

**MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH**

Tagungsbericht 08/2000

**Darstellungstheorie endlichdimensionaler Algebren**

20.02. - 26.02.2000

The meeting was organized by I Reiten (Trondheim) and C. M. Ringel (Bielefeld).

The main theme of the meeting was the theory of wild algebras, and several talks were related to various aspects of this topic. Other topics covered, some of which have connections with wild algebras, were quiver representations and their geometry, homological methods, including  $A_\infty$ -categories, Schur algebras, Lie algebras and quantum groups.

The stimulating talks presented many results and open questions, suggesting directions for further research. They initiated interesting discussions and conversations, leading to solutions of some problems posed in the lectures.

The talks reflected the rich development and the liveliness of the field, and its interplay with other branches of mathematics.

The inspiring atmosphere of both the location and the meeting, the growth as well as the deepening of contacts were very stimulating for all participants, and will certainly contribute to further progress.

## Abstracts of the talks given at the meeting

### On the Flat Cover Conjecture

LIDIA ANGELERI HÜGEL

This is a report on some recent results due to Eklof and Trlifaj [1], [2]. In 1981, Enocks conjectured that every module over an arbitrary ring has a flat cover. This conjecture remained open till summer 1999, when two different proof were obtained, one by Enocks, the other by Bican and El-Bashir [3]. Enocks' proof relies on a result in [1] and was then further generalized in [2].

In the talk, I report on the generalized version of Eklof and Trlifaj [2] by presenting the following results.

**Theorem 1.** [1] *Let  $R$  be a ring,  $\mathcal{S}$  a set of  $R$ -modules,  $\mathcal{C} = \mathcal{S}^\perp$  and  $\mathcal{F} = {}^\perp \mathcal{C}$ . Then for every module  $X_R$  there are exact sequences  $0 \rightarrow X \xrightarrow{f} Y \rightarrow Z \rightarrow 0$  and  $0 \rightarrow A \rightarrow B \xrightarrow{g} X \rightarrow 0$  with  $Y, A \in \mathcal{C}$ ,  $Z, B \in \mathcal{F}$ .*

**Corollary.** *Let  $M \in \text{Mod } R$ . Then every module has a special  $M^\perp$ -preenvelope.*

**Theorem 2.** [2] *Let  $\mathcal{M} \subset \text{Mod } R$  be a class consisting of pure-injective modules. Then every module has a special  ${}^\perp \mathcal{M}$ -cover.*

Observe that Theorem 2 implies that every module has a flat cover, just by taking for  $\mathcal{M}$  the class of all pure-injective  $R$ -modules.

Here for  $\mathcal{X} \subset \text{Mod } R$ ,  $\mathcal{X}^\perp = \{A \mid \text{Ext}^1(X, A) = 0 \forall X \in \mathcal{X}\}$  and  ${}^\perp \mathcal{X} = \{A \mid \text{Ext}^1(A, X) = 0 \forall X \in \mathcal{X}\}$

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### One parameter families of modules for wild algebras

RAYMUNDO BAUTISTA

Let  $Q$  be any finite quiver,  $k$  an algebraically closed field. By a remark of C. M. Ringel done at the end of the talk we have:

**Lemma.** *Take  $Q'$  a subquiver of  $Q$  having the same vertices as  $Q$ . Then for  $M, N$  in  $\text{mod } kQ$ :*

$$\delta_{Q'} = \dim_k \text{Ext}_{kQ}(M, N) - \dim_k \text{Ext}_{kQ'}({}_{kQ'}M, {}_{kQ'}N) \geq 0.$$

Now for  $X \in \text{mod } kQ'$  consider the family

$$\mathcal{S}_{Q', X}(i) = \{M \in \text{ind } kQ \mid {}_{kQ'}M \in \text{Add}(X) \text{ and } \delta_{Q'}(M, M) = i\}.$$

With this notation we obtain the following.

**Theorem.**

- (a)  $\mathcal{S}_{Q', X}(0)$  is a discrete family.
- (b)  $\mathcal{S}_{Q', X}(1)$  is a tame family.

## An example of wild behaviour

THOMAS BRÜSTLE, BIELEFELD

The aim of this talk is to illustrate a "typical behaviour" of wild algebras by studying one particular example, the local commutative algebra

$$A = k[X, Y]/(X^3, X^2, Y^2)$$

over an algebraically closed field  $k$ . By elementary methods one can classify the  $A$ -modules in small dimensions. It turns out that for our algebra  $A$ , the indecomposable modules of dimension at most 5 occur in a finite number of one-parameter families, whereas in dimension 6 there exists an affine plane  $\mathcal{E}$  of pairwise non-isomorphic indecomposable  $A$ -modules, given by  $6 \times 6$ -matrices  $X$  and  $Y$  as follows:

$$X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, Y_{s,t} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & 0 & 0 \\ 0 & t & 1 & 0 & 0 & 0 \end{bmatrix}, s, t \in k.$$

Viewing the parameters  $s, t$  as indeterminates, this family naturally yields a  $k\langle s, t \rangle$ - $A$ -bimodule (free of rank 6 over  $k\langle s, t \rangle$ ), and thus an exact functor

$$F_{\mathcal{E}} : \text{mod } k\langle s, t \rangle \rightarrow \text{mod } A.$$

This functor sends the one-dimensional  $k\langle s, t \rangle$ -modules onto the plane  $\mathcal{E}$  (thus proving that  $A$  is not tame), but it fails to be a representation embedding on the higher-dimensional modules. This effect is due to the existence of some commutator action  $t \mapsto t + (sa - as)$  on the parameters  $s$  and  $t$  that is not visible in the field  $k$ , but has strong impact for higher-dimensional modules.

