TAGUNGSBERICHT 04/2000

The Mathematics of Porous Media

23.01.-29.01.2000

The meeting was organized by C.J. van Duijn (Amsterdam) and P. Knabner (Erlangen) and was attended by 44 scientists from Germany (# 21), USA (# 8), the Netherlands (# 7), France (# 3), Belgium (# 1), Brasil (# 1), Italy (# 1), Israel (# 1) and Sweden (# 1). Most of the participants were mathematicians, amended by a few theoretically oriented scientists from the geosciences.

According to the new format, 5 participants have been invited to give survey lectures and 13 participants to give longer lectures to provide a backbone to the conference in addressing one of the following subjects:

(1) Interfaces versus Fingering.
(2) Upscaling.
(3) Multiphase Flow.
(4) Reactive Flow.
(5) Buoyancy Driven Flow.
(6) Identification.

In addition, 16 shorter lectures were given. To meet the requirements of the wide-ranging scope of the conference the talks were ordered according to the subjects above and also along the lines modeling – analysis – numerical analysis and simulation. This format was rated very positive by the participants.

The organisors and participants thank the 'Mathematische Forschungsinstitut Oberwolfach' to make the conference possible is the usual optimal setting. The abstracts follow according to the schedule.

Monday, January 24th, 2000

FROM THEOREMS TO PRACTICAL APPLICATIONS ... AND BACK TO THEOREMS (long lecture)

ALAIN BOURGEAT

We consider scaling up of 2-phases flow through heterogeneous porous media, for global Peclet (or Capillary) number of order 1. Not only absolute permeability and porosity are oscillating, but also the capillary curve and the relative permeability curves are changing according to the different facies.

We presented the necessary adaptation we had to make in order to apply our theorem (1995) to real field studies. For this we did:

1. a zoning (Geological Pattern Recognition) finding all the subdomains of the reservoir to which a constant absolute permeability could be associated;
2. a change of unknown and an approximation of the relative permeability curves tensor i.e. a lumping by modification of the auxiliary problem boundary conditions on the representative elementary volume - for using the standard numerical simulators;
3. deconnecting the subregions for scaling up the absolute permeability from the bigger subregions where the curves will be scaled up latter- mainly for giving "'enough time"' to the capillary forces to appear in case of non small Peclet number.

Finally we present a comparison with the methods (PVW, Dominant Facies, Kytte Berry, Stone, ...) used in the petroleum engineering literature. Clearly our method has the advantage to give directly the scaled up curves with the expected properties (monotonicity for instance) and to give very few such curves making the usual classification step non necessary.

This work is a joint work with A. Badea and A. Hidani and M. Jurak. It was supported by the Direction of research of ELF-France.

EFFECTIVE, EQUIVALENT AND APPARENT PROPERTIES OF POROUS FORMATION (long lecture)

GEDEON DAGAN

We consider flow of water and solute transport in heterogeneous porous formations. The hydraulic conductivity is modelled as a random space function.

The velocity and pressure head are related by Darcy's Law, and are both random.

The solute concentration is also random. The quantity of major interest is their expected value. The common approach is to replace the heterogeneous formation by a fictitious one of homogeneous effective properties, leading to solution of the flow and transport problems identical with the mean values of head, velocity and concentrations. This approach applies in the case of average uniform flow and unbounded domains.

The effective properties, depending only on the heterogeneous structure, do not exist in the case of bounded formations and/or strongly nonuniform flows. An important example of the latter is well flow. It is suggested to define in such cases additional properties, equivalent and apparent. Their properties are analyzed in the talk.

ON THE INTERFACE BOUNDARY CONDITION OF BEAVERS, JOSEPH AND SAFFMAN (long lecture)

ANDRO MIKELIĆ

We consider the laminar viscous channel flow over a porous surface. It is supposed, as in the experiment by Beavers and Joseph, that a uniform pressure gradient is maintained in the longitudinal direction in both the channel and the porous medium. After studying the corresponding boundary layers, we obtain rigorously Saffman's modification of the interface condition observed by Beavers and Joseph. It is valid when the pore size of the porous medium tends to zero. Furthermore, the coefficient in the law is determined through an auxiliary boundary-layer type problem (see the article with same title by W. Jäger and A. Mikelić in SIAM J. Appl. Maths. 2000).

HOMOGENIZATION OF A FLUID PROBLEM WITH A FREE BOUNDARY

BEN SCHWEIZER

We study a homogenization problem for the Stokes equations. It is well-know that, if a fluid has to pass a periodic structure of obstacles, then, in the limit of small obstacles, the flow is best described by a Darcy law. Our interest lies in the study of a free boundary value problem for the Stokes flow.
The geometry is as follows: The obstacle consists of a periodic array of vertical cylinders of size and distance $O(\varepsilon)$. The upper boundary of the fluid domain is a free boundary; It is given as the graph of a function on a two-dimensional perforated domain.
We study the limit $\varepsilon \to 0$ and derive equations for the two-scale limit of the stationary solutions. The limiting system is a free boundary system. In the interior of the (a priori unknown) fluid domain there holds a Darcy law. The condition on the upper surface is given by a formula connecting pressure and height via a nonlinear

elliptic operator. This nonlinear operator is a homogenized version of the mean curvature operator. Depending on the equation for the contact-angle the static pressure is in general unbounded.

