# Mathematisches Forschungsinstitut Oberwolfach 

Tagungsbericht 25/2000

## Topics in Classical Algebraic Geometry

18.06.-24.06.2000

The conference was organized by David Eisenbud, Joe Harris and Frank-Olaf Schreyer with an emphasis on topics around rationality questions and explicite equations of algebraic varieties. In selecting the speakers (as well as the participants) precedence was given to the bright young people in the field, for example Hans-Christian von Bothmer, Andreas Gathmann, Tom Graber, Mircea Mustata and Jason Starr.

The number of the talks was kept to four per day, each of 50 minutes to allow plenty of time for discussion and to encourage questions at the end of the talks. Perhaps partly because of these policies, the attendance at the talks was very high. There were also many lively discussions among the members between the talks, and several research projects moved forward in this time.

The enthusiasm of the participants, the level of activity in discussions among them, and the quality of the talks, made us feel that this was a highly successfull conference.

## Abstracts

## Factorization of birational maps

Dan Abramovich

Let $X_{0}$ and $X_{\infty}$ be complexe projective manifolds, and $\phi: X_{0}-\rightarrow X_{\infty}$ a birational map.
Theorem. There exist complex projective manifolds $X_{1}, X_{2}, \ldots, X_{n}=X_{\infty}$, $X_{01}, X_{12}, \ldots, X_{n-1, n}$ and a diagram

where $X_{i, i+1} \rightarrow X_{i}$ and $X_{i, i+1} \rightarrow X_{i+1}$ are blowings-up with nonsingular center and the composite rational map $X_{0} \rightarrow X_{n}=X_{\infty}$ is $\phi$.

Two proofs are known, one by Jarosław Włodarczyk and one by Abramovich-Karu-Matsuki-Włodarczyk.

The proofs have the ingredients:
(a) toric factorization (Włodarczyk, Morelli)
(b) birational cobordism: a $\mathbb{C}^{*}$ - variety $B$ with certain open sets $B_{+}, B_{-}$such that $B_{-} / \mathbb{C}^{*} \simeq X_{0}$ and $B_{+} / \mathbb{C}^{*} \simeq X_{\infty}$
(c) decomposition of birational cobordisms into elementary pieces (closely related to variation of GIT quotient or symplectic reduction).

Włodarczyk's proof relies also on a new theory of stratified toroidal embeddings and the torific ideal of Abramovich- de Jong.

## Geometric Syzygies of canonical Curves

Hans-Christian v. Bothmer

For any canonical curve $C \subset \mathbb{P}^{g-1}$ and any linear System $|D|$ of Cliffordindex $k$ with $\operatorname{dim}|D| \geq 1$ and $\operatorname{dim}|K-D| \geq 1$ Green and Lazarsfeld construct certain "geometric" Quadrics/Syzygies in the $(g-k)^{t h}$ step of the resolution of the canonical curve $C$.

In this context Green's Conjecture reads:

> no Green-Lazarsfeld
> Syzygies in step $k$$\Longleftrightarrow$ no $k^{\text {th }}$ Syzygies at all

An obvious generalisation ist

$$
\operatorname{span}\binom{\text { Green-Lazarsfeld }}{\text { Syzygies in step } k}=\text { All } k^{\text {th }} \text { Syzygies }
$$

This is known for the $1^{\text {st }}$ step (i.e Quadrics) of all curves by Green and shown for the $2^{\text {nd }}$ step (i.e $1^{\text {st }}$ Syzygies) of the general curve in this talk.

## Deformation and Semiregularity

> Ragnar-Olaf Buchweitz (with H.Flenner)

We first discussed the general problem how to bound from below the dimension of the base space of a semiuniversial deformation of a deformation problem such as embedded deformations of subspaces or deformations of coherent sheaves. We presented various bounds

1. in terms of an obstruction theory, if such exists
2. in terms of curvilinear deformations
3. through the dimension of the kernel of a natural transformation from an obstruction theory to a left exact functor.

