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This meeting was organized by Michel Boileau (Toulouse), Klaus Johannson (Knoxville) and Heiner Zieschang (Bochum).

The main field of this conference has been different aspects of classical three–dimensional topology. In addidtion, some studies and results of two–dimensional topology and geometric group theory have been presented.

Abstracts

A proof of the Orbifold Theorem

Daryl Cooper
(joint work with C. Hodgson and S. Kerckhoff)

A **geometric orbifold** is the quotient of a homogeneous Riemannian manifold by a discrete group of isometries. A (topological) **orbifold** has the local structure of a geometric orbifold. It is locally modelled on the quotient of Euclidean space by a finite group of isometries. These models fit together in a way that resepects the local group actions. A 3–orbifold is **orbifold–irreducible** if every 2–suborbifold with positive orbifold Euler characteristic bounds the quotient of a 3–ball by a finite group. A closed 3–orbifold is **orbifold–atoroidal** if there are no orbifold–essential Euclidean 2-suborbifolds.

The Orbifold Theorem

A closed locally orientable, orbifold–irreducible, orbifold–atoroidal 3–orbifold with non–empty 1–dimensional singular locus is geometric.

The proof of the Orbifold theorem splits into two parts depending on whether or not the orbifold fundamental group is finite or infinite. The case of infinite orbifold fundamanetal group was completed during 1999. A geometric 3-orbifold with finite orbifold fundamental group is spherical. The the proof in this case has only been completed this year. The proof of the orbifold theorem is a deformation argument. One of the possible outcomes of the deformation is a Euclidean cone metric on the orbifold with cone angles which are smaller than the required orbifold angles. One must show that such an orbifold has either a spherical or $S^2 \times R$ structure. If, in addition, the orbifold has finite orbifold fundamental group it is called **pre–spherical**.

The last issue in the proof of the Orbifold Theorem has been to show that a pre–spherical orbifold is spherical. In the case that the singular locus is a 1–manifold, this was done using Hamilton's theorem on Ricci flow. However when the singular set is a graph there are technical problems (smoothing a singular Euclidean metric to obtain one with non–negative Ricci curvature) which seem to make this approach difficult.

The smoothing problem in the case the singular set is a 1–manifold is basically solved using a process which smoothes a metric on the a 2–dimensional disc with a cone point. One just "rounds" the metric at the cone point with sandpaper. This provides a way to smooth a product metric (disc with cone pt) x interval.

The next step is to observe that this smoothing is invariant under an involution which reverses the interval and is a reflection on the disc factor. In this way one is able to smooth *certain* Euclidean cone metrics at *certain* vertices. A 3–orbifold is **dihedral** if every vertex has local group a dihedral group. This leads to a proof that pre–spherical orbifolds are spherical in the dihedral case. One now deduces the non–dihedral case from the dihedral case by the technique of *re–labelling.*

The Orbifold Theorem

JOAN PORTI
(joint work with M. Boileau and B. Leeb)

The purpose of this talk is to announce a complete proof of the Orbifold Theorem and to focus on the final part of the proof. This talk is coordinated with the previous talk by D. Cooper, who obtained a different proof in a joint work with C. Hodgson and S. Kerckhoff.

The situation at the end of the proof of the orbifold theorem is the following: we have an orbifold $\mathcal{O}$ and a sequence C_n of hyperbolic cone manifolds with the same combinatorial type as $\mathcal{O}$ and increasing cone angles. The sequence C_n collapses in a uniform way: it can be re-scaled to converge to a compact Euclidean cone manifold with the same combinatorial type, and its cone angles are less than the orbifold angles. We prove that in this situation $\mathcal{O}$ is spherical. The difficult case occurs when $\mathcal{O}$ has a singular vertex whose isotropy group is not dihedral. We give an argument that uses comparison techniques and the variety of representations into $SU(2) \times SU(2) \cong Spin(4)$.

Non–integral toroidal surgeries and unknotting number one tangle sums

CAMERON GORDON
(joint work with John Luecke)

M. Eduave–Muñoz has constructed infinitely many knots K such that $K = T_1 \cup T_2$, T_i nonsplit tangles, $i = 1, 2$ and unknotting number $u(K) = 1$. Passing to the 2–fold branched cover gives hyperbolic knots in S^3 with $m/2$–Dehn surgeries giving toroidal manifolds. We show that any knot with this porperty must look very much like Eduave–Muñoz's examples.

