On the Baum Connes Conjecture: Dual Dirac method
15:30 - 16:30	Vincent Lafforgue	On the Baum Connes Conjecture: Banach method
16:45 -18:15	Tom Farrell	On the Isomorphism Conjecture in algebraic K- and L-theory

Wednesday

9:30 - 10:30	Holger Reich	On the Atiyah Conjecture I
11:00 - 12:00	Peter Linnell	On the Atiyah Conjecture II
13:30		Excursion

Thursday

9:30 - 10:30	John Roe	Approximation properties and coarse geometry
11:00 - 12:00	Martin Olbrich	L^2-invariants of locally symmetric spaces
16:00 - 17:00	Dan Burghelea	Witten-Helffer Sjöstrand theory in the presence of symmetry and applications to L^2-topology
17:15 - 18:15	John Lott	On the homotopy invariance of higher signatures for manifolds with boundary

Friday

9:30 - 10:30	Michael Farber	On the zero-in-the-spectrum conjecture
11:00 - 12:00	Rostislav Grigorchuk	On the spectrum of Hecke type operators related to some fractal groups
16:00 - 17:00	V. Mathai	On the spectral teory of the Harper operator and the quantum Hall effect
17:15 - 18:15	Mikhael Rørdam	Classifications of C^*-algebras by their K-theory

Abstracts

Witten Hellfer Sjöstrand theory in the presence of symmetry

Dan Burghelea

For a closed G-manifold, G a compact Lie group, we consider Morse-Smale pairs, $\tau = (h, g)$. Here h is a special G-Morse function, g an invariant Riemannian metric satisfying some conditions. For a Morse Smale pair τ and an irreducible G-representation ξ, we describe a finite collection of positive numbers $\{\lambda^q_{i,\xi}\}$, which depends only on critical orbits and their indices.

(i) We prove that Morse Smale pairs exist.

(ii) We describe a cochain complex of G-representations $(C^*(M,\tau), D^*)$, whose ξ component $(C^*(M,\tau)_\xi, D^*_\xi)$ is finite dimensional.

(iii) If we denote by $\Delta(t)^*$ the Laplacian obtained from g and the differential operator $d^*(t) = e^{-th} \cdot d^* \cdot e^{th}$ and by $\Delta(t)_\xi$ its $\xi-$component we show that the spectrum of $\Delta(t)_\xi$, for t large, decomposes in two parts: one which converges exponentially fast to the numbers $\{\lambda^q_{i,\xi}\}$, the other linearly fast to ∞. This implies a decomposition of the ξ component $(\Omega^*(M)_\xi, d^*(t)_\xi)$ as $(\Omega^*(M)_\xi, d^*(t)_\xi)_{sm} \oplus ((\Omega^*(M,\tau)_\xi, d^*(t))_\xi)_{la}$ with the $la-$part acyclic.

(iv) We prove that the $sm-$part is asymptotically isometric to $((C^*(M,\tau)_\xi, D^*)_\xi)$ after a canonical rescaling.

Applications to G-topology and to L^2 invariants are announced.

L^2-homology of Coxeter groups

Michael Davis

This is a report on joint work with Boris Okun. Associated to any finite flag complex L there is a right-angled Coxeter group W_L and a cubical complex Σ_L on which W_L acts properly and cocompactly. Its two most salient features are that 1) the link of each vertex of Σ_L is L and 2) Σ_L is contractible. It follows that if L is a triangulation of S^{n-1}, then Σ_L is a contractible n-manifold. I describe a program for proving the Dodzuik-Singer Conjecture (on the vanishing of the reduced L_2-homology except in the middle dimension) in the case of Σ_L where L is a triangulation of S^{n-1}. The program succeeds when $n = 4$. A corollary is that every closed 4-manifold with a nonpositively curved piecewise Euclidean cubical structure has nonnegative Euler characteristic. Our methods suggest the following two generalizations of the Dodzuik-Singer Conjecture. Conjecture: If X is a compact n-dimensional polyhedron with spherical links in codimensions $\leq 2k+1$, then the reduced L^2-homology of its universal cover vanishes in dimensions $\geq n-k$. Conjecture: If a discrete group G acts properly on a contractible n-manifold, then its L^2-Betti numbers $b_i^{(2)}(G)$ vanish for $i > n/2$.

