MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 41/1999

Graph Theory

17.10.-23.10.1999

The organizers of this meeting on graph theory were Reinhard Diestel and Paul Seymour. Besides the normal formal lectures, the meeting included a number of "informal sessions." Each session was concerned with a particular area of graph theory, and anyone interested was welcome to attend. During these informal meetings, participants presented results and open problems concerning the topic and the audience was encouraged to interrupt with questions, counter-examples, proofs, etc. These sessions resulted in the resolution of a number of conjectures as well as stimulating collaboration outside the structure of the conference. The following is a summary of the sessions followed by a collection of abstracts of the formal talks.

Session on Infinite Graphs
Convenor: Reinhard Diestel

1. Cycle space in locally finite infinite graphs

Bruce Richter asked how the fact that the fundamental cycles of a finite graph form a basis of its cycle space can be adapted appropriately to infinite graphs. In the discussion it emerged that end-faithful spanning trees would play a significant role here, and various models based on these were discussed. Richter's objective was to prove a uniqueness-of-embedding theorem for 3-connected locally finite graphs with suitable compactification such as one point at infinity for every class of ends pairwise not separated by a finite cycle. This led to further informal collaboration later in the week.

2. Transitive graphs and Cayley graphs

Recalling Woess's problem of whether every locally finite connected vertex-transitive graph is quasi-isometric to a Cayley graph, Leader presented a limit construction of transitive non-Cayley graphs obtained jointly with Diestel, whose limit they conjecture not to be quasi-isometric to a Cayley graph. The limit graph admits a simple direct definition. Details will appear in a paper by Diestel and Leader soon to be available in preprint form.

3. Arbitrarily vs. infinitely many disjoint substructures of a given type

Andreae posed the following general problem: for which infinite graphs H is it true that every graph containing k disjoint copies of H for every $k \in \mathbf{N}$ also contains infinitely many disjoint copies of H? This was proved by Halin in the 1960s for H a ray, and in the 1970s by Andreae for various trees. Andreae conjectures that if the containment relation is weakened from "subgraph" to "topological minor" then this should hold for all trees. If containment is weakened further to the minor relation, he conjectures that this may hold for all locally finite connected H. He presented an uncountable graph H without this property (for minors), but no countable counterexample for minors appears to be known.

4. Classifying the $(\aleph_0, \aleph_1)$-regular bipartite graphs

Diestel briefly presented his recent characterization with Leader of the graphs not admitting a normal spanning tree. The characterization is by two types of forbidden minor, and one of these types are the $(\aleph_0, \aleph_1)$-regular bipartite graphs (A, B). They asked how these graphs might be classified up to minor equivalence. They know of two types of such graphs: the "ever-splitting" binary tree with tops (where A is the tree and B the set of tops), and a "non-splitting" example in which the subsets A' of A that have uncountably many neighbours in B each sending infinitely many edges to A' form a non-principal ultrafilter on A. Neither of these two types has a minor of the other type, but it is an open problem whether every $(\aleph_0, \aleph_1)$-regular bipartite graph has a minor of one of these two types. A preprint is available from Diestel and Leader.

5. Hamiltonian double rays in planar graphs

Yu indicated his recent proof of Nash-Williams's conjecture that every 4-connected infinite planar graph with at most two ends contains a spanning double ray. A preprint is available from the author.

Session on Coloring and the Colin de Verdière Number
Convenor: Alexander Schrijver

1. Relation between Colin de Verdière's μ and the Hadwiger number on graphs with $\alpha(G) = 2$.

Andreĭ Kotlov asked if the equation $1 + \mu(G) = \eta(G)$ held for graphs G with no independent set of size 3 where μ is Colin de Verdière's graph invariant and η is Hadwiger's number. (The order of the largest clique minor of G.) He noted that the inequality $1 + \mu(G) \geq \eta(G)$ holds for all G and equality holds for cliques. It was discussed that graphs with $\alpha(G) = 2$ must have chromatic number at least $|V(G)|/2$ and so may give a counterexample to Hadwiger's conjecture which states in this case that such graphs have a clique minor on at least $|V(G)|/2$ vertices. Kotlov sketched a proof that such graphs have a clique minor of size $\frac{|V(G)|}{3} + O(\sqrt{|V(G)|})$. Bojan Mohar provided a counterexample to $1 + \mu(G) = \eta(G)$ when $\alpha(G) = 2$ by considering compliments of triangle-free planar graphs.

2. Equitable k-colorings

A. Kostochka introduced equitable k-colorings and began with the general question of when a graph with maximum degree k has an equitable k-coloring. He presented various results concerned with such colorings. He also indicated methods to obtain such colorings and gave conditions under which these methods worked.

3. The graph invariant λ

Heine Van der Holst gave a definition of his graph-invariant λ. He then defined the k-closure of a graph and defined a graph to be k-flat if k is even and the $k/2$-closure of G can be embedded in $\mathbf{R}^k$. This notion generalizes planar as a graph is 2-flat iff it is planar. Heine asked if it was true that G being k-flat implies that $\lambda(G)$ is at most $k + 1$. He also asked if being k-flat implies $\mu(G)$ is at most $k + 1$. Both of these are true in the case that G is 2-flat. It was pointed out by Alexander Schrijver that 4-flat is weaker than being linklessly embeddable in $\mathbf{R}^3$ and indicated that the relationship between linkless-embeddable and 4-flat is the same as that between outer-planar and planar.

Session on Graphs on Surfaces
Convenor: Bojan Mohar

1. Chromatic numbers of graphs of girth at least five embeddable on surfaces

Robin Thomas recalled that Thomassen proved that if Σ is the torus or projective plane and G is a graph with girth at least 5 which can be embedded in Σ, G is 3-colorable. Thomas stated that he and Barret Walls proved a similar result for the case when Σ is the Klein bottle. Thomas conjectured that for any surface Σ there are only finitely many 4-critical graphs of girth at least 5 embeddable on Σ.

2. Large independent sets and fractional chromatic number

Robin Thomas announced that he and Christopher Heckman have proved a conjecture of Albertson that every planar sub-cubic triangle-free graph has an independent set containing at least 3/8 of the vertices. Based on this result, Thomas conjectured that every sub-cubic triangle-free planar graph has fractional chromatic number less than 8/3. Thomas also conjectured that every sub-cubic triangle-free graph has fractional chromatic number less than 14/5 (based on a result of Staton that such graphs have an independent set of size at least 5/14). This led to collaboration throughout the week.

3. Generalization of Dirac's Map-Coloring Theorem

Riste Škrekovski recalled that Dirac showed that if G is embedded on a surface Σ that is neither the plane nor the projective plane, the chromatic number of G is less than the chromatic number of the surface $\chi(\Sigma)$ iff G does not contain $K_{\chi(\Sigma)}$ as a subgraph. He then announced a new theorem generalizing this result. If s is an integer and G is embedded on a surface Σ of genus $g \geq f(s)$ ($f(s)$ is given explicitly), the chromatic number of G is at most $\chi(\Sigma) - s - 1$ iff G does not contain a sub-graph which is $\chi(\Sigma) - s$-critical on at most $\chi(\Sigma)$ vertices.

