MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/2000

Functional Analysis and Partial Differential Equations

19.03. - 25.03.2000

The meeting "Functional Analysis and Partial Differential Equations" was organized by W. Arendt (Ulm), Ph. Benilan (Besançon), Ph. Clément (Delft) and J. Prüß (Halle). There were 49 participants from 13 countries and 4 continents.

Two topics were at the centre of this year's meeting: Regularity Theory and Qualitative Theory of Evolution Equations. The meeting was organized as follows. Each morning started with an invited 50-minute survey lecture. The other contributions of 20 or 30 minutes were organized in the form of sessions on special topics. They had various styles: There were overviews with descriptions of open problems, sessions of two talks illuminating a subject from different sides, presentations of joint work by two mathematicians and outlines of particularly interesting proofs. Several young mathematicians presented important work of their own. This meeting left a considerable amount of free time which was widely used by the participants to exchange ideas, to discuss new developments or to start joint research.

The high quality of the talks, the enthusiasm of the participants, the dynamics of the subject "Evolution Equations", and above all, the extraordinary atmosphere of Oberwolfach made it a most successful meeting.

List of abstracts

Herbert Amann
Strong solvability of the Navier-Stokes equations

We report on some new results concerning the solvability of the Navier-Stokes equations with initial data in Besov spaces of negative order. By means of semi-group and interpolation-extrapolation methods we are, in particular, able to prove an analogue to a recent result of M. Cannone, where we can consider bounded domains.

Charles J. K. Batty
Asymptotic behaviour of solutions of Cauchy problems

This was a survey talk describing results obtained by many people during the last few years, on the subject of asymptotic behaviour of C_0-semigroups and mild solutions of inhomogeneous Cauchy problems. The talk also included connections with the general theory of Laplace transforms. For example the inequality $\omega_1(T) \leq s_0(A)$, relating the exponential growth bound $\omega_1(T)$ of classical solutions of a well-posed homogeneous Cauchy problem (i.e. a C_0-semigroup T) to the pseudo spectral bound $s_0(A)$ of the generator A, is now known to be an instance of a general result about Laplace transforms of exponentially bounded functions. Some precise results relating growth bounds to spectral bounds for Hilbert spaces and positive semigroups on L^p-spaces, were described. There was also a summary of countable spectral conditions, non-resonance conditions and other sufficient conditions for certain types of asymptotic behaviour to occur.

Assia Benabdallah
Exponential decay for a von Karman system with thermal effects

The von Karman system with thermal effects modelizes the movement of the plate subject to an unknown temperature when we consider the von Karman relations for the strain tensor and linear relation for the thermostrain. This leads to a system of 5 equations for which we add boundary conditions. The von Karman nonlinearity is a quadratic term of the gradient of the vertical displacement. The main results are: Let γ denote the coefficient of the rotational inertia:

$\gamma > 0$

$\exists!$ of finite energy solutions and exponential decay

$$E_\gamma(t) \leq C(E_\gamma(0))e^{-\omega_\gamma t}$$

where $E_\gamma(t)$ denotes the energy of the full system.

$\gamma = 0$

- $\exists!$ of finite energy solution.
- Additional regularity (due to the analyticity of the linear thermoelastic plates).
- exponential decay.

Simon Brendle

Asymptotic behaviour of delay differential equations

We consider delay differential equations of the form

$$(*)\qquad \dot{u}(t) = Au(t) + \int_{-1}^{0} \varphi(s)u(t+s)ds$$

where $\varphi \in L^1([-1,0], \mathcal{L}(X))$ and A is the generator of a C_0-semigroup on a Banach space X. Using the critical spectrum, we characterize uniform exponential stability ot the solution of $(*)$.

José Carrillo

Entropy solutions for first order equations in bounded domains

We define entropy solutions for the problem

$$(\mathrm{P}(u_0, f))\qquad \begin{cases} \dfrac{\partial u}{\partial t} + \operatorname{div} \Phi u = f & \text{in } (0,T)\times\Omega \\ \qquad u(0) = u_0 & \text{in } \Omega \\ \quad + \text{Boundary Conditions}, & \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^n$, with Lipschitz continuous boundary.

We have an existence and uniqueness result, for entropy solutions, for any $u_0 \in L^\infty(\Omega)$ and any $f \in L^1(0,T; L^\infty(\Omega))$.

Ralph Chill

Asymptotic behaviour of non-autonomous Cauchy problems

We consider the asymptotic behaviour of solutions of the problem

$$(\mathrm{nACP})\qquad \begin{cases} \dot{u}(t) = A(t)u(t), & t \geq s \geq 0 \\ u(s) = x. & \end{cases}$$

Assuming that (nACP) is well posed, i.e. that there exists a strongly continuous evolution family $(U(t,s))_{t\geq s\geq 0}$ solving (nACP), we give several characterisations of the property

$$\lim_{t\to\infty} \|U(t,s)x\| = 0 \qquad \forall s \geq 0, \forall x \in X$$

in terms of complete trajectories of the dual evolution family, of stability of corresponding evolution semigroups and of their generators on function spaces $L^p(\mathbb{R}_+, X)$. This is joint work with C. Batty and Y. Tomilov.

