MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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"Mengenlehre"

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The meeting "Set Theory" was organized by Ronald Jensen (Berlin) and Menachem Magidor (Jerusalem). It belongs to the series of Set Theory meetings at Oberwolfach which take place roughly every other year.

The new developments presented in 28 talks brought several important results and refinements of the existing methods, as well as embarking on new methods. The discussion focused on the following, mutually interrelated topics:

1. Forcing
2. Inner model theory and fine structure
3. Descriptive set theory
4. Infinite games and determinancy
5. Infinitary combinatorics
6. Cardinal characteristics
7. Cardinal arithmetic and pcf theory
8. Large cardinals
9. Set theoretic topology
10. Applications of the above in other fields of mathematics

The presented results on forcing comprise both applications in the analysis of cardinal characteristics of the continuum as well as those in infinitary combinatorics and large cardinals. Among the former fall measuring the complexity of various so called adequate relations in terms of Turing reducibility to various canonical generic reals, analysing of sets of canonical generic reals in terms of Baire property and Lebesque measurability, results on automorphism groups of Boolean algebras or developing new forcing techniques such as multidimensional Mathias forcing. Here belongs also the work related to Borel conjecture on σ-ideals and their relationship to certain special ideals such as the ideal of meager sets, null sets and strong measure zero sets.

Among the latter are developing new techniques in connection with Woodin's $\mathbb{P}_{\max}$ forcing and their application in combinatorics of posets such as separating Knaster's conditions of different levels, isolating a special forcing-like principle for the class of semiproper ω_1-trees and work on indestrictibility of various large cardinals and universal indestructibility principle.

Various ideals, mainly over the real numbers, are often studied in the ZFC context or in ZFC plus some special axiom. One such example is the Open Coloring Axiom OCA – several its new consequences in connection with Borel liftings of automorphisms of quotients of $\mathcal{P}(\mathbb{N})$ over P-ideals were presented. Another result on P-ideals gives a ZFC characterization of those P-ideals which are Borel reducible to one of Frèchet's ideals. A related topic is the study of ideals of compact sets of reals in connection with cofinal G_δ-sets and Tukey reducibility to the ideal of the meager compact sets. Finally we mention here also some results on dichotomies concerning P-trees which are not sensitive to the status of the Continuum Hypothesis, but some of them are closely related to principles of considerable large cardinal strength.

The central question in the inner model theory is building the core model which reflects as much of the large cardinal structure of the universe as possible; such constructions using mere ZFC have been known under rather restrictve assumptions. The main result presented along these lines is a construction of such a model and a proof of so called weak covering lemma under much less restrictive assumption, namely that the iterations which arise in the comparison process are "almost linear" in a precise sense.

Applications of the inner model theory are related mainly to descriptive set theory, infinitary combinatorics and cardinal arithmetic. Here a new, generalized construction of fine structural inner models has been developed; these methods allow to give a uniform proof of the Moschovakis scales propagation theorem for any reasonable pointclass using purely inner model theory. Another construction yields a fine stuctural analysis of the model constructible from Martin-Solovay tree under appropriate determinancy hypothesis.

The results on the infinitary combinatorics brought a detailed analysis of extenders and the internal structure of fine structural models constructed relative to coherent extender sequences; this lead to a complete characterization of $\square$-like principles in such models. One more related topic is the theory of indiscernibles for the core model and reflection of large cardinal properties; the techniques from this area yiels several new results in connection with clubs of regular cardinals of the core model.

Infinitary combinatorics is also closely related cardinal arithmetic, pcf theory and forcing. Along these lines we mention an interesting improvement of Shelah's theorem on unboundedness of the power of the first fixpoint of the $\aleph$ function in the ZFC context. Another topic which falls in this area is a detailed analysis of various notions of stationarity and their relationship to classical combinatorial principles (e.g. the principle $\square$). Finally new results were obtained in the theory of partition relations on countable ordinals using purely combinatorial methods.

The theory of infinite games and determinancy is known to be related mainly to descriptive set theory and inner model theory, however two applications outside these areas were presented. One of them is related to Gower's work on separable Banach spaces – it proposes a reformulation of his notion of weakly Ramsey set in terms of infinite games, this is used to establish special properties of definable sets of block bases in such spaces. The other application is in set theoretic topology and yields a characterization of countable ordinals which are minimal winning lengths for certain canonical games associated with topological spaces.

