

Tagungsbericht 01 / 2000

Combinatorics

02.01. – 08.01.2000

The meeting was organized by László Lovász (Redmond) and Hans Jürgen Prömel (Berlin). The 38 talks that were delivered during the meeting covered many different aspects of combinatorics. In the following we include the abstracts in alphabetical order.

Maximal number of constant weight vertices of the unit n -cube contained in a k -dimensional subspace

Rudolf Ahlswede (Bielefeld)

We introduce and solve a seemingly basic geometrical extremal problem. For the set $E(n, w) = \{x^n \in \{0, 1\}^n : x^n \text{ has } w \text{ ones}\}$ of vertices of weight w in the unit cube of \mathbb{R}^n we determine $M(n, k, w) = \max\{|U_k^n \cap E(n, w)| : U_k^n \text{ is a } k\text{-dimensional subspace of } \mathbb{R}^n\}$.

The motivation for the study of the interplay between the properties “constant weight” and “linearity” comes from Information Theory (c.f. the problem stated Aug. 1993 in the Book of Problems).

Here is our complete solution.

Theorem. For $k, w, n \in \mathbb{N}$ with $k \leq n$ and $w \leq \frac{n}{2}$

(a) $M(n, k, w) = M(n, k, n - w)$

$$(b) \quad M(n, k, w) = \begin{cases} \binom{k}{w}, & \text{if (i) } 2w \leq k \\ \binom{2(k-w)}{k-w} 2^{2w-k}, & \text{if (ii) } k < 2w < 2(k-1) \\ 2^{k-1}, & \text{if (iii) } k-1 \leq w. \end{cases}$$

The sets giving these values are (up to isomorphies)

- (i) $\mathcal{S}_1 = E(k, w) \times \{0\}^{n-k}$
- (ii) $\mathcal{S}_2 = E(2(k-w), k-w) \times \{10, 01\}^{2w-k} \times \{0\}^{n-2w}$
- (iii) $\mathcal{S}_3 = \{10, 01\}^{k-1} \times \{1\}^{w-k+1} \times \{0\}^{n-k-w+1}$.

An auxiliary result of independent interest is the

Lemma. *Let $X = X_1 \dot{\cup} X_2 \dot{\cup} \dots \dot{\cup} X_s$ with $|X_i| = n_i$ for $i = 1, 2, \dots, s$ and let $\mathcal{A} = \{A \subset X : |A| \leq w\}$ be a family with the property that for any $A, B \in \mathcal{A}$ and $j = 1, 2, \dots, s$ the sets $A \cap \left(\bigcup_{i=1}^j X_i\right)$, $B \cap \bigcup_{i=1}^s X_i$ are incomparable, if they are different. Then $|\mathcal{A}| \leq \max_{\sum_{i=1}^s w_i \leq w} \prod_{i=1}^s \binom{n_i}{w_i}$ and this bound is best. (The case $(i = 1, w = n_1)$ is Sperner's well-known result.)*

An evaluation of this bound leads to the trichotomy in the Theorem.

We also present an extension of the results to multi-sets, explain an analogy to the (higher dimensional) Erdős–Moser problem and mention results and conjectures for subspaces over $\text{GF}(2)$.

This is joint work with H. Aydinian and L. Khachatryan.

Ramsey graphs

Noga Alon (Tel Aviv)

The problem of constructing explicitly Ramsey graphs, that is, graphs which contain neither large cliques nor large independent sets, received a considerable amount of attention. I will describe some recent and less recent constructions, mention several open problems and discuss some related information theoretic questions.

New 2-intersection sets in finite projective planes

Aart Blokhuis (Eindhoven)

A 2-intersection set, or an (m, n) -set, is a collection of points in a finite projective plane, such that every line contains either m or n points of the set. Classical examples are unions of disjoint Baer subplanes $(m, \sqrt{q} + m)$, unitals $(1, \sqrt{q} + 1)$, hyperovals $(0, 2)$ in planes of even order, or more generally Denniston arcs $(0, 2^h)$ in $PG(2, 2^n)$. In odd order planes all examples had $n - m = \sqrt{q}$ until Dover and Batten found a $(4, 9)$ -arc in $PG(2, 5^3)$. It consists of the points in an orbit of a subgroup of index 19 in the Singer cycle. In a very interesting paper C. White and B. Schmidt (“All two-weight irreducible cyclic codes”) found a (possibly complete) list of 2-intersection sets of this form including ones in $PG(2, 17^{11})$ and $PG(2, 41^{27})$ obtained by taking subgraphs of the Singer cycle of index 67 resp. 163.

Concerning $(m, \sqrt{q} + m)$ sets it was proved by Storme, Szőnyi and myself that any set of $m(q + \sqrt{q} + 1)$ points with at least m points on a line is the union of m Baer subplanes provided that $m < q^{1/6}$. An example of such a set (an $(m, \sqrt{q} + m)$ -set) of this size that is not the union of Baer was constructed by Michel Lavrauw and me for $m = q^{1/4} + 1$.

The Interlace Polynomial - a new graph polynomial

Belá Bollobás (Memphis and Cambridge)

We report on results from three papers, with Arratia, Coppersmith, and Sorkin; Arratia and Sorkin; Balister, Riordan and Scott.

The problem of DNA-sequencing by hybridization leads to problems concerning the number of Euler circuits in 2-in 2-out graphs defined by pairings. An n -pairing is a sequence of length $2n$ with n symbols, each symbol occurring twice, as in $ABBCAC$. An n -pairing S defines a 2-in 2-out graph $D(S)$ on the set of symbols, with the directed edges being the successive symbols of the pairing. An n -pairing S defines a graph $H(S)$ as well, the *interlace graph* of S : the vertices are the symbols and two symbols A and B are joined by an edge if they are interlaced in S , i.e. they occur as $\dots A \dots B \dots A \dots B \dots$. Among other results, we show that the number of Euler circuits of $D(S)$ is determined by the interlace graph $H(S)$.

More importantly, we define a pivot operation for every graph G and every edge $ab \in E(G)$. Set $V_1 = \{c \in V(G) \setminus \{a, b\} : ac, bc \in E(G)\}$, $V_2 = \{c \in V(G) \setminus \{a, b\} : ac \in E(G), bc \notin E(G)\}$, $V_3 = \{c \in V(G) \setminus \{a, b\} : ac \notin E(G), bc \in E(G)\}$, $V_4 = \{c \in V(G) \setminus \{a, b\} : ac, bc \notin E(G)\}$.

Now the pivot graph G^{ab} has vertex set $V(G)$, and $E(G^{ab})$ is obtained from $E(G)$ by toggling the edges between every two of the first three classes.