Our treatment streamlines and generalizes varous earlier results in this area by Ran, Kawamata, Fantechi-Manetti and others. As a major application of the general theory we construct a general semiregularity map

$$
\left(\sigma_{n}\right)_{n \in \mathbb{N}}: E x t_{X}^{2}(\mathcal{F}, \mathcal{F}) \rightarrow \prod_{n \geq 0} H^{n+2}\left(X, \Lambda^{n} \mathbb{L}_{X}\right)
$$

where $X$ is a complex space and $\mathcal{F}$ a coherent $\mathcal{O}_{X}$-module, $\Lambda^{n} \mathbb{L}_{X}$ the indicated exterior power of the cotangent complex. The component $\sigma_{0}$ is the trace $E x t_{X}^{2}(\mathcal{F}, \mathcal{F}) \rightarrow H^{2}\left(X, \mathcal{O}_{X}\right)$ and the higher components are obtained from the Atiyah-Chern character $\exp (-a t(\mathcal{F}))$, where $\operatorname{at}(\mathcal{F}) \in \operatorname{Ext}_{X}^{1}\left(\mathcal{F}, \mathcal{F} \otimes \mathbb{L}_{X}\right)$ is the Atiyah class of $\mathcal{F}$. The resulting application to the semiuniversal deformation of $\mathcal{F}$ generalizes results by Artamkin-Mukai and in case $\mathcal{F}=\mathcal{O}_{Z}$ for a closed subspace $Z \subset X$, results of Severi, Kodaira-Spencer, Bloch, Ran, Kawamata.

## Focal Loci of Algebraic Varieties

Fabrizio Catanese (with Cecilia Trifogli)
The talk was devoted to illustrate some example of interplay between extrinsic differential and algebraic geometry, in particular

## 1. Focal Loci of Algebraic Varieties

2. Theory of Dual Varieties and Cayley Forms.

Given $X^{\prime} \subset \mathbb{C}^{m}$, an Euclidean space with a non degenerate quadric form $Q_{\infty}$, (and associated scalar product $<,>$ ) one defines the Euclidean normal bundle, if $X^{\prime}$ is smooth, as

$$
\left\{(x, y) \mid x \in X^{\prime},(y-x) \text { is a normal vector to } X \text { at } x\right\}
$$

One defines the Euclidean normal bundle of $X \subset \mathbb{P}^{M}, N X$ as the closure of the above locus, where $X^{\prime}=\left(X-\mathbb{P}_{\infty}^{m-1}\right)_{\text {smooth }}$. Since $N X \subset X \times \mathbb{P}^{m}$ the second projection

$$
\epsilon: N X \rightarrow \mathbb{P}^{m}
$$

is a morphism between varieties of the same dimension, and one defines $\varphi_{X}$ as its ramification locus, finally

$$
\Sigma_{X}=\text { Focal Locus of } X:=\epsilon\left(\varphi_{X}\right) .
$$

This notion extends the classical notion of evolute of plane curves, for which an algebraic theory was given by Cayley, and one century later by Fantechi. In general $\Sigma_{X}$ is expected to be a hypersurface. We classify the degenerate cases where

1. $\epsilon$ is not surjective
2. some component of $\Sigma_{X}$ is not a hypersurface.

The theorems are too long to reproduce, as well as the formula for $\operatorname{deg}\left(\Sigma_{X}\right)$ when $\Sigma_{X}$ is a hypersurface. However we have the simple

Theorem. Let $X$ be orthogonally general ( $X$ smooth, $X$ transversal to $\mathbb{P}_{\infty}^{m-1}$ and $Q_{\infty} \subset$ $\left.\mathbb{P}_{\infty}^{m-1}\right)$. Then $\operatorname{dim} \Sigma_{X}<m-1 \Longleftrightarrow X$ is a linear space of $\operatorname{dim}>0$.

We gave the proof and remarked that if $X$ is smooth, then for a general $g \in P G L(m+1)$, $g X$ is orthogonally general, and exposed many related results.

## Kummer surfaces, old and new

## Igor Dolgachev

Using geometry of Kummer surfaces we construct a rational self-map of degree 16 of the moduli space $\mathcal{M}_{2}$ of curves of genus 2 . This map assigns to a curve of genus 2 another curve of genus 2 together with one of a pair of points $\left(p, p^{\prime}\right)$ such that $\left|p+p^{\prime}\right|=K,|3 K-5 p| \neq \emptyset$, $p \neq p^{\prime}$. The number of such pairs is 16 .

The construction uses two different interpretations of the moduli space of principally polarized abelian surfaces as the moduli space of lattice polarized K3-surfaces. The first moduli space corresponds to Kummer surfaces, the second one to K3-surfaces with Picard lattices
isomorphic to $U \perp E_{8} \perp E_{7}$. We explained a construction of the latter surfaces as double covers of a quadric ramified along a genus 2 curve of degree 6 with peculiar configuration of two cusps.