Author of the report: Martin Zeman

ABSTRACTS

Uri Abraham, Ben Gurion University

Hausdorff gaps on a stationary set

This is a joint work with Saharon Shelah. For a stationary subset S of ω_1 an S-Hausdorff gap is a pair $\{(a_i \mid i \in \omega_1), (b_j \mid j \in \omega_1)\}$ of subsets of ω such that $a_i \subseteq^* a_j \subset b_j \subseteq^* b_i$ for $i < j$, and such that for all $\delta \in S$ (on a club) and $j \in \omega_1 \setminus \delta$, for every ω-sequence δ_k converging to δ, for every n, there is a K such that if $k \geq K$ then $a_k \setminus n \not\subset b_j$.$\omega_1$-Hausdorff is the usual notion of a Hausdorff gap. We investigate this notion and prove the consistency of: There is a stationary set S such that every gap is S-Hausdorff, and yet not every gap is Hausdorff.

Joan Bagaria, Universitat de Barcelona

Weakly Ramsey sets in Banach spaces

Given an infinite-dimensional, separable Banach space X with a Schauder basis we introduce a two players game $\partial_\sigma[Y]$ for a normalized block base and a block subspace Y of X. Player I starts playing a block vector $x_1^{(1)} \in Y$, player II responds by playing either a block vector $y_1 \in [x_1^{(1)}]$ or 0. If II plays a vector I goes on by playing $x_1^{(2)} \in Y$, otherwise I must play a vector $x_2^{(1)}$ with support above that of $x_1^{(1)}$ and so on. II wins iff she produces a sequence from σ. Using this notion of game we redefine Gowers' notion of weakly Ramsey set: σ is weakly Ramsey if for every $\Delta > 0$ there is a normalized block base Y such that either $[Y] \cap \sigma = \varnothing$ or II has a winning strategy for $\partial_{\sigma(\Delta)}[Y]$, where $\sigma(\Delta)$ is the set of all block subspaces Z of X such that $d(W, Z) \leq \Delta$ for some $W \in \sigma$. In a joint work with López-Abad we show that every analytic set of normalized block bases is weakly Ramsey and extend the result to $\mathbf{\Sigma}^1_2$-sets using a strong form of Martin's axiom. We further show that in the model obtained by Levy collapsing a Mahlo cardinal over $\mathbf{L}$, every projective set is weakly Ramsey and that the same conclusion follows from Projective Determinancy. On the negative side, we show that Martin's axiom implies the existence of non-weakly Ramsey sets and that in $\mathbf{L}$, there is a non-weakly Ramsey set which is Δ^1_2.

Tomek Bartoszynski, Boise State University

Borel conjecture

Let $\mathcal{J}$ be a σ-ideal of subsets of reals and NON($\mathcal{J}$) be the collection of all $X \subset \mathbb{R}$ such that $F''X \in \mathcal{J}$ for every continuous $F : X \to \mathbb{R}$). Let $\mathcal{N}$, $\mathcal{M}$ be the ideal of measure and meager sets respectively and $\mathcal{SN}$ be the ideal of all X satisfying

$$(\forall g \in {}^\omega\omega)(\exists f \in {}^\omega({}^{<\omega}\omega))[(\forall n)(f(n) \in 2^{g(n)}) \ \&\ (\forall x \in X)(\exists^\infty n)(x \upharpoonright g(n) = f(n))].$$

We show (jointly with Shelah):

1) ZFC $\vdash \mathrm{NON}(\mathcal{M}) \neq [\mathbb{R}]^{\leq\omega}$.

2) It is consistent that $\mathrm{NON}(\mathcal{N}) = [\mathbb{R}]^{\leq\omega}$.

3) It is consistent that for a σ-ideal $\mathcal{J}$, $\mathrm{NON}(\mathcal{J}) = [\mathbb{R}]^{\leq\omega}$ iff $\mathrm{non}(\mathcal{J}) < 2^\omega$, where $\mathrm{non}(\mathcal{J})$ is the minimal size of a set which is not in $\mathcal{J}$.

4) It is consistent that for every σ-ideal $\mathcal{J}$, $\mathrm{NON}(\mathcal{J}) \subset \mathrm{NON}(\mathcal{SN})$ if $\mathrm{non}(\mathcal{J}) < 2^\omega$.

5) If $b = \omega_1$ then there exists an uncountable set $X \subset \mathbb{R}$ such that $X + F$ is meager for $F \in \mathcal{M}$.

6) If $d = \omega_1$ then there is an uncountable set $X \subset \mathbb{R}$ such that $X + G$ is null for $G \in \mathcal{N}$.