The main result, proved with Arratia and Sorkin, is that there is an *interlace polynomial* $q : \mathcal{G} \rightarrow \mathbb{Z}[X]$ such that

$$q(E_n) = X^n \quad \forall n \quad \text{and} \quad q(G) = q(G - a) + q(G^{ab} - b) \quad \forall ab \in E(G).$$

Rather little is known about $q(G)$, but we do know that $q(H(S))(1)$ is the number of Euler circuits of $D(S)$.

Counting Euler circuits also leads to an extremely simple proof of Bankwitz' theorem, a weak version of Tait's conjecture about alternating links, that an alternating link K with n crossings satisfies $|A_K(-1)| \geq n$, where $A_K(t)$ is the Alexander polynomial.

Random proper colorings of regular trees

Graham Brightwell (London)

We consider the space $Hom(T_r, K_q)$ of all proper q -colorings of the $(r + 1)$ -regular tree T_r ; we wish to study measures on this space with certain properties. A motivating example is the measure μ_{RW} generated as follows: color a root with one color, chosen

uniformly at random; then work out, at each stage coloring a site with some color chosen uniformly independently at random from those not used at the “parent”.

We wish our measure to be (a) a Gibbs measure, (b) simple - conditional on the color at one site of T_r , the colorings on different branches are independent, (c) invariant under the automorphism group of T_r , or at least “semi-invariant” – invariant under parity-preserving automorphisms.

This is related to the antiferromagnetic Potts model at zero temperature.

Our results include the following: (1) There is always a unique simple invariant Gibbs measure – namely μ_{RW} . (2) For $q < r + 1$, there is always more than one simple semi-invariant Gibbs measure. (3) For $q = r + 1$, there is only one simple semi-invariant Gibbs measure, but others appear if the “activities” of the colors are non-constant.

This is joint work with Peter Winkler.

Normal spanning trees, Aronszajn trees, and excluded minors

Reinhard Diestel (Hamburg)

We prove that a connected infinite graph has a normal spanning tree (the infinite analogue of a depth-first search tree) if and only if it has no minor obtained canonically from either an (\aleph_0, \aleph_1) -regular bipartite graph or an order-theoretic Aronszajn tree. This disproves Halin’s conjecture that only the first of these obstructions was needed to characterize the graphs with normal spanning trees. As a corollary we deduce Halin’s further conjecture that a connected graph has a normal spanning tree if and only if all its minors have countable colouring number.

The precise classification of the (\aleph_0, \aleph_1) -regular bipartite graphs remains an open problem. One such class turns out to contain obvious infinite minor-antichains, so as an unexpected corollary we reobtain Thomas’s result that the infinite graphs are not well-quasi-ordered as minors.

This is joint work with I. Leader.

Connectivity and supermodular functions

András Frank (Budapest)

We overview developments of the past decade concerning connectivity preserving and increasing of graphs. The main emphasis is on the use of supermodular functions. A starting point is Lovász’ splitting-off theorem asserting that, given an undirected graph $G = (V + s, E)$ with $d(s)$ even so that $\lambda(x, y) \geq k, \forall x, y \in V$, then the edges incident with s can be paired and split off so that the resulting graph is k -edge-connected. [$\lambda(x, y)$ denotes the minimum cardinality of a cut separating x and y .] This theorem is equivalent to a characterization of graphs G which may be extended to a k -edge-connected graph by adding a degree-specified graph to G . Lovász’ theorem has several extensions due to Mader, Bang-Jensen and Jackson, Benczúr and Frank, Jordan and Szégeti. A main

open question of the area is the optimal node connectivity augmentation problem: given a graph G add a minimum number of edges to G so as to obtain a k -node-connected graph. The problem is solved for $k = 1$ (trivial), $k = 2$ (Eswaran and Tarjan), $k = 3$ (Watanabe and Nakamura). Here we provide a solution for arbitrary k in the special case when G is the complement of a bipartite graph.

**Turán type problems in graphs, multigraphs, weighted graphs,
hypergraphs, etc.**

Zoltán Füredi (Urbana and Budapest)

Let $ex(n, F)$ be the maximum number of edges of an n -vertex graph can have without a subgraph isomorphic to F . For example (Turán 1941, Mantel 1907) $ex(n, K_3) = \lfloor n^2/4 \rfloor$. We discuss new developments, like $ex(n, K_{3,3}) = (\frac{1}{2} + o(1))n^{5/3}$ and generalizations for multigraphs, and weighted graphs (obtained together with André Kündgen). These density results lead to the solution of a problem of V. Sós, namely $ex_3(n, \text{Fano}) = (\frac{3}{4} + o(1))\binom{n}{3}$.

This means, that for some $c > 0$ if $\mathcal{F} \subseteq \binom{[n]}{3}$ is a triple system on n vertices, with $|\mathcal{F}| > \frac{3}{4}\binom{n}{3} + cn^2$, then it contains 7 triples forming a Fano plane, and on the other hand an example of Sós (the set of triples meeting both points of an $n/2 - n/2$ partition) shows that $ex_3(n, \text{Fano}) \geq \binom{n}{3} - \binom{\lfloor n/2 \rfloor}{3} - \binom{\lceil n/2 \rceil}{3}$. This bound is conjectured to be exact (for $n > n_0$). The above result is joint work with D. de Caen.

Branch width and well-quasi-ordering in matroids

Bert Gerards (Amsterdam and Eindhoven)

We prove that a class of matroids representable over a fixed finite field and with bounded branch width is well-quasi-ordered under taking minors. The result implies Robertson and Seymour's result that graphs with bounded tree width are well-quasi-ordered under taking minors.

This is joint work with Jim Geelen and Geoff Whittle.

Entropy, independent sets and antichains

Jeff Kahn (New Brunswick)

"Dedekind's Problem" of 1897 asks for the number, say $f(n)$, of antichains in the Boolean algebra of subsets of $[n]$.

In 1969 Kleitman showed that $\log(f(n))$ is asymptotic to the middle binomial coefficient (call it $b(n)$), and a 1975 improvement by Kleitman and Markowsky showed that the error term is not more than $O(\log n/n)b(n)$. Then Korshunov (1981) and later Sapozhenko (1989) determined the asymptotics of $f(n)$ itself.

Proofs of the preceding results range from difficult to impenetrable. Our main goal in this talk will be to sketch an entropy-based "book" proof of the Kleitman-Markowsky bound. What we actually prove is an exact bound for general graded partial orders, which, somewhat curiously, specializes to essentially K-M in the case of a Boolean algebra.