STABILITY OF THE EQUILIBRIA FOR SPATIALLY PERIODIC FLOWS IN POROUS MEDIA

JOACHIM ESCHER

Of concern are the stability properties of the equilibria of two moving boundary problems describing one-phase flows in porous media, which are of Hele-Shaw and Stefan type, respectively. The determination of appropriate conserved quantities leads to a suitable reduction of these flows, and by the principle of linearized stability for fully nonlinear parabolic evolution equations the exponential stability of the equilibria is established.

SOLVABILITY OF MIXED FORMULATION FOR DARCY–FORCHHEIMER FLOW IN POROUS MEDIA

GERHARD SUMM

We consider gas flow through a porous medium governed by the Darcy–Forchheimer equation, the continuity equation and the ideal gas law as equation of state. Using the mass flux density $\mathbf{m}$ and an auxiliary variable $S = |p|p$, where p is the pressure, we can reformulate the problem in the following form:

$$\begin{aligned} (\alpha + \beta|\mathbf{m}|)\,\mathbf{m} + \nabla S &= 0\,, \\ \phi\,\partial_t \rho(S) + \operatorname{div}(\mathbf{m}) &= 0\,, \qquad \rho(S) = S/(\gamma\sqrt{|S|})\,. \end{aligned}$$

We study the mixed formulation of the corresponding steady state and time-discretized problem under minimal, physically realistic assumptions on the regularity of the coefficient functions $\alpha, \beta, \gamma, \phi$. Using monotonicity techniques of nonlinear operator theory we prove the existence and uniqueness of a solution $(\mathbf{m}, S) \in W^3(\operatorname{div};\Omega) \times L^{3/2}(\Omega)$, where $W^3(\operatorname{div};\Omega) = \left\{\mathbf{v} \in (L^3(\Omega))^n \;\middle|\; \operatorname{div}(\mathbf{v}) \in L^3(\Omega)\right\}$.

FRACTALS AS POROUS MEDIA: HOMOGENIZATION AND NUMERICAL APPROXIMATION

VOLKER METZ

Like the Cantor set many fractal sets are constructed by recursive self-similar cutouts. Therefore, they are models for extremely inhomogeneous porous media. The properties of the resulting material can be studied via homogenization, that is, we avoid the complicated local structure by considering only global (or effective) properties. This idea will be made mathematically precise using Dirichlet forms, the Dirichlet principle and a metric on Dirichlet forms. Numerically, we will coarsen fractal grids by local Schur complements.

HOMOGENIZATION AND A MODEL FOR THE WATER UPTAKE IN PLANTS (long lecture)

STEPHAN LUCKHAUS

The problem is to give a model for the interaction between evapotranspiration of a plant (say a tree) and the flow of water in the soil. The capacity of the soil to act as storage and conduit for the water is modelled with a dual porosity approach. Mathematically the model consists of a periodic array of finely porous cells in a large pore matrix. Since one has to model the partially saturated zone there is a dependence of the scaling for capillary pressure and permeability. The equations read before homogenization:

$$\begin{aligned}
&\partial_t((1-\chi_\varepsilon)s(p)+\chi_\varepsilon s(\rho_\varepsilon p)) = \\
&\operatorname{div}([(1-\chi_\varepsilon)k(s(p))+\chi_\varepsilon\rho_\varepsilon^2 k(s(\rho_\varepsilon p))](\nabla p+\vec{g})) \\
&\chi_\varepsilon = \sum_{\mathbf{Z}^d}\chi_{\varepsilon(i+Y)}\ , \qquad Y \text{ the cell.}
\end{aligned}$$

The plant is modelled as having a root in each cell and applying the same "'sucking"' pressure in space which is physiologically limited, this leads to the variational inequality

$$\begin{aligned}
&p_{\max}\le p_s(t), \\
&p\le p_s(t), \partial_\nu p+\nu\vec{g}\ge 0 \quad \text{on} \quad \partial Z, \quad Z\subset Y \quad \text{the roots} \\
&\sum \int_{\partial(\dot{\varepsilon}+\varepsilon Z)} \rho_\varepsilon^2 k(\partial_\nu p+\nu\vec{g}) =: j\le j(t), \\
&(p-p_s(t))(\partial_\nu p+\nu\vec{g})\equiv 0, \quad (p_s(t)-p_{\max})(j-j(t))\equiv 0.
\end{aligned}$$

In the case that ε and ρ_ε scale like $\rho_\varepsilon/\varepsilon\to 0, \frac{\varepsilon^2}{\rho_\varepsilon}\rho\to 0$ that is in the whole range $\rho_\varepsilon=\varepsilon^\gamma, \frac{1}{2}<\gamma<1$.
One gets by homogenization

$$\begin{aligned}
&\partial_t((1-|Y|)s_1+|Y|s_2) = \operatorname{div}(\tilde{K}k(s_1)(\nabla p+\vec{g}))+f \\
&f=c(t)\chi_{s_1>0}, \quad s_1=s(p) \\
&\partial_t s_2(x)((\max_x s_2)-s_2(x))\equiv 0, \quad \partial_t s_2(\max_x s_1)\equiv 0 \\
&c(t)|\{s_1>0\}|+|Y|\int\partial_t s_2=g(t) \quad \text{or} \quad s_2=s_{\min}.
\end{aligned}$$

$\tilde{K}$ is the usual anisotropic matrix.