Andreas Blass, University of Michigan

Recursive aspects of cardinal characteristics

For sets A_-, A_+ and a relation $A \subset A_- \times A_+$ (usually Borel in $\mathbb{R}$ call a set $X \subset A$ *adequate* if $(\forall y \in A)(\exists x \in X) yAx$. The associated cardinal characteristics $||(A_-, A_+, A)||$ is the smallest cardinality of an adequate set. To measure not only the cardinality but the complexity needed for adequate sets, we define a real a to be *needed* for (A_-, A_+, A) if for every adequate X there is an $x \in X$ such that $a \leq_T x$ (where $\leq_T$ means Turing reducibility). That a is needed for a cardinal means it is needed for the natural relation defining that cardinal. Only the recursive reals are needed for the splitting number, additivity, covering, bounding number and uniformity characterizations of measure and category. A real is needed for the dominating number iff it is hyperaritmetic. The same goes through for the cofinality of category and we conjecture the same for measure (the "if" direction of the conjecture is proved). Any real needed for the unsplitting number is hyperaritmetic, but the converse is not known. Several of the proofs depend on knowing which ground model reals are recursive in some or all generic reals of various sorts. Results of this sort include:

	Cohen	random	Hechler	Sacks	Miller	Laver	Mathias
some	R	R	H	A	A	A	A
all	R	R	H	R	R	H	H

Here R,H,A means recursive, hyperaritmetic, arbitrary, respectively.

Jörg Brendle, Kobe University

How small can the set of generics be?

We investigate the "size" of the set of generics over M with respect to either Cohen or random forcing in some model $N \supset M$. Judah-Shelah proved that adding first a random and then a Cohen real over M gives a model with $\mathrm{Ra}(M)$ (= random reals over M) of measure zero. We show that, in fact, the following holds: If $M \subset N$ are models of ZFC then in $N[c]$, $\mathrm{Ra}(M)$ is always measurable, and $\mu(\mathrm{Ra}(M)) = 1$ iff $\mu(\mathrm{Ra}(M)) = 1$ in N (otherwise $\mu(\mathrm{Ra}(M)) = 0$). A result similar to the Judah-Shelah result cannot be obtained for Cohen forcing: If $\mathrm{Co}(M) \neq \varnothing$ then $\mathrm{Co}(M)$ is non-meager (here $\mathrm{Co}(M)$ is the set of Cohen reals over M.)

James Cummings, Carnegie Mellon University

Changing cofinalities

A sequence $\langle S_n; 1 \leq n < \omega \rangle$ is said to be *mutually stationary* iff

$$\{N \prec H_\theta;\ (\forall n)(\sup(N \cap \aleph_n) \in S_n)\}$$

is stationary for some (equivalently: any) large θ. N is *tight* iff $N \cap \prod \aleph_n$ is cofinal in the product $\prod(\aleph_n \cap N)$ and $\langle S_n; 1 \leq n < \omega \rangle$ is *tightly stationary* iff

$$\{N \prec H_\theta;\ N \text{ is tight } \ \&\ (\forall n)(\sup(N \cap \aleph_n) \in S_n)\}$$

is stationary for some (any) large θ. We show that for $1 \leq k < \omega$ it is consistent that there exists $\langle S_n; 1 \leq n < \omega \rangle$ such that

a) $S_n \subset \aleph_n \cap \mathrm{cof}(\aleph_k)$ for $n > k$,
b) $\langle S_n; 1 \leq n < \omega \rangle$ is mutually stationary,
c) $\langle S_n; 1 \leq n < \omega \rangle$ is not tightly stationary.

$\Box_{\kappa,\omega}$ is the weak square principle with $\leq \omega$ clubs at each level. In a joint result with Schimmerling we show that $\Box_{\kappa,\omega}$ holds in any Prikry extension. This can be shown to apply in more general situations.

Ilijas Farah, City University of New York, CSI

Open Coloring Axiom

OCA is a Ramsey-type axiom with a strong effect on quotients over Polish spaces. For example, it implies that the structure of $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$ (i.e., the power-set of the integers, taken modulo its ideal of finite sets) is very canonical. Under OCA, all of its automorphisms have Borel liftings, and the gap-spectrum is the simplest possible (both of these two properties may fail under different set-theoretic assumptions, such as the Continuum Hypothesis). We show that all automorphisms of quotients over other P-ideals also have Borel liftings but that some new phenomena occur in their gap-structure. Namely, there exist "new" Hausdorff gaps in quotients over certain analytic P-ideals. The latter result can also be stated as: The Lebesgue decomposition theorem for measures fails for lower semicontinuous submeasures on $\mathbb{N}$. This result does not assume OCA or any other additional axioms.