Giuseppe Da Prato
Elliptic operators with unbounded coefficients

We consider the elliptic operator

$$N\varphi = \frac{1}{2}\Delta\varphi + \langle b(x), D\varphi\rangle \tag{1}$$

on $\mathbb{R}^n$, where b is a vector field of class C^1 and such that

$$\langle b(x), x\rangle \leq a(1+|x|^2), \quad x \in \mathbb{R}^n.$$

We study several properties of the semigroup

$$P_t\varphi(x) = \mathbb{E}[\varphi(X(t,x))], \quad \varphi \in C_b(\mathbb{R}^n)$$

where $dX = b(X)dt + dW_t$, $X(0) = x$, as existence and uniqueness of invariant measures, convergence to the equilibrium, spectral gap, logarithmic Sobolev inequality.

Robert Denk
On parameter-dependent boundary value problems with non-homogeneous symbols

In the classical theory of elliptic boundary value problems the operators are either parameter-independent, or parameter-elliptic in the sense of Agmon and Agranovich–Vishik. Some operators, however, do not belong to one of these two classes; as examples one can take equations of singular perturbation theory, the resolvent of Douglis–Nirenberg systems, or operators appearing in the description of the Stefan problem. All these operators depend on a parameter, but either this parameter has no definite weight (i.e. the corresponding symbol is not quasi-homogeneous) or the condition of ellipticity with parameter is not satisfied.

It is possible to define ellipticity for such boundary value problems, using the concept of the so-called Newton polygon. Here ellipticity for a partial differential operator $A(x, D, \lambda)$ depending on the complex parameter λ is defined by an inequality of the form

$$|A(x,\xi,\lambda)| \geq \text{ const } W_P(\xi,\lambda)$$

where the right-hand side W_P is given in terms of the Newton polygon and, in general, is not quasi-homogeneous in (ξ, λ).

On closed manifolds this type of ellipticity is equivalent to the validity of an a priori estimate which is uniform with respect to the parameter and which holds in appropriate parameter-dependent norms. In the case of boundary value problems the main difficulty is to find the "correct" conditions on the boundary operators. For scalar operator pencils depending polynomially on the parameter and for Douglis–Nirenberg systems parameter-dependent versions of the classical Shapiro–Lopatinskii condition can be found which lead to (or are equivalent to) uniform estimates on the solution. In particular, it is possible to obtain a priori estimates for singularly perturbed boundary value problems depending on a small parameter.

The presented results are based on joint work with L. Volevich (Moscow).

Giovanni Dore
Estimates for irregular boundary value problems

We consider the irregular boundary value problem

$$\begin{cases} \lambda\, u - u'' + Au &= f \qquad \text{on } [0,T] \\ \alpha\, u'(0) + \beta\, u'(1) + \gamma\, u(0) + \delta\, u(1) &= f_1 \\ \alpha\, u(0) - \beta\, u(1) &= f_2 \end{cases}$$

in a Hilbert space where A is a sectorial operator. We prove existence and uniqueness of a L^p-solution provided that $f \in L^p(0,1;D(A^{1/2}))$, $f_1 \in D(A;1-1/2p,p)$, $f_2 \in D(A,3/2-1/2p,p)$ with natural estimates. Due to irregularity of the boundary value problem it is not possible to have existence for f, f_1 and f_2 in the "natural" spaces. (Joint work with S. Yakubov)

Joachim Escher
Stability of the equilibria for spatially periodic flows in porous media

The mathematical models for flows through porous media consist in moving boundary problems of Hele-Shaw and Stefan type, depending whether the porous medium is rigid or not.

Identifying appropriate conserved quantities, we prove the exponential stability of the equilibria of these moving boundary problems in the spatially periodic setting. As a main tool we use the principle of linearized stability for fully nonlinear parabolic evolution equations.

Djairo G. de Figuereido
On nonlinear elliptic systems

In this lecture we discuss the question of existence of positive solutions for a system of nonlinear elliptic equations of the form

$$\begin{cases} -\Delta u = f(x,u,v,\nabla u,\nabla v) \\ -\Delta v = g(x,u,v,\nabla u,\nabla v) \end{cases} \tag{1}$$

subject to Dirichlet boundary conditions. Since the system is not variational, in general, we use topological methods; more precisely results on fixed point index for mappings in cones. The great difficulty in the use of such methods lies in getting a priori bounds. Here we show how to obtain such bounds by three distinct methods:

1. Moving Planes techniques. In 1992 P. Clément, E. Mitidieri and I applied this method with success to systems of the form

$$-\Delta u = v^p, \quad -\Delta v = u^p.$$

2. Hardy-type inequalities. In 1996 P. Clément, E. Mitidieri and I applied this method to system (1) but dependence on the gradients being bounded.