THEOREM. *Let K be a hyperbolic knot in S^3 such that $K(m/l)$ is toroidal for some $l \geq 2$. Then*

(1) $l = 2$,
(2) *K arises from the 2–fold branched covering construction outlined above,*
(3) $K(m/2) = X^1 \cup_{\widehat{T}} X_2$, *$\widehat{T}$ an incompressible torus such that* $|\ \widehat{T} \cap$ *surgery core* $| = 2$,
(4) *X_i is a Seifert fiber space $/D^2$ with two singular fibers with multiplicities q_i, q_i', $i = 1, 2$ and*
(5) *q_1 is 2 or 3.*

COROLLARY. *If $K = T_1 \cup T_2$, with $T_i, i = 1, 2$ nonsplit tangles and $u(K) = 1$, then T_i is a sum $R_i + R_i'$ of rational tangles R_i, R_i' for $i = 1, 2$ and $R_1 = 1/2$ or $R_1 = 1/3$.*

An interesting open problem is to eliminate $q_1 = 3$ from (5).

The topology of limit sets

BRIAN BOWDITCH

A "Kleinian Group" is a group that acts isometrically and properly discontinually on hyperbolic n-space $\mathbb{H}^n$. Such a group acts by homeomorphism on its limit set $\Lambda\Gamma \subset \partial\mathbb{H}^n$. In fact, the action on $\Lambda\Gamma$ is a "convergence action" in the sense of Gehring and Martin. We describe some recent results which relate the algebraic structure of such groups to the dynamics of such actions on the topology of $\Lambda\Gamma$. Of particular interest is the case where Γ is geometrically finite. This fits into the more general context of "(relatively) hyperbolic groups" in the sense of Gromov.

The theorem states that a hyperbolic group can be characterized as a converge group acting on a perfect metrizable compactum such that every point is a canonical limit point.

Suppose that Γ is a hyperbolic group (e.g. a geometrically finite Kleinian group without parabolics). In this context Stalling's theorem tells us that $\partial\Gamma$ is connected iff Γ does not split over any finite subgroup. In this case a result of Bestvina–Mess tells us that $\partial\Gamma$ is locally connected if $\partial\Gamma$ has no global cut point.

Using ideas of Levitt and Swarup, one can show that certain convergence actions on continua have the property that every global cut point is a parabolic point. Putting this together with Bestvina–Mess, the fact that a hyperbolic group has no parabolics, we deduce that $\partial\Gamma$ is always locally connected. With some additional work, one can deduce a similar result in the relative case. In particular this applies to geometrically finite Kleinian groups.

Minimal surfaces in hyperbolic 3–manifolds

Hyam Rubinstein

By recent work of Gabai, Meyerhoff and Thurston it is known that a hyperbolic metric which is complete and finite volume is a homotopy invariant on a 3–manifold. So closed minimal surfaces embedded or immersed in 3–manifolds with such hyperbolic metrics give rise to interesting invariants (area, second fundamental form, etc.).

In work with T. Pitts (Texas A&M), a minimax method was developed to construct unstable minimal surfaces in the isotopy classes of strongly irreducible Heegard splittings, for 3–manifolds with bumpy metrics. Using techniques of B. White, this can be used to get such minimal surfaces in the case of hyperbolic metrics. $\mathbb{Z}_2$–homology can give rise to nonorientable surfaces with multiplicity 2, so must be avoided.

The monotonicity formula can then be used to obtain a lower bound for the Heegard genus of a closed hyperbolic 3–manifold in terms of the injectivity radius. For cusped manifolds or those with short closed geodesics, the injectivity radius can be computed over a 2–complex (the Ford domain).

Thurston has suggested that closed hyperbolic 3–mainfolds may all have immersed surfaces with principal curvature ≤ 1, which are π_1–injective. A polyhedral approach (work with I. Aitchison and S. Matsumoto) was given to the figure eight knot space, showing that surfaces which are immersed, normal and do not have adjacent quadrilaterals can be smoothed to have principal curvatures ≤ 1. B. Andrews (ANU) has recently shown that such surfaces flow to unique minimal surfaces.