Note that $(1 - s_2)$ plays the role of the latent heat in the Stefan problem for the advancing front $\partial\{s_1 > 0\}$. Most of this result is part of the Ph. D. thesis of T. Canarius.

EVOLUTION OF MICROSTRUCTURE IN TWO-PHASE FLOW (survey lecture)

FELIX OTTO

We consider the sharp-interface model proposed by Saffmann and Taylor for immiscible two-flow in a Hele-Shaw cell. The two phases have different mobilities and densities. As a consequence of the Saffman-Taylor instability, the phase distribution is thought to develop a microstructure in form of multiple fingers, so that its evolution is effectively unpredictable.
We identify the constraints on the coarse-grained quantities, like the averaged volume fraction of the phases, and show that these constraints allow to derive some predictions on how the macroscopic quantities change over time. Furthermore, we investigate a class of closure hypothesis, which complement these constraints and thereby determine an evolution of the macroscopic quantities themselves, by analyzing the stability of this evolution. This allows us to discriminate between the different closure assumptions within this class.
Our goal is to identify mathematically reasonable closure hypotheses *without* adding assumptions on the physics. Our rigorous analysis uses a combination of tools from nonlinear scalar conservation laws and ideas from the theory of effective moduli.

Tuesday, January 25th, 2000

MODELLING REACTIVE TRANSPORT IN THE SUBSURFACE USING THE LAGRANGIAN STOCHASTIC ADVECTION REACTION (LaSAR) FRAMEWORK (long lecture)

VLADIMIR CVETKOVIC

Reactive subsurface transport takes place in 3D geological media with randomly varying flow and reaction parameters. Our goal is to simplify the transport problem by focusing on random advection and reactions. We describe the transport with a system of Lagrangian 1D hyperbolic balance equations with follow random streamtubes. The fieldscale transport variables (such as concentration, tracer flux, tracer discharge) are obtained by appropriate statistical averaging. One particular problem discussed is reactive solute transport through discrete fractures, where the hyperbolic system can be solved analytically. Transport in discrete fractures provides a unique example of direct coupling between random flow and diffusion/sorption. We show the significance of flow variability for diffusive mass transfer using simulations and also comparison with field data. Another example where analytical solutions can be obtained for the hyperbolic system, is modeling colloid-facilitated tracer transport. The mass transfer from the aqueous phase to the particulate phase is assumed to be kinetically controlled and irreversible, where the colloids may be subject to filtration. Several other examples where the LaSAR methodology yields relatively simply solutions have been considered, in the particular the wide class of linear mass transfer reactions for tracer sorption-desorption. The LaSAR framework can provide a number of relatively simple modeling tools that can conveniently combine analytical (exact or approximate) results with numerical simulations for quantifying transprt of reactive contaminants in the subsurface.

FLUID INTERFACES, INSTABILITIES AND VISCOUS FINGERING (survey lecture)

YANIS C. YORTSOS

Flow and displacement patterns in porous media are affected by both microscale and macroscale processes. True fluid interfaces can be defined at the microscale. Interfaces at the macroscale are iso-contours of macroscopic fields, obtained by suitable averaging over the microscale. The microscale is controlled by Stokes flow, single-pore and pore-network geometry effects. The macroscale involves Darcy flow and permeability variations. In either case, differences in fluid viscosity and density give rise to instabilities. In this lecture we discuss the development, stability and patterns of fluid interfaces at three different scales: single-pore, pore-network and porous medium.

In single-pore geometries (Hele-Shaw cells, capillaries) we discuss the Saffman-Taylor problem, governed by Darcy's law, and displacements at high rates governed by Stokes equations. We discuss issues of stability, patterns and shape selection. At the pore-network scale, we discuss the phase diagram of fully-developed immiscible displacements, using arguments from Invasion Percolation in a Gradient. The results are equivalent to a stability analysis at the pore network scale and serve to delineate the conditions under which the conventional Buckley-Leverett approach is valid (Yortsos, et al., 1997). At the continuum scale we examine the stability of base states for a variety of problems, including miscible and immiscible displacements in homogeneous media, and immiscible displacements in heterogeneous media. We probe the interplay between instability and heterogeneity. Finally, we discuss patterns of fully-developed regimes in stable and unstable displacements. For the first problem, we propose a weakly non-linear model, which extends the KPZ equation to account for non-local effects. For the second problem, we propose an approach based on the method of moments to describe viscous fingering in porous media, with objective to interpret empirical models (such as the Koval and Todd-Longstaff).

SOME MATHEMATICAL PROBLEMS IN REACTIVE FLOW (survey lecture)

STEVEN L. BRYANT

We present mathematical aspects of certain computational problems arising in simulation of reactive flows and of a novel class of chromatographic waves.