3. Blow-up techniques. This method was used by M. A. Saito (1992), M. Montenegro (1998) in their Doctoral Dissertations at UNICAMP. Recently I used this method to treat systems where f and g depend on the gradients as powers larger than 1 and smaller than 2.

GISÈLE RUIZ GOLDSTEIN

The n-dimensional Heat Equation with Generalised Wentzell Boundary Conditions

Of concern is the Cauchy problem for the heat equation on $D \subset \mathbb{R}^n$

$$\begin{aligned} u_t &= \Delta u \qquad && x \in D, t > 0 \\ u(x,0) &= f(x) \qquad && x \in D \end{aligned}$$

with generalised Wentzell boundary conditions

$$\Delta u + a\frac{\partial u}{\partial n} + bu = 0 \quad \text{for} \quad x \in \partial D, t > 0.$$

The usual Wentzell boundary condition corresponds to $(a,b) = (0,0)$. We present an L^p theory for this problem, which gives rise to a C_0 contraction semigroup on X_p, $1 \le p < \infty$. The novel feature of this work is that X_p is not $L^p(D)$, but rather an L^p space based on functions on D and ∂D, with a measure on ∂D based on the boundary conditions.

This is joint work with Angelo Favini, Jerry Goldstein and Silvia Romanelli.

Jerome A. Goldstein

Nonlinear Wave Equations with Acoustic Boundary Conditions

We solve

$$\begin{aligned} u_{tt} - M\left(\int_\Omega u^2 dx\right)\Delta u + C|u_t|^\alpha u_t &= 0 && x \in \Omega \subset\subset \mathbb{R}^n \\ u &= 0 && \text{on } \Gamma_0, \quad \partial\Omega = \Gamma_0 \cup \Gamma_1, \text{meas}(\Gamma_0) > 0 \\ \rho u_{tt} + n\delta_{tt} + \lambda\delta_t - k\delta &= 0 && \text{on } \Gamma_1 \\ \partial u/\partial n &= \delta_t && \text{on } \Gamma_1 \\ u(x,0) = u_0(x), u_t(x,0) &= u_1(x) && x \in \Omega \end{aligned}$$

Existence holds if $M \geq m_0 > 0$, $\alpha > 1$ (and other hypotheses). Uniqueness and continuous dependence requires $n \leq 3$ and $\alpha \leq 2$ if $n = 3$. We may let Γ_0 shrink to a point. (Joint work with Cicero Frotz).

We also (jointly with Gisèle Goldstein) give an extension in an abstract setting of an old result of Lax-Phillips: For compactly supported initial data, solutions of

$$u_{tt} = \Delta u \qquad (x \in \mathbb{R}^n, t \in \mathbb{R})$$

exhibit equipartition of energy from a finite time onward if n is odd (and not when n is even).

Jean-Pierre Gossez

A nodal domain property for the p-Laplacian

The classical nodal domain theorem of Courant implies that if λ_2 is the second eigenvalue of $-\Delta$ on $H_0^1(\Omega), \Omega$ a bounded domain in $\mathbb{R}^N$, and if u is an associated eigenfunction, then u has exactly two nodal domains. In this talk we present an extension of this result to the p-Laplacian. Our approach does not rely upon unique continuation (which is still a largely open question for the p-Laplacian). A similar result also holds for the Fučik spectrum of the p-Laplacian: any nontrivial solution of the Fučik equation corresponding to a point on the first curve of that spectrum has exactly two nodal domains. (Joint work with M. Cuesta (Calais) and D. de Figueiredo (Campinas).)

Gustaf Gripenberg

Nonsmoothing in a conservation law with memory

It is shown that a discontinuity in the initial value $u_0(x)$ of the solution of the equation

$$\frac{\partial}{\partial t}\left(u(t,x)+\int_0^t k(t-s)\left(u(s,x)-u_0(x)\right)\,ds\right)+\sigma(u)_x(t,x)=0,$$

where $t>0$ and $x\in R$, is not immediately smoothed out provided the nonlinearity σ is strictly convex, even for memory kernels k for which the linear problem has a solution which is continuous for $t>0$.