Universal covering spaces of closed 3–manifolds are simply–connected at infinity ($\pi_1^\infty \widetilde{M}^3 = 0$)

Valentin Poénaru

The main theorem of the talk is already announced in the title. Here is a very sketchy idea about the methods of proof.

Theorem 0. *Let V^3 be an open simply-connected 3–manifold. Assume that for some $n \in Z_+$ we can find a geometrically simply–connected smooth manifold X^{n+3} such that*

$$V^3 \times 0 \subset X^{n+3} \underset{i}{\hookrightarrow} V^3 \times B^3,$$

the inclusion i being PROPER. *Then $\pi_1^\infty V^3 = 0$.*

[Here g.s.c. means without handles of index $\lambda = 1$.] For any 3-manifold N^3, with $\partial N^3 = \emptyset$, we denote

$$N_h^3 \stackrel{\text{def}}{=} N^3 - \{\textit{a closet } \textbf{\textit{tame}} \textit{ totally disconnected subset}\}. \tag{1}$$

*The first ingredient of the proof is the construction of an "S_u-structure" for an arbitrary V^3 (like in theorem 0). This is a certain $X^2 \xrightarrow{f} V^3$ with certain properties not to be developed here. Anyway, the open **regular** neighbourhood Nbd(fX^2) is well defined, and for $|S_u V^3| \stackrel{\text{def}}{=}$ Nbd(fX^2) $\times B^n$ (n high) we get a diagram*

$$V_h^3 \times 0 \subset |S_u V^3| \underset{\text{PROPER}}{\hookrightarrow} V_h^3 \times B^n. \tag{2}$$

A second structure $S_b V^3$ is introduced, very closely related to $S_u V^3$. An $(n+3)$-dimensional manifold $|S_b V^3|$ is canonically associated to $S_b V^3$. For open simply-connected V^3's, this $|S_b V^3|$ is automatically g.s.c. but, generally speking, it fails to satisfy something like (2) above

THEOREM 1. If $V^3 = \widetilde{M}^3$ the $S_u \widetilde{M}^3$ (and hence the associated $S_b \widetilde{M}^3$) can be choosen to be $\pi_1 M^3$-equivariant.

Hence $S_u \widetilde{M}^3$, $S_b \widetilde{M}^3$ descend to $S_u M^3$, $S_b M^{\pounds}$.

THEOREM 2. *Because M^3 is compact*

$$|S_u M^3| \stackrel{\text{DIFF}}{=} |S_b M^3|. \tag{3}$$

Our constructions are functorial and so

$$|S_u M^3|^{\sim} = |S_u \widetilde{M}^3|, \; |S_b \widetilde{M}^3| = |S_b M^3|^{\sim}.$$

Hence, (3) imples that

$$|S_u \widetilde{M}^3| = |S_b \widetilde{M}^3|. \tag{4}$$

The $(n+3)$-manifold appearing in (4) above is now g.s.c. (because $\widetilde{M}^3$ is open simply connected) and satisfies (2). Theorem (0) implies then that $\pi_1^\infty \widetilde{M}^3 = 0$.

Arithmetic knots in closed 3–manifolds

MARK D. BAKER
(joint work with A. Reid (U. Texas–Austin))

Let M be a closed, orientable 3–manifold. A link $l \subset M$ is called *arithmetic* if $M \backslash l \cong \mathbb{H}^3/\Gamma$ where $\mathbb{H}^2$ is hyperbolic 3–space and $\Gamma \subset PSL_2(\mathbb{C})$ is a torsion–free subgroup commensurable with a Bianchi–group $PSL_2(\mathcal{O}_m)$.

Since the figure eight knot complement is both arithmetic an universal, it follows that M contains an arithmetic link. A natural question therefore is: does every closed orientable 3–manifold contain an arithmetic knot? One motivation for this question is that an affirmative answer would imply the Poincaré Conjecture!

In our talk we prove that this is not the case (alas) by giving a classification of the odd order lens spaces that contain an arithmetic knot:

THEOREM. *Let L be a lens space with $|\pi_1(L)| = r$ odd. If L contains an arithmetic knot K, then $r = 5$ and $L \backslash K$ is homeomorphic to either the sister of the figure eight complement or the double cover of the figure eight knot complement.*

The proof uses the topology of the principal convergence manifolds $\mathbb{H}_3/\Gamma(I)$ as well as the subgroup structure of $PSL_2(\mathbb{F}_p)$.