Qi Feng, Academia Sinica/Beijing

A cofinal branch principle

Consider a rather classical question: which trees of height ω_1 have a cofinal branch? We propose a cofinal branch principle which actually maximizes such an answer:

CBP: Every semiproper tree of height ω_1 has a cofinal branch.

(All trees are assumed to be normal.) CBP is an immediate consequence of SPFA, Semiproper Forcing Axiom. It has strong consequences, like e.g. Strong Reflection Principle of Todorčević, Souslin Hypothesis or $\mathrm{MA}^+(\sigma\text{-closed})$.

The basic analysis comes from a characterization of (ω,∞)-distributive posets: $\mathbb{P}$ is (ω,∞)-distributive iff for every $p \in \mathbb{P}$, the set

$$S_p = \{N \in [H_\kappa]^\omega;\ \mathbb{P}, p \in N\ \&\ (\exists q \le p)(q \text{ is a strong master condition for } N)\}$$

is stationary in $[H_\kappa]^\omega$. Call $\mathbb{P}$ strongly Baire if for every $p \in \mathbb{P}$, S_p is projective stationary, i.e. for any stationary $T \subset \omega_1$, the set $S_p^T = \{N \in S_p;\ N \cap \omega_1 \in T\}$ is stationary. Then $\mathbb{P}$ is strongly Baire iff $\mathbb{P}$ is (ω,∞)-distributive and $\mathbb{P}$ is "almost semiproper", where the latter means "$\mathbb{P}$ preserves stationary subsets of ω_1".

We conclude that CBP is equivalent to any of the following two statements:

- Every strongly Baire tree of height ω_1 has a cofinal branch.
- Every almost semiproper tree of height ω_1 has a cofinal branch.

Moti Gitik, Tel Aviv University

No bound for the power of the first fixpoint

If δ is singular and such that $\aleph_\delta > \delta$ is strong limit, then by Galvin-Hajnal and Shelah, $2^{\aleph_\delta} < \min(\aleph_{(2^{|\delta|})^+}, \aleph_{|\delta|^{+4}})$. The question addressed – is there a bound for the power of the least fixpoint, i.e. the least δ such that $\aleph_\delta = \delta$? We show that for an $\aleph_0$-cofinal singular cardinal κ satisfying $\{\alpha < \kappa;\ o(\alpha) \ge \alpha^{+\tau}\}$ for every $\tau < \kappa$ and any λ there is a forcing extension such that:

a) κ is the least repeat point,
b) GCH holds below κ,
c) all the cardinals above κ are preserved,
d) $2^\kappa \ge \lambda$.

This improves a previous result of Shelah that the power set of the first fixpoint or order omega is unbounded below an inaccessible. The assumptions used are optimal in the GCH situation. Without GCH below κ probably "$(\forall n)(\{\alpha < \kappa;\ o(\alpha) \ge \alpha^{+n}\})$ is unbounded" may be sufficient.

Joel David Hamkins, City University of New York
Universal indestructibility

The well-known Laver preparation makes any supercompact cardinal κ indestructible by $(<\kappa)$-directed closed forcing. By our recent work on gap forcing, no measurable and partially supercompact cardinal below κ becomes indestructible after Laver preparation. This leads to question the possibility of the *universal indestructibility principle:* Every measurable and partially supercompact cardinal γ is indestructible by $(<\gamma)$-directed closed forcing. The main result (jointly with A. Apter) is that the existence of a high-jump cardinal guarantees the existence of a transitive ZFC model with a supercompact cardinal in which universal indestructibility holds. The method of proof is trial-by-fire. At stage γ in a reverse Easton iteration, one destroys as much of the supercompactness of γ as possible with $(<\gamma)$-directed closed forcing. Any amount of supercompactness that survives is indestructible by further $(<\gamma)$-directed closed forcing. The large cardinal assumption is used to show that in fact something does survive the iteration. Modifications produce models of a supercompact cardinal in which every supercompact, partially supercompact, measurable, Ramsey, weakly compact and Mahlo cardinal is indestructible. Univesal indestructibility is inconsistent with the existence of two supercompact cardinals.