Alain Haraux

A new functional setting for the HUM method of J. L. Lions

A new functional approach is devised to establish an equivalence between the null-controllability of a given initial state and a certain individual observability property involving a momentum depending on the state. For instance if one considers the abstract second order control problem $y''+Ay=Bh(t)$ in time T by means of an control function $h\in L^2(0,T,H)$ with $B\in\mathcal{L}(H),\quad B=B^*\geq 0$, a necessary and sufficient condition for null-controllability of a given state $[y^0,y^1]\in D(A^{1/2}\times H)$ is that the image of $[y^0,y^1]$ under the symplectic map lies in the dual space of the completion of the energy space with respect to a certain semi-norm. A similar property is derived for a general class of first order systems including the transport equation and Schrödinger equations. When A has compact resolvent the necessary and sufficient condition can be formulated by some conditions on the Fourier components of the initial state in a basis of "eigenstates" related to diagonalization of the quadratic form measuring the observability degree of the system under B.

Matthias Hieber

A characterization of the growth bound of a semigroup via Fourier multipliers

Let $\omega_0(T)$ be the growth bound of a C_0-semigroup on a Banach space X with generator A. We show that

$$\omega_0(T)=\inf\{\mu>s(A);\sup_{\alpha\geq\mu}||R(\alpha+i\cdot,A)||_{\mathcal{M}_p}<\infty\}$$

where $R(\alpha+i\cdot,A)$ denotes the resolvent of A. This generalizes a known result of J. Prüß in Hilbert spaces to arbitrary Banach spaces.

Peer Christian Kunstmann

L_p-spectral properties of the Neumann Laplacian on horns, comets and stars

We study L_p-spectral properties of Neumann Laplacians on some planar domains and show by calculation that the essential spectrum of the Neumann Laplacian on certain horns depends on

p. The proof uses ideas due to E.B. Davies and B. Simon for the reduction to one-dimensional operators and techniques involving Gaussian bounds. For domains looking like comets or stars, i.e. having countably many horn-shaped outlets, we prove a decoupling-reduction result. These results are used to construct planar domains for which the Neumann Laplacian has maximal L_p-spectrum in the class of generators of symmetric submarkovian semigroups.

Philippe Laurençot

Global solutions to viscous Hamilton-Jacobi equations with irregular or singular initial data

We consider the Cauchy problem

$$\begin{aligned} u_t - \Delta u + |\nabla u|^p &= 0 \quad \text{in } (0,+\infty) \times \mathbb{R}^N, && (1) \\ u(0) &= u_0 \quad \text{in } \mathbb{R}^N, && (2) \end{aligned}$$

where $p \in (0;+\infty)$. Existence and uniqueness of non-negative weak solutions are shown when $1 < p < (N+2)/(N+1)$, the initial datum u_0 being in the space of bounded and non-negative measures. Within the same range of values of p the existence and uniqueness of a very singular solution at the origin (in the sense of Brezis, Peletier and Terman) are also obtained. Some issues related to the large time behaviour of the solutions to (1)–(2) will be presented, together with the extinction in finite time of non-negative and integrable solutions when $0 < p < N/(N+1)$. These results are joint works with S. Benachour (Nancy), H. Koch (Heidelberg) and D. Schmitt (Nancy).

Alessandra Lunardi

Stability in parabolic free boundary problems

We consider a class of free boundary parabolic problems, such as for instance (unknowns: the open sets $\Omega_t \subset \mathbb{R}^{\mathbb{N}}$, $t > 0$ and the function u)

$$\begin{aligned} u_t(t,\eta) &= \Delta u(t,\eta) + f(u(t,\eta), Du(t,\eta)) && t \geq 0, \eta \in \Omega_t \\ u(t,\eta) &= g_0(\eta) && t \geq 0, \eta \in \partial\Omega_t \\ \frac{\partial u}{\partial \nu}(t,\eta) &= g_1(\eta) && t \geq 0, \eta \in \partial\Omega_t \end{aligned}$$

with initial condition $(\Omega_0, u(0,\eta))$. Under a transversality assumption on data g_0, g_1 ($g_1 \neq \partial g_0/\partial\nu$ on $\partial\Omega_0$) we describe a method, developed in collaboration with C.-M. Brauner and others, which gives local existence and uniqueness of regular solutions for initial data near stationary solutions (or near travelling wave solutions in the case of unbounded domains) and lets us study the stability of such stationary or travelling wave solutions. This method has been applied to some popular mathematical models in combustion theory, such as the DDT (= Deflagration-Detonation Transition) model and the Sivashinsky model.

Alexander Mielke

On the dynamics of nonlinear parabolic problems on unbounded domains

We consider problems on domains $\Omega_n \subset \mathbb{R}^d$, where $\Omega_n \to \mathbb{R}^d$ in the sense $B(0, R_n) \subset \Omega_n$ with $R_n \to \infty$. Our parabolic systems are assumed to define global semiflows $(S_t^{\Omega_n})_{t\geq 0}$ which admit a global attractor $\mathcal{A}^{\Omega_n} \subset L^\infty(\Omega_n)$. Using weighted norms with decaying weights we obtain upper semicontinuity $\mathrm{dist}_{\mathrm{weighted}}(\mathcal{A}^{\Omega_n}, \mathcal{A}^{\mathbb{R}^d}) \to 0$ for $n \to \infty$.