Circle packings on $\mathbb{CP}^1$–surfaces

SADAYOSHI KOJIMA
(joint work with Ser Peow Tan (Singapore National Univ) and Shigeru Mizushima (Tokyo Institute of Technology))

A $\mathbb{CP}^1$-surface, or better known as a Riemann surface with a projective structure, is by definition a surface locally modeled on the Riemann sphere with the action of projective transformations. A $\mathbb{CP}^1$-structure is not a Riemannian structure, however, since a projective transformation maps a circle to a circle, the circle does make sense.

Given a triangulation τ on a surface Σ_g of genus $g \geq 2$. When τ is realized as a nerve of the packing on some $\mathbb{CP}^1$-surface, assign to each edge e a real number coming from the cross ratio of vertices of two triangular regions touched to e. We show that the set of such cross ratio parameters, denoted by $\mathcal{C}_\tau$, is a real algebraic variety of dimension $\geq 6g-6$. If τ contains only one vertex, in other words, a corresponding packing consists of single circle, $\mathcal{C}_\tau$ is nonsingular and of dimension $= 6g-6$.

We do not know $\mathcal{C}_\tau$ injects to the space of all $\mathbb{CP}^1$-structures on Σ_g which is homeomorphic to the Euclidean space of dimension $12g-12$, however a further composition to the space of $\mathrm{PSL}_2(\mathbb{C})$-representations by assigning holonomy is shown to be a regular map. When $g=1$, more detailed description including the relation with the uniformization can be obtained.

Acylindrical Splittings of finitely generated Groups

RICHARD WEIDMANN

Abstract: A splitting $\mathbb{A}$ of a group G is called k-acylindrical if no non-trivial element of G fixes a segment of length greater than k in the associated Bass-Serre tree. Z. Sela showed that the number of vertex groups of a k-acylindrical splitting of a finetly generated group G is bounded by a constant depending on k and the group G itself.

We use the theory of foldings as develloped by Stallings, Bestvina and Feighn, and Dunwoody to show that k-acylindrical splittings of a finitely generated group G have at most $2k(\text{rank } G - 1) + 1$ vertex groups.

We also indicate how to show stronger results namely rank formulaes for splittings of groups that take into account the ranks of the vertex and edge groups. In the case of a 1-acylindrical amalgamated product this yields the following:

THEOREM *Let $G = A *_C B$ with $C \neq 1$ malnormal in G. Then*

$$\textit{rank } G \geq \frac{1}{3}(\textit{rank } A + \textit{rank } B - 2\textit{rank } C + 5)$$

It should be noted that the assumption that C is malnormal cannot be dropped; in general the are examples of amalgamated products of type $G = A *_C B$ with rank $G =$ rank $C = 2$, rank $A \geq n$ and rank $B \geq n$ where $n\mathbb{N}$ is arbitrary.

Gromov–norm minimizing chains and surfaces in 3–manifolds

Thilo Kuessner

We study fundamental cycles (of a compact, orientable manifold M) of l^1-norm close to the simplicial volume $\| M \|$, especially how they behave with respect to codimension one objects. The applications are:

- for a codimension 1 submanifold $F \subset M$ we want to compare $\| M_F \|$ to $\| M \|$, where M_F is M cut along F,
- to quantify the branching of a codimension 1 foliation (or lamination) $\mathcal{F}$, we consider its Gromovnorm $\| M \|_{\mathcal{F}}$

In particular, we consider finite-volume hyperbolic manifolds of dimensions ≥ 3, where we extend results of Jungreis and Calegari as follows:

Theorem 1. *If int (M^n) is hyperbolic of finite volume, $n \geq 3$, F^{n-1} a closed geodesic hypersurface, then $\| M_F \| > \| M \|$.*

Theorem 2. *If int (M^3) is hyperbolic of finite volume, M^3 not Gieseking-like (i.e, there is no regular ideal triangulation with vertices in cusps of M), and $\mathcal{F}$ is an asymptotically separated lamination, then $\| M \|_{\mathcal{F}} > \| M \|$.*

Theorem 2, together with several results of Calegari, gives strong support to the following conjecture: If $\mathcal{F}$ is a foliation of a finite-volume hyperbolic 3-manifold, then the covering foliation $\widetilde{\mathcal{F}}$ of $\widetilde{M} = H^3$ branches in both directions if and only if $\| M \|_{\mathcal{F}} > \| M \|$.