In many simulators of large-scale reactive flow and transport for applications such as environmental contamination and remediation, a class of reaction sub-problems arises. For example, it is often required to compute the state of thermodynamic equilibrium for a set of chemical species. This computation is a batch problem in the sense that the total concentrations of chemical components (the chemical 'building blocks' of the system) are given, and the distribution of these components among the various species must be determined. One formulation of this problem is as a set of nonlinear equations: the batch mass balances and the equilibrium expressions relating concentrations of various sets of species. For mineral phases, however, the equilibrium expression is an equality only if the mineral is actually present; if the mineral is not present in the equilibrium assemblage, it provides only an inequality constraint on the aqueous phase concentrations. Consequently the correct set of nonlinear equations cannot be determined a priori, and a procedure for computing the equilibrium state in this manner remains an outstanding problem. An alternative method is to minimise the total Gibbs free energy of the system, subject to the batch mass balance constraints. The Interior Point method is an effective scheme for this problem, since it readily handles the inequality constraints arising from the appearance/disappearance of phases. This approach is robust but its performance has not yet been optimised, and in particular the computational work required (number of iterations) is not known in general. This leads to problems with balancing the computational work on a parallel machine, because the chemical variations may be concentrated in a few parts of the spatial domain, and these locations may vary with time. While schemes for dynamically balancing the chemical computations have been implemented which provide excellent scalability, the algorithm for determining when and where such balancing is needed is only retrospective. A predictive method for estimating the amount and distribution of chemical work, possibly by taking advantage of method of characteristics approaches, would be of great utility. Finally, it often the case that some species will be controlled by reaction kinetics, while others will be constrained by thermodynamic equilibrium. This 'mixed' system presents computational complications, and it would be of interest to determine the feasibility and the error estimates associated with recasting the mixed problem as a set of stiff ODEs, in which the equilibrium reactions are modeled as very fast kinetic reactions.

Motivated by field and laboratory observations of unusual wave propagation, we have investigated the behavior of simple model problems involving transport with competitive adsorption of H^+ and a metal cation. In the absence of diffusion/dispersion, the model problem yields two shocks with the velocities expected from classical theory. In the presence of diffusion/dispersion, the solution exhibits an additional feature, a pulse of metal ion concentration that moves rapidly and independently of the metal ion shock. This 'fast wave' is associated with the pH shock in the simplest model problem studied here. Theoretical analysis of this problem yields a jump condition which numerical experiments confirm. Along with diffusion/dispersion, proton sorption/desorption appears to be prerequisite for this phenomenon. This extension of classical chromatography theory may explain several field and laboratory observations. One practical implication of this finding is the importance of accurate handling of diffusion and dispersion in numerical simulation of reactive transport problems. Another is that estimates of species migration based on simple theory, such as retardation factors, may fail to capture important features of the actual behavior.

INTERPLAY BETWEEN CHEMICAL REACTIONS AND VISCOUS FINGERING IN POROUS MEDIA (long lecture)

ANNE DE WIT

Viscous fingering is an ubiquitous hydrodynamical instability happening when a fluid with low viscosity is displacing another more viscous fluid. Fingering leads to a deformation of the planar interface between the two fluids into "fingers" of the more mobile phase that invade the other phase. This is usually undesirable as it leads to a mixing of the two phases, a phenomenon that engineers want to avoid in applications related to porous media where fingering is observed. Such applications range from chromatographic separations, fixed bed chemical processing, reactive infiltration in geochemical settings to chemical treatment of oil-bearing formations. In these different applications, chemical reactions may take place and affect the properties of viscous fingers.

In this context, we have studied the problem of viscous fingering in miscible solutions in the presence of simultaneous chemical reactions. The two-dimensional flow is governed by the Darcy equations, with a concentration-dependent viscosity $\mu = \mu(c)$. The concentration field $c(x, y, t)$ in turn obeys a reaction-convection-diffusion equation in which the rate of chemical reaction is taken to be a function of the concentration of a single chemical species and admits two dimensionless stable equilibria $c = 0, 1$ separated by an unstable one $c = d$. The resulting nonlinear partial differential equations are solved by numerical simulation. We find that viscous fingering of miscible fluids flowing in porous media is strongly influenced by chemical reactions, leading to new interactions and pattern formation mechanisms. The main new effect is the formation of droplets of the less viscous phase that disconnect from the bulk and invade the more viscous solution when chemical reactions take place. Chemical reactions also strongly enhance tip splittings that occur even for lower values of the Peclet number for which no tip splitting is observed in standard viscous fingering. We also note that, in absence of reactions, dispersion leads to an interface smoothly joining the stable steady states while the interface remains sharp at any time when reactions are taken into account. All these new characteristics observed can be understood in terms of the properties of the hydrodynamics and of the chemical kinetics chosen.

RESEARCH ON POROUS MEDIA AT ITWM

HELMUT NEUNZERT

The research on porous media conducted at my Institute for Industrial Mathematics is driven by problems posed by industry. These refer mainly to multiphase flow in woven or nonwoven fabrics: Water in filts, sweat and vapour through textiles, oil in filters etc. Since very often measurements of characteristic quantities as permeabilities are not available, we use auxiliary microscopic problems (arising from 2-scale Ansatz) to compute these quantities. Therefore our problems are mainly computational: How do we solve multiphase Stokes or Navier-Stokes in periodicity cells, which have a very complex fibre structure? We need flow codes, which allow to compute the formation of drops etc. i.e. the flow field of each phase, when the topology changes.