## Complexity of ideal sheaves

Lawrence Ein

Let $X$ be a smooth complex variety and $I \subset \mathcal{O}_{X}$ be a coherent sheaf of ideal.
Theorem (Ein, Lazarsfeld, Smith). Let $e=\max _{p \in A s s \mathcal{O}_{X / I}}\left\{\operatorname{dim}\left(\mathcal{O}_{X / I}\right)_{p}\right\}$. Then $I^{(k e)} \subset$ $I^{k}$ for each positive integer $k$.

Definition. Let $X$ be an irreducible projective variety and $H$ be an ample Cartier divisor on $X$. Suppose that $I \subset \mathcal{O}_{X}$ be a coherent sheaf of ideals. Let $f: B l_{I} X \rightarrow X$ be the blowup and we denote by $E$ the exceptional divisor. The s-invariant of $I$ with respect to $I$ is defined as

$$
s_{H}(I)=\inf \left\{t \mid \pi^{*} t H-E \text { is } n e f\right\} .
$$

Theorem (Cutkosky, Ein, Lazarsfeld). Let X be an irreducible projective variety over an infinite field $k$ and $H$ a very ample divisor on $X$. Then

$$
\lim _{p \rightarrow \infty} \frac{\operatorname{Reg}_{H}\left(I^{p}\right)}{p}=\lim _{p \rightarrow \infty} \frac{d_{H}\left(I^{p}\right)}{p}=s_{H}(p)
$$

where $\operatorname{Reg}_{H}\left(I^{p}\right)=\min \left\{k \mid H^{i}\left(I^{p} \otimes \mathcal{O}_{X}((k-i) H)\right)=0\right.$ for alli $\left.>0\right\}$ and $d_{H}\left(I^{p}\right)=\min \left\{k \mid I^{p} \otimes\right.$ $\mathcal{O}_{X}((k H))$ is generated by global sections $\}$.

## Absolute and relative Gromov-Witten invariants

## Andreas Gathmann

For a smooth hypersurface $Y$ in a smooth projective variety $X$, the relative GromovWitten invariants are the (possibly virtual) numbers of curves in $X$ that intersect $Y$ with given multiplicities and satisfy some additional incidence conditions with subvarieties. For the case of a very ample hypersurface and curves of genus zero, we sketch an algebrogeometric construction of these invariants and show how they are related to the (absolute) Gromov-Witten invariants of $X$ and $Y$. These relations are always sufficient to compute the Gromov-Witten invariants of the hypersurface from those of the ambient space in a straightforward way. This establishes a new and entirely geometric proof of the "mirror principle" in the case of hypersurfaces with non-positive canonical bundle, and indicates how one can try to generalize this mirror transformation to arbitrary hypersurfaces (and, in the best of all worlds, to higher genus of the curves).

# Hurwitz Numbers and Hodge integrals 

Tom Graber (with Ravi Vakil)

We prove a formula discovered by Eckedahl, Lando, Shapiro and Vainshtein expressing Hurwitz numbers in terms of integrals over the moduli space of pointed curves. Specifically, if we set

$$
H_{g}^{\alpha}=\#\left\{\begin{array}{c}
\text { branched covers of } \mathbb{P}^{1} \text { with ramification of type } \alpha \text { at } \infty \\
\text { and simple branching at } r \text { other specified points }
\end{array}\right\}
$$

then

$$
H_{g}^{\alpha}=\frac{r!}{\# \operatorname{Aut}(\alpha)} \cdot \prod \frac{\alpha_{i}^{\alpha_{i}}}{\alpha_{i}!} \int_{\bar{M}_{g, n}} \frac{c(\mathbb{E})}{\prod\left(1-\alpha_{i} \psi_{i}\right)}
$$

Here $\mathbb{E}$ denotes the Hodge bundle and $\psi_{i}$ is the first Chern class of the $i^{\text {th }}$ cotangent line bundle. Our proof is based on virtual localization on the moduli space of stable maps.

## Regularity of Curves in $\mathbb{P}^{3}$

Shigeru Mukai

A curve $C$ in $\mathbb{P}^{3}$ is called $m$ - regular if $H^{1}\left(\mathbb{P}^{3}, \mathcal{I}_{C}(m-1)\right)=H^{2}\left(\mathbb{P}^{3}, \mathcal{I}_{C}(m-2)\right)=0$. This implies among all that the homogeneous ideal of $C$ is generated by its component of degree $\leq m$. Hence $C \subset \mathbb{P}^{3}$ is an intersection of surfaces of degree $m$ and has no $(m+1)$ - secant lines. In extremal case the converse holds:

Theorem (Castelnuovo,1893). $C$ is $(d-1)$ - regular if $C$ is not planar where $d=$ $\operatorname{deg}\left[C \subset \mathbb{P}^{3}\right]$.