On the "zero-in-the-spectrum" conjecture

Michael Farber

In the talk I gave a negative answer to the "zero-in-the-spectrum" conjecture in its form suggested by J. Lott. More precisely, I showed that for any $n > 5$ there exists a closed n-dimensional manifold M such that zero does not belong to the spectrum of the Laplace-Beltrami operator Δ_p acting on the space of square integrable forms of all degrees on the universal covering $\tilde{M}$. The proof uses surgery and the technique of extended L^2-homology. This is a joint result with S. Weinberger.

On the Isomorphism Conjecture in algebraic K and L-theory

Tom Farrell

This talk was on a conjecture made by L.E. Jones and myself which proposes a formula calculating the Whitehead groups $Wh(G,n)$ of an arbitrary (discrete) group G in terms of a spectral sequence whose $E(2;p,q)$ term is the p-th homology group of the orbit space (under the action of G) of a G-space which is universal for the class of virtually cyclic subgroup of G. The coefficients in these homology groups are $Wh(G(x),q))$ where x is an arbitrary point in the universal G-space and $G(x)$ is the subgroup of G which stabilizes it. The group $Wh(G,1)$ is the abelian group defined by Whitehead; $Wh(G,0)$ is the projective class group of the ring $\mathbb{Z}G$; $Wh(G,n)$ is $K(\mathbb{Z}G,n)$ when $n < 0$ and is closely related to $K(ZG,n)$ when $n > 1$. Jones and I have proven this conjecture when G is a discrete cocompact subgroup of a virtually connected Lie group (or any subgroup of such a group). We also proved it for any discrete torsion free subgroup of $GL(n,\mathbb{R})$. We have an analogous Conjecture and Theorem in L-theory.

On the spectrum of Hecke type operators related to some fractal groups

R.I.Grigorchuk (Steklov Institute of Mathematics, Moscow)

We give the first example of a connected 4-regular graph whose Laplace operator's spectrum

is a Cantor set, as well as several other computations of spectra following a common "finite approximation" method. These spectra are simple transforms of the Julia sets associated to some quadratic maps. The graphs involved are Schreier graphs of fractal groups of intermediate growth, and are also "substitutional graphs". We also formulate our results in terms of Hecke type operators related to some irreducible quasi-regular representations of fractal groups and in terms of the Markovian operator associated to noncommutative dynamical systems via which these fractal groups were originally defined.
In the computations we performed, the self-similarity of the groups is reflected in the self-similarity of some operators; they are approximated by finite counterparts whose spectrum is computed by an ad hoc factorization process.
This results are obtained in collaboration with L.Bartholdi.

The Baum-Connes conjecture and the γ-element method
Pierre Julg

The work of Kasparov and Kasparov-Skandalis have introduced the so-called Dirac-dual Dirac method, or γ-element method for the Baum-Connes conjecture. They show, for a large class of reasonnable groups (containing all discrete subgroups of Lie groups and p-adic groups), the Baum-Connes assembly map is split injective. Its image is the image of an idempotent γ of the ring $KK_G(\mathbf{C},\mathbf{C})$ acting on the abelian group $K_*(C_r^*G)$. The proof of the conjecture (i.e., in this case, of surjectivity) is thus equivalent to $\gamma = 1$ in the ring of endomorphisms of $K_*(C_r^*G)$. however in general, $\gamma \neq 1$ in $KK_G(\mathbf{C},\mathbf{C})$. We tried to review three different approaches of the homotopy $\gamma = 1$: the complementary series approach (Kasparov, Fox-Haskell, Julg-Kasparov, Chen), the approach using the Haagerup property or a-T-menability (Higson-Kasparov, Tu) and the Banach spaces approach (Lafforgue).