Kai Hauser, Humboldt Universität zu Berlin
Toward a Fine Structural Martin Solovay Tree

About thirty years ago Martin and Solovay introduced a tree construction which has turned out to be a basic tool in the analysis of the third level of the projective hierarchy. The smallest model of set theory containing the Martin-Solovay tree T_2 is also of intrinsic interest. (It consists of all sets constructible from T_2 and is denoted by $\mathbf{L}[T_2]$.) Despite its concrete definition, the combinatorial structure of $\mathbf{L}[T_2]$ remained a mystery. Similiarly, there was no descriptive set theoretic identification of its real numbers. Assuming all games with pay-off set in the smallest model of set theory containing the real numbers are determined, the following can be shown:

1) $\mathbf{L}[T_2]$ is of the form $\mathbf{L}[E^\sharp]$ where $\mathbf{L}[E]$ is the limit of a directed system of fine structure models in which $\aleph_\omega = lh(E)$ is a Woodin cardinal.

2) The reals constructible from T_2 are closed under the $(\mathbf{L}^{y_0})^\sharp$ operation.

3) In $\mathbf{L}[T_2]$, the first uncountable cardinal of V is measurable.

The proof of 3) uses generic elementary embeddings of $\mathbf{L}(\mathbb{R})$. These lead to various strengthenings of 1) and 2). For example:

4) $\mathbf{L}[T_2]$ is closed under the $(\mathbf{L}^{y_0})^\sharp$ operation up to $\aleph_\omega$ in the codes.

The above results form part of a larger program with the aim of achieving a complete synthesis of the theory of large cardinal axioms and their canonical models with methods from descriptive set theory. In the present context, the hope is to re-organize $\mathbf{L}[T_2]$ as a fine structure model. This would clarify its internal theory and show in particular that it satisfies the generalized continuum hypothesis.

Stephen Jackson, North Texas State University
Some remarks on determinancy and p.c.f.

We investigate the relationship between the cardinal structure of V and that of $L(\mathbb{R})$ by combining the determinacy theory of $L(\mathbb{R})$, Shelah's p.c.f. theory, and Woodin's theory of the non-stationary ideal on ω_1. One reult is:

Theorem *Assume the non-stationary ideal on ω_1 is ω_2 saturated + large cardinals (ω Woodin cardials + a measureable). Then either a) there is an ordinal κ such that $L(\mathbb{R}) \models \kappa$*

is regular and $\kappa < \aleph_{\aleph_2}$ but $V \models \kappa$ is not a cardinal or b) there is a set A of regular cardinals of size $\aleph_1$ with $|p.c.f.(A)| > |A|$.

The consequence of Woodin's theory that is used is that, under these hypotheses, every set $B \subseteq \lambda < \Theta^{L(\mathbb{R})}$ of size $\aleph_1$ can be covered by a set in $L(\mathbb{R})$ of size $\aleph_1$.

Vladimir Kanovei, MCCME Moscow
On Ulam's partial representation problem

We present the following partial result: If I is a Borel P-ideal Borel reducible to one of Frèchet's ideals Fin^{α} for $\alpha < \omega_1$, then I is either Fin or Fin $\times$ 0.

Menachem Kojman, Ben Gurion University
Collapse algebras

It is proved in ZFC that if μ is a singular cardinal of countable cofinality, then forcing with $\mathcal{P}_{\mu}(\mu)$, the quotient of $\mathcal{P}(\mu)$ modulo the ideal of subsets of μ of cardinality $< \mu$, collapses $\mu^{\aleph_0}$ to $\aleph_1$. This settles affirmatively a conjecture of Balcar and Simon.

Let $\mathrm{Col}\,(\omega_1, \mu^{\aleph_0})$ be the standard forcing for collapsing $\mu^{\aleph_0}$ to ω_1 (countable functions from ω_1 to $\mu^{\aleph_0}$). Then in fact $V^{\mathrm{Col}\,(\omega_1, \mu^{\aleph_0})}$ is always an intermediate universe between V and the generic extension $V^{\mathcal{P}_{\mu}(\mu)}$, obtained by forcing with closed conditions in the original forcing.

Jean A. Larson, University of Florida
Quadrilaterals

We show that $\omega^{\omega^{\beta}} \longrightarrow (\omega^{\omega^{\beta}}, 4)^2$ for all countable indecomposable limit ordinals β. Here $\alpha \longrightarrow (\alpha, 4)^2$ means that for any graph G with vertex set α, either there is an independent set $X \subset \alpha$ of order type α (independent means no pair from X is joined in G), or there is a complete quadrilateral (4 points, all joined in G). The proof is a modification of one developed by Rene Schipperus, who proved the corresponding result for triangles. However, his limit structures have been replaced by ladderly trees. This result is a part of a project called by Paul Erdös, who asked for a characterization of those ordinals $\alpha < \omega_1$ and those $m < \omega$, for which $\alpha \longrightarrow (\alpha, m)^2$. He was particularly interested in those α for which $\alpha \longrightarrow (\alpha, 3)^2$. The following two problems are open:

1) Does $\omega^{\omega^{3}} \longrightarrow (\omega^{\omega^{3}}, 3)^2$?
2) Is there $\alpha < \omega_1$ with $\alpha > \omega^{\omega}$ so that $\alpha \longrightarrow (\alpha, m)^2$ for all $m < \omega$?