Theorem (Gruson, Lazarsfeld, Peskine 1983). $C$ is $(d-2)$ - regular if $C$ has no ( $d-$ 1) - secant lines.

In general, non-existence of $(m+1)$ - secant lines is not sufficient but we have the following
Theorem. Assume $n \leq \frac{d}{2}-1$ and
(A) $C \subset \mathbb{P}^{3}$ has no $(d-n+1)-$ secant lines,
(B) $C$ has no $g_{n-1}^{2}$, i.e. linear not of degree $n-1$, and
(C) the number of $g_{n}^{2}$ is finite and there are only 1 -dimensional families of $g_{n+1}^{2}$.

Then $C \subset \mathbb{P}^{3}$ is $n$ - regular.

Theorems of Castelnuovo and GLP are special cases of $n=1,2$, respectively.

## Jet spaces of l.c.i. rational singularities

Mircea Mustata

The $m^{\text {th }}$ jet space of a variety $X$ parametrises the $k[t] /\left(t^{m+1}\right)$-valued points of $X$. We give a proof of the following:

Theorem. If $X$ is a l.c.i. variety $/ \mathbb{C}$, then $X_{m}$ is irreducible for every $m \geq 1$ iff $X$ has rational singularities.

The idea of the proof is to embed $X$ in a smooth variety $Y$, take an embedded resolution of singularities $\tilde{Y} \rightarrow Y$ and compare suitable motivic integrals on $Y$ and $\tilde{Y}$.
We discuss applications of Eisenbud and Frenkel to the case when $X$ is a nilpotent cone of a simple Lie algebra.

## Trigonal Curves and $\operatorname{Spin}(8)$ Bundles

William Oxbury

The Problem: to give a moduli interpretation of a unique Heisenberg invariant quartic $Q \subset|2 \Theta|=\mathbb{P}^{15}$ with the property that $\operatorname{Sing}(Q)$ contains embedded $S U_{C}(2)$, the moduli space of rank 2 vektor bundes with trivial determinant, where $C$ is a curve of genus 4 (Oxbury-Pauly 1999). $Q$ is an analogue of the Kummer $(g=2)$ and Coble $(g=3)$ quartics.
The Canidate: moduli space $\mathcal{N}_{C} . C$ a trigonal curve (i.e. any curve of genus 4) studied in joint work with S. Ramanan. $\mathcal{N}_{C}$ is a moduli variety for "Chevalley bundles", that is $\operatorname{Spin}(8)$ bundles on the Galois-Closure $C!\rightarrow \mathbb{P}^{1}$ equivariant for the $S_{3}$ acts on $C!$ and by triviality.

Theorem. There exists an inclusion $S U_{C}(2) \hookrightarrow \mathcal{N}_{C}$ as semistable boundary, and away from $S U_{C}(2) \mathcal{N}_{C}$ is smooth of dimension $7 g-14$.
Moreover there exists a natural $J_{C}[2]$-action on $\mathcal{N}_{C}$, and for any nonzero $\eta \in J_{C}[2]$ the fixed point set is two copies of $S U_{R_{\eta}}$, where $R_{\eta}$ is the Recillon curve. There exists a commutative diagramm


An Observation: When one projects $Q \subset \mathbb{P}^{15}$ from $\mathcal{O} \oplus \mathcal{O} \in K_{n m}$, one obtains cubics $\left|\mathcal{I}_{C}^{2}(3)\right|$ where $C \stackrel{\left|K^{2}\right|}{\hookrightarrow} \mathbb{P}^{3 g-4}$. They can be identified as coming by pull-back from the secant varieties of the Severi variety $\mathbb{P}^{2}(\mathbb{A}), \mathbb{A}=\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ over $k=\mathbb{C}$. This suggests:
bicanonical curve $=$ linear section of $\bigcirc \subset \mathbb{P}^{26}$

## Toric Hilbert Schemes

Irena Peeva

This is a joint work with M. Stillman. We introduce toric Hilbert schemes. Such a scheme $H_{T}$ parametrizes all ideals with the same multigraded Hilbert function as a given toric ideal $T$. We show that $T$ lies on exactly one component of $H_{T}$ and $[T]$ is a smooth point. If $T$ has codimension 2 we prove that $H_{T}$ has one component, is $2-$ dimensional and smooth; it follows that in this case $H_{T}$ is the toric variety of the Groebner fan of $T$.