4. Counting perfect matchings in 3-dimensional grids

Martin Loebl was concerned with finding a formula for counting the number of perfect matchings in a general 3-dimensional grid. He defined acceptable orientations of the edges of such grids and defined their determinants. He then presented a formula relating the number of perfect matchings to a certain sum involving the determinants of acceptable orientations. He noted that this gives a relation between the number of perfect matchings and the average of functions depending linearly on the determinants of orientations. This is an improvement over previous results which use quadratic dependence.

5. Cycles containing many vertices in graphs embedded on surfaces

Yu began by recalling Whitney's result that 4-connected planar triangulations are Hamiltonian. A similar result does not hold for 3-connected planar graphs. However, Jackson and Wormald have shown that in this case there is a cycle containing at least $cn^{0.2}$ where c is a constant and n is the total number of vertices. It has been conjectured that the correct answer is $cn^{log(2)/log(3)}$ and examples show that this would be tight. Yu announced that he and Chen have proved this conjecture and similar results for graphs on surfaces of small genus.

6. Non-contractible separating cycles in embedded graphs

Bruce Richter stated that Zha conjectured that 3-representable graphs on a surface of genus at least 2 have a non-contractible, separating cycle (separating the surface, not the graph). Richter announced a theorem that there are such cycles in 6-representable graphs in the case of orientable surfaces and 5-representable in the case of non-orientable. Richter conjectures that the correct answer is 5 for all surfaces.

7. A conjecture of Randby

Neil Robertson mentioned a conjecture of Randby that if G is triangle-free, 4-connected, and more than 4-representable on a surface, G contains a topological K_5 minor.

8. Progress on Negami's conjecture

Petr Hliněný stated Negami's conjecture that a graph G has a planar cover (map from a planar graph to G which preserves neighborhoods) iff G can be embedded in the projective plane. Hliněný indicated that the reverse direction is easy and the forward direction uses the forbidden minors for the projective plane. To prove the conjecture, it only remains to prove that $K_{1,2,2,2}$ has no planar cover. To do so has proven difficult. Hliněný announced a theorem joint with Robin Thomas that there are at most 16 graphs which are counterexamples to Negami's conjecture.

9. Isomorphism testing for graphs embedded on a surface

Martin Grohe described a polynomial-time algorithm to test if two graphs embeddedable on a surface are isomorphic. He began by describing a color refinement algorithm that colors the vertices in stages based on the colors of the neighbors in the previous stage. He announced a theorem that this process stabilizes after a determined number of steps for graphs embeddable on a surface (depending on the surface) and determines the graph up to isomorphism. He mentioned another theorem that this algorithm also works in polynomial-time for graphs of bounded tree-width.

Session on Odd Minors
Convenor: Bruce Reed

1. Excluding an odd K_5 minor

Bert Gerards began the session by introducing odd minors (minors with parity conditions) and presenting a way of thinking of them as minors of a graph with signed edges and rules about which edges may be contracted and how the signs of edges may be switched. He then gave the complete list of graphs which do not contain an odd K_4 minor. He presented a list of graphs which do not contain an odd K_5 minor, but it is not known if this list is complete.

2. Erdős-Posa property and odd cycles

Bruce Reed presented the family of Escher walls as an example showing that a graph may not have two vertex-disjoint odd cycles while still requiring an unbounded number of vertices to hit all odd cycles. However, he did announce the following theorem. For every u there exists $f(u)$ such that G has either u vertex-disjoint odd cycles or a large odd minor (Reed said he preferred the term "parity minor".) Escher wall, or a set $X \subseteq V(G)$ with $|X| \leq f(u)$ such that $G \backslash X$ is bipartite.

3. Disjoint odd cycles and odd cycle covers

While the Escher walls show that the Erdős-Posa property does not hold in general for odd cycles, Dieter Rautenbach recalled that Thomassen has proved that for any integer k, a 2^{39k}-connected graph either has a set $X \subseteq V(G)$ with $|X| \leq 2k-2$ such that $G \backslash X$ is bipartite or G has k vertex-disjoint odd cycles. Rautenbach announced that he and Reed have improved the bound on connectivity to $2000k$, which is essentially best possible. He introduced "auxiliary" graphs and indicated their use in the proof.

Session on Flows
Convenor: Luis Goodyn

1. Packing and hitting theorems for even cycle matroids

Bertrand Guenin began the session by describing the even cycle matroid of a graph. The cycles of this matroid are the even cycles of the graph together with pairs of odd cycles that share at most one vertex. He then presented packing and hitting theorems that held fractionally by LP-duality but which could also be shown to hold integrally. He also considered the even cut matroid and obtained similar results.

2. Flows in bi-directed graphs

Matt DeVos described how local tensions (related to colorings and equal in the plane) are dual to flows in graphs embedded in orientable surfaces. In the case of non-orientable surfaces, the dual of local tensions are bidirected flows where some edges (the ones that are signed in the embedding) are directed both ways. For an ordinary (uni-directed) edge, sending a flow of x along an edge removes x amount of flow from its tail and deposits x at its head. But with bi-directed edges assigning x to an edge deposits x amount of flow at both ends. (So, flow is not generally conserved.) DeVos stated that Bouchet conjectured that any bi-directed graph which has a nowhere-zero flow (Every edge gets some non-zero flow.) has a nowhere-zero 6-flow. (Every edge gets an integer flow between -5 and 5 excluding 0.) Examples show this would be best possible. DeVos has proven this holds with 6 replaced by 12, improving the previous best bound of 30. In case the graph is 4-edge-connected, DeVos can find a nowhere-zero 4-flow, improving the previous bound of 18.

3. Counterexample to a Conjecture of Goodyn

Andreas Huck produced a counterexample to the following conjecture of Goodyn: Let G be a cubic, cyclically 4-connected graph which has a chordless cycle dominating all the vertices of G. Then G is 3-edge colorable. The counterexample was obtained by starting with copies of I_5 and the Petersen and applying the dot-product combining operation.

4. A result connected to Tutte's 4-flow conjecture

Robin Thomas recalled Tutte's conjecture that if G is a bridgeless graph with no Petersen minor, then G has a nowhere-zero 4-flow. Robertson, Seymour, Sanders, and Thomas proved the conjecture in the case of cubic graphs. For the general case, Thomas announced a theorem with Jan Thomson that if G is bridgeless and has no minor isomorphic to the Petersen minus an edge, then G has a nowhere-zero 4-flow. This is based on their theorem that if G has minimum-degree 3, girth at least 5, and is non-planar, then G has a minor isomorphic to the Petersen minus an edge.

5. Special colorings and Anti-Flows

Nešetřil considered the problem of determining the minimum number of colors needed to color the vertices of a graph so that no cycle is colored with at most 2 colors. He also recalled the problem of determining the minimum number of colors needed to color any orientation of a graph such that all edges between any two color classes go in the same direction. He noted one could similarly ask for the coloring to be a map from the vertices of G to those of a circulant with the directions on edges between any two color classes preserved. He also described the anti-flow problem. This problem asks if there is a fixed k such that every 3-edge connected graph with orientations on the edges has a flow such that every edge receives a non-zero integer flow between $-k$ and k but such that no edge receives the negative of another edge. This problem led to further discussion throughout the week and appears to have been answered in the affirmative by DeVos, Johnson, and Seymour.