Research has been done and is on the way with respect to numerical methods as level set methods, immersed interface methods and Lattice Boltzmann methods. Technical difficulties have partly been removed, comparisons, combinations and new versions of these methods have been tested for these flows and similar paradigmatic cases like Hele-Shaw flows.

Wednesday, January 26th, 2000

THE LOCAL DISCONTINUOUS GALERKIN METHOD FOR CONTAMINANT TRANSPORT

CLINT DAWSON

In this talk, we discuss the so-called Local Discontinuous Galerkin (LDG) method for approximating convection-diffusion equations arising in contaminant transport. This method uses discontinuous approximating spaces on each element, and allow for both h and p (polynomial order) adaptivity. Some recent theoretical results on the convergence and stability of the method are discussed, and numerical results are presented. The application of the method to competitional sorption is also given.

UNIFORM ERROR ANALYSIS FOR LAGRANGE–GALERKIN APPROXIMATIONS OF CONVECTION-DOMINATED DIFFUSION PROBLEMS

MARKUS BAUSE

Characteristic-based methods are appropriate approximation schemes for convection-dominated diffusion problems. Lack of the existing error analyses for these approaches is that the error constants normally depend on norms of the solution or even reciprocally on the small diffusion parameter ε. Thus, the error constants may explode in the hyperbolic limit and the analyses have no meaning for small diffusion parameters. In this talk a rigorous ε-uniform error analysis for Lagrange–Galerkin approximations of convection-dominated diffusion problems is presented. Here, the error constants depend on norms of the data and not of the solution and do not tend to infinity in the hyperbolic limit. First order convergence with respect to the discretization in time and a scale of estimates for the spatial discretization are presented.
The error analysis is heavily based on ε-uniform a priori estimates for the solution of the continuous problem which are derived first. This will be done in a Lagrangian framework by transforming the convection-diffusion equation completely into subcharacteristic coordinates.

APPLICATION OF THE MIXED FINITE ELEMENT METHOD TO THE RICHARDS EQUATION

ECKHARD SCHNEID

The Richards equation in its pressure formulation is a model for saturated and unsaturated flow of water in a porous medium. This nonlinear partial differential equation is degenerate. Its type is elliptic in the saturated and parabolic in the unsaturated region. We apply fully implicit time discretization and lowest order Raviart-Thomas elements (RT_0) on triangles to the discrete variational formulation. The resulting system of equations is further enlarged by hybridization of RT_0. Static condensation reduces the global system of nonlinear equations to Lagrangian multipliers only. This system is solved by Newton's Method. The linear subproblems are solved by a multigrid method . The correspondence between the Mixed FEM and a Nonconforming FEM is used to define the grid transfer operators. The residual of the Darcy Law in its variational formulation is used as an error indicator for grid refinement. The mass conservation equation on each element describes the discrete evolution of the pressure. An error indicator, useful to control the time stepsize, can be evaluated after each calculated time step. Application of the adaptive strategies within convergence tests exhibit the same asymptotic behaviour as with homogeneous discretization parameters but with less numerical effort.

FINITE VOLUME METHODS FOR SYSTEMS OF VISCOUS NONLINEAR CONSERVATION LAWS WITH APPLICATIONS TO TWO PHASE AND THERMOCONVECTIVE FLOWS

JÜRGEN FUHRMANN

We present a setting for the description of systems of viscous nonlinear conservation laws discretized by time-implicit, Voronoi box based finite volume schemes which is based on a flux function formulation similar to that used for the description of explicit schemes for hyperbolic problems. Based on this formulation, it is possible to derive stability and existence results in the scalar case.

At the same time, this setting gives rise to an Application Programming Interface which is able to handle a rather large range of problems in the class described above, including Richards Equation, two-phase flow problems in different formulations and thermoconvective flow problems.

SOME ASPECTS OF NUMERICAL SIMULATION OF TWO-PHASE FLOW THROUGH PERMEABLE MEDIA (survey lecture)

TODD ARBOGAST

After presenting the equations governing two-phase flow, we reformulate them in a form more amenable to mathematical analysis. Existence and minimal regularity results for solutions are presented. The solution is not very smooth; in particular, the time derivative of the saturation may only be L^2 in time and H^{-1} in space. Nevertheless, we show that near optimal approximation of the saturation may be obtained by using a certain mixed finite element method. Practical subsurface simulation requires solving problems of enormous size and on many scales. We present two numerical techniques that facilitate such simulation. To better model large-scale features such as faults, irregular domain boundaries, and lithographic strata, we introduce multi-block gridding strategies for which the grids need not match between blocks (i.e. subdomains). The equations of flow are posed on each block separately, and a 'mortar' space is introduced between the blocks to couple the systems.
A discretization based on mixed finite elements is presented and shown to converge (for single-phase flow). For better small to intermediate-scale resolution, we describe a numerical upscaling technique that can be used in conjunction with the mortar technique on the blocks or subdomains. This upscaling technique allows improved sub-grid scale resolution by modifying the form of the coarse-scale discrete operator

rather then by introducing averaged parameters or pseudo-functional relationships. The technique produces optimal approximation of single-phase flow. Numerical results demonstrate its efficacy for two-phase flow.