On the Baum-Connes conjecture III - The Banach method
Vincent Lafforgue

Let G be a closed subgroup of a semisimple Lie group or of a reductive p-adic group. Then there is $\gamma \in KK_G(\mathbb{C},\mathbb{C})$ such that the image of the Baum-Connes map is the image of $j_r(\gamma)$ where $j_r : KK_G(\mathbb{C},\mathbb{C}) \to \mathrm{End}(\mathrm{K}_*(\mathrm{C}_\mathrm{r}^*(\mathrm{G})))$ is the descent map. There is a homotopy between γ and 1 in (some variant of) $KK_G^{ban}(\mathbb{C},\mathbb{C})$: this theory is the same as Kasparov theory but we replace unitary representations in Hilbert spaces by isometric representations in Banach spaces. Let $A(G)$ be the completion of $C_c(G)$ by a norm such that $||f||_A$ depends only on $g \mapsto |f(g)|$. Then we have a descent map $KK_G^{ban}(\mathbb{C},\mathbb{C}) \to \mathrm{End}(\mathrm{K}_*(\mathrm{A(G)}))$ and then it is possible to prove an anlogue of the Baum-Connes conjecture for $A(G)$, and in particular for $L^1(G)$. This gives also the usual Baum-Connes conjecture when G is a semi-simple Lie group or a reductive p-adic group or a cocompact lattice in $SL_3(\mathbb{F})$ with $\mathbb{F}$ a local field.

On the Atiyah Conjecture II
Peter A. Linnell

Let G be a group. Then we have the following series of inclusions:

$$\mathbb{C}G \subseteq C_r^*(G) \subseteq \mathcal{N}(G) \subseteq L^2(G) \subseteq \mathcal{U}(G)$$

Every element of $\mathcal{U}(G)$ can be written in the form $\alpha\beta^{-1}$ with $\alpha, \beta \in \mathcal{N}(G)$ and β a nonzero divisor in $\mathcal{N}(G)$.
Now suppose G is torsion free. Then we will consider the following conjectures.

Conjecture 1. If $0 \neq \alpha \in \mathbb{C}G$ and $0 \neq \beta \in L^2(G) \setminus 0$, then $\alpha\beta \neq 0$.

Conjecture 2. There is a division ring D such that $\mathbb{C}G \subseteq D \subseteq \mathcal{U}(G)$.

Conjecture 2 is often known as the Atiyah conjecture, or perhaps with $\mathbb{C}G$ replaced by $\mathbb{Q}G$ in the statement. It is not difficult to see that Conjecture 2 implies Conjecture 1. It is unknown whether Conjecture 1 implies Conjecture 2. Conjecture 1 is true in the case G is left orderable.
Let $\mathcal{D}(G)$ denote the division closure of $\mathbb{C}G$ in $\mathcal{U}(G)$ $(= \operatorname{div}(\mathbb{C}G, \mathcal{U}(G)))$, that is the smallest subring of $\mathcal{U}(G)$ containing $\mathbb{C}G$ which is closed under taking inverses in $\mathcal{U}(G)$. Then Conjecture 2 is equivalent to $\mathcal{D}(G)$ is a division ring. Conjecture 2 is true if G is free or free abelian. More generally it is true if G has a normal subgroup F such that G/F is elementary amenable and F is a direct product (i.e. all but finitely many coordinates 1) of free groups. Furthermore Thomas Schick has proved that if G has a normal subgroup H such that G/H is free and H satisfies Conjecture 2, then so does G. He has also proved under mild technical conditions that if G has a sequence of subgroups $H_1 \supset H_2 \supset \cdots$ such that $\bigcap_i H_i = 1$ and G/H_i is torsion free and satisfies Conjecture 2 for all i, then G satisfies Conjecture 2. Interesting classes of groups for which Conjecture 2 is still open are lattices in Lie groups of rank at least 2, hyperbolic groups, and right angled Coxeter groups. Also we have the following result.

Theorem. Let G be a free group. Then $\mathcal{D}(G) \cap \mathcal{N}(G) = \operatorname{div}(\mathbb{C}G, C_r^*(G))$.