Paul Larson, Kobe University
Chain conditions in maximal models

Given a partial order $(P, \leq)$, a subset $X \subset P$ is n-linked if for every $a \in [X]^n$ there is a $p \in P$ such that $p \leq q$ for every $q \in a$. Knaster's forcing axiom K_n is the statement that if $(P, \leq)$ is c.c.c. then every $X \in [P]^{\omega_1}$ contains an n-linked subset of size ω_1. Veličković and Todorčević conjectured that K_2 does not imply K_3. In a joint work with Todorčević we present two variations of Wodin's $\mathbb{P}_{\mathrm{max}}$ forcing which we hope to use to separate statements that fall in between Martin's Axiom and Souslin's hypothesis. In one of these, we preserve a particular coherent Souslin tree and, after obtaining every Π_2-sentence consistent with its existence, force with it. K_3 fails in the resulting model, and we have made some progress towards showing that K_2 holds there. In the other, we preserve an unfilled tower of subsets of omega at length ω_1. The full application of this variation is still awaiting the discovery of the right type of tower, however.

Alain Louveau and Boban Velickovic, Université Paris VI / Université Paris VII – C.N.R.S.
A note on Borel equivalence relations

Given two forcing notions $\mathcal{P}$ and $\mathcal{Q}$ let us say that $\mathcal{Q}$ is stronger than $\mathcal{P}$ and write $\mathcal{P} \leq \mathcal{Q}$ if forcing with $\mathcal{Q}$ produces a $\mathcal{P}$-generic filter over the ground model. If Σ is a class of forcing notions we say that Σ_0 is a *basis* for Σ if for every $\mathcal{Q} \in \Sigma$ there is $\mathcal{P} \in \Sigma_0$ with $\mathcal{P} \leq \mathcal{Q}$ (we exclude trivial, i.e. atomic forcings). In this talk I will describe the progress and current research on the problem of finding a basis for the class of all ccc forcing notions. Prikry conjectured that consistently Cohen and Random forcing form such a basis. We will show some partial results going in this direction.

Heike Mildenberger, Hebrew University
n-dimensional Mathias forcing

Let $D_0, \ldots, D_{n-1}$ be pairwise not Rudin-Keisler equivalent ultrafilters on ω. We consider the following notion of forcing $\mathbb{Q}[D]$. Conditions are pairs $(\omega, A) \in [\omega]^{<\omega} \times [\omega]^{\omega}$ such that: If $\langle k_j;\, j \in \omega\rangle$ is the increasing enumeration of A, then $A_i := \{k_j;\, j \equiv i \pmod n\} \in D_i$ for $i < n$. If $\langle l_j;\, j < m\rangle$ is the increasing enumeration of w, then $w_i := \{\ell_j;\, j \equiv i \pmod n\}$. A condition (v, B) is stronger than (w, A) (i.e. $(v, B) \leq (w, A)$) iff $(\forall i < n)(B_i \subset A_i \;\&\; v_i \trianglerighteq w_i \;\&\; v_i - w_i \in A_i)$. The following is a joint result with Shelah: If $r \in \mathbf{V}^{\mathbb{Q}(D)} \cap [\omega]^{\omega}$ and $U = \{X \in \mathbf{V} \cap [\omega]^{\omega};\, r \subset^* X\}$ is an ultrafilter on $[\omega]^{\omega} \cap \mathbf{V}$, then there is some $\ell < n$, $g \in \mathbf{V}$, $g : B \to \omega$ injective, $B \in D_\ell$ such that g witnesses that D_ℓ is Rudin-Keisler equivalent to U. This strengthens a recent result of Shelah and Spinas.

Bill Mitchell, University of Florida
Preserving cardinals and club sets

Let E be the class of inaccessible cardinals of the core model $\mathbf{K}$ and suppose that $\kappa \cap E$ contains a club for a cardinal $\kappa > \omega_1$. We analyze the relation between the size of κ in $\mathbf{V}$ and $\mathbf{K}$. The older results by Prikry, Gitik and Gitik-Mitchell show that if κ is a cardinal, inaccessible resp. measurable in $\mathbf{V}$ then in $\mathbf{K}$, $\mathrm{o}(\kappa) = 1$, $\{\gamma < \kappa;\, \mathrm{o}(\gamma) \geq \beta\}$ is stationary for every $\beta < \kappa$, resp. $\mathrm{o}(\kappa) = \kappa$.