6. Open problems involving cycles and flows

Luis Goodyn presented a list of unsolved problems dealing with flows and cycles in graphs and matroids. It appears that some of them can be solved using partial results of Goodyn and Seymour.

Session on Matroids
Convenor: James Oxley

1. Steps toward a structure theorem for (binary) matroids

Paul Seymour began the session by describing the recent attempts of Johnson, Robertson, and Seymour to describe the structure of (binary) matroids G which do not contain a fixed binary matroid H as a minor. Seymour stated that the kind of theorem they hoped to prove was that G could be obtained by "gluing" together graphic and co-graphic matroids which themselves do not have H as a minor. Seymour indicated that the new proof by Diestel, Gorbunov, Jensen, and Thomassen that large tree-width implies having a large grid minor for graphs seems to be applicable to the matroid case. However, at least in the non-binary case, it seems one needs to consider "gridles" and "girdles" in addition to grids when characterizing the obstructions to small tree-width.

2. Matroid matching

Jim Geelen described the matroid matching problem in which a representable matroid has its elements bijectively assigned to the vertices of some graph and it is to be determined if there is a basis of the matroid which corresponds to a set of vertices covered in some matching of the graph. Geelen reduced the problem to determining if some matrix of the form $A + T$ has full rank where A is skew-symmetric and T is the

Tutte matrix of the graph. Geelen presented a method of writing such matrices as the sum of smaller rank matrices such that the sum of the ranks equals the rank of the original matrix.

3. Spine/cospine transformation

Dirk Vertigan introduced the spine and cospine transformation. Given a matroid M, he considered an element e such that $(A \cup e, B)$ and $(A, B \cup e)$ are separations of the same order in M. This means that e is either in the "guts" (in the case (A, B) is a separation of the same order in $M \backslash e$) or the "coguts" (in the case (A, B) is a separation of the same order in M/e) of (A, B). In the former case, Vertigan considered the matroid N such that $M \backslash e = N/e$ and the dual equation in the later. N is said to be obtained from M by performing a "spine/cospine transformation." Vertigan presented some results concerning such transformations, including a result that the number of representations of M and N over a field P are equal.

4. Sharp bounds on the size of connected matroids

James Oxley presented some results concerning the maximum size of connected matroids given the size of the largest circuit and cocircuit they contain. After a brief history of bounds previously obtained, Oxley announced the new result that $|E(M)| \leq cc^*/2$ where c is the size of the largest circuit in a connected matroid M and c^* is the size of a largest cocircuit. Similarly, $|E(M)| \leq (c_e - 1)(c_e^* - 1) + 1$ where c_e and c_e^* are respectively the sizes of a largest circuit and cocircuit containing a given element e. Oxley presented some corollaries of these results and also mentioned previously known results giving lower bounds on the size of a connected matroid in terms of the size of the smallest circuit and cocircuit.

Session on Connectivity
Convenor: Wolfgang Mader

1. Splitting vertices to maintain edge-connectivity

Tibor Jordán began the session by considering the problem of splitting off edges from a vertex s (two edges us, vs are replaced by the edge uv) so that the edge-connectivity between any pair of vertices disjoint from s does not decrease. He presented a theorem of Mader that says if the degree of s is even and s is not incident to a cut-edge, there is a pair of edges which may be split off from s to maintain all connectivities as described. Jordán wanted to extend this result to the case when edges are "detached" instead of just split off. (So, edges come off in bundles still incident to a common vertex instead of in pairs.) Given a degree sequence $(d_1, ..., d_n)$ with $d(s) = \Sigma d_i$, Jordán gave a theorem which exactly characterizes when the edges at s can be detached into bundles of degree d_i.

2. S-T Connectors

Alexander Schrijver presented a theorem which generalizes both Nash-Williams's theorem on edge-disjoint spanning trees and Kőnig's Theorem. Given a graph G with vertices partitioned into sets S, T, Schrijver defined an S-T connector as a subset $F \subseteq E(G)$ such that each component of $(V(G), F)$ intersects both S and T. Schrijver's theorem states that there are k edge-disjoint S-T connectors iff for every sub-partition of S or T into classes, the number of edges in G leaving at least one class is at least k times the number of classes.

3. Finding vertex-disjoint even cycles and paths in digraphs

James Geelen considered the problem of finding in a given digraph the maximum number of edges that can be partitioned into vertex-disjoint even cycles and paths. He explained that this problem is related to matching and S-T paths in undirected graphs but is NP-hard since it is also related to determining if a graph is Hamiltonian. Geelen defined symmetric digraphs as those which have an edge from a to b if they have an edge from b to a. Weakly symmetric digraphs were defined as digraphs with symmetric strong components. Geelen can solve the problem in the case of weakly symmetric digraphs, which generalizes the matching and S-T paths problems.

4. k-connected dominating sets

Given a fixed k, Yuster considered the problem of finding k-connected dominating sets in k-connected graphs. As $\delta(G)$ grows, perhaps there is such a dominating set D with $|D| \leq \frac{n log(\delta)(1+O_\delta(1))}{\delta}$. This has been proven for $k = 0$ by Lovász and for $k = 1, 2$ by Yuster and others. There are lower bounds of the same form as well. The problem is open for $k \geq 3$.

Session on Matroids, Part 2
Convenor: James Oxley

1. Hadamard Conjecture

Martin Loebl began the session by discussing the Hadamard conjecture. After defining the Hadamard matrix, he presented some results toward solving the conjecture. Loebl is interested in using simplicial complexes to define binary codes and conversely answer which binary codes come from such complexes. A related question is which $(0,1)$ matrices are incidence matrices of simplicial complexes.

2. *Extent to which even cycle space determines underlying graph*

Bert Gerards asked what one could conclude about two graphs if one knew they had the same even cycle space. He presented a list of operations that may be performed on a (signed) graph which leaves its even cycle space unchanged. The hope is that one may go between any two graphs with the same even cycle space by performing a series of operations in the list, but this is not known.

3. *Finding large grids in (binary) matroids of large tree-width*

Thor Johnson continued Paul Seymour's presentation of progress that has been made in showing matroids of large tree-width have large grid minors (or gridles or girdles in the case of non-binary). Johnson outlined the first half of a proof that if two pairs of subsets of elements are highly linked, either the linkages can be made "disjoint" or the matroid has a large grid or girdle or gridle minor.

4. *Graph Minors for Matroids*

Neil Robertson outlined a general scheme for applying the techniques used in the Graph Minors series (Robertson and Seymour) to obtain similar results in the category of matroids. The presentation included a statement of the types of theorems he hoped could be proven using these methods along with the main "stepping stone" theorems that would need to be developed. Robertson's presentation was continued at a subsequent informal meeting.

Session on Classical Minors
Convenor: Robin Thomas

1. *Complete minors of the hypercube*

Andreĭ Kotlov began the session by asking what is the largest complete graph which is a minor of the d-dimensional hypercube Q_d. Kotlov presented a lemma which implies that Q_d contains $K_{\sqrt{2}^d}$ as a minor. He presented an upper bound of $\sqrt{d2^d}$ which is obtained essentially by counting edges. While the lower bound holds for small values of d, it is not tight for larger values.