We discuss a counterexample for lower semicontinuity.

The reported results are partially joint work with G. Schneider (Bayreuth) and Th. Gallay (Orsay).

Enzo Mitidieri

Sharp results on blow-up solutions of parabolic inequalities

We discuss various recent results on blow-up solutions for degenerate (singular) parabolic and hyperbolic inequalities.

In particular sharp results are obtained for some well known interesting models. For instance for a special case effect consider non negative solutions (weak) of the following equation ($n \geq 1$)

$$\begin{cases} u_t - \operatorname{div}\left(\dfrac{Du}{\sqrt{1+|Du|^2}}\right) = u^p & \text{in } \mathbb{R}^n \times (0,\infty) \\ \qquad\qquad\qquad\qquad u \geq 0 & \text{on } \mathbb{R}^n \times (0,\infty) \end{cases}$$

It is well known that if $1 < p < 1 + \frac{2}{n}$ then blow-up takes place and if $p > 1 + \frac{2}{n}$ there exist global solutions if the initial value decays sufficiently fast at infinity.

We establish that $p = 1 + \frac{2}{n}$ belongs to the blow-up case.

Other problems of the form (no assumptions on the sign of u)

- $u_t - |x|^\tau \Delta u \geq |u|^q \qquad 0 \leq \tau \leq 2$

and

- $u_{tt} - |x|^\tau \Delta u \geq |u|^q \qquad 0 \leq \tau \leq 2$

are discussed. This is part of a series of joint papers with Stanislav Ivanovich Pohozaev (Steklow Institute - Moskow).

Tetsuro Miyakawa

Navier-Stokes equations and Hardy spaces

I would like to talk about some recent progress on the space-time asymptotic behaviour of weak and strong solutions to the Navier-Stokes equations in the whole space and in the half-space. In the ordinary functional-analytic approach, one usually deals with the solutions of evolution PDE's as a curve in specific function spaces, thereby ignoring the intuitive behaviour with respect to space variables. However, in the case of the whole space and the half-space, the kernel functions of the Stokes semigroup have concrete representations, and so one can describe the behaviour of solutions of the nonstationary Navier-Stokes equations with respect to space and time variables.

For instance, consider the Cauchy problem. If the initial data a satisfy some specific conditions, then the corresponding strong solutions behave like

$$|x|^{-\alpha}t^{(\gamma-\alpha)/2}$$

where $0 \leq \alpha \leq \gamma$ is arbitrary and $1 \leq \gamma \leq n+1$ is a fixed number determined by a. Furthermore, if $\gamma = n+1$ and if $(1+|x|)|a| \in L^1$, then one can expand, up to and including order n (the space dimension), the solutions u in terms of *derivatives* of Gaussian-like functions. (This expansion holds in $L^1 \cap L^\infty$.) Even when we assume only that $a \in L^2$ and $(1+|x|)|a| \in L^1$, one can deduce the first-order expansion (in L^2 norm) of the type described above for weak solutions. This result enables us to give an answer to the problem of Schonbek concerning the lower bound of rates of energy decay.

It is also possible to deduce the first-order expansion for weak and strong solutions of the Navier-Stokes equations in the half-space. We hope that the result will be applicable to investigating the asymptotic behaviour in more detail, as in the case of the Cauchy problem.

The works described above are done jointly with Y. Fujigaki (Kobe), or M. E. Schonbek (Santa Cruz).

Sylvie Monniaux
Existence and uniqueness of mild solutions of the Navier-Stokes system in Lipschitz domains

The proof of uniqueness of mild solutions (in the critical space) for the Navier-Stokes system is given. The main tool of the proof is the maximal L^p-regularity for the Stokes operator, together with other regularising properties, comparable to the Dirichlet-Laplacian ones.

Jan M. A. M. van Neerven
The stochastic abstract Cauchy problem in Banach spaces

We formulate necessary and sufficient conditions for existence and uniqueness of weak solutions for the problem

$$(*) \qquad \begin{cases} dX_t = AX_t dt + BdW_t^H, t \geq 0 \\ X_0 = x \end{cases}$$

where A generates a C_0-semigroup $\{S(t)\}_{t\geq 0}$ on a separable real Hilbert space, H is a separable real Hilbert space, $(W_t^H)_{t\geq 0}$ is a standard cylindrical Wiener process with values in H, and $B \in \mathcal{L}(H, E)$ is a bounded linear operator.