Time permitting, we will also say a little about the proof of a conjecture of Benjamini, Haggstrom and Mossel on the range of a "cube-indexed random walk."

How general is the Upper Bound Theorem (and some diversions)

Gil Kalai (Jerusalem)

The upper bound theorem (UBT) asserts that among all the d -polytopes with n vertices the cyclic polytopes $C(d, n)$ have the maximum number of k -faces for every k between 1 and $d - 1$.

This was conjectured by Motzkin and proved by McMullen. Stanley proved the UBT for all simplicial $(d - 1)$ -dimensional spheres. Novik proved the UBT even for large classes of $(d - 1)$ -dimensional simplicial manifolds.

The cyclic polytopes consist of the convex hull of n points on the moment curve (t, t^2, \dots, t^d) . As Vera Sós said, studying the cases of equality in an extremal problem is as important as proving the inequality. The cyclic polytopes are not the only examples for equality for the UBT. Equality holds for all neighborly polytopes. (Polytopes for which every $\lfloor d/2 \rfloor$ vertices determine a face.) There are many such polytopes and they are quite mysterious.

But the moment curve is (in even dimension) the only $\lfloor d/2 \rfloor$ -neighborly embedding of R^1 in R^d ; "only" in the sense of order types (or oriented matroids). So this was an opportunity to promote the study of order types of non-discrete subsets in R^d . In this direction I diverged to consider Perles question on k -dimensional r -neighborly manifolds in R^n , it is easy to see that $n \geq (k + 1)r$ and Vassiliev showed that n must be actually $\geq 2kr - d(r)$, where $d(r)$ is the number of "1"s in the binary expansion of r . Wigderson and I gave examples where n is polynomial in k and r .

With these diversions it is no wonder that I could not bring McMullen's proof but its especially simple presentation in terms of simple polytopes is available e.g. in Mulmuley's book on computational geometry. Nor did I even mention my proof with Noga Alon, the recent continuous extensions by Ulli Wagner and Welzl and other related things.

In the talk I explained Stanley's general approach in the language of Gröbner bases (or algebraic shifting). For every simplicial polytope P with n vertices (or a subcomplex of a simplicial polytope) one can associate a set of monomials $B(P)$ in the variables y_1, y_2, \dots . The number of k -faces of P can easily be read from the number of monomials of degree $k + 1$ of $B(P)$ and the crucial fact is that $B(P)$ is always a *subset* of the set of monomials associated to the cyclic polytopes.

This inclusion implies a very strong extension (that we refer to as GUBT) (of Kruskal-Katona type) which give an upper bound for the number of k -faces given the number of $(k - 1)$ -faces.

It is believed that the GUBT applies not only to simplicial spheres but also to simplicial manifolds with vanishing middle homology (in particular, to odd-dimensional simplicial manifolds). And even to a large class of pseudomanifolds with vanishing middle (intersection) homology (the so called Witt spaces). I also speculated that the UBT is a manifestation of the Van-Kampen theorem and is related to the impossibility to embed certain complete skeleta of simplices in the space in question. (So the nonplanarity of K_5 is the starting point for the UBT.)

Moreover, it is also conjectured that the GUBT applies to general polytopes (simplicial or not). This will show (using an argument of Anders Björner) that the face numbers of all d -polytopes are unimodal in the lower and upper quarters of the face numbers. (Namely, $f_0 \leq f_1 \leq \dots \leq f_{d/4}$.) (Gluing together a cyclic polytope and its dual shows that you cannot expect unimodality in the range between $d/4$ and $3d/4$.)

The GUBT for arbitrary polytopes will settle also the following conjecture of Imre Barany which (shamefully) we cannot answer: The number of k -faces of a d -polytope is at least the minimum of the numbers of vertices and facets.

On the largest member of a regular intersecting family

Gyula O.H. Katona (Budapest)

Let X be a finite set of n elements. If $F \subset X$ then e_F denotes the characteristic vector of F . A family $\mathcal{F} \subset 2^X$ is balanced if there are non-negative real numbers λ_F such that $\sum_{F \in \mathcal{F}} \lambda_F e_F = e_X$. A family is regular if every element of X is contained in the same number of members $F \in \mathcal{F}$. A regular family is always balanced. Suppose that any t members ($t \geq 2$) of a balanced family on X have at least k common elements. It is proved that the largest member of the family has at least $k^{1/t} n^{1-1/t}$ elements. The estimate is asymptotically sharp when t, k are fixed, n is large.

This is joint work with A. Idzik and R. Vohra.

Random matchings which induce Hamilton cycles, and Hamilton decompositions of random regular graphs

Jeong Han Kim (Redmond)

Select four perfect matchings of $2n$ vertices, independently at random. We find the asymptotic probability that each of the first and second matchings forms a Hamilton cycle with each of the third and fourth. This is generalized to embrace any fixed number of perfect matchings, where a prescribed set of pairs of matchings must each produce Hamilton cycles with suitable restrictions on the prescribed set of pairs. We also show how the result with four matchings implies that a random d -regular graph for fixed even $d \geq 4$ asymptotically almost surely decomposes into $d/2$ Hamilton cycles. This completes a general result on the edge-decomposition of a random regular graph into

regular spanning subgraphs of given degrees together with Hamilton cycles, and verifies conjectures of Janson and of Robinson and Wormald.

This is joint work with Nick Wormald.

Extremal set theory via information theory

János Körner (Rome)

It came as a surprise when L. Tolhuizen proved in a paper yet to appear in the IEEE Trans. Information Theory that the Frankl-Füredi asymptotic upper bound for the maximum size of cancellative set families is tight. The Tolhuizen paper has put a well-known problem from extremal set theory into a purely information-theoretic framework and, in a way, has underlined the relevance of information-theoretic methods in extremal combinatorics.

In a series of papers to appear in Combinatorics, Probability and Computing, N. Alon, E. Fachini, A. Monti and the author have studied several related problems, and in particular, the maximum size of locally thin set families. A family of subsets of an n -set is called k -locally thin if for every k distinct members of the family, at least one point of the ground set is contained in exactly one of them. We denote by $M(n, k)$ the maximum size of such a set family and define

$$t(k) = \limsup_{n \rightarrow \infty} \frac{1}{n} \log M(n, k)$$

Obviously, $M(n, 2) = 2n$ and $t(2) = 1$, but already the determination of $t(3)$ is an extremely difficult problem concerning strong Δ -systems and one doesn't even know whether $t(3) < 1$. It is tempting to conjecture that $t(k)$ is monotonically decreasing in k , but this is open. On the other hand, one easily sees that $t(k+l) \leq \max\{t(k), t(l)\}$. We can prove that $t(k)$ goes to 0 with increasing k . We prove that $0.26 < t(4) < 0.496$ and $0.19 < t(5) < 0.57$. As the best lower bounds are obtained by routine random choice, the interesting part of the results is the method used to obtain upper bounds. We interpret our problems as graph and hypergraph covering and then use the sub-additivity of graph entropy to bound the minimum number of graphs (hypergraphs) of a given form needed to cover a fixed graph (hypergraph).