We finally show how one can use Drozd's proof of the tame-wild theorem to *construct* a representation embedding by composing  $F_{\mathcal{E}}$  with another functor  $F_{\mathcal{M}} : \text{mod } k\langle u, v \rangle \rightarrow \text{mod } k\langle s, t \rangle$  which is induced by some affine plane  $\mathcal{E}$  of 7-dimensional  $k\langle s, t \rangle$ -modules.

## Large Infinitely generated tilting modules

FLÁVIO U. COELHO (JOINT WORK WITH LIDIA ANGELERI HÜGEL)

We extend Miyashita's notion of a tilting module of finite projective dimension to infinitely generated modules over an arbitrary ring  $R$  and characterize the classes  $\mathcal{X} \subset \text{Mod } R$  induced by such tilting modules in terms of the existence of  $\mathcal{X}$ -preenvelopes [1]. This extends results of Auslander-Reiten [4] and Angeleri-Tonolo-Trlifaj [3].

We also study the existence of complements to a partial tilting module over an arbitrary ring. As a consequence of our study, we show that a finitely generated partial tilting module over an artin algebra admits always a (possibly infinitely generated) complement [2].

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## Semi-invariants for quiver representations

HARM DERKSEN (JOINT WORK WITH JERZY WEYMAN)

Each  $n$ -tuple  $(\lambda_1, \lambda_2, \dots, \lambda_n)$  with  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$  and  $\lambda_i \in \mathbb{Z}$  corresponds to an irreducible representation  $V_\lambda$  of  $\mathrm{GL}_n$ . For which triples  $(\lambda, \mu, \nu)$  does  $V_\nu$  appear in  $V_\lambda \otimes V_\mu$ ? Klyachko proved that there exists a positive integer  $N$  such that  $V_{N\nu} \subset V_{N\lambda} \otimes V_{N\mu}$  if and only if a certain explicit set of inequalities is satisfied. Moreover, recently Knutson and Tao proved the saturation problem, i.e., if  $V_{N\mu} \subset V_{N\nu} \otimes V_{N\lambda}$  for some  $N \geq 1$  then  $V_\nu \subset V_\lambda \otimes V_\mu$  (the converse is easy).

In this talk I will give an alternative proof of the results of Klyachko and Knutson–Tao using semi-invariants of quiver representations. Let  $Q$  be a quiver without oriented cycles. For a dimension vector  $\alpha$  we denote the ring of semiinvariants by  $\mathrm{SI}(Q, \alpha)$ . Schofield introduced special semi-invariants  $c^V \in \mathrm{SI}(Q, \alpha)$  for representations  $V$  of  $Q$ . We prove that these semi-invariants span  $\mathrm{SI}(Q, \alpha)$ . A consequence of this is that the set  $\Sigma(Q, \alpha)$  of all weights  $\sigma$  with  $\mathrm{SI}(Q, \alpha)_\sigma \neq 0$  is given by homogeneous linear inequalities. In particular this set is saturated. Applied to the triple flag quiver, the Klyachko/Knutson–Tao results follow.

## Canonical Decomposition for Quivers

HARM DERKSEN (JOINT WORK WITH JERZY WEYMAN)

Let  $Q$  be a quiver without oriented cycles. Kac described the set of dimension vectors  $\alpha$  for which there exists an indecomposable representation. This set can be identified with the positive roots of a Kac-Moody Lie algebra (corresponding to the diagram you obtain by forgetting the orientation of the arrows). Kac distinguishes real and imaginary roots. For real roots there exists exactly one indecomposable representation, where for imaginary representations there exist infinitely many indecomposable representations. A dimension vector  $\alpha$  is called a Schur root if the generic representation of dimension  $\alpha$  is indecomposable. Some slides will clarify the structure of real and imaginary roots, and of real and imaginary *Schur* roots.

Following Kac, we call  $\alpha = \alpha_1 \oplus \alpha_2 \oplus \dots \oplus \alpha_r$  the canonical decomposition of  $\alpha$  if a generic representation  $V$  decomposes into indecomposables as  $V = \bigoplus_i V_i$  with the dimension of  $V_i$  equal to  $\alpha_i$  for all  $i$ . Schofield found algorithms for computing the canonical decomposition. I will describe a new very efficient algorithm for computing such a canonical decomposition. Our algorithm uses the braid group action on so-called exceptional sequences and does not use recursion (i.e., the canonical decomposition for many smaller dimension vectors).

## Tame biextensions of derived tame hereditary algebras

PETER DRÄXLER

If  $B$  is a derived tame hereditary algebra (i.e. there is a tame hereditary algebra  $H$  such that  $D^b(B) = D^b(H)$ ), then every object  $Y$  in  $D^b(B)$  (in particular any  $B$ -module) can be decomposed as  $Y = \bigoplus_{\nu \in \mathbb{Z}} Y[\nu]$  such that each indecomposable direct summand of  $Y[\nu]$  is of the form  $X[\nu]$  for some indecomposable  $H$ -module  $X$ . Moreover, in  $D^b(H)$  we find all the shifts  $\mathcal{R}[\nu]$  of the exact abelian subcategory  $\mathcal{R}$  of  $H$ -mod formed by the regular modules.

Let  $C', B, C$  be finite-dimensional algebras,  $R'$  a  $B$ - $C'$ -bimodule and  $R$  a  $B$ - $C$ -bimodule. The associated *biextension algebra*  $[C', R']B[R, C]$  is the triangular matrix algebra

$$\begin{pmatrix} C & 0 & 0 \\ R & B & 0 \\ D(R') \otimes_B R & D(R') & C' \end{pmatrix}$$

endowed with the obvious addition and multiplication. Any  $C'$ - $C$ -subbimodule  $W$  of  $D(R') \otimes_B R$  yields an ideal  $J(W)$  of the algebra  $[C', R']B[R, C]$ . We call the factor algebra  $[C', R']B[R, C]/J(W)$  the *truncated biextension algebra* with respect to  $W$ .