Thursday, January 27th, 2000

FLOW OF FRESH AND SALT WATER (long lecture)

HANS WILHELM ALT

The flow is described by the system

$$\partial_t u + \operatorname{div}(uq - \delta \nabla u) = 0$$
$$\operatorname{div}(q) = 0 \ , \quad q = -(\nabla p + ue)$$

with a small $\delta > 0$. We consider the limit $\delta \to 0$. Under certain circumstances it is possible to prove existence of a sharp interface solution in the limit $\delta = 0$. We recall some joint work with C.J. van Duijn in the stationary case. We then consider measure valued solutions in the limit $\delta \to 0$, satisfying corresponding entropy inequalities. In particular 1D self-similar measure valued solutions are classified. They correspond to fingering in the direction of gravity.

A MIXED FINITE ELEMENT METHOD FOR ELLIPTIC SYSTEMS OF FIRST ORDER

MANFRED DOBROWOLSKI

I consider second order elliptic equations which are written in the form $u^\alpha = b_i^\alpha(x, p, \nabla p)$, $\alpha = 1, \dots, n$, $\partial_i c_i(x, u, p) = f(x, p, \nabla p)$ in $\Omega \subset \mathbb{R}^n$, $p = 0$ on $\partial\Omega$. We use conforming finite element spaces $X_h \subset H^{1,2}(\Omega)^n$, $Y_h \subset H_0^{1,2}(\Omega)$, for example continuous piecewise linears. Since it is well known that a direct approximation of the system with these spaces in general does not converge we use a stabilized method: Find $(u_h, p_h) \in X_h \times Y_h$ such that

$$(u_h, v_h) = (b_i(\cdot, p_h, \nabla p_h), v_h) \quad \forall\, v_h \in X_h \ .$$

$$-(c_i(\cdot, u_h, p_h), \partial_i q_h) \ - \omega\{(u_h, B(p_h)q_h) - (b^\alpha(\cdot, p_h, \nabla p_h), B(p_h)q_h)\}$$
$$= \ (f(\cdot, p_h, \nabla p_h), q_h) \quad \forall\, q_h \in Y_h$$

where

$$b_i^\alpha = \frac{\partial}{\partial r_i} b^\alpha(x, p, r) \ , \quad B(p_h)q_h = (b_i^\alpha(x, p_h, \nabla p_h)\partial_i q_h)_{\alpha=1,\dots,n} \ .$$

A DARCY FLOW MODEL WITH A DYNAMIC PRESSURE SATURATION RELATION

JOOST HULSHOF

We consider a model for groundwater flow which differs from previous models in that the saturation-pressure relation is extended with a dynamic term, namely the time derivative of the saturation. It also allows a natural modification to take hysteresis effects into account. The resulting model equation is of nonlinear degenerate pseudo-parabolic type.

INFILTRATION IN POROUS MEDIA WITH CAPILLARY PRESSURE: TRAVELLING WAVES

CARLOTA CUESTA

We consider a model for non-static groundwater flow where the saturation-pressure relation is extended by a dynamic term. This approach together with a convective term due to gravity, results in a pseudo-parabolic Burgers type equation. We give a rigorous study of global travelling wave solutions, with emphasis on the role played by the dynamic term and the appearance of fronts.

MODELING PERMEABILITY HYSTERESIS IN TWO-PHASE FLOW VIA RELAXATION (long lecture)

BRADLEY J. PLOHR

Two-phase flow in a porous medium can be modelled, using Darcy's law, in terms of the relative permeability functions of the two fluids (say, oil and water). The relative permeabilities generally depend not only on the fluid saturations but also on the direction in which the saturations are changing. During water injection, for example, the relative oil permeability k_{r0} falls gradually until a threshold is reached, at which stage the k_{r0} begins to decrease sharply. This stage is termed imbibition. If oil is subsequently injected, then k_{r0} does not recover along the imbibition path, but rather increases only gradually until another threshold is reached, whereupon it rises sharply. This second stage is called drainage, and the type of flow that occurs between the imbibition and drainage stages is called scanning flow. Changes in permeability during scanning flow are approximately reversible, whereas changes during drainage and imbibition are irreversible. Thus there is hysteresis, or memory, exhibited by the two-phase flow in the porous medium.

In our lecture, we described two models of permeability hysteresis. Common to both models is that the scanning flow regime is modelled with a family of curves along which the flow is reversible. In the Scanning Hysteresis Model (SHM), the scanning curves are bounded by two curves, the drainage and imbibition curves, where the flow can only occur in a specific direction. This model is consistent with experiments but does not have have a nice mathematical specification. For instance, the algorithm for constructing solutions of Riemann problems involves several ad hoc assumptions.

The second model augments the first by (a) allowing the scanning flow to extend beyond the drainage and imbibition curves and (b) treating these two curves as attractors of states outside the scanning region. The attraction, or relaxation, occurs on a time scale that corresponds to the redistribution of phases within the pores of the medium driven by capillary forces. By means of a formal Chapman-Enskog expansion, we showed that the SHM arises from the augmented model in the limit of vanishing relaxation time, provided that the diffusion associated with capillarity exceeds that induced by relaxation. Through a rigorous study of travelling waves, we showed that the shock waves used to solve Riemann problems in the SHM are precisely those that have diffusive profiles. Numerical experiments confirm our analysis.