On the equilateral dimension of the rectilinear space

Monique Laurent (Amsterdam)

It has been conjectured by Kusner (1983) that there exist at most $2k$ equidistant points in the k -dimensional rectilinear space. This conjecture has been verified for $k \leq 3$ (by Bandelt, Chepoi and Laurent, Discrete and Computational Geometry, 19 (1998)). We show here its validity in dimension $k = 4$; the proof is based on a reformulation of the problem in terms of set systems and a strengthening of the conjecture. We also discuss a number of related questions.

For instance, what is the maximum number $h(k)$ of equidistant points lying in the hyperplane: $\sum_{i=1}^k x_i = 0$? If this number would be equal to k , then the above conjecture would follow.

Other related parameters are $s(k)$, the maximum number of pairwise touching translates of the k -dimensional simplex, and $a(n)$, the maximum cardinality of an antichain in a design \mathcal{B} on n points; a multiset \mathcal{B} on $[1, n]$ being a design if there exist integers r, λ such that each point $i \in [1, n]$ belongs to exactly r members of \mathcal{B} and any two distinct points $i, j \in [1, n]$ belong to exactly λ common members of \mathcal{B} . The following relations hold:

$$h(k) = s(k - 1), \quad h(k) \geq n \iff a(n) \leq k.$$

Clearly, $a(n) \leq n$ and equality would imply the original conjecture. More precisely, if $a(n) \geq 2k$, then there are at most n equidistant points in the rectilinear k -space. However, it is known that $a(n) \leq n - 1$ for $n \geq 5$, $h(k) = s(k - 1) \geq k + 1$ for $k \geq 4$.

This is joint work with Jack Koolen and Lex Schrijver.

Sparse Deuber sets

Imre Leader (London)

We prove a conjecture of Deuber that for any m, p, c there exists a subset of the natural numbers such that the mpc -sets contained in it form a hypergraph of large girth and large chromatic number. This extends the sparse van der Waerden theorem of Prömel and Voigt and the sparse Rado theorem of Nešetřil and Rödl.

The main tools are amalgamation and a Ramsey theorem on products of trees that may be of independent interest.

On Heilbronn's problem in higher dimensions

Hanno Lefmann (Dortmund)

Heilbronn conjectured that given arbitrary n points from the 2-dimensional unit square, there must be three points which form a triangle of area at most $O(1/n^2)$. This conjecture was disproved using random arguments by Komlós, Pintz and Szemerédi. They showed that for every n there is a configuration of n points in the unit square where all triangles have area at least $\Omega(\log n/n^2)$. Considering a generalization of this problem to dimensions $d \geq 3$, recently Barequet proved the existence of n points in the d -dimensional unit-cube such that the minimum volume of any simplex spanned by $(d + 1)$ of these points is at least $\Omega(1/n^d)$. In this talk we showed how to improve on this lower bound by the logarithmic factor $\Theta(\log n)$.

Some questions and results about the girth

Nati Linial (Jerusalem)

The main question I have addressed is: How large can the girth of a graph be in a d -regular graph with n vertices - $g(d, n)$. The upper bound follows from a straightforward counting argument, whereas the best lower bound comes from the Ramanujan Graphs that were constructed by Margulis and Lubotzky-Phillips-Sarnak. Some attempts on both bounds were described: A recent spectral method (jointly with A. Amit and S. Hoory) that ties in the Markov Moment Problem. The concept of random lifts of graphs (developed jointly with A. Amit) seems like a promising tool with which to improve the lower bound on $g(d, n)$.

A graph theory of crystal structures

Martin Loebel (Prague)

We study enumeration of perfect matchings and edge-cuts of toroidal square grids and of $3d$ cubic lattices. We present some observations on how degeneracy of toroidal grids and cubic lattices depends on frustration of basic building blocks of these lattices. We also present a new expression for the enumeration of perfect matchings of the cubic lattice, which is also known as $3d$ dimer problem. We prove that it may be computed easily from average determinant of matrices associated with certain orientations of the $3d$ lattice.

Szemerédi's Regularity Lemma and Ramsey Theory

Tomasz Łuczak (Poznań)

We discuss two applications of Szemerédi's Regularity Lemma to Ramsey Theory. Thus, we show that for an odd n , the Ramsey number for a cycle C_n is given by

$$R(C_n, C_n, C_n) = (4 + o(1))n,$$

which settles in the affirmative a conjecture of Bondy and Erdős. Then we present a result of Haxell, Łuczak and Tingley, and prove that if T is a tree with bipartition (V_1, V_2) , where $|V_2| = t_2 \geq |V_1| = t_1$, and $\Delta(T) = o(t_2)$, then

$$R(T, T) = (1 + o(1)) \max\{2t_1 + t_2, 2t_2\}.$$

On the chromatic number of Kneser hypergraphs

Jiří Matoušek (Prague)

Let \mathcal{S} be a system of subsets of a finite set X . The *Kneser r -hypergraph* $\text{KG}_r(\mathcal{S})$ has \mathcal{S} as the vertex set, and an r -tuple (S_1, S_2, \dots, S_r) of sets in \mathcal{S} forms an edge if $S_i \cap S_j = \emptyset$ for all $i \neq j$. Kneser conjectured in 1955 that $\chi(\text{KG}_2(\binom{[n]}{k})) \geq n - 2k + 2$. This was proved in 1978 by Lovász as one of the earliest and most spectacular applications of topological methods in combinatorics. Further proofs and extensions were obtained e.g. by Bárány, Schrijver, Walker, Alon, Frankl, Lovász, Dolnikov, Sarkaria, and Kříž. Here we consider a lower bound for the chromatic number of $\text{KG}_r(\mathcal{S})$ for an arbitrary set system \mathcal{S} due to Kříž from 1992 (for $r = 2$, the result was obtained earlier by Dolnikov in 1988). We recall that a mapping $c : V \rightarrow [m]$ is a (*proper*) *coloring* of a hypergraph $\mathcal{H} = (V, E)$ if none of the edges $e \in E$ is monochromatic under c . The *chromatic number* $\chi(\mathcal{H})$ of \mathcal{H} is the smallest m such that a proper coloring $c : V \rightarrow [m]$ exists. We define the *r -colorability defect* $\text{cd}_r(\mathcal{H})$ as the minimum cardinality of $Y \subseteq X$ such that there is a partition $X \setminus Y = A_1 \cup \dots \cup A_r$ with no $e \in E$ satisfying $e \subseteq A_i$ for some $i = 1, 2, \dots, r$. The Kříž–Dolnikov theorem states

$$\chi(\text{KG}_r(\mathcal{S})) \geq \frac{1}{r-1} \cdot \text{cd}_r((X, \mathcal{S}))$$

for any set system (X, \mathcal{S}) and any $r \geq 2$.