2. *A splitter theorem for internally 4-connected graphs*

Thor Johnson presented a result joint with Robin Thomas which concerns building an internally 4-connected graph G from an internally 4-connected minor H. The steps involved in the construction are either one element extensions or very restricted extensions of size two. Moreover, the intermediate graphs are "nearly 4-connected" and Johnson indicated why maintaining internal 4-connectivity is not possible.

3. *Connectivity vs. $K_{a,k}$ minors*

Bojan Mohar was interested in what kind of connectivity for sufficiently large graphs implies having a $K_{a,k}$ minor for fixed a and increasing k. (The connectivity may depend on a but not k) Mohar presented a result that every large 7-connected graph contains $K_{3,k}$ as a minor. He presented an example showing that 6-connectivity was not sufficient. Mohar also presented a general theorem that for every a, there exists $c(a)$ such that every sufficiently large $c(a)$-connected graph of bounded tree-width contains $K_{a,k}$ as a minor.

Session on Extremal Graph Theory
Convenor: Hans Prőmel

1. *The Prague dimension of Kneser graphs*

Zoltán Füredi pointed out another connection between the Prague dimension of graphs and the dimension theory of partially ordered sets by giving a very short proof of a theorem of Poljak, Pultr and Rödl. He showed that the dimension of the Kneser graph is bounded as $\dim_{\mathrm{P}}(K(n,k)) < C_k \log\log n$, where C_k depends only on k.

2. *Optimal H-packings*

Raphael Yuster defined an optimal H-packing of K_n as a maximum set of edge-disjoint copies of H in K_n and denoted the cardinality of an optimal packing by $P(H,n)$. He considered an optimal H-packing, L, of K_n. He let $\chi(L)$ denote the chromatic number of the intersection graph of L (the graph whose vertices are the $P(H,n)$ members of L and whose edges connect two members sharing at least one vertex of K_n). He defined the resolution number $\chi(H,n)$ as the minimum of $\chi(L)$ taken over all possible optimal H-packings of K_n. This corresponds to the minimum number of layers of vertex-disjoint copies of H whose union forms an optimal packing. Yuster can prove that if H is any fixed graph with h vertices and m edges then $\chi(H,n) = (n-1)\frac{h}{2m}(1+o_n(1))$.

3. Nice graphs

Alexander Kostochka defined an oriented graph G as "nice" if for every pair of vertices u,v and every orientation of the edges of the path P of length k, there is a path in G from u to v with the same edge orientations as those given to P. Similarly, an undirected graph G with its edges colored with c colors is "nice" if "coloring" replaces "orientation" in the previous definition. In a joint paper, Kostochka has shown that graphs are nice iff they do not contain "black holes". He has also shown that "niceness" of graphs is a special case of a natural notion involving "nilpotency of semigroups of endomorphisms of certain algebraic structures."

4. f-connectivity

Given a function $f : \mathbf{N} \to \mathbf{N}$, Reinhard Diestel calls a graph G f-connected if for every separation (A, B) of G with $|A| < |B|$ there are at least $f(|A - B|)$ vertices in $A \cap B$. (Thus, k- connectedness amounts to f-connectedness for f having constant value k, but the $n \times n$ grid (say), which is only 2-connected, is f-connected for some f growing like the root function.) Diestel presented the following problem: Is there a function $f : \mathbf{N} \to \mathbf{N}$ tending to infinity such that for every $n \in \mathbf{N}$ there is a $k \in \mathbf{N}$ such that every graph of chromatic number at least k has an f-connected subgraph of order at least n? (Diestel remarks that such an f will be particularly interesting if replacing "chromatic number" with "average degree" makes the statement false, at least for this f.)

5. A characterization of triangle-free graphs

Using a clever inductive counting argument Erdős, Kleitman and Rothschild showed that almost all triangle-free graphs are bipartite, i.e., the cardinality of the two graph classes is asymptotically equal. In this talk we investigate the structure of the few triangle-free graphs which are not bipartite. Using similar techniques as Erdős, Kleitman and Rothschild we prove that with high probability these graphs are bipartite up to a few vertices. This means that almost all of them can be made bipartite by removing just one vertex. Almost all others can be made bipartite by removing two vertices, and then three vertices and so on.

Session on Digraph Minors
Convenor: Paul Seymour

1. Digraphs with dicycles of only one length

William McCuaig began the session by asking the question (originally asked by Paul Seymour) of when all dicycles in a digraph have the same length. He was also interested in the more general question of when a digraph has an arc-weighting such that all dicycles have the same non-zero weight. McCuaig defined an exact transversal as a set of arcs such that every dicycle uses exactly one edge in the set. Finally, McCuaig defined butterfly minors and considered the class of graphs which do not have double dicycle butterfly minors. It is easy to see that if a digraph has an exact transversal, it has an arc-weighting so that all dicycles have the same length. Also, if there is such a weighting, a digraph does not have a double dicycle butterfly minor. McCuaig announced a theorem that actually all these are equivalent, i.e. digraphs with no double dicycle butterfly minor have exact transversals.

2. Eulerian digraph immersion

Thor Johnson defined digraph immersion and asked the general question of the structure of Eulerian digraphs which do not immerse a fixed Eulerian digraph H. Following the general outline of the Graph Minors series of Robertson and Seymour, Johnson defined tree-width for Eulerian digraphs and showed that having small tree-width is an obstruction to immersing H. According to Johnson, as in the case of Graph Minors, having large tree-width implies immersing a certain type of grid from which a structure theorem can be built. Johnson stated that it appears the main results of Graph Minors will hold in the category of Eulerian digraphs and will yield, among other things, a polytime algorithm for solving the k edge-disjoint linkage problem for fixed k.

3. Directed tree-width and havens

Robin Thomas presented a proof that the directed tree-width of a digraph (defined in his talk) is tied to the size of the largest haven in the digraph. This is the first step in proving the conjecture that directed tree-width is tied to containing a certain directed grid as a butterfly minor.

Talks

Problems on Infinite Graphs
R. Halin (Hamburg)

The intention of the talk was to give an introduction to a recent collection of problems concerning infinite graphs, which, in my brief announcement, include the following topics.