For a sequence $\vec{B}$ with $B_\gamma \in U_\gamma = U(\gamma, 0)$ for all masurables γ let

$$D^{\vec{B}} = \{\lambda;\, (\forall\beta < \lambda)(\exists\gamma)(\mathrm{o}(\gamma) \geq \beta \;\&\; \lambda \in B_\gamma\} \cup \{\lambda;\, \mathrm{o}(\lambda) \geq \lambda\}.$$

Then for κ Mahlo, weakly compact resp. Ramsey we have $(\forall\vec{B})(D^{\vec{B}}$ is stationary), $(\forall\vec{B})(D^{\vec{B}}$ is in the weakly compact filter), resp. for all $f : [\kappa]^{<\omega} \to 2$ and all

$$\mathcal{B} : \text{Measurables} \times [\kappa]^{<\omega} \longrightarrow \text{Measure one sets}$$

there is a set I of indiscernibles for f such that $I \subset D^{\vec{B}}$ where $B_\gamma = \bigcup\{B(\gamma, \vec{I});\, i \in [I - \gamma]^{<\omega}\}$. It is open whether the following, apparently weaker condition is sufficient for Ramsey: “For all f and $\vec{B}$ there is a set $I \subset D^{\vec{B}}$ of indescernibles for f.”

Itay Neeman, Humboldt Universität zu Berlin/Harvard University
Mice and scales

We introduce a construction which, given a mouse $\mathcal{M}$ over a real x and a second real z, produces a mouse $\mathcal{M} \wr z$ over the real z. A *smallnes condition* is a Π_1-formula Ψ such that for every mouse $\mathcal{M}$ over a real x and for every $z \leq_T x$: $\mathcal{M} \wr z \models \Psi \;\longrightarrow\; \mathcal{M} \models \Psi$. $\mathcal{M}$ is Ψ-small iff $\mathcal{M} \models \Psi$. A smallness condition is *cofinal* if for every real x and every Ψ-small mouse $\mathcal{M}$ over x: given $\hat{x} \geq_T x$, there is a Ψ-small mouse $\hat{\mathcal{M}}$ over $\hat{x}$ such that $\mathcal{M} \lhd \hat{\mathcal{M}} \wr x$. For a smallness condition Ψ and a Σ_1-formula Θ we define a set of reals $A(\Psi, \Theta)$ by

$$A(\Psi, \Theta) = \{x \in \mathbb{R};\text{ there is a }\Psi\text{-small mouse }\mathcal{M}\text{ over }x\text{ such that }\mathcal{M} \models \Theta.\}$$

Let $\Gamma(\Psi)$ be the pointclass generated by the sets $A(\Psi, \Theta)$, where Θ is a Σ_1-formula. Our main result is that $\Gamma(\Psi)$ has the scale property whenever Ψ is a cofinal smallness condition. Applying this result varying smallness conditions one obtains all known scaled pointclasses via a single, uniform proof.

Ernest Schimmerling, Carnegie Mellon University
Analysis of extenders

Extenders are certain directed systems of filters and core models are universes constructed from sequences of extenders. Jensen style fine structure and its generalizations have been used to prove that some core models satisfy well-known combinatorial principles. In this direction, the optimal results on $\square_\tau$ are due to Schimmerling and Zeman. Some of the chief concerns that arise in the proofs can be described immediately after giving the definition of an extender, that is, without getting into too many of the technicalities of core models. Aspects of this analysis seem likely to be of interest outside of inner model theory.

Ralf-Dieter Schindler, Universität Wien
The core model for almost linear iterations

We introduce $0^{\P}$ ("zero handgrenade") as the least mouse $\mathcal{M}$ having a measurable κ such that $\mathcal{J}_\kappa^{\mathcal{M}} \models$ "There is a proper class of strong cardinals". Every normal iteration of a premouse below $0^{\P}$ is almost linear in a sense made precise. By this observation, $\mathbf{K}^c$, the preliminary version of the core model $\mathbf{K}$ we are heading towards, can be built using ω-completeness as the criterion for putting extenders on the sequence. The main result is the proof of weak covering for $\mathbf{K}^c$ (for every countably closed singular cardinal κ, $\kappa^{+\mathbf{K}^c} = \kappa^+$) which was the last bit missing for a proof of:

$$\text{ZFC} + \neg 0^{\P} \vdash \text{"}\mathbf{K}\text{ exists"}.$$

Sławomir Solecki, Indiana University
Analytic Ideals

We present several results on ideals of compact subsets concerning

1) cofinal G_δ subsets (which give a common generalization of theorems of Zafrany, Kechris-Louveau-Woodin, and Todorčević and the author)

2) Tukey reducibility to the ideal of meager compact sets (which show that under some mild assumptions, G_δ ideals of compact sets and their cofinal subsets are simpler, in a precise sense, than the ideal of meager compact sets and its cofinal sets).