We outline a proof of this result; the basic approach is similar to that of Kříž, but our proof is simpler and more accessible to non-specialists in topology.

Concentration for independent permutations

Colin McDiarmid (Oxford)

We discuss an extension of some concentration inequalities of Talagrand. This extension concerns both independent random variables and independent random permutations. It is particularly useful for analysing randomised methods for graph colouring.

Combinatorics of mappings (homomorphisms)

Jaroslav Nešetřil (Prague)

Given two graphs $G = (V, E), G' = (V', E')$ a homomorphism $f : G \rightarrow G'$ is any mapping $f : V \rightarrow V'$ which preserves the edges: $[x, y] \in E \Rightarrow [f(x), f(y)] \in E'$.

Homomorphisms generalize coloring problems ($\chi(G) \leq k$ iff $G \rightarrow K_k$) and form a category. On the other side they induce a quasiorder \leq on the class of all graphs: $G \leq H$ iff $G \rightarrow H$. The quasiorder \leq is universal.

Given a class \mathcal{K} of graphs we say that a pair (G, H) is a gap in \mathcal{K} if $G, H \in \mathcal{K}$, $G < H$ and there is no $F \in \mathcal{K}$ with $G < F < H$.

Welzl (1980) characterized all gaps for undirected graphs and Nešetřil, Tardif (1998) characterized all gaps for directed graphs and even all the gaps for general relational structures of a given type.

An H -coloring of a graph G is any homomorphism $G \rightarrow H$. We say that an H -coloring problem permits a Finitary Homomorphism Duality (FHD) if there are graphs F_1, \dots, F_t , $F_i \rightarrow H$, such that for any G either $G \rightarrow H$ or $F_i \rightarrow G$ for some i . Nešetřil and Tardif proved a 1-1 correspondence between gaps and FHD and characterized all FHD for classes of graphs and even relational systems of a given type. This and related results were surveyed in the lecture.

Combinatorics, probability and computation on finite groups

Igor Pak (New Haven)

I will give a somewhat biased review of recent results on theoretical and practical methods for generating random group elements. We will start by introducing the algorithms and discuss problems from various fields as they arise.

Recent Progress on the Generalized Baues Problem

Jörg Rambau (Berlin)

A polyhedral subdivision of a point configuration \mathcal{A} is a polyhedral complex that covers the convex hull of \mathcal{A} . (Polyhedral complex means that the intersection of any two cells is a face of each.)

Given a projection π of the n vertices of a d' -polytope P onto a point set \mathcal{A} in the d -dimensional Euclidean space, we call a polyhedral subdivision π -induced if all its cells are projections of faces of P under π .

The Generalized Baues Problem asks for which $n > d' > d$ the set of all π -induced polyhedral subdivisions of \mathcal{A} , partially ordered by refinement, has the homotopy type of a $(d' - d - 1)$ -sphere. (The homotopy type of a poset is given by the homotopy type of the simplicial complex of all chains.)

This problem is related to many other problems formulated before: flip connectivity of triangulations in computational geometry (a generalization of the concept that two-dimensional triangulations are connected by flipping diagonals); extension space conjecture in the theory of oriented matroids, counting roots in sparse polynomial systems, etc.

In this talk selected results in this area are presented, mounting in the most recent achievement by Santos showing that there is a triangulation of a six-dimensional point configuration without any flip. This result has been double checked by the software package TOPCOM, available at <http://www.zib.de/rambau/TOPCOM.html>.

An instance of Hilbert’s paradigm

Bruce Reed (Paris)

In 1900, Hilbert gave his famous address at the International Congress of Mathematicians and presented his list of 23 problems for the 20th century. We recall some of his remarks and present an example of a problem which appears to have influenced mathematics in the way he suggests a good problem should.

Minimizing a submodular function

Lex Schrijver (Amsterdam)

It was proved by Grötschel, Lovász, and Schrijver in 1981 that the minimum value of a submodular function can be determined in polynomial time. The algorithm is based on the ellipsoid method, and is therefore highly impractical. The question remained to find a combinatorial method. In 1999 two such algorithms were found, one by Iwata, Fleischer, and Fujishige, and one by the present author. In our lecture we present a description of our algorithm.

Quasi-random graphs

Miklós Simonovits (Budapest)

It is an important and interesting question, how the randomlike graphs “can be characterized”. Several works are centered around this question. We mention here only the papers of Wilson, Thomason, Chung-Graham-Wilson, . . .

Sós and Simonovits proved the strong connection between quasirandomness and Szemerédi’s Regularity Lemma. This gave rise to a new proof technique which made the whole theory more transparent. Recently we applied these methods to prove that certain counting properties which are originally not quasirandom properties, they turn into quasirandom ones if we extend them into hereditary properties: if we assume them not only for the whole graph G_n but all its sufficiently large subgraphs. (It is natural to extend the properties in question to hereditary properties since being a random graph is a hereditary property: all large subgraphs of random graphs behave in a randomlike manner.) The properties discussed here are “counting properties”: counting the number of certain subgraphs. When counting not necessarily induced subgraphs, “everything behaves” as it should, yet, counting *induced subgraphs* some strange counterexamples may occur.

This is joint work with Vera T. Sós.

Hypergraph Problems

Vera T. Sós (Budapest)

Let $t^r(n; L^r) = \max_{L^r \subseteq G_n^r} e(G_n^r)$ and let $RT^r(n; L^r, f(n))$ denote the maximum number of edges an r -uniform hypergraph G_n^r can have if $L^r \not\subseteq G_n^r$ and the stability number $\alpha(G_n^r) < f(n)$. Put $c_L = \lim_{n \rightarrow \infty} \frac{1}{n^r} t^r(n; L^r)$ and $c_L(f) = \lim_{n \rightarrow \infty} \frac{1}{n^r} RT^r(n; L^r, f(n))$.

There are several results for c_L and for $c_L(f)$ if $f(n) = o(n)$ in the case $r = 2$. (Turán resp., Ramsey-Turán theory.)