LARGE SCALE SIMULATIONS OF MULTIPHASE FLOW IN POROUS MEDIA (long lecture)

PETER BASTIAN

The talk is concerned with the fast resolution of nonlinear and linear algebraic equations arising from a fully implicit vertex-centered finite volume discretization of two-phase flow in porous media. The method can handle different capillary pressure functions in parts of the domain ('rock types'). It is shown how the transition conditions at the interfaces are correctly incorporated into the discretization. A Newton-Multigrid algorithm on unstructured meshes is employed in two and three space dimensions to solve the system of nonlinear algebraic equations. The discretized operator is used for the coarse grid systems in the multigrid method. Problems with jumps in the permeability coefficient are avoided by the use of a matrix-dependent restriction operator. We show experimentally a reasonable behaviour including water-gas flow, two-phase flow in fractured porous media and multiphase/multicomponent nonisothermal flow.

THE MATHEMATICAL THEORY OF THREE-PHASE FLOW (survey lecture)

DAN MARCHESIN

In this talk, we discuss the mathematical modelling of immiscible three-phase (e.g. oil/water/gas) flow in a porous medium and one of its applications.

Standard models for immiscible three-phase flow, for example Stone's model, exhibit unusual behaviour associated with loss of strict hyperbolicity. Anomalies were at one time thought to be confined to the region of nonhyperbolicity, where the purely convective form of the model is ill-posed. However, recent abstract results have revealed that diffusion terms, which are usually neglected, can have a significant effect. The delicate interplay between convection and diffusion determines a larger region of diffusive linear instability. For artificial and numerical diffusion, these two regions usually coincide, but in general they do not. The existence of such an instability region in three-phase flow models is established mathematically using results show that several generic phenomena are related:

a) occurrence of arbitrarily weak nonclassical viscous shock waves;
b) simultaneous loss of genuine nonlinearity and linearized bounded L^2 stability;
c) occurrence of 2-cycles of shock waves;
d) nonuniqueness of solutions of Riemann problems
e) nonuniqueness of asymptotic states for the parabolic form of the conservation laws.

For certain three-phase flow models, however, the instability region is shrunk to a point. As an application this type of model, high-resolution simulations with negligible numerical diffusion have been employed to understand the immiscible flow of idealized oil, gas, and water in a long horizontal thin core, initially containing a high oil saturation, when water and gas are injected alternately (Water-Alternating-Gas, or WAG, recovery). The simulations indicate three important features in the flow before breakthrough.

Closest to the production end is an oil bank, across which the oil saturation decreases substantially. Surprisingly, this oil bank can be followed by a second shock wave, with nearly the same speed, which also decreases the oil saturation. Whereas the oil bank is a two-phase Buckeley-Leverett fast shock wave (the saturation of one of the injected phases is almost constant), the second shock wave is of a kind only recently understood, called a transitional shock wave. Such shock waves occur in models where characteristic speeds can coincide; their structure depends sensitively on diffusion. (Physical diffusion originates from capillary pressure, but the simulation of these waves is also affected by numerical diffusion.) Behind the transitional shock wave is the injection region, where the oil saturation decreases smoothly to zero at the injection well while the water and gas saturations oscillate. High oil recovery can result from the combined effect of the Buckely-Leverett and transitional shock waves. We discuss how the analysis of the nonlinear waves in three-phase flow can be used to devise mathematically optimal WAG strategies for oil recovery.

Friday, January 28th, 2000

NUMERICAL PARAMETER ESTIMATION FOR FLOW IN POROUS MEDIA (long lecture)

JEROMÉ JAFFRÉ

Due to the difficulty of acquiring data from the subsurface, parameter estimation plays an important role in modelling flow in the subsurface, for petroleum reservoir simulation as well as for environmental problems like water management, soil remediation or monitoring pollution etc...

The method we are considering is the least square minimization of a data misfit function. Important features of these problems is that the calculation of this data misfit function is expensive since it necessitates to solve linear or nonlinear PDEs, and that the number of parameters can be large (from 20 to several hundreds), even though the amount of data is small in comparison.

In this lecture we present several parameter estimation techniques which have been successful or are under development for field problems and for laboratory experiments: the gradient method where the exact gradient is obtained through the calculation of the adjoint state, linear analysis which shows that saturation measurements are very useful in the estimation of relative permeabilities and capillary pressure, multiscale parameterization which allow for good minimization performance. We terminate by presenting ideas about the zoning problem where the unknown parameters are not only the values of a piecewise constant parameter but also the zones in which it is constant.

IDENTIFICATION PROBLEMS ASSOCIATED WITH POROUS FLOW (long lecture)

PAUL DUCHATEAU

Indirect determination of physical properties of porous media through identification of coefficients in partial differential equations leads to a class of inverse problems with an interesting abstract structure. Exploiting this structure, based on the introduction of appropriate adjoint problems, it is possible to systematically examine questions of:

- unique identifiability,
- sensitivity/stability,
- guidance for experimental design.

The approach is illustrated by a few simple examples which suggest how the ideas may extend to more complex applications. Finally, it is shown how the approach might lead to a numerical algorithm that is computationally less demanding than output least squares.