## Equations of modular curves

## Sorin Popescu (with L.Borisov and P.Gunuells)

Let $p \geq 5$ be a prime number, and let $X_{i}(p)=X\left(\Gamma_{1}(p)\right)$ be the modular curve for the congruence subgroup $\Gamma_{1}(p) \subset S L(2, \mathbb{Z})$. $X_{i}(p)$ parametrises "elliptic" curves with the choice of a non-trivial $p$-torsion point.
We show that the space of weight one Eisenstein series defines an embedding of $X_{1}(p)$ into $\mathbb{P}^{\frac{p-3}{2}}$ and show the image is scheme theoretically cut out by explicit quadrics

$$
\begin{aligned}
(p-4)\left(s_{a} s_{b}+s_{b} s_{c}+s_{c} s_{a}\right)= & 2\left(s_{a}^{2}+s_{b}^{2}+s_{c}^{2}\right)-\frac{4}{p-2} \sum_{k \neq 0} s_{k}^{2} \\
& +\sum_{k \neq 0, a} s_{k} s_{a-k}+\sum_{k \neq 0, b} s_{k} s_{b-k}+\sum_{k \neq 0, c} s_{k} s_{c-k}
\end{aligned}
$$

for all $a, b, c \in(\mathbb{Z} / p \mathbb{Z})^{*}$ with $a+b+c=0 \bmod p$ and where $\left\{s_{a}\right\}_{a \in(\mathbb{Z} / p \mathbb{Z})^{*}}$ with $s_{a}=-s_{a}$ denote the coordinates in $\mathbb{P}^{\frac{p-3}{2}}$.

## Varieties of Sums of Powers of Cubics

We consider varieties of presentations of cubic forms as a sum of $k$ cubes of linear forms for suitable $k$. More precisely we make a compactification in the Hilbertscheme:

$$
V S P(f, k)=\overline{\left.\left\{\left(l_{1}, \ldots, l_{k}\right) \in \operatorname{Hilb}_{k} \check{\mathbb{P}}^{n}\right\} \mid f=\sum_{i=1}^{k} l_{i}^{3}\right\}}
$$

where $f \in \mathbb{C}\left[x_{0}, \ldots, x_{n}\right]_{d}$.
Classically, it was known what is the minimal $k$ for which VSP is not empty when $n$ is small, also it was known what $V S P$ is in those cases $(n \leq 3)$. The classical methods of apolarity have been taken up recently by Mukai and others.

In the talk I explained a strategy that has given results for general cubic 3 - folds and 4folds, and some special results for cubic 5 - folds related to canonical curves of genus 9 . This is common work with Schreyer and with Iliev, and says that $\operatorname{VSP}(f, 8)$ for a cubic 3 - fold $F=V(f)$ is a 5 - dimensional Fano of index 1, while $\operatorname{VSP}(f, 10)$ for a cubic 4 fold is isomorphic to the variety of lines in another cubic 4 - fold. For cubic 5 - folds we show that those coming from canonical curves do not behave generically with respect to the $V S P$ question.

## The Rationality of some Non-abelian Torsors

Nick Shepherd-Barron

Torsors under algebraic tori over geometrically rational surfaces $S$ (over fields $k \neq \bar{k}$ ) have led to the construction by Beauville et al. of irrational such surfaces that are stably rational, and then to the construction of stably rational irrational 3 -folds over $\mathbb{C}$. The usual construction of these torsors is "from the bottom up", in terms of the Galois structure of $\operatorname{Pic}(S \otimes \bar{k})$. This talk described a "top down" construction of these and other (non-abelian) torsors, starting from a representation of the structure group $G$. This simplifies some of the known constructions, but still only leads to examples. It also raises the question of what the examples exemplify; in the abelian case, they are (essentially) universal in the sense of Colliot-Théline and Sensue, but in the non-abelian case things are less clear.