**Theorem.** *Let  $B$  be a derived tame hereditary algebra of type  $\tilde{\mathbf{A}}_n$  or  $\tilde{\mathbf{D}}_n$  and  $R'_1, \dots, R'_s, R_1, \dots, R_t$  be two sequences of indecomposable modules which are assumed to be of length 2 in some abelian subcategory  $\mathcal{R}[\nu]$  of the derived category  $D^b(B)$ . Moreover, the  $R'_j$  and  $R_i$  lie in non-homogeneous tubes in  $D^b(B)$  in the case  $\tilde{\mathbf{A}}_n$  and in one shift orbit of tubes of rank  $n - 2$  in the case  $\tilde{\mathbf{D}}_n$ .*

*We consider the  $B$ -module  $R' = \bigoplus_{j=1}^s R'_j$  as  $B$ - $C'$ -bimodule where  $C' = \prod_{\nu \in \mathbf{Z}} C'[\nu]$  and  $C'[\nu] = \text{End}_B(R'[\nu])$  for all  $\nu \in \mathbf{Z}$  and in the same way the  $B$ -module  $R = \bigoplus_{i=1}^t R_i$  as  $B$ - $C$ -bimodule where  $C = \prod_{\nu \in \mathbf{Z}} C[\nu]$  and  $C[\nu] = \text{End}_B(R[\nu])$  for all  $\nu \in \mathbf{Z}$ . Finally, we define  $W$  to be the  $C'$ - $C$ -subbimodule  $\bigoplus_{\nu \in \mathbf{Z}} D(R'[\nu + 1]) \otimes_B R[\nu]$  of  $D(R') \otimes_B R$ .*

*Then the truncated biextension  $[C', R']B[R, C]/J(W)$  is tame provided that none of the modules  $\tau^- R'_j[-1]$  is isomorphic to some  $R_i$ .*

This theorem generalises the main results of [3] and [2] and also of our talk at ICRTA 8.5 where only algebras with directed quivers were addressed.

The proof proceeds by transforming the module category into a bimodule problem which is brought into a standard form by using the derived category.

We cannot prove the tameness of the bimodule problem itself but we find a degeneration into a problem which, using the language of bushes from [1], translates immediately into a problem of clan type.

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### Boxes and Wildness

YURIY A. DROZD

This talk is a survey on the tame/wild dichotomy for boxes and their application to finite dimensional algebras. I emphasize three advantages of boxes:

- Possibility of “change-of-rings” operation which leads to an equivalence of the representation categories, that allows to construct reduction algorithms.
- Use of *free boxes*, when the set of representations of a fixed dimension is a total affine space (like in the case of quivers without relations).
- Diverse areas of applications: to representations of finite dimensional algebras as well as to Cohen-Macaulay modules, vector bundles, etc.

I give the necessary definitions related to boxes and explain the main features of the reduction algorithm used in the proof of the tame/wild dichotomy. In particular, I present three “minimal wild cases” arising during this algorithm.

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## On cubic functors

YURIY A. DROZD

Let  $\mathbf{fab} = \text{add } \mathbb{Z}$  be the category of finitely generated free abelian groups;  $\mathbf{Cub}(R)$  be the category of *cubic functors*  $F : \mathbf{fab} \rightarrow R \text{ mod}$ , i.e., the functors of degree  $\leq 3$  in the sense of [2]; write  $\mathbf{Cub}$  for  $\mathbf{Cub}(\mathbb{Z})$ , the category of *cubic modules*. We show that the problem of classification of cubic modules is *wild* in the usual sense of the representation theory. On the contrary, we give a complete classification of the cubic functors in the following cases, all of them being *tame*:

1. **2-divisible cubic modules**, i.e., those from  $\mathbf{Cub}(\mathbb{Z}[1/2])$ .
2. **Cubic vector spaces**, i.e., the functors from  $\mathbf{Cub}(\mathbf{k})$ , where  $\mathbf{k}$  is a field (certainly, the non-trivial case being of characteristic 2).
3. **Weakly alternative cubic modules**, i.e., such that  $F(\mathbb{Z}) = 0$ .
4. **Torsion-free cubic modules**, i.e., the functors  $F \in \mathbf{Cub}(\mathbb{Z})$  such that  $F(A)$  is torsion free for all  $A$ .

In case (1), the classification happens to be quite similar to that of *quadratic modules* given in [1]. The description is given in terms of generators and relations and fits the frame of “strings and bands.” The main observation is the following:

**Proposition 1.** *The category  $\mathbf{Cub}(\mathbb{Z}[1/2])$  is equivalent to the category  $\mathbf{A}\text{-mod}$ , where  $\mathbf{A}$  is the subring of  $\mathbb{Z}[1/2]^2 \times \text{Mat}(2, \mathbb{Z}[1/2])^2$  consisting of all quadruples*

$$\left( a, b, \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}, \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix} \right)$$

such that  $a \equiv c_1 \pmod{3}$ ,  $c_4 \equiv d_1 \pmod{3}$  and  $d_4 \equiv b \pmod{3}$ .

As this result is also analogous to that for quadratic modules, one can propose the following conjecture:

**Conjecture.** *Let  $R = \mathbb{Z}[1/(p-1)!]$ , where  $p$  is a prime number. The category of functors  $\mathbf{fab} \rightarrow R \text{ mod}$  of degree  $\leq (p-1)$  is equivalent to  $\mathbf{B}\text{-mod}$ , where  $\mathbf{B}$  is the subring of  $R^2 \times \text{Mat}(2, R)^{p-1}$  consisting of all tuples*

$$(a, b, C^1, \dots, C^{p-1}), \quad \text{where } C^k = \begin{pmatrix} c_1^k & c_2^k \\ c_3^k & c_4^k \end{pmatrix},$$

such that  $c_4^k \equiv c_1^{k+1} \pmod{p}$  for  $1 \leq k < p-1$ ,  $a \equiv c_1^1 \pmod{p}$  and  $b \equiv c_4^{p-1} \pmod{p}$ .