We can define a semigroup $\{P(t)\}_{t\geq 0}$ on the space of bounded Borel-measurable functions f on E by

$$P(t)f(x) = \mathbb{E}(f(X_t(x))), \quad t \geq 0, x \in E,$$

where $(X_t(x))_{t\geq 0}$ is the weak solution of $(*)$. A proof of the following uniqueness result for invariant Gaussian measures for $\{P(t)\}_{t\geq 0}$ is sketched: If the minimal invariant measure μ_∞ in nondegenerate and the adjoint orbit $t \mapsto S^*(t)x^*$ is bounded for all $x^* \in \bigcap_{n\geq 0} D(A^{n*})$, then μ_∞ is the unique Gaussian invariant measure for the semigroup $\{P(t)\}_{t\geq 0}$.

El-Maati Ouhabaz
Gaussian upper bounds for heat kernels and spectral multipliers

A well known result of Hörmander gives a sufficient condition on a function $f : [0, \infty) \to \mathbb{C}$ such that the operator $f(-\Delta)$ extends from $L^2(\mathbb{R}^d)$ to all $L^p(\mathbb{R}^d)$ with $1 < p < \infty$. We extend this result to all self-adjoint operators (acting on any domain of a space of homogeneous type) whose heat kernel satisfies a Gaussian estimate.

Michel Pierre
Second shape derivatives and stability of critical shapes

In calculus of variations, it is classical to study the positivity of the second derivative to decide whether a critical point is a minimum point. In shape optimization where the variable is a subset of

$\mathbb{R}^N$, specific difficulties arise due to some norm-incompatibility: the coercivity-norm is often weaker than the differentiability-norm. We will discuss this problem and show how it can be overcome on significant examples. In particular, we will give an estimate of the variation of the second derivative around a critical shape which is interesting for itself.

Carolus J. Reinecke

Stationary solutions to an elliptic system of FitzHugh-Nagumo type

We consider the following system of semilinear elliptic equations:

$$(\mathrm{P}_\lambda) \qquad \begin{cases} -\Delta u &= \lambda\,(f\,(u) - v) & \text{in } \Omega; \\ -\Delta v &= \lambda\,(\delta u - \gamma v) & \text{in } \Omega; \\ u &= v \,=\, 0 & \text{on } \partial\Omega. \end{cases}$$

In this system the parameters λ, δ and γ are assumed to be positive and Ω is a bounded domain in $\mathbb{R}^N$ with a C^3 boundary. The nonlinearity is given by $f\,(u) = u\,(1-u)\,(u-a)$ with $0 < a < 1/2$. This system is motivated by the FitzHugh-Nagumo equations

$$\begin{cases} u_t &= u_{xx} + f\,(u) - v & (x,t) \in I \times \mathbb{R}^+; \\ v_t &= \delta u - \gamma v & (x,t) \in I \times \mathbb{R}^+, \end{cases}$$

with appropriate boundary and initial conditions prescribed and with I an interval in $\mathbb{R}$.

Our main result for system (P_λ) is the existence, under some conditions on the parameters δ and γ, of a smooth curve, parameterised by λ, of nondegenerate positive solutions to the system. These solutions have boundary layers of $o(\lambda^{-1/2})$. Using the nondegenerateness of the solutions we also prove stability and uniqueness results.

A first step in our analysis is to show that under natural conditions on γ and δ, the system can be transformed and the nonlinearity f can be modified in order to obtain a quasimonotone system. Solutions to this new system, in an appropriate range then correspond to solutions of the original system. The main point about this transformation is that quasimonotone systems satisfy a maximum principle, similar to that of scalar equations. One then uses, among other things, sub- and supersolutions and sweeping principle type arguments to prove the theorems.

Roland Schnaubelt

Exponential dichotomies of parabolic evolution equations

We study the long–term behaviour of the parabolic evolution equation

$$(\mathrm{CP}) \qquad u'(t) = A(t)u(t) + f(t), \quad t > s, \qquad u(s) = x,$$

which is assumed to be asymptotically autonomous in the sense that $f(t) \to f_\infty$ and $(w - A(t))^{-1} \to (w - A)^{-1}$ as $t \to \infty$ for a sectorial operator A. Moreover we suppose that $\sigma(A) \cap i\mathbb{R} = \emptyset$. It is shown that the evolution family solving the homogeneous problem has exponential dichotomy and the solution of (CP) converges to 'stationary solution at infinity', i.e.

$$u(t) \to u_\infty := -A^{-1} f_\infty, \qquad u'(t) \to 0, \qquad A(t)u(t) \to Au_\infty$$

as $t \to \infty$.

Frédérique Simondon
Convergence for degenerate parabolic equations

We consider the problem:

$$\begin{cases} u_t = \Delta(\varphi(u)) - f(u) & \text{in } \Omega \times (0,\infty) \\ u = 0 & \text{on } \partial\Omega \times (0,\infty) \\ u(0) = u_0 \geq 0 \end{cases} \tag{1}$$

Question: Does any positive, bounded, global solution converge when $t \to +\infty$.