For $r > 2$ just a few exact results are known for c_L and even less for $c_L(f)$. In this lecture we *put emphasis on phenomena* which are very different for $r = 2$ and for $r > 2$. We also mention some open questions.

On the structure of $K_{\ell+1}$ -free graphs

Angelika Steger (München)

In 1976 Erdős, Kleitman and Rothschild showed that almost all triangle-free graphs are bipartite, i.e., that the cardinality of the two graph classes is asymptotically equal. In this talk we investigate the structure of the “few” remaining triangle-free graphs which are not bipartite. As it turns out, with high probability these graphs are bipartite up to a few vertices. More precisely, almost all of them can be made bipartite by removing just one vertex. Almost all others can be made bipartite by removing two vertices, and then three vertices, and so on.

We also show that similar results hold if we replace “triangle-free” by “ $K_{\ell+1}$ -free” and “bipartite” by “ ℓ -colorable”.

This is joint work with H. J. Prömel and T. Schickinger.

Blocking sets in projective planes and spaces

Tamás Szőnyi (Budapest)

A k -blocking set in $PG(n, q)$, $q = p^h$, is a set of points intersecting every $(n - k)$ -dimensional subspace. Extending previous results known for the case $n = 2$, with Zs. Weiner we proved that a 1-blocking set of size less than $\frac{3}{2}(q + 1)$ intersects every hyperplane in 1 modulo p points. Using geometric arguments it immediately implies the same result for 2-blocking sets. A corollary of this result is that 1-blocking sets in $PG(h, p^h)$ which are not contained in a hyperplane are precisely the subgeometries $PG(h, p)$.

On the evolution of triangle-free graphs

Anusch Taraz (Berlin)

In this talk we address the question whether a random triangle-free graph with n vertices and m edges is bipartite or not. Classical results from the theory of random graphs imply that the first threshold (from bipartite to non-bipartite) occurs when $m = n/2$. Prömel and Steger (1996) showed that almost surely such a random graph remains non-bipartite for $m = O(n^{3/2})$ and is bipartite again when $m = \Omega(n^{7/4} \log n)$.

We complete the picture by showing that the second threshold (from non-bipartite to bipartite) occurs when $m = \frac{\sqrt{3}}{4} n^{3/2} \sqrt{\log n}$. The idea of the proof is to partition the probability space before applying large deviation inequalities, and makes also use of a result recently obtained by Łuczak.

This is joint work with Deryk Osthus and Hans Jürgen Prömel.

Tutte's edge 3-coloring conjecture

Robin Thomas (Atlanta)

Tutte conjectured in 1966 that every 2-connected cubic graph with no minor isomorphic to the Petersen graph is edge 3-colorable. The conjecture implies the Four Color Theorem by a result of Tait.

In the first part of the lecture I will discuss related results and problems. In the second part I will outline a proof of Tutte's conjecture obtained in joint work with N. Robertson, D.P. Sanders and P.D. Seymour.

The extremal function for complete minors

Andrew Thomason (Cambridge)

Let $c(t)$ be the minimum number c such that every graph G with $e(G) \geq c|G|$ contracts to a complete graph K_t . Mader in 1968 showed that $c(t)$ exists, and Kostochka and I proved in the early '80s that $c(t)$ is of order $t\sqrt{\log t}$.

Here we show that

$$c(t) = (\alpha + o(1)) t \sqrt{\log t}$$

where $\alpha = 0.319\dots$ is an explicit constant. The extremal graphs are (more or less) disjoint unions of pseudorandom graphs of a certain size and density. No explicit extremal graphs are known.

Correlation, enumeration and sorting

William T. Trotter (Tempe)

The following theorem, stated in terms of conditional probability was first proved by Shepp [4] using the FKG inequality and a clever definition of a distributive lattice on a particular set of functions.

Theorem 1 *Let x, y and z be distinct points in a poset P with $\text{Prob}[x > z] > 0$. Then*

$$\text{Prob}[x > y] \leq \text{Prob}[x > y | x > z].$$

□

Theorem 1 is known as the XYZ-theorem; it asserts that the events $x < y$ and $y < z$ are positively correlated in a poset P . The result was originally posed by Rival and Sands who in fact conjectured that the inequality is strict when x, y and z form a 3-element antichain. This stronger result does not follow from Shepp's argument. Subsequently, P. Fishburn [2] used repeated applications of the Ahlswede/Daykin four functions theorem [1] to prove the "strong" version of the XYZ theorem.

Theorem 2 *Let $\{x, y, z\}$ be a 3-element antichain in a poset P . Then*

$$\text{Prob}[x > y] < \text{Prob}[x > y | x > z].$$

□

It is easy to see that Theorem 2 yields Shepp's version as a corollary. Of course, the FKG inequality is a special case of the Ahlswede/Daykin inequality, i.e., it is just the case when all four functions have unit weight. However, Fishburn's proof also required some complex variations in the definitions of auxiliary distributive lattices.

In this talk, we first outline a proof of the strong version of the XYZ theorem which avoids completely any use of the Ahlswede/Daykin theorem—or any of its derivative versions.

Let P be a finite poset with ground set X , and let $x \in X$ be a fixed element of P . Define a sequence h_1, h_2, \dots, h_n , where $n = |X|$ by

$$h_i = \{L \in E(P) : h_L(x) = i\}.$$

This sequence is called the *height sequence* of x .

The following theorem was originally proved by Stanley [5] using the Alexandrov/Fenchel inequalities for mixed volumes (in fact, a much stronger result is proved).

Theorem 3 *Let P be a poset with ground set X and set $n = |X|$. Then for each $x \in X$, the height sequence h_1, h_2, \dots, h_n of x is log-concave, i.e.,*

$$h_i h_{i+2} \leq h_{i+1}^2$$

for all $i = 1, 2, \dots, n - 2$. □

Stanley's proof of this result is both compact and elegant. However, the algebraic machinery obscures the structural properties of the poset, so for example, nothing is known about the following natural questions.

Question 4 *Under what circumstances is it true that the inequality $h_i h_{i+2} \leq h_{i+1}^2$ is tight?*

Question 5 *If the inequality $h_i h_{i+2} \leq h_{i+1}^2$ is strict, what is the minimum size ϵ_n of the error term?*

To date, we have only been able to make marginal progress in providing a combinatorial proof of Theorem 3. Specifically, we can settle the special case when the height sequence contains exactly 3 non-zero terms. Even for this case, the argument is quite complex and requires the full power of the Ahlswede/Daykin theorem *plus* some new ideas emanating from the combinatorial approach to the XYZ theorem. Also, we can prove the inequality when $i = 1$ or when $i + 2 = n$, i.e., when x is either a maximal point or a minimal point. In this case, we do not need to know that the height sequence has only three non-zero terms. However, the techniques used to prove these special cases do not seem likely to extend to a proof for the general case, and we consider this effort a major challenge. Our efforts to provide a combinatorial proof for Stanley's log-concavity result were originally motivated by a question posed to us by Jeff Kahn [3].