## Rational Curves on Hypersurfaces

Jason Starr

For a general hypersurface $X \subset \mathbb{P}_{\mathbb{C}}^{n}$ of degree $d \leq \frac{n+1}{2}$ and $n \geq 6$, one has the following
Theorem (Harris, Roth, Starr). For every $e \geq 1$ the space $R_{e}(X)$ parametrizing smooth rational curves of degree e lying on $X$ is an integral, local complete intersection scheme of dimension $(n+1-d) e+n-4$

Additionally, for every smooth cubic hypersurface $X \subset \mathbb{P}_{\mathbb{C}}^{4}$, one has the following

Theorem (Harris, Roth, Starr). For every $e \geq 1$ the space $R_{e}(X)$ is an integral, LCI scheme of dimension $2 e$

Both of these results are proved using a "deformation and specialization" argument. The chief tools used in the proof are

1. the Kontsevich moduli space of rational curves ,
2. Mori's bend-and-break-lemma, and
3. a detailed study of the space of lines on $X$.

One uses the Kontsevich space to study the deformation theory of curves on $X$. One uses Mori's bend-and-break-lemma to prove that the general member of an irreducible component of $\bar{M}_{0,0}(X, e)$ specializes to a reducible curve. Repeatedly applying this argument one reduces to the study of "configurations of lines" on $X$.

## Eisenbud-Levine-Theorem and Singular Curves in $\mathbb{P}_{2}(\mathbb{R})$

## Duco van Straten

The classical Eisenbud-Levine theorem states that the degree $\operatorname{deg}(F, 0)$ of a finite map $\operatorname{germ} F:\left(\mathbb{R}^{n}, 0\right) \rightarrow\left(\mathbb{R}^{n}, 0\right)$ is equal to the signature $\operatorname{Sign}\left(B_{\phi}\right)$, where $B_{\phi}: \mathcal{A} \times \mathcal{A} \rightarrow \mathbb{R}$ is the bilinear form $\phi(a \cdot b)$ and $\mathcal{A}=\mathbb{R}\left[\left[x_{0}, \ldots, x_{n}\right]\right] / J, J=\left(f_{0}, \ldots, f_{n}\right), \phi: \mathcal{A} \rightarrow \mathbb{R}$ with $\phi(h)>0\left(h=\operatorname{det}\left(\partial_{i} f_{j}\right)\right)$. An isolated hypersurface singularity $f \in \mathbb{R}\left[\left[x_{0}, \ldots, x_{n}\right]\right]=: P$ gives such a situation where $f_{i}:=\partial_{i} f$. For singularities with 1-dimensional singular locus one can define an Artinian Gorenstein module $I / J$ where $I=\left(J: m^{\infty}\right)$ is the saturation of $J$ with respect to the maximal ideal. If $h \notin I J$ it seems that the composition

$$
I / J \times I / J \rightarrow I^{2} / I J \hookrightarrow P / I J \xrightarrow{\phi} \mathbb{R}
$$

$(\phi(h)>0)$ expresses the self-duality of $I / J$. In some cases we can show

$$
\operatorname{sign} B_{\phi}=\chi(F>0)-\chi(F<0)
$$

$F \quad \in \quad \mathbb{R}[X, Y, Z]_{2 k} \quad$ defining a singular curv



Proofs at the moment use disentanglements.

# Hilbert-Kunz multiplicity, McKay correspondence and good ideals in 2-dimensional Rational Double Points 

Kei-ichi Watanabe

In this talk we prove:
Theorem. Let $(A, m)$ be F-rational double point of dimension 2 in char $p>0$. Let $f: X \rightarrow \operatorname{Spec}(A)$ be the minimal resolution with $Z_{0}=\sum_{i=1}^{r} n_{i} E_{i}$ fundamental cycle. $\left(m \mathcal{O}_{X}=\mathcal{O}_{X}\left(-Z_{0}\right)\right)$. Then we have:

1. If $I$ is a good ideal $\left(: \Longleftrightarrow\right.$ integrally closed and $I=\mathcal{O}_{X}(-Z)$ invertible $Z=$ $\left.\sum_{i=1}^{r} a_{i} E_{i}\right)$ then $e_{H K}(I)=l_{A}(A / I)+\sum_{i=1}^{r} a_{i} n_{i} / N$, where $N$ is the order of the "group" attached to this singularity in characteristic 0. ( $e_{H R}$ is the Hilbert-Kunz multiplicity of I)
2. If $I \subset A$ is any integrally closed m-primary ideal of $A$, then $e_{H K}(I)-l_{A}(A / I)=$ $e_{H K}\left(I^{g}\right)-l_{A}\left(A / I^{g}\right)$, where $I^{g}$ is the smallest good ideal (good closure) containing $I$.

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