A consequence of this is that every element of $\mathcal{D}(G)$ can be written in the form $\alpha\beta^{-1}$ with $\alpha, \beta \in C_r^*(G)$. One can also consider the case $p > 2$. Define

$$L^p(G) = \{\sum_{g \in G} a_g g \mid \sum_{g \in G} |a_g|^p < \infty\}$$

If $L^2(G)$ is replaced by $L^p(G)$ with $p > 2$, then Conjecture 1 becomes false. Here are some examples (these are due to Mike Puls and Mike Puls/Linnell).

(i) Let G be free abelian on $\{x, y\}$. Then $(2xy - x + y - 2)\beta = 0$ where $0 \neq \beta \in L^p(G)$ and $p > 4$.

(ii) Let G be free abelian on $\{x_1, \dots, x_d\}$ where $d > 1$. Then

$$\left(\frac{2d-1}{2} - \frac{1}{2}\sum_{i=1}^{d}(x_i + x_i^{-1})\right)\beta = 0$$

where $0 \neq \beta \in L^p(G)$ and $p > 2d/(d-1)$. Also in this case one can give necessary and sufficient conditions for $\alpha \in \mathbb{C}G$ to be a zero divisor in $L^p(G)$.

(iii) G free on $\{x_1, \dots, x_d\}$ where $d > 1$. Then $(x_1 + x_1^{-1} + \cdots + x_d + x_d^{-1})\beta = 0$ where $0 \neq \beta \in \bigcap_{p>2} L^p(G)$.

(iv) G free on $\{x_1, \dots, x_d\}$, where d is even and $d > 3$. Then $(x_1 + \cdots + x_d)\beta = 0$, where $0 \neq \beta \in \bigcap_{p>2} L^p(G)$.

Here is a sketch proof of Conjecture 2 for the case G is the free group on two generators. We have a well known Fredholm module

$$P\colon L^2(G) \to L^2(G) \oplus L^2(G) \oplus \mathbb{C}$$

with the property that $g - P^{-1}gP$ has finite rank for all $g \in G$ (i.e. $\dim_{\mathbb{C}}(\mathrm{im}(g - P^{-1}gP)) < \infty$). Thus $\alpha - P^{-1}\alpha P$ has finite rank for all $\alpha \in \mathbb{C}G$.
We now have two traces. If $\alpha \in \mathcal{N}(G)$ and $\alpha = \sum_{g\in G} \alpha_g g$ with $\alpha_g \in \mathbb{C}$, then $\mathrm{tr}\alpha = \alpha_1$. Suppose $\alpha \in \mathcal{N}(G)$ and $\alpha - P^{-1}\alpha P$ has finite rank. Define $\mathrm{Trace}\,\alpha$ to be the trace of the bounded linear operator $\alpha - P^{-1}\alpha P\colon L^2(G) \to L^2(G)$ with respect to some Hilbert basis of $L^2(G)$ (the fact that α has finite rank means that $\mathrm{Trace}\,\alpha$ is well defined and independent of the choice of Hilbert basis). These traces have the following property. If $\alpha \in \mathcal{N}(G)$ and $\alpha - P^{-1}\alpha P$ has finite rank, then $\mathrm{tr}\alpha = \mathrm{Trace}\,\alpha$.
Now let $\alpha \in \mathbb{C}G \setminus 0$ and suppose $\alpha\beta = 0$ for some $\beta \in L^2(G) \setminus 0$. Let e be the projection of $L^2(G)$ onto $\ker\alpha$ $(= \gamma \in L^2(G) \mid \alpha\gamma = 0)$. Then $e \neq 0, 1$ and $e \in \mathcal{N}(G)$. Furthermore one can show that $e - P^{-1}eP = e_1 - e_2$, where e_1, e_2 are projections with finite rank. If e_i is a projection with finite rank, then $\mathrm{Trace}\,e_i = \dim_{\mathbb{C}}(\mathrm{im}e_i)$. This last number is obviously an integer (even a nonnegative integer), so it follows that $\mathrm{tr}e = \mathrm{Trace}(e_1 - e_2) = \mathrm{Trace}(e_1) - \mathrm{Trace}(e_2) \in \mathbb{Z}$. A theorem of Kaplansky states that if e is a projection in $\mathcal{N}(G)$ and $e \neq 0, 1$, then $0 < \mathrm{tr}e < 1$. This contradicts the fact that $\mathrm{tr}e$ is an integer and Conjecture 1 follows.
This argument extends easily to prove Conjecture 2. One needs to do the same argument with matrices. An important ingredient here is that if $\alpha \in \mathcal{D}(G)$, then for some positive integer n,