1. R. Schmidt classified the rayless graphs by introducing an ordinal valued order function. How can his theory be extended to larger classes of graphs?
2. W.T. Tutte showed that the cycle space of every 3-connected finite graph is generated by its non-separating circuits. To what extent can Tutte's theorem be carried over to infinite graphs?
3. In "tree partitions of infinite graphs" (Halin 1991) pseudo-trees and quasi-trees were introduced. Open problem: Characterize the pseudo-trees which are not quasi-trees.
4. Motivated by the fact that every connected locally finite graph is a quasi-tree, the author introduced the term "infinity type" (1989). Characterize the infinity types of locally finite graphs, which would yield a classification of the locally finite graphs.
5. Search for a spanning tree T of a graph G in which every end $\mathcal{E}$ is represented by a given number $k(\mathcal{E})$ of ends (of T), when $0 \leq k(\mathcal{E}) \leq m_1(\mathcal{E})$ (= max. number of pairwise disjoint rays of $\mathcal{E}$).
6. Which configuration must be present if a graph contains an end $\mathcal{E}$ in which $m_1(\mathcal{E}) \geq \aleph_1$?
7. Characterize the connected graphs which admit a normal rooted spanning tree (in Jung's sense) by forbidden minors.
8. Lattice theoretical problems in connection with Erdős' conjecture concerning Menger's theorem for infinite graphs.
9. Study *vertex* minimization of infinite n-connected graphs analogously to R. Halin, Minimization problems for infinite n-connected graphs (1993).
10. In a note in JGT (1983) the author extended the notion of Hamiltonicity to graphs with more than two ends. The question whether Thomassen's extension of Fleischner's theorem also holds if this notion of Hamiltonicity is assumed is still open.
11. a) Which groups of order $\aleph$ are isomorphic to the automorphism group of a countable graph? b) If G is locally finite, connected and $|\text{Aut } G| = \aleph_0$, must then Aut G contain a translation? c) If G is countable with $|\text{Aut } G| = \aleph_0$, must then G contain a double ray?
12. A vertex pair (x, y) in G is tight if the Menger number of (x, y) is infinite. G^* arises from G by adding all edges xy for tight (x, y), G_* is G minus all edges xy with tight (x, y). G is *collapsing* if G contains at least one edge xy with tight (x, y) such that (x, y) does not remain tight in G_*. It is shown that every collapsing graph contains 2 ground configurations each having T_ω as a topological minor. Several open problems arise in this context.

For details see R. Halin, Miscellaneous problems for infinite graphs.

Eigenvalues and Embeddings of Graphs
Alexander Schrijver (CWI & University of Amsterdam)
Joint work with László Lovász

In 1990, Y. Colin de Verdière introduced the parameter $\mu(G)$ of a graph G, being the maximum corank of any $V \times V$ symmetric matrix $M = (m_{u,v})$ having exactly one negative eigenvalue (of multiplicity 1), and with $m_{u,v} < 0$ if u and v are adjacent, and $m_{u,v} = 0$ if u and v are nonadjacent, and satisfying the Strong Arnold Hypothesis. This parameter was motivated by studies of the spectrum of Schrödinger operators. Colin de Verdière showed that $\mu(G) \leq 3$ if and only if G is planar. We showed that $\mu(G) \leq 4$ if and only if G is linklessly embedded. In the proof we use a Borsuk theorem for antipodal links, which may be interesting in its own right. In the lecture, we give an introduction to the above, and we explain the methods.

Some Algebraic Methods in Connection with the Chromatic Number
Carsten Thomassen (Technical University, Denmark)

Finding the chromatic number is one of the notoriously hard problems in graph theory. We discuss three algebraic methods.

1. The color matrix (joint work with Tommy Jensen) whose rows consists of all k-colorings where the colors are elements in a fixed field. Our main result says that the 3-color matrix of a planar triangle-free graph has full rank.

2. The chromatic polynomial, introduced in 1913 by Birkhoff. We describe a sufficient condition, in terms of the roots of the chromatic polynomial, for a graph to contain a Hamiltonian path and discuss the (unsolved) counterpart for Hamiltonian cycles.

3. Toft conjectured in 1976 that every 4-chromatic graph contains a totally odd K_4-subdivision. This was proved recently independently by Wenan Zang and myself. It is pointed out how the cycle space is used in the solution.

Graphs on Surfaces
Bojan Mohar (University of Ljubljana)

Three recent results about graphs on surfaces have been presented. For each of them, there may be a more general theory leading to important results.

1. Geometric structure of embeddings has been illustrated by the following results (joint work with Neil Robertson): For each surface S, there is a finite number of patch structures so that every planar graph embedded in S can be turned into one of these patch structures after a series of Whitney $2-$, $3-$, and $4-$flippings.

2. Combinatorial structure of graphs embedded with large face-width. A recent theorem (joint work with Thomas Bőhme and Carsten Thomassen) shows that every 4-connected graph embedded with sufficiently large face-width contains two cycles C_1, C_2 each of which contains more than 99.9% of the vertices ("almost Hamiltonian") and $C_1 \cup C_2$ covers the whole graph.

3. Coloring graphs on surfaces: A recent result is that locally bipartite graphs on nonorientable surfaces with sufficiently large edge-width have chromatic number 2, 3, or 4 and those with chromatic number 4 are completely characterized. Such examples cannot occur on orientable surfaces. (Joint result with Paul Seymour.)

Graphs and Curves on Surfaces
Alexander Schrijver (CWI & University of Amsterdam)

From a theorem on minimizing crossings of systems of closed curvers on a compact surface S by Reidemeister moves, we derive a number of theorems on decomposing graphs on surfaces, on finding circulations of given homotopy types in a graph, on characterizing homotopies of systems of curves by their crossing numbers, and on minimal graphs of given crossing-functions ("kernels").

Counting Problems Related to the Tutte Polynomial
Dominic Welsh (Oxford)

The Tutte polynomial of a graph, or more generally a matrix or matroid, is a two variable polynomial $T(G;x,y)$ which contains as specialisations a host of different objects: These include

1. Along $xy = 1$ the Jones polynomial of the alternating link determined by G

2. The partition functions of the Ising/Potts models of statistical physics on G

3. The chromatic and flow polynomials of G

In 1990, with Jaeger and Vertigan we proved that evaluating $T(G;x,y)$ is $\#P$-hard except when $(x-1)(y-1) = 1$ or when (x,y) is one of $(1,1)$, $(-1,0)$, $(0,-1)$, $(-1,-1)$, $(^{+}_{-}i,^{-}_{+}i)$ and $(^{+}_{-}j,^{-}_{+}j)$ where i,j are complex square and cube roots of unity. Since 1990 I have been trying to find fully polynomial approximation schemes for the Tutte polynomial in the region $x \geq 0$, $y \geq 0$. As far as we are aware there is no obstacle to such a scheme existing but existing results are at best fragmentary. For example with Alon and Frieze (1994) we show such schemes exist for $x \geq 1$, $y \geq 1$ for "dense" graphs. Two points of apparent simplicity but on which progress has been remarkably slow are $(2,0)$ where T counts the number of acyclic orientations and $(2,1)$ where T counts the number of forests. With Bartels and Mount (1998) we used the random walk volume approximation method to attack the forest problem. This involves finding a random integer point in the base of the polyhedron $\Sigma x_i \leq e(U)$ where $\phi \subseteq U \subseteq V$ and U runs through all the subsets of V. The vertices of the polyhedron correspond bijectively with acyclic orientations. However we hit exactly the same denseness barrier as with the earlier methods.

Decomposition of Perfect Graphs, Balanced Matrices, and Ideal Binary Clutters
Gérard Cornuéjols (Carnegie Mellon University)

In this talk, we survey recent decomposition results.

Conforti, Cornuéjols, Gasparyan, and Vuskovic prove a special case of Chavatal's skew partition conjecture and introduce new perfection preserving decompositions. Conforti and Cornuéjols use this result to show that WP-free graphs can be decomposed into bipartite graphs and line graphs of bipartite graphs, using perfection preserving operations.