We consider the case where $\varphi(u) = u^m$ and f satisfies

- $f \in C^1(\mathbb{R}^+) \cap C^\infty(0,\infty)$, $f(0) \leq 0$
- f can be extended by an analytic function on a sector $S = \left\{ s \in \mathbb{C}; 0 < |s| < b, \arg s \in (-\alpha(b), \alpha(b)), \alpha(b) \in (0, \frac{\pi}{2}) \right\}$

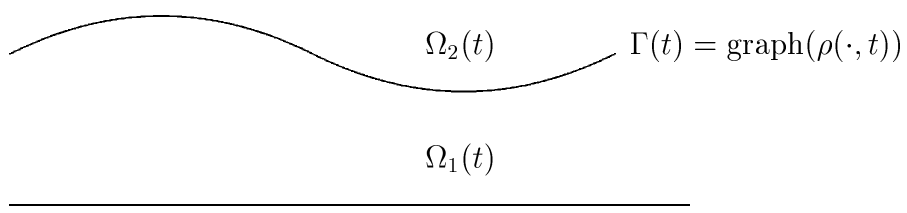
Then for any positive bounded global solution of (1), $u(t)$, there exists w solution of

$$\begin{cases} \Delta(\varphi(w)) - f(w) = 0 & \Omega \\ w = 0 & \partial\Omega \end{cases} \tag{2}$$

such that $u(t) \to w$ in $C(\overline{\Omega})$ for $t \to \infty$.

Gieri Simonett

On the Stefan problem with surface tension



We consider the two-phase Stefan problem with surface tension:

$$(*) \qquad \begin{cases} \partial_t u - \Delta u = 0 & \text{in } \mathbb{R}^{n+1} \setminus \Gamma(t) \\ u + \operatorname{div}\left(\dfrac{\nabla \rho}{\sqrt{1 + |\nabla \rho|^2}} \right) = 0 & \text{on } \Gamma(t) \\ V - [\partial_\nu u] = 0 & \text{on } \Gamma(t) \\ u(0) = u_0 & \text{in } \Omega(0) := \mathbb{R}^{n+1} \setminus \Gamma_0 \\ \Gamma(0) = \Gamma_0 & \end{cases}$$

where V is the normal velocity of $\Gamma(t)$ and $[\partial_\nu u]$ is the jump of the normal derivative of u across $\Gamma(t)$.

Theorem (Escher, Prüß, Simonett 1999). *Suppose that*

- $p > 3 + n$, $T > 0$ *are fixed,*
- $\rho_0 \in W_p^{4-\frac{3}{p}}(\mathbb{R}^n), u_0 \in W_p^{2-\frac{2}{p}}(\Omega(0))$,
- $u_0|_{\Gamma_0} =$ *mean curvature of* Γ_0, $[\partial_\nu u_0] \in W_p^{2-\frac{6}{p}}(\Gamma_0)$,
- ρ_0, u_0 *satisfy some appropriate smallness conditions.*

Then, problem $(*)$ *has a unique solution* (u, ρ) *and* (u, ρ) *are analytic functions in space and time.*

Pavel E. Sobolevskii
Well-posedness of elliptic difference equations

The elliptic difference equations of the second order

$$v_{i,j} - [(v_{i+1,j} - 2v_{i,j} + v_{i-1,j})h^{-2} + (v_{i,j+1} - 2v_{i,j} + v_{i,j-1})h^{-2}] = f_{i,j} \tag{1}$$

($i, j = \overline{-\infty, +\infty}, 0 < h < 1$) is considered as operator equation

$$v^h - (D_1^{h,2} v^h + D_2^{h,2} v^h) = f^h \tag{2}$$

in the Banach space C^b of bounded scalar functions

$$\varphi^h = (\varphi_{i,j}; i, j = \overline{-\infty, +\infty}) \tag{3}$$

with norm

$$\|\varphi\|_{C^b} = \sup_{i,j=\overline{-\infty,+\infty}} |\varphi_{i,j}| \tag{4}$$

It is established that for any $f^h \in C^b$ equation (2) has the unique solution $v^h \in C^b$ and

$$\sup_{f^h \in C^b, \|f\|_{C^b}=1} \{\|v^h\|_{C^b} + \|D_1^{h,2} v^h\|_{C^b} + \|D_2^{h,2} v^h\|_{C^b}\} = m \tag{5}$$

for some $0 < m < +\infty$ not depending of $0 < h \leq 1$. This result leads us to the almost exact estimate of convergence rate of approximate solutions of elliptic differential equation

$$v(x_1, x_2) - \partial^2 v(x_1, x_2)/(\partial x_1)^2 - \partial^2 v(x_1, x_2)/(\partial x_2)^2 = f(x_1, x_2)$$

on the plane $\mathbb{R}^2$ in the coercive difference norm.