It would imply an analogous description of such functors.

Here are some corollaries of Proposition 1 and of the description of cubic 2-divisible modules.

**Corollary.** *For every  $F \in \mathbf{Cub}(\mathbb{Z}[1/2])$*

1. *If  $F_p \simeq F'_p$  for every odd prime  $p$ , then  $F \simeq F'$ .*
2.  *$\text{pr. dim } F \in \{0, 1, \infty\}$ .*
3.  *$F$  has a periodic projective resolution of period 6 starting from the second term.*
4. *If  $F$  is indecomposable and non-projective,  $T$  is its torsion part, then  $F/T$  is either 0 or a direct sum of at most 2 irreducible torsion free modules.*

Cases (2), (3) also fit the frame of strings and bands, though in case (2) the resulting description is more intricate. All of them rely upon the so called “Gelfand matrix problems” (Crawley-Boevey’s clans or Bondarenko–Nazarova–Roiter’s bunches of chains).

Case (4) is the simplest one: we show that  $\text{Cub}(\mathbb{Z}_2)$  is equivalent to  $B - \text{mod}$ , where  $B$  is a Bäckström order and its quiver is tame (indeed, the union of  $\tilde{D}_4$ ,  $D_4$  and  $A_3$ ).

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### On Ringel duality of Schur algebras

KARIN ERDMANN (JOINT WORK WITH ANNE HENKE)

Let  $K$  be an infinite field of characteristic  $p > 0$ , and let  $E$  be an  $n$ -dimensional vector space over  $K$ . Then  $E^{\otimes r}$  is a permutation module for the symmetric group  $\mathcal{S}_r$ . The Schur algebra  $S(n, r)$  can be defined as

$$S(n, r) = \text{End}_{\mathcal{S}_r}(E^{\otimes r}).$$

$S(n, r)$ -modules are the same as  $r$ -homogeneous polynomial modules for  $GL(n, K)$ .

The algebra  $S(n, r)$  is quasi-hereditary, so it has a Ringel dual  $S(n, r)'$ . We determine completely all degrees  $r$  for which  $S(2, r)'$  and  $S(2, r)$  are Morita equivalent.\* For  $r > p^2$ , this is the case if and only if  $r$  is of the form  $ap^k - 2$  or  $ap^k - 2 \pm 1$  where  $2 \leq a \leq p$  and  $k \geq 2$ .

It is known that  $S(n, r)'$  is Morita equivalent to  $K\mathcal{S}_r/I_n$  for  $n < p$ , here  $I_n$  is the kernel of the action on  $E^{\otimes r}$ .

It follows that for any  $r$ , the quotient  $K\mathcal{S}_r/I_2$  is Morita equivalent to an algebra of the form  $eS(2, m)e$  where  $e$  is a ‘good’ idempotent of  $S(2, m)$  for some  $m \geq r$  such that  $S(2, m)$  is Ringel self-dual. This gives new insight to representations of symmetric groups corresponding to 2-part partitions, in particular also a new proof of a theorem by Kleshchev and Sheth.

\* (the very last step was only done after Oberwolfach)

### Blocks of enveloping algebras

IAIN GORDON (JOINT WORK WITH A. PREMET)

Let  $G$  be a connected, reductive algebraic group over  $K$ , an algebraically closed field of positive characteristic, and let  $\mathfrak{g} = \text{Lie}(G)$ .

The enveloping algebra of  $\mathfrak{g}$ ,  $U(\mathfrak{g})$ , has a central subalgebra isomorphic to the algebra of regular functions on  $\mathfrak{g}^*$ ,  $\mathcal{O}(\mathfrak{g}^*)$ . Given  $\chi \in \mathfrak{g}^*$  and  $\mathfrak{m}_\chi \in \mathcal{O}(\mathfrak{g}^*)$  its corresponding maximal ideal, we let

$$U_\chi = \frac{U(\mathfrak{g})}{\mathfrak{m}_\chi U(\mathfrak{g})},$$

a *reduced enveloping algebra* of  $\mathfrak{g}$ . Recently, these algebras have become popular objects in Lie theory thanks to the confirmation, by Premet, of a long-standing conjecture and some fresh conjectures of Lusztig on their representation theory. In particular, in the case  $\chi = 0 \in \mathfrak{g}^*$ , one recovers Lusztig’s original conjecture on the characters of simple  $G$ -modules.

For some time it has been hoped that a mixture of induction, restriction and deformation would yield a good understanding of the representations of  $U_\chi$ .

We study the blocks of the algebras  $U_\chi$  and use degenerations of algebras to prove a general result describing the cohomology (specifically support varieties) of the blocks. As a consequence we determine the representation type of any block. Besides using degenerations of algebras and cohomology, we need to study coinvariant algebras of Weyl groups, the orbits of the action of  $G$  on  $\mathfrak{g}^*$  and  $\mathbb{Z}$ -graded representation theory.

In certain cases the blocks of  $U_\chi$  are related to the deformed preprojective algebras studied by Crawley-Boevey and Holland, amongst others. This last paragraph is joint work with D. Rumynin.

### **Controlled wild algebras**

YANG HAN

The controlled wild algebras, which are not only wild but also of Corner's type, are introduced. A covering criterion for an algebra to be controlled wild, which is very effective for many wild algebras, is given. This criterion is applied to wild radical square zero algebras, wild local algebras and wild group algebras of non-trivial finite  $p$ -groups over any algebras. Usually, a controlled wild algebra is controlled by only finite indecomposable modules, this leads to the definition of controlling index. It is proved that the controlling indices of the above algebras are bounded by five, fourteen and nineteen respectively.