Conjecture 6 *Let $\mathbf{P} = (X, P)$ be a poset with $|X| = n$, and let x and y be distinct elements of X . If $|\{z \in X : z \leq x \text{ or } z \leq y\}| = m$, then*

$$\max\{h(x), h(y)\} \geq m - 1.$$

Kahn noted that if $n = m$, then the conjecture is true. This assertion follows easily from the log-concavity of the height sequence plus the fact that $\max\{\text{Prob}[x > y], \text{Prob}[y > x]\} \geq \frac{1}{2}$. However, when $n > m$, log-concavity seems to allow the maximum of the two heights to fall all the way down to $m \log 2$. Secondly, as Kahn also noted, the question may be generalized in a natural fashion to $k \geq 2$ points. In general it is natural to believe that the extremal value is provided by taking disjoint and pairwise incomparable chains of approximately equal length.

- [1] R. Ahlswede and D. E. Daykin, An inequality for the weights of two families of sets, their unions and intersections, *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete* **43** (1978), 183–185.

- [2] P. C. Fishburn, A correlational inequality for linear extensions of a poset, *Order* **1** (1984), 127–137.
- [3] J. Kahn, personal communication.
- [4] L. A. Shepp, The FKG inequality and some monotonicity properties of partial orders, *SIAM J. Alg. Disc. Meth.* **1** (1980), 295–299.
- [5] R. P. Stanley, Two applications of the Alexandrov/Fenchel inequalities, *J. Combinatorial Theory (A)* **31** (1981), 56-65.

Generalizations of Davenport-Schinzel Sequences, and their applications

Pavel Valtr (Prague)

We say that a sequence s contains a sequence n if s has a subsequence isomorphic to n (two sequences are isomorphic if one of them can be obtained from the other one by renaming the symbols, e.g. 11231, 22432 are isomorphic).

For a sequence n over $\{1, \dots, k\}$ and for $n \geq 1$, we define (n, n) -GDS sequence (generalized Davenport-Schinzel sequence) as any sequence over $\{1, \dots, n\}$ not containing n and having the property that any two occurrences of the same integer are at distance at least k from each other. The maximum length of such a sequence, $f_n(n)$, has been studied in several papers. Klazar showed that $f_n(n) = \mathcal{O}(n\beta(n))$, where $\beta(n) \rightarrow \infty$ very slowly. It is also known that $f_n(n) = \mathcal{O}(n)$ holds for some sequences, e.g. $n = abcdcbabcd$. As a consequence it can be shown that any geometric graph on n vertices with no k pairwise crossing edges has at most $c_k \cdot n \log n$ edges. It is an open problem whether or not the $\log n$ factor is needed. Another application is due to Alon and Friedgut. They have shown that the number of permutations from S_n avoiding a fixed permutation $\tau \in S_k$ is bounded by $c^{n\gamma^*(n)}$ where $\gamma^*(n)$ is an extremely slowly growing function. Up to the $\gamma^*(n)$ -term this settles a conjecture of Stanley and Wilf. The bound c^n conjectured by Stanley and Wilf was shown by Alon and Friedgut for a certain class of permutations τ , using the generalized Davenport-Schinzel sequences again.

Small complete arcs in projective planes

Van H. Vu (Redmond)

An *arc* of a projective plane is a set of points with no three on a line. The arc is *complete* if no other point from the plane could be added to it without violating this property. The notion of arcs and complete arcs was developed by B. Segre in the 50's and 60's. Given a projective plane, determining the size of the smallest complete arc is one of the major open questions in discrete geometry. Let $n(P)$ denote the size of a smallest complete arc. For any projective plane of order q , a lower bound $n(P) > \sqrt{2q}$ was already shown in the 50's by Lunelli and Sce, but no close upper bound has been

known. For a Galois plane, which is a special projective plane, the best upper bound was $n(P) < cq^{3/4}$. The proof (due to Szőnyi) made use of Hesse-Weil's theorem and therefore depends on the structure of the fields.

Practically nothing has been known for general planes. In this paper, we will show that there is some constant c such that $n(P) < q^{1/2} \log^c q$ for any projective plane P . This matches the lower bound within a polylogarithmic factor.

Our proof uses a variant of the probabilistic method known as the semi-random method or Rödl nibble. This is a quite surprising application of a probabilistic method in this area, where algebra seems to dominate. Central to the proof is a new concentration result which is of independent interest.

This is joint work with J.H. Kim.

Coloring Hamming Graphs, and the 0/1-Borsuk Problem

Günter M. Ziegler (Berlin)

The 0/1-Borsuk problem asks whether any set of 0/1-vectors in \mathbb{R}^d can be partitioned into at most $d + 1$ sets of smaller diameter. This is known to be false in high dimensions (in particular for $d \geq 561$, due to Kahn & Kalai, Nilli, and Raigorodskii), and yields the known counterexamples to Borsuk's problem from 1933.

Here we ask whether there might be counterexamples in low dimension as well, and we show – using results about the chromatic numbers of Hamming graphs as well as some coding theory bounds – that there is no counterexample to the 0/1-Borsuk conjecture in dimension $d \leq 9$. (In contrast, the general Borsuk conjecture is open even for $d = 4$.)