Surface homeomorphisms, Reidemeister classes and automatic structure

John Guaschi

Let $\phi \colon G \to G$ be an endomorphism of a group G. Two elements $g_1, g_2 \in G$ are said to be *ϕ-conjugate* if there exists $h \in G$ satisfying: $g_2 = \phi(h) \cdot g_1 \cdot h^{-1}$. We wish to be able to decide whether or not two given elements of G are ϕ-conjugate. If one is able to solve this problem in general then one may calculate the Nielsen number of self-maps of polyhedra.

We consider the connection with *Artin's braid groups* B_n, $n \geq 1$, and look for topological ϕ-conjugacy invariants. Given $\beta \in B_n$, let $\phi \in \mathrm{Aut}(\mathbb{F}_n)$ be the associated free group automorphism, and define $B^n_{n+1} = \{\beta \in B_{n+1} \mid \sigma(\beta)(n+1) = n+1\}$, where $\sigma(\beta)$ is the induced permutation. Set $U_{n+1} = \{T_i \mid 1 \leq i \leq n\}$, where T_i is the braid represented geometrically by a full twist of the $(n+1)^{\text{st}}$ string about the i^{th} string. Then $x_i \mapsto T_i$ defines an isomorphism $\xi : \mathbb{F}_n \cong U_{n+1}$. Given $w \in \mathbb{F}_n$, define the *w-extension* β_w of β by $\beta_w = \iota(\beta) \cdot \xi(w) \in B^n_{n+1}$, where $\iota \colon B_n \to B^n_{n+1}$ denotes inclusion. Then:

Theorem 1. *With β, ϕ as above, let $u, v \in \mathbb{F}_n$. Then u, v are ϕ-conjugate if and only if β_u and β_v are conjugate in B^n_{n+1} via an element of U_{n+1}.*

Now given $u \in \mathbb{F}_n$, we define $g_u \colon (\mathbb{D}^2, A) \to (\mathbb{D}^2, A)$ to be the Thurston homeomorphism within its isotopy class rel. A such that the geometric braid obtained by suspending g_u is β_u.

Theorem 2. *If u and v are ϕ-conjugate then g_u and g_v are topologically conjugate.*

Universal bounds for hyperbolic Dehn filling

Steve Kerckhoff
(joint work with C. Hodgson)

Let M^3 be a 3–manifold with boundary whose interior has a complete, finite volume hyperbolic structure. For any notrivial simple curve on $T^2 = \partial M$, one can form the Dehn–filled manifold M_γ by attaching a solid torus to T^2 so that γ bounds a disk.

Thurston proved that for all but finitely many γ's M_γ is again hyperbolic. However, the proof was not effective; it did not indicate which γ's could be exceptional or wether there is a bound on the number which is independent of M.

The end of M_γ is foiliated by flat, horospherical tori. There is one for which the shortest geodesic has length at least 1. We show that if the geodesic length of γ on this torus is sufficiently long (roughly ≥ 24, and not the shortest curve), then M_γ is hyperbolic. It follows that the number of exceptional surgeries is bounded, independent of M.

On the geometry of cone manifolds of dimension 3 with cone angles $< \pi$

Bernhard Leeb
(joint work with M. Boileau and J. Porti)

We prove that cone manifolds of dimension 3, constant curvature $k \in \mathbb{R}$ on the smooth part, an upper cone angle bound $< \pi$ and a lower diameter bound $D_0 > 0$ admit a decomposition of their thin part into disjoined components which belong to a short list of geometric models, all of them rigid.

Important consequences are that these cone manifolds

(i) are thick (with few execptions if $k \geq 0$),
(ii) have, in the case of finite volume, a compact core with horospherical boundary,
(iii) are geometrically stable, i.e. the space of these core manifolds is compact in the pointed Gromov–Hausdorff topology.