### **Is wild type axiomatizable?**

STANISŁAW KASJAN

Fix a number  $d$  and consider the class of all  $d$ -dimensional associative algebras with 1 over algebraically closed fields of fixed characteristic  $p$ . The class of all algebras of tame representation type is characterized by a set of first order sentences in the language algebra.

The question whether the same is true for the class of wild algebras seems to be open. It is shown that the positive answer is equivalent to the following assertion: for every algebraically closed field  $K$  of characteristic  $p$  the class of tame algebras induces a Zariski-open subset in the variety of  $d$ -dimensional  $K$ -algebras.

### **$A$ -infinity categories in representation theory**

BERNHARD KELLER

J. Stasheff [9] invented  $A_\infty$ -spaces and  $A_\infty$ -algebras at the beginning of the sixties as a tool in the study of 'group-like' topological spaces. In the subsequent two decades,  $A_\infty$ -structures found applications and developments [5] [1] in homotopy theory; their use remained essentially confined to this subject (cf. however [6]). This changed at the beginning of the nineties when the relevance of  $A_\infty$ -structures in algebra, geometry and mathematical physics became more and more apparent (cf. e.g. [3], [10]). Of special influence was M. Kontsevich's talk [4] at the International Congress in 1994: Inspired by K. Fukaya's preprint [2] Kontsevich gave a conjectural interpretation of mirror symmetry as the 'shadow' of an equivalence between two



triangulated categories associated with  $A_\infty$ -categories. His conjecture was proved in the case of elliptic curves by A. Polishchuk and E. Zaslow [8].

In these talks, we will motivate the introduction of  $A_\infty$ -structures by a problem from homological algebra: that of reconstructing a complex from its homology. We will see how the  $A_\infty$ -formalism allows us to write the set of quasi-isomorphism classes of complexes with given homology as the set of orbits under a group action. Under suitable assumptions, it is even an algebraic group action. We will then introduce the derived category of an  $A_\infty$ -algebra and more generally of an  $A_\infty$ -category. Each algebraic triangulated category is equivalent to such a derived category. In particular, the derived category of coherent sheaves on a smooth projective variety may be described by an  $A_\infty$ -category. For varieties that admit a mirror symmetric dual, one choice of  $A_\infty$ -category is made explicit by Kontsevich's conjecture.

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## Factorisations of morphisms for wild hereditary algebras

OTTO KERNER

If  $A$  is a connected tame hereditary algebra,  $X$  is a preprojective and  $Y$  a preinjective module and  $\mathcal{T}$  any regular tube, then each homomorphism  $f: X \rightarrow Y$  factorises through  $\text{add } \mathcal{T}$ . For wild hereditary algebras, a much stronger factorisation property holds:

**Theorem.** *Let  $H = kQ$  be a finite dimensional connected wild hereditary algebra,  $X_1 \neq 0$  a preprojective,  $X_2 \neq 0$  a regular and  $X_3 \neq 0$  a preinjective module. If  $R \neq 0$  is regular, then one has.*

- (a) *Each homomorphism  $f: X_1 \rightarrow X_2$  factorises through  $\tau^{-m}R$  for  $m \gg 0$ .*
- (b) *Each homomorphism  $g: X_2 \rightarrow X_3$  factorises through  $\tau^m R$  for  $m \gg 0$ .*
- (c) *Each homomorphism  $h: X_1 \rightarrow X_3$  factorises through  $\tau^m R$  for  $|m| \gg 0$ .*

The proof is strongly related with the existence of monomorphisms respectively epimorphisms. With the same notation as in the Theorem one additionally gets the following statements (and their dual ones):

- (a') *There exists a monomorphism  $X_1 \rightarrow \tau^m X_2$  for  $|m| \gg 0$ .*
- (b') *There exists a monomorphism  $X_i \rightarrow \tau^m X_3$  for  $m \gg 0$  and  $i = 1, 2$ .*

## Some examples of injectively copresented modules

STEFFEN KÖNIG

Let  $\mathfrak{g}$  be a semisimple complex Lie algebra and let  $\mathcal{O}$  be the BGG–category. Enright’s relative completion (in Mathieu’s version) is an endofunctor of a certain subcategory  $\mathcal{O}'$  of  $\mathcal{O}$  which is a composition of a localisation with restriction and taking a locally finite part. This can be reinterpreted as an approximation functor with respect to the category of modules having an injective copresentation by injective modules of a special kind. A consequence of this new interpretation is a very easy proof of Enright’s conjecture – asserting that relative completion functors satisfy braid relations – which had been proven before independently by Bouaziz and by Deodhar.

Using Auslander’s ‘Representation theory of Artin algebras, I’, the category of (relatively or absolutely) complete modules is identified with the category of  $eAe$ –modules where  $A$  is the algebra of the fixed block of  $\mathcal{O}$  and  $e$  is a suitably chosen idempotent. Such an algebra  $eAe$  always is standardly stratified, of infinite global dimension, and its decomposition numbers can be computed by a Kazhdan–Lusztig type combinatorics.

Projective  $A$ –modules are complete. Thus the algebra  $A$  is QF–3 and has dominant dimension at least two. In particular there is a double centraliser property  $A \simeq \text{End}_{fAf}(Af)$  where  $fA$  is the unique indecomposable projective–injective  $A$ –module. This reproves Soergel’s ‘Struktursatz’ for  $\mathcal{O}$ .

The last result is joint work with I.H.Slungård and C.C.Xi. Everything else is joint work with V.Mazorchuk.