MATHEMATICAL THEORY AND SHARP ERROR ANALYSIS FOR MULTIPHASE FLOW IN POROUS MEDIA (long lecture)

ZHANGXIN CHEN

The governing equations that describe multiphase fluid flow in porous media are usually time-dependent, degenerate nonlinear coupled systems. Also, the number of the equations is not a priori known at a given place of the porous media, and the capillary pressure functions relating pressures in different fluid phases are generally unbounded. Moreover, these capillary pressures often exhibit hysteresis. Due to these facts, the analysis of these governing equations is difficult. In this presentation, existence, uniqueness, regularity and stability of a weak solution to a degenerate elliptic-parabolic partial differential system describing two-phase incompressible flow in porous media will be discussed. Through a careful choice of unknown variables, we show the existence, uniqueness, regularity and stability of a weak solution to this system under reasonable assumptions on physical data. Also, a finite element approximation for solving this elliptic-parabolic system will be considered. The elliptic equation is approximated by mixed finite element methods, while the degenerate parabolic equation is approximated by characteristics-based finite element methods. Recently developed characteristics-based local methods will be reviewed. Sharp error estimates in energy norms are obtained. The error analysis respects the minimal regularity on the weak solution. Techniques based on local grid refinement will be also presented. Numerical results will be described.

LOCALLY IMPLICIT TIME STEPPING FOR CONVECTION PROBLEMS

WILLEM HUNDSDORFER

In flow problems in porous media, one often has to deal with convection problems $u_t+\nabla.(\vec{q}f(u)) = 0$ where the velocity $\vec{q}$ assumes large values near source points, such as wells. We consider numerical schemes where spatial discretization is performed with a monotone flux-limited finite-volume scheme of van Leer. This leads to a large ODE system in $\mathbf{R}^m$

$$w'(t) = F(t, w(t))$$

where the components $w_j(t)$ approximate the PDE solution $u(x_j, t)$ at grid points x_j. Time integration then gives fully discrete approximations $w_n = [w_j^n] \in \mathbf{R}^m$ at the time levels t_n. A fully implicit treatment of convection terms is not very efficient. This is mainly due to the fact that *monotonicity* properties of implicit time stepping schemes are in general comparable to those of their explicit counterparts (unlike the stability properties), see for example Bolley & Crouzeix. We consider BDF2 type schemes that are implicit only locally in space,

$$\tfrac{3}{2}w_n - 2w_{n-1} + \tfrac{1}{2}w_{n-2} = \Delta t\ F(t_n, \Theta w_n + (I-\Theta)(2w_{n-1} - w_{n-2}))$$

where $\Theta = \text{diag}(\theta_j)$ is chosen according to the local Courant number at grid point x_j. In regions with a low local Courant number we take $\theta_j = 0$ to obtain an explicit formula. With a large local Courant number we chose $\theta_j = 1$ or $\frac{3}{4}$ to avoid the small time steps that are necessary with fully explicit schemes. Along with implementation aspects of the implicit relations, we discuss accuracy and monotonicity properties of the schemes. Test results are presented for a 2D quarter of five spots problem.

ANALYSIS OF A NEW MODEL FOR UNSATURATED FLOW IN POROUS MEDIA INCLUDING HYSTERESIS AND DYNAMIC CAPILLARY EFFECTS

RUUD SCHOTTING

The relationship between capillary pressure and saturation is a very important element of two-phase flow theories. The present model capillary pressure saturation suffers from two major shortcomings. First the relationship is hysteretic. Explicit representation of full hysteresis with a myriad of scanning curves in models of multiphase flow has been a difficult problem. A second problem relates to the fact that $(P^c - S)$ relationships, determined under static conditions, are not necessarily valid when there is flow. There exists models which take into account either hysteresis or dynamic effects. But the combination of both effects has not been considered yet. In the presentation, we introduce a new model that includes these effects in an approximate fashion. The model is constructed around main inbibition and drainage curves and assumes that all scanning curves are vertical lines in the $(P^c - S)$-plane. The dynamic effect is taken into account by introducing a damping coefficient in the $(P^c - S)$-relation. Governing equations for the particular case of unsaturated flow in a porous medium are given. Two simple example problems are discussed to emphasize important properties of solutions of the new model.

NUMERICAL MODELLING OF BUOYANCY-INDUCED FLOWS IN POROUS MEDIA (long lecture)

PETER FROLKOVIĆ

In the lecture the typical features of buoyancy induced flow and transport in porous media are presented. New results concerning well-known benchmark example "Elder problem" were described that are contradictory to the results from previous efforts in this area. Important numerical issues like upwinding method, consistent velocity approximation, linearization techniques concerning the problem of coupled transport and flow, are discussed.

GEL PLACEMENT IN POROUS MEDIA

FRED VERMOLEN

In this talk we analyse advective transport of polymers, cross-linkers and gel, taking into account non-equilibrium gelation, gel adsorption and cross-linker precipitation. In absence of diffusion/dispersion the resulting model consists of hyperbolic transport-reaction equations. These equations are studied in several steps using mainly analytical techniques. For simple cases, we obtain explicit travelling wave solutions, whereas for more complicated cases we rely on analytical techniques to analyse the problem qualitatively. Finally, a numerical solution for the full system of equations is obtained. The results developed in this study can be used to validate numerical solutions obtained from commercial simulators.

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