These results are part of our proof of the Orbifold Theorem regarding the geometrization of 3–dimensional orbifolds, originally outlined by Thurston.

Linking the configuration space integrals of the Chern-Simons theory to the Kontsevich integral

Christine Lescop

The so–called *perturbative expansion of the Chern–Simons theory for knots* Z_{CS} is a universal Vassiliev invariant described by configuration space integrals that generalize the well-known Gauss integral which computes the linking number of two disjoint knots. It has been studied by many authors including Guadagnini, Martellini, Mintchev, Kontsevich, Bott, Taubes, Bar–Natan, Altschuler, Freidel, D. Thurston ...

We present Z_{CS} and its properties including the most recent ones due to Poirier and we discuss the following natural unsolved problem :

Is Z_{CS} equal to the Kontsevich integral ???

Recently, Sylvain Poirier made substantial progress towards the answer by proving that if the so-called Bott and Taubes *anomaly* was zero in degree greater than 2, then the answer would be YES. To do that, he defined a suitable limit of Z_{CS} that he extended to combinatorial tangles; and he showed that like the Kontsevich integral, his limit of Z_{CS}

is a monoidal functor with some symmetry properties. I have then characterized all the knot invariants that share the common properties of the Kontsevich integral and of the Poirier limit of Z_{CS}. These results allowed me to express Z_{CS} as a function of Z_K and of the anomaly, to improve the denominators of the Kontsevich integral and to give new information about the still unknown anomaly.

Endomorphisms of Kleinian groups

LEONID POTYAGAILO
(joint work with T. Delzaut)

A group G is called *cohopfian* if any injective endomorphism $f : G \to G$ is surjective. We study the cohopfian property of discrete (Kleinian) subgroups of the isometry group of the hyperbolic space $\mathbb{H}^n$. We prove that a geometrically finite, non-elementary, one-ended Kleinian group G without 2-torsion is cohopfian if and only the following two conditions are satisfied:

1) G does not split as $G = A *_C B$ or $G = A*_C$, where C is an elementary subgroup of G such that the unique maximal elementary subgroup $\widetilde{C}$ of G containing C is not conjugate into A or B.
2) G does not split as $G = A *_C B$ or $G = A*_C$, where B is elementary and the smallest subgroup $N(C)$ of B generated by C is an infinite index subgroup of B.[1]

On the finiteness of ∂–slopes of immersed surfaces of 3–manifolds

SHICHENG WANG

We will present some results on the finiteness of boundary slopes of essential surfaces in hyperbolic 3–manifolds with boundary either torus or totally geodesic.

Those results are obtained in the joint papers with Hass–Rubinstein–Wang and Hass–Wang–Zhou.

Fibered knots and monodromy

NORBERT A'CAMPO

A divide is the image P of a generic, relative immersion od a 1-manifold N in the unit disk D of $\mathbb{R}^2$. The link $L(P)$ of the divide P is the intersection in $T(\mathbb{R}^2) = \mathbb{R}^2 \times \mathbb{R}^2 = \mathbb{R}^4$ of the space of vectors

$$T(P) := \{(x,u) \in T(\mathbb{R}^2) \mid x \in P, u \in T_x(P)\}$$

that are tangent to P with the unit sphere

$$\{(x,u) \in T(\mathbb{R}^2) \mid ||x||^2 + ||u||^2 = 1\} = S^3.$$

The intersection is transversal, so $L(P)$ is union of embedded oriented circles in the oriented 3-sphere. We give $T(P)$ the natural orientation as tangent space, where we give the opposite orientation of the unit ball in $T(\mathbb{R}^2)$ to S^3.

Links of divides are very special. For instance if P is connected, the link $L(P)$ admits on its complement in S^3 a fibration over S^1. Links of plane curve singularities appear as links of divides. If $L(P)$ is a knot, i.e. if P is a generically relatively immersed interval in D, the 4-ball genus and the unknotting number of $L(P)$ are equal to the number of double points of P.

[1] *1991 Mathematics Subject Classification.* 57M50, 30F40, 20F32, 57M07

Mikami Hirasawa has identified the knots of the so called slalom divides as arborescent knots with all plumbing numbers equal to 2. He was able to extend the computation of the unknotting number to a larger class of arborescent knots.