## The trivial representation of a finite group generates a subcategory of tame type

HENNING KRAUSE

Let  $\mathcal{C}$  be a category of finite dimensional modules over some fixed algebra. Call  $\mathcal{C}$  of *tame type* (in the sense of Jensen/Lenzing) if every non-zero direct summand of a product of modules in  $\mathcal{C}$  has an indecomposable direct summand. I would like to advertize the following problem which is due to Jensen and Lenzing: Is a finite dimensional algebra  $\Lambda$  of tame representation type in the sense of Drozd if and only if the category of all finite dimensional  $\Lambda$ –modules is of tame type? In my lecture I shall discuss the following somewhat surprising joint result with Dave Benson: *Let  $G$  be a finite group and  $k$  be a field. Then  $\mathcal{C} = \{\Omega^i k \mid i \in \mathbb{Z}\}$  is of tame type.* The proof uses an embedding of the prime spectrum of the cohomology ring  $H^*(G, k)$  into the Ziegler spectrum of the group algebra  $kG$ ; it is based on methods from stable homotopy theory and provides a complete classification of the modules which arise as a direct summand of a product of modules of the form  $\Omega^i k$ .

## A criterion for concealed-canonical artin algebras

DIRK KUSSIN

The talk is on joint work of Z. Pogorzaly and the speaker. Let  $k$  be a field and  $A$  be a finite-dimensional  $k$ –algebra, denote by  $\text{mod}(A)$  the category of finitely generated right  $A$ –modules. We prove that  $A$  is concealed-canonical if and only if  $A$  is derived equivalent to a canonical  $k$ –algebra (in the sense of Ringel/Crawley-Boevey) and there is an omnipresent indecomposable  $M \in \text{mod}(A)$  such that  $M$  lies in a regular Auslander-Reiten component over  $A$  and the class of  $M$

in the Grothendieck group is fixed by some power of the Coxeter transformation. As application we get that  $A$  is tubular if and only if it is derived equivalent to a tubular algebra and there is an omnipresent indecomposable  $M \in \text{mod}(A)$  which lies in a semi-regular Auslander-Reiten component over  $A$ . The proof exploits the following characterization of concealed-canonical algebras by Lenzing/de la Peña:  $A$  is concealed-canonical if and only if there is an exceptional curve  $\mathbb{X}$  (in the sense of Lenzing) and a torsion-free tilting object in the category of coherent sheaves over  $\mathbb{X}$  whose endomorphism ring is isomorphic to  $A$ .

## Noncommutative geometry and quivers

LIEVEN LE BRUYN

One possible definition of "noncommutative geometry" is the study of families of commutative varieties, connected together by some suitable axioms, and controlled by a formally smooth noncommutative algebra (i. e. one having the lifting property for morphisms modulo nilpotent ideals). In this talk I proposed two such possible settings from the theory of quivers:

Moduli spaces of  $\theta$ -semistable representations of dimension vector  $\alpha$  of a quiver  $Q$ :  $M_\alpha^{ss}(Q, \theta)$ . They are connected as "sum-families", that is  $\oplus$  induces morphisms

$$M_\alpha^{ss}(Q, \theta) \times M_\beta^{ss}(Q, \theta) \xrightarrow{+} M_{\alpha+\beta}^{ss}(Q, \theta)$$

These varieties (projective if  $Q$  has no oriented cycles) are controlled locally by a formally smooth algebra, a suitable universal localization  $\mathbb{C}[Q]_\sigma$  determined by a determinantal semi-invariant. Moreover, this local description allows to study the étale local structure of these moduli spaces by reducing to a quotient variety of an associated quiver situation, similar to the case of semi-simple representations of quivers. This local description is often useful to determine the dimension vectors  $\alpha$  having a  $\theta$ -stable representation.

Another class of examples comes from quotient varieties  $iss_\alpha \Pi_\sigma$  of semi-simple  $\alpha$ -dimensional representations of deformed preprojective algebras  $\Pi_\sigma$ . As a first step one develops relative noncommutative differential forms and de Rham cohomology of  $\mathbb{C}\bar{Q}$ , the path algebra of the double of  $Q$ . Using an acyclicity result one proves an exact sequence of Lie algebras

$$0 \rightarrow \mathbb{C}^{vertQ} \rightarrow \frac{\mathbb{C}\bar{Q}}{[\mathbb{C}\bar{Q}, \mathbb{C}\bar{Q}]} \rightarrow Der_\omega \mathbb{C}\bar{Q} \rightarrow 0$$

where the last term are the  $\mathbb{C}^{vertQ}$ -derivations of  $\mathbb{C}\bar{Q}$  preserving the canonical 'moment'-element  $\sum[\alpha, \alpha^*]$ . The middle term is given the structure of a Lie algebra via the Kontsevich-Poisson bracket which follows from a precise form of the 1-forms. Then one can mimic a proof of V. Ginzburg on Calogero-Moser phase space to prove that  $iss_\alpha \Pi_\sigma$  is a coadjoint orbit for the dual Lie algebra  $\left(\frac{\mathbb{C}\bar{Q}}{[\mathbb{C}\bar{Q}, \mathbb{C}\bar{Q}]}\right)^*$  provided  $iss_\alpha \Pi_\sigma$  is a smooth variety. Combining Nakajima's hyper-Kähler structure induction with results of Crawley-Boevey one can show that this happens precisely when  $\alpha$  is a minimal dimension vector of a simple representation of  $\Pi_\sigma$ . This is joint work with my student Raf Bockland. In general, one expects that representation spaces of quivers and derived spaces such as those determined by (deformed) preprojective algebras are flat enough to have an infinite dimensional group acting transitively on them along strata determined by the same representation type. Having a description of the coadjoint orbits among the  $iss_\alpha \Pi_\sigma$  it would be interesting to find a procedure to package them together in "adelic-like" objects such as the adelic Grassmannian in the case of Calogero-Moser particles in the work of G. Wilson and V. Ginzburg.

## Vanishing of the First Hochschild Cohomology Group

SHIPING LIU (JOINT WORK WITH RAGNAR-OLAF BUCHWEITZ)

Let  $A$  be a finite dimensional algebra over a field  $k$ . We are interested in the problem when  $H^1(A)$  vanishes. If  $k$  is algebraically closed, then  $A$  is Morita equivalent to  $kQ_A/I$  with  $(Q_A, I)$  a bound quiver. It is relatively a long-standing problem if vanishing of  $H^1(A)$  implies the non-existence of oriented cycles in  $Q_A$ . We have recently solved this problem. First of all, the problem has a negative answer in general.