In a graph, a double star consists of two adjacent nodes and some of their neighbors. Conforti, Cornuéjols, and Rao show that a balanced 0,1 matrix is totally unimodular or its bipartite representation has a double star cutset. This result yields a polytime recognition algorithm for balancedness.

Seymour conjectured that a binary clutter is ideal if and only if it does not have F_7, O_{K_5}, or $b(O_{K_5})$ minors. Cornuéjols and Guenin recently proved Seymour's conjecture for the class of clutters that do not have Q_6^+ or $b(Q_6)^+$ minors. This implies the Edmonds-Johnson T-join theorem as well as Guenin's theorem on weakly bipartite graphs. The result is obtained by first showing that minimally nonideal binary clutters are 3-connected and internally 4-connected.

Matroid 4-Connectivity
Geoff Whittle (Victoria University of Wellington)

Traditionally results in matroid structure theory and matroid representation theory have relied crucially on 3-connectivity. Moreover there exists a collection of "solid tools" for obtaining such results. Examples include the Wheels and Whirls theorem of Tutte and Seymour's Splitter Theorem. However, for various technical reasons it is becoming clear that 3-connectivity is not enough. In particular there is hope for a 4-connectivity version of a conjecture of Kahn that fails for 3-connectivity. To do this analogues of the Wheels and Whirls or the splitter theorem are needed. The talk discussed one such analogue; namely a theorem whereby a chain theorem is developed for "sequentially 4-connected" matroids.

Extremal Graph Theory (A Survey)
Miklos Simonovits (Institute of Mathematics, Hungary)

Extremal graph theory is one of the wider areas of graph theory with many connections and possible applications to other fields of combinatorics and also to fields outside of combinatorics.

The basic problem is if one has a given family $\mathcal{L}$ of so called sample graphs, how many edges can a graph have if it has n vertices and does not contain any $L \in \mathcal{L}$. These type of problems were first investigated by Mantel, Turán, Erdős and developed into a wide area in the sixties, seventies.

The main topics

1. Historical remarks: Turán's theorem, Erdős' application of graph theory to number theory, geometry, etc.
2. The general theory
3. Degenerate extremal graph problems
4. Finite geometric constructions
5. Product conjecture
6. Ramsey-Turán theorem
7. Hypergraph extremal problems
8. Excluding topological subgraphs

9. Extremal subgraphs of random graphs
10. Erdős-Kleitman-Rothschild theory and related results

and many other fields.

General Theory

The general theory started with a corollary of Erdős-Stone theorem (=the Erdős-Simonovits limit theorem) stating that if one has two families of excluded subgraphs, with the same minimum chromatic number, then their extremal numbers are the same, apart from an error term $o(n^2)$. Later Erdős and Simonovits proved that for every extremal graph problem the extremal graphs or the nearly-extremal graphs can be changed into Turán graphs by adding and removing $o(n^2)$ edges. If the minimum chromatic number is 2, i.e. there is a bipartite excluded graph in $\mathcal{L}$, then the extremal number is $o(n^2)$, and the above assertions are not too informative. This case will be called the theory of degenerate extremal problems. A large part of extremal graph theory is related to this question and some other part is to analyze how one can reduce complicated non-degenerate extremal graph problems to degenerate ones.

Degenerate extremal graph problems

Here we list just the most important bipartite graphs for which the corresponding extremal graph problems were investigated and completely or partly solved. The $K_2(p,q)$, C_{2k}, the cube C_8. Lower bounds were given using finite geometric constructions or other (even more) algebraic constructions.

The product conjecture

I will restrict myself to the simplest case. I conjecture that if the so called decomposition of an excluded L of chromatic number $p+1$ does not contain trees or forests, then there are always extremal graphs which can be obtained from a complete multipartite graph (of roughly equal classes) by only adding edges, never deleting. The condition on the decomposition can be rephrased: one cannot color L in $p+1$ colors so that the first two classes span a tree or a forest.

This conjecture means that in these cases the extremal graph problem can be reduced to degenerate ones.

Topological Minors and Girth
Wolfgang Mader (Hannover)

C. Thomassen noticed in 1983 that large girth has for the existence of minors the same effect as large degree. We study the same for topological minors. So, for instance, we show that for every graph H with maximum degree $\Delta(H) \geq 3$, there is a t_H such that every graph G with minimum degree $\delta(G) \geq \Delta(H)$ and girth $\tau(G) \geq t_H$ contains a subdivision of H. This can be generalized in the following main result. For every graph H with $\Delta(H) \geq 3$, $lim_{t\to\infty} f_t(H) = (\Delta(H)-1)/2$ holds, where $f_t(H){:=}inf\{c > 0 : ||G|| \geq c, |G| > 0$ and $\tau(G) \geq t \Rightarrow G$ contains a subdivision of $H\}$.

Furthermore, we show that every $2n$-connected graph of sufficiently large girth contains a subdivision with prescribed branch vertices of every graph H with $||H|| \leq n$, but without isolated vertices.

Tree-width
Bruce Reed (CNRS)

We discuss tree-width, a connectivity invariant of graphs defined by Robertson and Seymour. We present a duality result and a canonical decomposition theorem tied to this invariant. We also discuss a number of applications of these results, including Robertson and Seymour's Graph Minors Project.

On the Hanna Neumann Conjecture
Gábor Tardos (Rònni Institute of Mathematics, Budapest)

The history of this problem of combinatorial group theory is presented from the fifties up to the recent equivalent graph theoretic formulations. The H.N. conjecture states that if two subgroups U and V of a free group have ranks $k+1$ and $l+1$ then the rank of $U \cap V$ is at most $kl+1$. A bound of $2kl+1$ was proved by Hanna Neumann herself, and most of this factor 2 gap is still present between the best upper bound ($2kl-2k-2l+5$ of Dicks and Formanek) and the conjectured bound.

The following two graph theoretical conjectures are equivalent to the H.N. conjecture and a stronger form of it (due to Walter Neumann):

1. Conjecture: Let G,H be directed finite graphs, edges colored red/green such that for every vertex the in/out degree in either color is at most 1. Let $G \times H$ be the graph on the product of vertices with a red/green edge put in whenever the two projections have a red/green edge. Let K be a component of $G \times H$. Then, $c(K) - 1 \leq (c(G) - 1)(c(H) - 1)$ where c is the cyclomatic number.

2. Conjecture (due to Warren Dicks): Let G_1, G_2, G_3 be simple subgraphs of a finite bipartite graph. Suppose $G_1 \cap G_2 = G_2 \cap G_3 = G_1 \cap G_3 = G$. Consider the disjoint union of $G_1 \cup G_2$, $G_2 \cup G_3$, and $G_1 \cup G_3$ and suppose it has an even number of components and they can be paired such that the pairs are isomorphic (as bipartite graphs, i.e. "left" vertices map to "left" vertices, "right" vertices to "right" vertices). Dick's Conjecture: Then G has at most half as many edges as the complete bipartite graph on the same set of vertices. Note: G_i can have vertices and edges outside G. If G_i does not have edges outside G that have both endpoints in G (i.e. G is a spanned subgraph of G_i) then the conjecture is proved.