Otmar Spinas, Universität Kiel
Canonizing Borel funtions on superperfect rectangles

We give an outline of the following result: Let $f : (\omega^\omega)^2 \to \omega^\omega$ be Borel measurable. Every superperfect rectangle contains a superperfect subrectangle where f is either constant or one-to-one in at least one coordinate. Here a rectangle is called superperfect if it has superperfect sides.

Juris Steprāns, York University
Automorphisms of $\mathcal{P}(\mathbb{N})/[\mathbb{N}]^{\aleph_0}$

An automorphism Φ of $\mathcal{P}(\mathbb{N})/[\mathbb{N}]^{\aleph_0}$ is said to be trivial if there are cofinite sets $A, B \subset \mathbb{N}$ such that for fome bijection $\pi : A \to B$ holds: For every $X \subset A$, $\Phi([X]) = [\pi"X]$. It is shown consistent that $\underline{d} = \aleph_1$, there is a nontrivial automorphism of $\mathcal{P}(\mathbb{N})/[\mathbb{N}]^{\aleph_0}$ yet the cardinality of the automorphism group of $\mathcal{P}(\mathbb{N})/[\mathbb{N}]^{\aleph_0}$ is $2^{\aleph_0}$. Question: Is it consistent that there is a nontrivial automorphism of $\mathcal{P}(\mathbb{N})/[\mathbb{N}]^{\aleph_0}$ such that the group generated by Φ and all trivial automorphisms is the group of all automorphisms?

Stevo Todorčević, C.N.R.S. Paris
Trees and reals

We analyze several dichotomies about trees which do not depend on the status of the Continuum Hypothesis. One of them is the following chain-antichain dichotomy:

($*$) Every P-tree either contains an uncountable antichain or it can be covered by countably many chains.

(A tree T is a P-tree if its orthogonal is σ-directed modulo the ideal of finite sets.) It is interesting that the dichotomy ($*$) without the assumption that the tree is a P-tree is a consequence of Souslin's hypothesis which is not sensitive on the ststus of CH, nor large cardinals. However, the restriction to the class of P-trees allows the principle ($*$) to be lifted to a different level, the class of statements with considerable large cardinal strength:

($**$) Every P-tree of the form $T = (\theta, <_T)$ where θ is an ordinal can either be covered by countably many chains or there is an uncountable closed set $C \subset \theta$ which can be covered by countably many antichains of T.

Jindřich ZAPLETAL, Darthmouth College
Transfinite open-point games

Given a topological space X, we consider a game of length α where I plays open sets O_n and II plays $x_n \in O_n$. I wins a play if $\{x_n; n < \alpha\}$ is dense in X. We give a partial answer to the question of Berner and Juhász, asking for a characterization of ordinals α which are minimal length for some space X in that we show that every such ordinal must be indecomposable and, on the other hand, that there is a model of ZFC in which every indecomposable countable ordinal is a minimal winning length for some X. It is open whether the latter is provable in ZFC. Our model uses essentially a sort of club-guessing on ω_1 which, in turn, uses a form of CH. The spaces are essentially refinements of Euclidean topology.

Martin Zeman, Universität Wien
Square

We give a characterization of the principle $\Box_\kappa$ in the fine structural model $\mathbf{L}[E]$ for a coherent extender sequence E. The extenders of E are represented in the Friedman-Jensen style, i.e. each extender is the corresponding ultrapower map restricted to the power set of its critical point. We show that in $\mathbf{L}[E]$, $\Box_\kappa$ holds iff there are nonstationarily many superstrong indices between κ and κ^+. The implication "if" is due to Schimmerling and Zeman (independently) and "only if" due to Jensen. The same sort of characterization holds for the weaker $\Box_{\kappa,<\kappa}$ (and thus also for $\Box_{\kappa,\lambda}$ where $\lambda < \kappa$). Moreover, we show that the principle global $\Box$ holds in $\mathbf{L}[E]$ at singular cardinals without any limitation. These results are proved abstractly with no direct reference to iterability, using only three of its consequences, namely condensation, solidity and universality of the standard parameter and Dodd solidity.

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