**Theorem 1.** *Let  $Q$  be the quiver consisting of three vertices  $a, b$  and  $c$ , two arrows  $\alpha : a \rightarrow b$ ,  $\gamma : b \rightarrow c$  and a loop  $\beta$  at  $b$ . Then there exist finite dimensional  $k$ -algebras  $kQ/I$  such that  $H^1(kQ/I) = 0$ . Moreover, these algebras are necessarily of wild representation type.*

Secondly the problem has an affirmative answer for representation-finite algebras. In fact, combining with some of Happel's results, we have the following:

**Theorem 2.** *Let  $k$  be algebraically closed and let  $A$  be representation-finite. Then  $H^1(A) = 0$  if and only if  $A$  is simply connected.*

## Non-homogeneous modules over non-strictly wild algebras

HIROSHI NAGASE

Crawley-Boevey showed that for any tame algebra  $A$ , almost all indecomposable  $A$ -modules are homogeneous and he gave a problem that the converse is true, i.e. "for any wild algebra  $A$ , there exist infinitely many non-homogeneous indecomposable  $A$ -modules of the same dimension."

On the other hand, Ringel conjectured that wild algebras are controlled wild algebras, which have a functor

$$F : \text{mod } k(\bullet \rightrightarrows \bullet) \longrightarrow \text{mod } A$$

controlled by a full subcategory  $\mathcal{C}$  of  $\text{mod } A$ . Therefore I'm interested in finding infinitely many non-homogeneous indecomposable  $A$ -modules of the same dimension in  $\text{Im } F$  and I showed that the Crawley-Boevey's problem is true if the number of indecomposable modules in  $\mathcal{C}$  is finite.

## Stable representations of quivers

JOSE ANTONIO DE LA PEÑA (JOINT WORK WITH LUTZ HILLE)

Let  $Q$  be a quiver without oriented cycles and  $k$  an algebraically closed field. We consider representations of  $Q$  over  $k$ .

For a vector  $d \in \mathbb{N}^{Q_0}$  we consider the set  $\mathbb{H}(d)$  of weights with respect to  $d$  as those linear functions  $\theta : \mathbb{Z}^{Q_0} \rightarrow \mathbb{Z}$  with  $\theta(d) = 0$ . A representation  $M$  is  $\theta$ -stable (resp.  $\theta$ -semistable =  $\theta$ -ss) if  $\theta(M) = 0$  and for every subrepresentation  $N$  of  $M$ ,  $\theta(N) < 0$  (resp.  $\theta(N) \leq 0$ ). A slope  $\mu = \theta/\kappa$  is the quotient of a linear map  $\theta : \mathbb{Z}^{Q_0} \rightarrow \mathbb{Z}$  by another with  $\kappa(M) > 0$  for  $M \in \text{rep } Q$ . We have the concept of  $\mu$ -stable and  $\mu$ -ss.

Given a slope  $\mu$  we consider the Harder - Narasimhan filtration  $0 = M_0 \subset M_1 \subset \dots \subset M_r = M$  which satisfies that  $\mu(M_i/M_{i-1}) > \mu(M_{i+1}/M_i)$  and  $M_i/M_{i-1}$  is  $\mu$ -ss,  $i = 1, \dots, r$ . We construct slopes for tame quiver algebras and for wild quivers we consider the case of distinguished slopes of the form  $\mu = (b^-\theta^- - b^+\theta^+)/ (a^+\theta^+ + a^-\theta^-)$  where  $\theta^+ = \langle -, y^+ \rangle$  and  $\theta^- = \langle -, y^- \rangle$  are associated to the eigenvectors  $y^+$  and  $y^-$  of the Coxeter transformation  $\phi$

of  $kQ$  such that  $\phi y^+ = \rho y^+$  and  $\phi y^- = \rho^{-1} y^-$  for the spectral radius  $\rho$  of  $\phi$ . Such a slope, in case it exists, satisfies that  $\mu(P) < \mu(R) < \mu(I)$  for any preprojective  $P$ , regular  $R$  and preinjective  $I$ . Moreover,  $\mu(\tau M) > \mu(M)$  for every regular, a fact which is shown using that Harder – Narasimhan filtration of  $M$ .

Finally we consider the concept of walls in the space  $\mathbb{H}(d)$  and discuss necessary conditions for inner and outer walls.

## Composition series of representations of quivers and combinatorics of words

MARKUS REINEKE

Let  $Q$  be a finite quiver without oriented cycles, and let  $k$  be an algebraically closed field of characteristic 0.

We define a monoid structure on "families of  $k$ -representations of  $Q$ "; more precisely:

For (Zariski-) closed, irreducible,  $\mathrm{GL}(d)$ -stable (resp.  $\mathrm{GL}(e)$ -stable) subvarieties  $\mathcal{A} \subset R(Q, d)$ ,  $\mathcal{B} \subset R(Q, e)$  of the representation varieties corresponding to dimension vectors  $d$  (resp.  $e$ ), we define  $\mathcal{A} * \mathcal{B}$  as the subvariety of  $R(Q, d + e)$  consisting of extensions of representations in  $\mathcal{A}$  by representations in  $\mathcal{B}$ .

This defines a structure of an associative monoid on the set  $\mathcal{M}(Q)$  of all closed, irreducible,  $\mathrm{GL}(d)$ -stable subvarieties of all  $R(Q, d)$ . We consider in some detail the submonoid  $\mathcal{C}(Q)$  spanned by the simple representations  $S_i$  of  $Q$ , called the composition monoid of  $Q$ .