$$\begin{pmatrix} \alpha & 0 \\ 0 & I_n \end{pmatrix} = X^{-1}AX$$

where I_n is the identity $n \times n$ matrix, X is an $n+1 \times n+1$ invertible matrix over $\mathcal{U}(G)$, and A is a matrix with entries in $\mathbb{C}G$.
It seems plausible that this argument should extend from free groups to hyperbolic groups. There one still has a Fredholm module, except it has the weaker property that $g - P^{-1}gP$ is *compact*, rather than has finite rank. The finite rank property is used crucially in the proof, so to prove Conjecture 2 for hyperbolic groups, a further trick will be required.

On the homotopy invariance of higher signatures for manifolds with boundary
John Lott

We show that if M is a compact oriented manifold-with-boundary whose fundamental group is virtually nilpotent or Gromov-hyperbolic then the higher signatures of M are oriented-homotopy invariants.

Introduction
Wolfgang Lück

We present some well-known theorems in group theory, Riemannian geometry and K-theory whose statements do not involve L-invariants but whose proof use L^2-methods. For instance we mention the result of Cheeger and Gromov that the Euler characteristic of a group G with finite classifying space BG vanishes if G contains an infinite normal amenable subgroup. We briefly explain how L^2-methods enter in the proof. We present and explain the main conjectures about L^2-invariants such as the Atiyah Cojecture and the Singer Conjecture and explain how they are related to the Zero-divisor Conjecture, the Baum-Connes Conjecture and the Isomorphism Conjectures in algebraic K and L-theory of Farrell and Jones.

On the spectral theory of the Harper operator and the quantum Hall effect
Varghese Mathai

The first part will be concerned with some qualitative aspects of the Harper operator, which is defined as the Random walk operator on the Cayley graph of a discrete group in the presence of a "magnetic field". I will list some open conjectures here.
The second part is concerned with applications to the quantum Hall effect. This is based on some joint work with Carey, Hannabuss, Marcolli.

L^2-invariants of locally symmetric spaces

Martin Olbrich

Let $X = G/K$ be a Riemannian symmetric space of the noncompact type, $\Gamma \subset G$ a discrete, torsion-free, cocompact subgroup, and let $Y = \Gamma\backslash X$ be the corresponding locally symmetric space. In this talk I explain how the Harish-Chandra Plancherel Theorem and results on $(\mathfrak{g}, K)$-cohomology can be used in order to compute the L^2-Betti numbers $b_p^{(2)}(Y)$, the Novikov-Shubin invariants (with respect to the Laplacian) $\alpha_p(Y)$, and the L^2-torsion $\tau^{(2)}(Y)$. The final results, due to Borel, Lott, Hess, Schick, and me, are

Theorem 1. Let $n = \dim Y$ and $m = \mathrm{rk}_{\mathbb{C}}G - \mathrm{rk}_{\mathbb{C}}K$ be the fundamental rank of G. Then

(i) $b_p^{(2)}(Y) \neq 0 \Leftrightarrow m = 0$ and $p = \frac{n}{2}$.

(ii) $\alpha_p(Y) \neq \infty^+ \Leftrightarrow m > 0$ and $p \in [\frac{n-m}{2}, \frac{n+m}{2}]$. In this range $\alpha_p(Y) = m$.

(iii) $\tau^{(2)}(Y) \neq 0 \Leftrightarrow m = 1$.