Parity and Connectivity
András Frank (Eőtuős University, Budapest)

Connectivity (paths,trees,flows) and parity (matchings) are two big branches of graph theory, with many similar results. In an attempt to understand better their features in common, we study problems concerning *both* connectivity and parity. For example, L. Nebesley characterized those undirected graphs which have an orientation so that the in-degree of every node is odd and so that every node is reachable by a directed path from a specified root-node. Our main open problem is finding a characterization of graphs having a strongly connected orientation so that the in-degree of every node is odd. What we have is the following. We call a subset $T \subseteq V$ of nodes of an undirected graph $G = (V, E)$ *compliant* if $|T| \equiv |E| \pmod 2$. An orientation of G is called T-*odd* if the in-degree of $v \in V$ is odd for $v \in T$ and even for $v \in V - T$.

Thm: For a graph $G = (V, E)$ the following are equivalent.

1. G has a k-edge-connected, T-odd orientation for every compliant subset $T \subseteq V$.

2. For every partition P of V into at least two non-empty parts, the number $i(P)$ of edges connecting distinct parts of P is at least $(k + 1)t - 1$.

3. G can be built from a node by a sequence of the following operations:(i) add a new edge connecting existing nodes (it may be a loop) (ii) choose a set F of k (distinct) edges, subdivide each element of F by a new node, identify the k new nodes into one, denoted by v, and connect v with an existing node.

The proof of implications (1) $\Rightarrow$ (2) and (3) $\Rightarrow$ (1) is rather straightforward. The difficulty lies in proving (2) $\Rightarrow$ (3). By using an old result (from 1980), we first prove that a graph has a (k, l)-edge-connected orientation iff $i(P) \geq k|P| - k + l$ for every partition P of V. Here, $0 \leq l \leq k$, and a digraph is called (k, l)-*edge-connected* if it has a node s so that there are k edge-disjoint paths from s to v and l edge-disjoint paths from v to s for every $v \in V$. Second, we provide a constructive characterization of $(k, k - 1)$-edge-connected digraphs. Another recent result of similar vein:

Thm: An undirected graph $G = (V, E)$ contains k edge-disjoint spanning trees after removing any of its edges if and only if G can be built from a node by the following two operations:

1. add a new edge

2. choose j existing edges ($1 \leq j \leq k - 1$), subdivide each, identify the j subdividing nodes with a new node z and add $k - j$ new edges incident to z.

Directed Tree-Width
Robin Thomas (Georgia Institute of Technology)

The notion of tree-width for undirected graphs has found application in both theory and practice. For instance, it is fundamental in the Graph Minors series of Robertson and Seymour. Also, many problems which are NP-complete in general (coloring, Hamilton cycle, etc.) may be solved in polynomial (often linear) time on graphs of bounded tree-width. Finally, besides giving theoretically fast algorithms, tree-width has been used in real-world programs that are applied in industry. With these benefits in mind, it seems useful to have a notion of tree-width for directed graphs.

Finding an appropriate definition has required some work. One explaination for this difficulty comes in the form of a game. For undirected graphs, there is a certain cops and robbers game which is related to tree-width. It turns out that there is either a particularly nice strategy for the cops to catch the robber or a nice strategy for the robber to elude the cops. However, for directed graphs and the corresponding game, this is no longer true. In particular, there are digraphs in which the cops can win but do not have a monotonic search strategy. (They must revisit some vertices they previously occupied and left.)

Despite these difficulties, we have formulated a definition of tree-width that seems correct. We have the following justifications for this claim:

1. Our definition is monotone under taking directed minors (the so-called "butterfly minor")

2. The tree-width of an undirected graph G is the same as the tree-width of the digraph obtained from G by replacing every undirected edge by two oppositely directed edges

3. The tree-width of an Eulerian digraph is within a factor (depending on the maximum degree) of the tree-width of the underlying undirected graph

4. For undirected graphs, there is a notion of a haven, and a graph has small tree-width iff it does not have a large haven. (This is related to the cops and robbers game.) There is a natural generalization of havens to directed graphs, and we can show a digraph has small tree-width iff it has no large haven

5. Many NP-complete problems (directed Hamilton cycle, directed linkages) can be solved in polynomial time in digraphs of bounded tree-width.

E-Mail Addresses of The Participants

Thomas Andreae <andreae@math.uni-hamburg.de>
Stephan Brandt <brandt@math.fu-berlin.de>
Gérard Cornuéjols <gc0v+@andrew.cmu.edu>
Matt DeVos <matdevos@math.Princeton.EDU>
Reinhard Diestel <diestel@math.uni-hamburg.de>
András Frank <Frank@cs.elte.hu>
Zoltán Fűredi <z-furedi@math.uiuc.edu>
Jim Geelen <jfgeelen@jeeves.uwaterloo.ca>
Bert Gerards <bgerards@cwi.nl>
Luis Goddyn <goddyn@cs.sfu.ca>
Martin Grohe <Martin.Grohe@mathematik.uni-freiburg.de>
Bertrand Guenin <guenin@math.gatech.edu>
Petr Hliněný <hlineny@math.gatech.edu>
Hein Van der Holst <Hein.van.der.Holst@cwi.nl>
Andreas Huck <huck@math.uni-hannover.de>
Bill Jackson <w.jackson@gold.ac.uk>
Thor Johnson <carlj@math.Princeton.EDU>
Tibor Jordán <jordan@daimi.au.dk>
Heinz-Adolf Jung <jung@math.tu-berlin.de>
Andreĭ Kotlov <andrei.kotlov@cwi.nl>
Alexander Kostochka <sasha@math.nsc.ru>
Daniela Kűhn<osthus@informatik.hu-berlin.de>
Imre Leader <I.Leader@dpmms.cam.ac.uk>
Martin Loebl <loebl@kam.ms.mff.cuni.cz>
Wolfgang Mader <mader@math.uni-hannover.de>
Bill McCuaig <wmccuaig@netcom.ca>
Bojan Mohar <mohar@uni-lj.si>
Jarik Nešetřil <nesetril@kam.ms.mff.cuni.cz>
James Oxley <oxley@marais.math.lsu.edu>
Hans Prőmel <proemel@informatik.hu-berlin.de>
Dieter Rautenbach <rauten@math2.rwth-aachen.de>
Bruce Reed <reed@ecp6.jussieu.fr>
Bruce Richter <brichter@math.carleton.ca>
Neil Robertson <robertso@MATH.MPS.OHIO-STATE.EDU>
Alexander Schrijver <lex@cwi.nl>
Alex Scott <A.D.Scott@dpmms.cam.ac.uk>
Paul Seymour <pds@math.Princeton.EDU>
Miklos Simonovits <miki@math-inst.hu>
Vera Sos <h1014sos@ella.hu>
Angelika Steger <steger@informatik.tu-muenchen.de>
Michael Stiebitz <stiebitz@mathematik.tu-ilmenau.de>
Gábor Tardos <tardos@math-inst.hu>
Robin Thomas <thomas@math.gatech.edu>
Carsten Thomassen <C.Thomassen@mat.dtu.dk>
Eberhard Triesch <triesch@math2.rwth-aachen.de>
Dirk Vertigan <vertigan@marais.math.lsu.edu>
Dominic Welsh <dwelsh@maths.ox.ac.uk>
Geoff Whittle <Geoff.Whittle@vuw.ac.nz>
Anders Yeo <yeo@math.uvic.ca>
Xingxing Yu <yu@math.gatech.edu>
Raphael Yuster <raphy@macam98.ac.il>