The relations of this monoid to the geometry of the representation varieties, to quantized enveloping algebras, and to the combinatorics of the category of  $k$ -representations of  $Q$  are discussed in the talk.

For a word  $\omega = (i_1 \dots i_k)$  in the alphabet of vertices of  $Q$ , the product  $\mathcal{E}(\omega) = S_{i_1} * \dots * S_{i_k}$  is the subvariety of representations possessing a composition series of type  $\omega$ . Various questions about the geometry of these subvarieties are posed.

Associated to  $Q$ , there is a Kac-Moody Lie algebra  $\mathfrak{g}$ . It is shown that  $\mathcal{U}_0(\mathfrak{g}^+)$ , the  $q = 0$ -specialization of the (twisted) quantized enveloping algebra of the positive part of  $\mathfrak{g}$ , maps onto the monoid ring  $\mathbb{Q}\mathcal{C}(Q)$ . This comparison map is an isomorphism if and only if  $Q$  is of Dynkin type.

It is shown that using work of A. Schofield, the algebraic structure of  $\mathcal{C}(Q)$  can be analyzed. For example, the relation  $R(d) * R(e) = R(d + e)$  holds in  $\mathcal{C}(Q)$  if and only if  $\mathrm{Ext}^1(E, D)$  vanishes generically for  $D \in R(d)$ ,  $E \in R(e)$ . Moreover, the canonical decomposition of a dimension vector, as considered by V. Kac, provides a first approach to a normal form for elements of  $\mathcal{C}(Q)$ .

## Moduli spaces of representations and vector bundles

AIDAN SCHOFIELD

I talked about the moduli spaces of representations of quivers. The idea was to give an outline of a proof that such a moduli space is always birational to a suitable number of matrices up to simultaneous conjugacy.

There are 3 steps to this proof. The first step is to show that this result is true for generalised Kronecker quivers. These are quivers with two vertices and a number of arrows from the first

vertex to the second. The second step is to give a reduction for an arbitrary Schur root over a quiver to the case of a two vertex quiver such that there are no arrows from the second to the first vertex. This is accomplished by showing that if  $\alpha$  is a Schur root then there are indivisible Schur roots  $\beta$  and  $\gamma$  such that  $\alpha = m\beta + n\gamma$  and  $\alpha$  has a unique subrepresentation of dimension vector  $n\gamma$ . The final step is to show that these two vertex cases can then be handled by combinatorial techniques and the results of the first step.

I concentrated on some of the details of the first step. Let  $\alpha$  be an indivisible Schur root for a generalised Kronecker quiver. I constructed two representations  $S$  and  $T$  such that  $\text{hom}(S, R) = 1 = \text{hom}(T, R)$  and  $\text{ext}(S, R) = 0 = \text{ext}(T, R)$  for a general representation  $R$  of dimension vector  $\alpha$ . Further,  $\text{hom}(S, T) = 1 + p$  where  $p = \dim^1(R, R)$  which is the dimension of the moduli space of representations of dimension vector  $\alpha$ . Then the functor  $\text{Hom}(S \oplus T, \_)$  sends a general representation of dimension vector  $n\alpha$  to a representation of dimension vector  $(n \ n)$  for the  $1 + p$ th generalised Kronecker quiver and induces a birational map between their moduli spaces. However this second moduli space is well-known to be birational to  $p \ n$  by  $n$  matrices up to simultaneous conjugacy.

## Nilpotent matrices

JAN SCHRÖER

Let  $k$  be an algebraically closed field,  $M_n(k) =$  the set of  $n \times n$ -matrices,  $\text{rk } A =$  rank of a matrix  $A$ .

It is a well-known problem of H. Kraft to classify the irreducible components of the affine variety

$$V_n = \{(A, B) \in M_n(k) \times M_n(k) \mid AB = BA = A^n = B^n = 0\}.$$

(See Kraft, "Geometric methods in representation theory", SLN 944 (1980)).

As an answer we get the following

**Theorem.** *Let  $n \geq 2$ . The irreducible components of  $V_n$  are*

$$\{(A, B) \in V_n \mid \text{rk } A \leq n - i, \text{rk } B \leq i\}.$$

*Each component has dimension  $n^2 - n + 1$ .*

Furthermore we study properties of the map

$$\begin{aligned} \Pi : V_n &\rightarrow \{A \in M_n(k) \mid A^n = 0\} \\ (A, B) &\longmapsto A. \end{aligned}$$

## Double Ringel-Hall algebras as a quantum group

JIE XIAO

This is joint work with B. Deng. First we verify that Ringel-Hall algebras also join the quantum group category that fit the solutions of quantum Yang-Baxter equations. After we get the decomposition of the double Ringel-Hall algebra  $D(\Lambda)$  induced by its skew-Hopf pairing, it is easy to see that  $D(\Lambda)$  is a restricted non-degenerate member of some datum in the sense of Green. As a consequence this shows that Ringel-Hall algebras are independent of the orientation of  $\Lambda$ . It is natural to define the highest weight module category  $\mathcal{O}$  and integrable modules over  $D(\Lambda)$ . The Ringel pairing  $\phi$  provides a  $R$ -matrix  $\Theta^+$ . The action of  $\Theta^+$  in  $\mathcal{O}$  induces the  $D(\Lambda)$ -module isomorphism  $M \otimes M' \simeq M' \otimes M$  for any  $M, M' \in \mathcal{O}$ . Furthermore, the operator

$\Theta^+$  satisfies a fundamental symmetry: quantum Yang-Baxter equation. We also confirm the complete reducibility and the well-known Weyl-Kac character formulae for the integrable weight modules with strongly dominant highest weights. Second, we present a theorem and its proof, due to Sevenhant-van den Bergh. It claims that the Drinfeld double of Ringel-Hall algebras is the quantified enveloping algebra of a generalized Kac-Moody algebra.

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