On the Atiyah conjecture I

Holger Reich

In the first part of the talk we introduced the algebra $\mathcal{U}\Gamma$ of operators affiliated to the group von Neumann algebra $\mathcal{N}\Gamma$. There is a dimension theory for modules over $\mathcal{U}\Gamma$ and this leads to an alternative definition of L^2-Betti numbers as the dimension of homology modules with twisted coefficients in $\mathcal{U}\Gamma$. These homology modules should be seen as an algebraic version of the reduced L^2-homology in the Hilbert space set-up. We presented vanishing results for derived functors of type $Tor_p^{\mathbb{Z}\Gamma}(-, \mathcal{U}\Gamma)$ and applied these to obtain results about Euler-characteristics of groups.
In the second part we gave an introduction to the Atiyah conjecture about the values of L^2-Betti numbers. We gave different equivalent formulations of the conjecture and explained Linnells strategy of attacking the conjecture by finding suitable intermediate rings of the ring extension $\mathbb{C}\Gamma \subset \mathcal{U}\Gamma$. We also indicated the relationship to the Isomorphism conjecture in algebraic K-theory.

Approximation properties and coarse geometry

John Roe

Let X be a coarse space. The C^*-algebra of X is a useful tool in the study of index theory and coarse versions of the Baum-Connes conjecture. In relating the coarse and usual versions of the Baum-Connes conjecture, the question arises what is $C^*|G|^G$, the algebra of G-fixed elements in the coarse C^*-algebra of a group G. It is asserted in the

literature, without adequate proof, that this is always equal to the reduced group C^*-algebra. We call groups G for which this is so groups with *property J*. We show that every amenable group, and even every *generalized amenable* group, has property J. By definition, G is generalized amenable if the reduced group C^*-algebra has the completely bounded approximation property .
This is joint work with Nigel Higson.

Classifications of C^*-algebras by their K-theory
Mikael Roerdam

Elliott conjectured in the late 1980's that it should be possible to classify (simple), nuclear, separable C*-algebras in a way analogous to his own classification of AF-algebras by their ordered K_0-group from the early 1970's. He substantiated his conjecture by a classification result for a class of C^*-algebras that arise as inductive limits of direct sums of matrix algebras over $C(\mathbb{T})$.
The talk addressed a theorem obtained by Kirchberg that confirms Elliott's conjecture for a certain class of very infinite C^*-algebras that are called *purely infinite*. A simple, purely infinite, separable, nuclear C^*-algebra is called a *Kirchberg* algebra. The classification theorem says that two Kirchberg algebras A and B are stably isomorphic if and only if they are KK-equivalent. By the Universal Coefficient Theorem, if A and B are KK-equivalent to Abelian C^*-algebras, then they are KK-equivalent if and only if $K_0(A) \cong K_0(B)$ and $K_1(A) \cong K_1(B)$ (as Abelian groups). For all pairs of countable Abelian groups (G_0, G_1) there is a Kirchberg algebra A (that is KK-equivalent to an Abelian C^*-algebra) such that $K_0(A) \cong G_0$ and $K_1(A) \cong G_1$.
It is an open problem if each nuclear C^*-algebras is KK-equivalent to an Abelian C^*-algebra. It is also an open problem if each (nuclear) simple, C^*-algebra that is infinte, in the sense of admitting no densely defined trace, is also purely infinite.

What do we hear on forms at the infinity of some nilpotent Lie groups?
Michel Rumin

Let G be a rational filtered nilpotent Lie group. The differential forms spectrum near zero is encoded in the shrinking family of Hilbert cones $C_\varepsilon = \{\alpha \in \Omega^p M/\overline{imd},\ ||d\alpha|| \leq \varepsilon||\alpha||\}$. In particular, the asymptotic Γ-linear thickness of C_ε give the exponents of decay of heat at large time, which are called the Novikov-Shubin numbers of G.
We first shown that the family C_ε is shrinking around a natural embedded subcomplex of the de Rham's one. This subcomplex itself turns out to be conjugated through a projection

to a differential complex acting only on the bundles $H(g)$ of Lie algebra cohomology of G. The main result is that it is an hypoelliptic complex. This allows to give estimations of the Novikov-Shubin numbers of G from the knowledge of the various homogeneous weights of $H(g)$.
For instance, for a r-steps stratified nilpotent group of homogeneous dimension N, one has

$$1 \leq minweightH^2(g) \leq \beta_1 \leq maxweightH^2(g) \leq r.$$

where $\beta_1 = \frac{N}{2}\alpha_1$ is the renormalized Novikov-Shubin number on 1-forms. In particular, 2-steps nilpotent groups with a lot of legendrian planes has $\beta_1 = 1$, and in the opposite direction the free r-steps nilpotent groups has $\beta_1 = r$.