For more information, please consult the papers, also at `http://xxx.lanl.gov/`

[AC1] Norbert A'Campo, *Real deformations and complex topology of plane curve singularities*, Annales de la Faculté des Sciences de Toulouse, (1999).

[AC2] Norbert A'Campo, *Generic immersions of curves, knots, monodromy and gordian number*, Publ. Math. IHES (1998).

[AC3] Norbert A'Campo, *Planar trees, slalom curves and hyperbolic knots*, Publ. Math. IHES (1998).

Nielsen root numbers of mappings betweeen surfaces

Elena Kudryavtseva
(joint result of Daciberg Gonçalves (Brasil), Heiner Zieschang (Germany) and the author [GKZ])

Let $f\colon M_1 \to M_2$ be a continuous map between closed (not necessarily orientable) surfaces, and $c \in M_2$ a point in the target. Let $MR[f]$ denote the minimal number $|g^{-1}(c)|$ of roots of the mappings g homotopic to f, and let $NR[f]$ denote the number of essential Nielsen classes of f. The so called *root problem* arises: to calculate the numbers $NR[f]$ and $MR[f]$, and to find all mappings f for which the general inequality $MR[f] \geq NR[f]$ becomes equality.

We say that the map f has *Wecken property* if it satisfies the condition

$$MR[f] = NR[f].$$

We give a criterion which maps f have Wecken property.

For a map $f\colon M_1 \to M_2$ denote by $A(f)$ its *absolute degree*, see [Epst,Hopf]. For instance, $A(f) = |\deg f|$ if the both surfaces M_1 and M_2 are orientable. Put $\ell(f) = [\pi_1(M_2) : f_\#(M_1)]$. A map $f\colon M_1 \to M_2$ is called *orientation-true* if orientation preserving curves are sent to orientation preserving ones and orientation reversing curves are sent to orientation reversing ones. According to Kneser's result [K], $NR[f] = 0$ if $A(f) = 0$, and $NR[f] = \ell(f)$ otherwise.

Theorem. [GKZ] *The map $f\colon M_1 \to M_2$ between two closed surfaces has the Wecken property if and only if one of the following conditions is fulfilled:*

1. *The surface M_2 is either the sphere S^2 or the projective plane $\mathbb{R}P^2$.*
2. *M_2 is different from S^2 and $\mathbb{R}P^2$, that is $\chi(M_2) \leq 0$, and $A(f) \leq \frac{\ell(f)-\chi(M_1)}{1-\chi(M_2)}$.*

In particular, all mappings which are not orientation-true, have the Wecken property.

The analogical result in the case, when both surfaces M_1 and M_2 are orientable, was proved in [GZ]. In the same case the number $MR[f]$ was explicitely calculated in terms of numbers $\deg f$ and $\ell(f)$, see [BGZ].

In the free groups $F_{2g} = \langle a_1, b_1, \ldots, a_g, b_g\rangle$ and $F_{g+1} = \langle a_0, a_1, \ldots, a_g\rangle$ certain quadratic equations

$$\prod_{j=1}^{h}[z_{2j-1}, z_{2j}] = B \quad \text{and} \quad \prod_{j=0}^{h} z_j^2 = B$$

closely related to the root problem are solved.

[BGZ] *S. Bogatyi, D.L. Gonçalves, H. Zieschang*, The minimal number of roots of surface mappings and quadratic equations in free groups, Math. Z. (to appear).
[Epst] *D.B.A. Epstein*, The degree of a map, Proc. London Math. Soc. (3) **16** (1966), 369-383.
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Unknotting tunnels and Seifert surfaces

Martin Scharlemann

Let K be a knot with an unknotting tunnel γ and suppose that K is not a 2-bridge knot. There is an invariant $\rho = p/q \in \mathbb{Q}/2\mathbb{Z}$, p odd, defined for the pair (K, γ). Although ρ is defined abstractly, it is naturally revealed when $K \cup \gamma$ is put in thin position.

We show that if $\rho \neq 1$ then there is a minimal genus Seifert surface F for K such that the tunnel γ can be slid and isotoped to lie on F. One consequence is that if $\rho(K, \gamma) \neq 1$ then $genus(K) > 1$. This confirms a conjecture of Goda and Teragaito for pairs (K, γ) with $\rho \neq 1$.

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