Hermann Sohr
Regularity problems of the Navier-Stokes equations

The known partial regularity results of the Navier-Stokes equations for bounded domains can be extended to (smooth) unbounded domains. This requires to prove the strong energy inequality for unbounded domains. The proof rests on maximal regularity properties for the Stokes evolution equation in a new class of Banach spaces.

Kazuaki Taira

Feller Semigroups and Elliptic Boundary Value Problems

This talk provides a careful and accessible exposition of the functional analytic approach to the problem of construction of Markov processes with boundary conditions in probability theory. Our approach is distinguished by the extensive use of the ideas and techniques characteristic of the recent developments in the theory of partial differential equations. In this talk we construct a Feller semigroup corresponding to such a diffusion phenomenon that a Markovian particle moves both by jumps and continuously in the state space until it "dies" at the time when it reaches the set where the particle is definitely absorbed. The following diagram gives a bird's eye view of Markov processes, semigroups, boundary value problems and how these relate to each other:

Probability	Functional Analysis	Partial Differential Equations
Markov process (X_t)	Feller semigroup $T_t f(\cdot) = \int p_t(\cdot, dy) f(y)$ $p_t(x, dy)$ Markov transition function	Infinitesimal generator A $T_t = \exp[tA]$
Markov property Starting afresh property	Semigroup property $T_{t+s} = T_t \cdot T_s$	Waldenfels operator Ventcel' boundary condition

Vincenzo Vespri

Application of semigroup theory to finance

We considered equations of the type

$$u_t = \sum_{i,j} a_{ij}\, D_{ij} u + \sum_i b_i\, D_i u - \gamma^2\, u$$

where a_{ij}, b_i, γ^2 are polynomials. Under suitable conditions we are able to prove existence, regularity, uniqueness of the solution. Such kind of equations arise in option prising.

Lutz Weis

Maximal L_p-regularity of parabolic evolution equations

We give a characterization of maximal L_p-regularity in terms of $\mathcal{R}$-bounded operator sets: $\mathcal{T} \subset B(X)$ is called $\mathcal{R}$-bounded, if for all $x_i \in X, T_i \in \mathcal{T}$

$$\int \left\| \sum r_n(s) T_n x_n \right\| ds \leq C \int \left\| \sum r_n(s) x_n \right\| ds$$

Theorem 1. *For a bounded analytic semigroup T_t on X the following are equivalent*

(i) *A has maximal L_p-regularity*

(ii) $\{tR(it, A) : t \in \mathbb{R}\}$ *is $\mathcal{R}$-bounded*

(iii) $\{T_{te^{i\varphi}} : t > 0, |\varphi| < \delta\}$ *is $\mathcal{R}$-bounded for some $\delta > 0$*

The known kriteria for L_p-regularity can be derived in a unified way from these conditions. A proof of Theorem 1 can be based on the following variant of the Dore-Venni theorem. (Joint work with Nigel Kalton.)

Theorem 2. *Assume that B has $H_\infty(\Sigma_{\theta_B})$-calculus and A is $\mathcal{R}$-sectorial with angle θ_A. If $\theta_A + \theta_B < \pi$, then $A + B$ is closed on $D(A) \cap D(B)$ and*

$$\|Ax\| + \|Bx\| \leq \|(A+B)x\| \qquad \textit{for } x \in D(A) \cap D(B).$$

The method of proof gives further properties, if X has property (α). Assume that A has a $H_\infty(\Sigma_\theta)$-calculus, then

- $\{f(A) : f \in H_\infty(\Sigma_\theta), \|f\|_\infty < 1\}$ is $\mathcal{R}$-bounded
- If A is $\mathcal{R}$-sectorial with angle $\varphi < \theta$, then A has a (better) $H_\infty(\Sigma_\varphi)$-calculus.

Petra Wittbold

Renormalized entropy solutions of scalar conservation laws on bounded domains

We study the problem

$$\text{(P)} \qquad \begin{cases} u_t + \operatorname{div} \Phi(u) = f & \text{on } Q = (0,T) \times \Omega \\ \quad + \text{ boundary and initial conditions} \end{cases}$$

where Ω is a bounded, open domain of $\mathbb{R}^N$, $N \geq 1$, with Lipschitz boundary, $\Phi \in C(\mathbb{R}; \mathbb{R}^N)$. In the case of bounded initial data, $f \in L^1(0, T; L^\infty(\Omega))$, well-posedness of the problem has been proved by J. Carrillo in the class of (bounded) entropy solutions. For unbounded data, in general, there does not exist an entropy solution. We introduce the new notion of renormalised entropy solution of (P). It is shown that this notion of solution is a generalisation of the notion of bounded entropy solution. Existence and uniqueness of a renormalised entropy solution is shown for arbitrary L^1-data.

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