Approximating L^2-Signatures

Thomas Schick

Suppose (X, Y) is a compact Poincaré duality pair and $\Gamma = \pi_1(X)$ is residually finite. That means we have a sequence of normal subgroups $\Gamma \supset G_1 \supset G_2 \supset \dots$ such that the quotient groups $\Gamma_k = \Gamma/G_k$ are finite and such that $\bigcap_k G_k = \{1\}$. Then we have corresponding finite coverings (X_k, Y_k) of (X, Y) with $\pi_1(X_k) = G_k$. We prove that

$$\operatorname{sign}_{(2)}(\tilde{X}, \tilde{Y}) = \lim_{k\to\infty} \frac{\operatorname{sign}(X_k, Y_k)}{|\Gamma_k|},$$

where $\operatorname{sign}_{(2)}$ is the L^2-signature.
This can be interpreted as "approximate" multiplicativity of the signatures for general Poincaré duality pairs. One should observe that in general, these signatures are not multiplicative.
If X has an amenable fundamental group, we prove a similar approximation theorem for $\operatorname{sign}_{(2)}(\tilde{X}, \tilde{Y})$ in terms of the signatures of a regular exhaustion of $\tilde{X}$. In this case we restrict our attention to Riemannian manifolds and work with an exhaustion by codimension zero submanifolds with boundary and with uniformly bounded geometry.

Knot concordance and L^2-signatures

Peter Teichner

With Tim Cochran and Kent Orr we construct many examples of non-slice knots in 3-space that cannot be distinguished from slice knots by previously known invariants. Using Whitney towers in place of embedded disks, we define a geometric filtration of the 3-dimensional topological knot concordance group. As special cases of Whitney towers of height less than four, the bottom part of the filtration exhibits all classical concordance invariants, including the Casson-Gordon invariants. Considering our entire filtration could lead to a 4-dimensional homology surgery theory. As a first step, we construct an infinite sequence of new obstructions that vanish on slice knots. These take values in the L-theory of Ore localizations of certain *rationally universal* solvable groups. Finally, we use the dimension theory of von Neumann algebras to detect the first unknown step in our obstruction theory by an L^2-signature. This provides a non-finitely generated subgroup of the knot concordance group on which all Casson-Gordon invariants vanish.

The Baum-Connes Conjecture I: General introduction
Alain Valette

To a countable group Γ, one may associate a purely topological object (the K-homology with Γ-compact supports of the classifying space for proper actions) and a purely analytical object (the K-theory of the reduced C*-algebra of Γ). It follows from results of Kasparov and Connes-Moscovici that the analytical group is the natural receptacle for indices of Γ-invariant elliptic operators on proper, Γ-compact manifolds.This led P. Baum and A. Connes to construct an index map (or analytical assembly map) from the topological group to the analytical one, and to conjecture that this map is an isomorphism. This conjecture is probably the most commutative part of Connes' non-commutative geometry programme. If true, it would imply results in topology (the Novikov conjecture on homotopy invariance of higher signatures), geometry (the Gromov-Lawson conjecture on manifolds with positive scalar curvature), and algebra/analysis (the Kaplansky-Kadison conjecture on the absence of idempotents in the reduced C*-algebra, when Γ is torsion-free). We observe that, when Γ is classified by a finite 2-complex, the conjecture also implies that every element in $K_1(C_r^*\Gamma)$ comes from a group element (something analogous to the vanishing of a Whitehead group). We conclude with a discussion of the present status of the conjecture (results by Julg-Kasparov, Higson-Kasparov, Lafforgue) and state some stability results due to Oyono and Tu (for groups acting on trees), and to Oyono (for short exact sequences).

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