Author: Anusch Taraz

E-mail addresses of participants

| | |
|-----------------------|---|
| Ahlswede, Rudolf | ahlswe@mathematik.uni-bielefeld.de |
| Aigner, Martin | aigner@math.fu-berlin.de |
| Alon, Noga | noga@math.tau.ac.il |
| Blokhuis, Aart | aartb@win.tue.nl |
| Bollobás, Béla | Bollobas@msci.memphis.edu, bollobas@dpmms.cam.ac.uk |
| Brightwell, Graham R. | graham@cdam.lse.ac.uk |
| Diestel, Reinhard | diestel@math.uni-hamburg.de |
| Frank, András | frank@cs.elte.hu |
| Füredi, Zoltán | z-furedi@math.uiuc.edu |
| Gerards, Bert | bert.gerards@cwi.nl |
| Kahn, Jeff | jkahn@math.rutgers.edu |
| Kalai, Gil | kalai@math.huji.ac.il |
| Katona, Gyula O.H. | ohkatona@math-inst.hu |
| Kim, Jeong Han | jehkim@microsoft.com |
| Körner, János | korner@dsi.uniroma1.it |
| Korte, Bernhard | dm@or.uni-bonn.de |
| Laurent, Monique | monique@cwi.nl |
| Leader, Imre B. | i.leader@ucl.ac.uk |
| Lefmann, Hanno | lefmann@ls2.informatik.uni-dortmund.de |
| Linial, Nathan | nati@cs.huji.ac.il |
| Loebl, Martin | loebl@kam.ms.mff.cuni.cz |
| Lovász, László | lovasz@microsoft.com |
| Łuczak, Tomasz | tomasz@mathcs.emory.edu, tomasz@amu.edu.pl |
| Matoušek, Jiří | matousek@kam.mff.cuni.cz |
| McDiarmid, Colin | cmcd@stats.ox.ac.uk |
| Nešetřil, Jaroslav | nesetril@kam.ms.mff.cuni.cz |
| Pak, Igor | paki@math.yale.edu |
| Prömel, Hans Jürgen | proemel@informatik.hu-berlin.de |
| Rambau, Jörg | rambau@zib.de |
| Reed, Bruce | reed@ecp6.jussieu.fr |
| Schrijver, Alexander | lex@cwi.nl |
| Simonovits, Miklos | miki@renyi.hu, miki@math-inst.hu |
| Sós, Vera T. | sos@renyi.hu, sos@math-inst.hu |
| Steger, Angelika | steger@informatik.tu-muenchen.de |
| Szónyi, Tamás | szonyi@cs.elte.hu |
| Taraz, Anusch | taraz@informatik.hu-berlin.de |
| Thomas, Robin | thomas@math.gatech.edu |
| Thomason, Andrew G. | A.G.Thomason@dpmms.cam.ac.uk |
| Trotter, William T. | trotter@asu.edu |
| Valtr, Pavel | valtr@kam.ms.mff.cuni.cz |
| Vu, Van H. | vanhavu@microsoft.com |
| Ziegler, Günter M. | ziegler@math.tu-berlin.de |

Tagungsteilnehmer

Prof. Dr. Rudolf Ahlswede
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
33501 Bielefeld

Prof. Dr. Graham R. Brightwell
Dept. of Mathematics
London School of Economics
Houghton Street
GB-London WC2A 2AE

Prof. Dr. Martin Aigner
Institut für Mathematik II (WE2)
Freie Universität Berlin
Arnimallee 3
14195 Berlin

Prof. Dr. Reinhard Diestel
Mathematisches Seminar
Universität Hamburg
Bundesstr. 55
20146 Hamburg

Prof. Dr. Noga Alon
Department of Mathematics
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv, P.O. Box 39040
Tel Aviv 69978
ISRAEL

Prof. Dr. Andras Frank
Department of Operation Research
Eötvös University
ELTE TTK
Muzeum krt. 6-8
H-1088 Budapest VIII

Prof. Dr. Aart Blokhuis
Department of Mathematics
Technische Universiteit Eindhoven
Postbus 513
NL-5600 MB Eindhoven

Prof. Dr. Zoltan Furedi
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall
1409 West Green Street
Urbana , IL 61801
USA

Prof. Dr. Bela Bollobas
Dept. of Mathematics
University of Memphis
Memphis , TN 38152
USA

Dr. A. Bert H.M. Gerards
CWI - Centrum voor Wiskunde en
Informatica
Postbus 94079
NL-1090 GB Amsterdam

Prof. Dr. Jeff Kahn
Dept. of Mathematics
Rutgers University
Busch Campus, Hill Center
New Brunswick , NJ 08903
USA

Prof. Dr.Dr.h.c. Bernhard Korte
Forschungsinstitut für Diskrete
Mathematik
Universität Bonn
Lennestr. 2
53113 Bonn

Dr. Gil Kalai
Institute of Mathematics and
Computer Science
The Hebrew University
Givat-Ram
91904 Jerusalem
ISRAEL

Dr. Monique Laurent
CWI
PO Box 94079
NL-1090 GB Amsterdam

Prof. Dr. Imre Leader
Department of Mathematics
University College London
Gower Street
GB-London , WC1E 6BT

Prof. Dr. Gyula O.H. Katona
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15
H-1364 Budapest

Dr. Hanno Lefmann
Fachbereich Informatik
Lehrstuhl II
Universität Dortmund
44221 Dortmund

Prof. Dr. Jeong Han Kim
AT & T Bell Laboratories
P.O. Box 636
600 Mountain Avenue
Murray Hill , NJ 07974-0636
USA

Prof. Dr. Nathan Linial
Institute of Mathematics and
Computer Science
The Hebrew University
Givat-Ram
91904 Jerusalem
ISRAEL

Dr. Janos Körner
Dept. of Computer Sciences
Universita "La Sapienza"
Via Salaria 113
I-00198 Roma

Prof. Dr. Martin LoebI
Department of Applied Mathematics
Charles University
Malostranske nam. 25
118 00 Praha 1
CZECH REPUBLIC

Prof. Dr. Laszlo Lovasz
Department of Computer Science
Yale University
P.O.Box 2158
Yale Station
New Haven , CT 06520-2158
USA

Prof. Dr. Igor Pak
Department of Mathematics
Yale University
Box 208 283
New Haven , CT 06520
USA

Prof. Dr. Thomas Luczak
Institute of Mathematics
A. Mickiewicz University
ul. J.Matejki 48/49
60-769 Poznan
POLAND

Prof. Dr. Hans Jürgen Prömel
Institut für Informatik
Humboldt-Universität Berlin
Unter den Linden 6
10044 Berlin

Prof. Dr. Jiri Matousek
Department of Applied Mathematics
Charles University
Malostranske nam. 25
118 00 Praha 1
CZECH REPUBLIC

Jörg Rambau
Konrad-Zuse-Zentrum für
Informationstechnik Berlin (ZIB)
Takustr. 7
14195 Berlin

Prof. Dr. Colin McDiarmid
Department of Statistics
University of Oxford
1 South Parks Road
GB-Oxford OX1 3TG

Prof. Dr. Bruce Reed
U.P.R. 175
Equipe Combinatoire, Case 189
CNRS, Un. Pierre et Marie Curie
4, Place Jussieu
F-75252 Paris Cedex 05

Prof. Dr. Alexander Schrijver
CWI Amsterdam
Postbus 94079
NL-1090 GB Amsterdam

Prof. Dr. Jaroslav Nesetril
Department of Applied Mathematics
Charles University
Malostranske nam. 25
118 00 Praha 1
CZECH REPUBLIC

Prof. Dr. Miklos Simonovits
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15
H-1364 Budapest