Compiled by Thor Johnson

Postal Addresses of The Participants

Prof. Dr. Thomas Andreae
Mathematisches Seminar
Universität Hamburg
Bundesstr. 55
20146 Hamburg

Dr. Stephan Brandt
Institut für Mathematik I
Freie Universität Berlin
Arnimallee 2-6
14195 Berlin

Prof. Dr. Gerard Cornuejols
Grad. School of Industr. Administr.
Carnegie-Mellon University
Schenley Park
Pittsburgh , PA 15213
USA

Prof. Dr. Matthew J. DeVos
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton , NJ 08544-1000
USA

Prof. Dr. Reinhard Diestel
Mathematisches Seminar
Universität Hamburg
Bundesstr. 55
20146 Hamburg

Prof. Dr. Andras Frank
Department of Operation Research
Eötvös University
ELTE TTK
Muzeum krt. 6-8
H-1088 Budapest VIII

Prof. Dr. Zoltan Furedi
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall
1409 West Green Street
Urbana , IL 61801
USA

Prof. Dr. Jim Geelen
Dept. of Combinatorics and
Optimization
University of Waterloo
Waterloo, Ontario N2L 3G1
CANADA

Dr. A. Bert H.M. Gerards
CWI - Centrum voor Wiskunde en
Informatica
Postbus 94079
NL-1090 GB Amsterdam

Prof. Dr. Luis Goddyn
Dept. of Mathematics & Statistics
Simon Fraser University
Burnaby , B.C. V5A 1S6
CANADA

Martin Grohe
Institut für Math.Logik und
Grundlagen der Mathematik
Universität Freiburg
Eckerstr. 1
79104 Freiburg

Prof. Dr. Bertrand Guenin
School of Mathematics
Georgia Institute of Technology
Atlanta , GA 30332-0160
USA

Prof. Dr. Rudolf Halin
Kaiser-Friedrich-Str. 4
23879 Mölln

Prof. Dr. Petr Hlineny
School of Mathematics
Georgia Institute of Technology
Atlanta , GA 30332-0160
USA

Prof. Dr. Hein van der Holst
Dennenlaan 1
NL-6711 RA Ede (Gld)

Andreas Huck
Voltmerstr. 41
30165 Hannover

Prof. Dr. Bill Jackson
Dept. of Mathematical Sciences
Goldsmiths' College
New Cross
GB-London , SE14 6NW

Carl D. Thor Johnson
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton , NJ 08544-1000
USA

Prof. Dr. Tibor Jordan
BRICS, Dept. of Computer Sciences
University of Aarhus
Ny Munkegade, Building 540
DK-8000 Aarhus C

Prof. Dr. Heinz Adolf Jung
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin

Prof. Dr. Alexandr. K. Kostochka
Institute of Mathematics
Siberian Branch of the Academy of Sciences
Universitetskiy Prospect N4
630090 Novosibirsk
RUSSIA

Prof. Dr. Andrei Kotlov
Centrum voor Wiskunde en Informatica
Kruislaan 413
NL-1098 SJ Amsterdam

Daniela Kühn
Rutnikstr. 20
10367 Berlin

Prof. Dr. Imre Leader
Department of Mathematics
University College London
Gower Street
GB-London , WC1E 6BT

Prof. Dr. Martin Loebl
Facultad de Ciencias Fisicas y Matematicas, Universidad de Chile
Dept. de Ingeniera Matematica
Casilla 170/3-Correo 3
Santiago
CHILE

Prof. Dr. Wolfgang Mader
Institut für Mathematik
Universität Hannover
Postfach 6009
30060 Hannover

Prof. Dr. Bill McCuaig
5268 Eglinton St.
Burnaby , B.C. V5G 2B2
CANADA

Prof. Dr. Bojan Mohar
FMF - Matematika
University of Ljubljana
Jadranska 19
1111 Ljubljana
SLOVENIA

Prof. Dr. Jaroslav Nesetril
Department of Applied Mathematics
Charles University
Malostranske nam. 25
118 00 Praha 1
CZECH REPUBLIC

Prof. Dr. James Oxley
Dept. of Mathematics
Louisiana State University
Baton Rouge , LA 70803-4918
USA

Prof. Dr. Hans Jürgen Prömel
Institut für Informatik
Humboldt-Universität Berlin
10099 Berlin

Prof. Dr. Dieter Rautenbach
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen
52056 Aachen

Prof. Dr. Bruce Reed
U.P.R. 175
Equipe Combinatoire, Case 189
CNRS, Un. Pierre et Marie Curie
4, Place Jussieu
F-75252 Paris Cedex 05

Prof. Dr. Bruce Richter
Department of Mathematics and
Statistics
Carleton University
710 Dunton Tower
Ottawa , Ont. K1S 5B6
CANADA

Prof. Dr. Neil Robertson
Department of Mathematics
Ohio State University
231 West 18th Avenue
Columbus , OH 43210-1174
USA

Prof. Dr. Alexander Schrijver
CWI Amsterdam
Postbus 94079
NL-1090 GB Amsterdam

Prof. Dr. Alex Scott
Department of Mathematics
University College London
Gower Street
GB-London WC1E 6BT

Dr. Paul D. Seymour
Applied Mathematics Department
Princeton University
Fine Hall, Washington Road
Princeton , NJ 08544
USA

Prof. Dr. Miklos Simonovits
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15
H-1364 Budapest

Dr. Riste Skrekovski
FMF - Matematika
University of Ljubljana
Jadranska 19
1111 Ljubljana
SLOVENIA

Prof. Dr. Angelika Steger
Institut für Informatik
TU München
80290 München

Prof. Dr. Gabor Tardos
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15
H-1364 Budapest

Prof. Dr. Robin Thomas
School of Mathematics
Georgia Institute of Technology
Atlanta , GA 30332
USA

Prof. Dr. Carsten Thomassen
Matematisk Institut
Danmarks Tekniske Universitet
Bygning 303
DK-2800 Lyngby

Prof. Dr. Eberhard Triesch
Lehrstuhl II für Mathematik
RWTH Aachen
Wüllnerstr. 5-7
52062 Aachen

Prof. Dr. Dirk L. Vertigan
Dept. of Mathematics
Louisiana State University
Baton Rouge
Louisiana 70803-4918
USA

Prof. Dr. Dominic J. A. Welsh
Merton College
Oxford University
GB-Oxford OX1 4JD

Prof. Dr. Geoff Whittle
School of Mathematical & Computing Sciences
Victoria University of Wellington
PO Box 600
Wellington
NEW ZEALAND

Prof. Dr. Anders Yeo
Dept. of Mathematics
University of Victoria
P. O. Box 1700
Victoria , B. C. V8W 3P4
CANADA

Prof. Dr. Xingxing Yu
School of Mathematics
Georgia Institute of Technology
Atlanta , GA 30332-0160
USA

Dr. Raphael Yuster
Department of Mathematics
University of Haifa
ORANIM
36006 Tivon
ISRAEl