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The conference was organized by Robion Kirby (Berkeley, USA), Wolfgang Lück (Münster, Germany), and Elmer Rees (Edinburgh, GB). It was attended by 45 mathematicians from Europe and North America.

There were 20 talks, among which a special series of 3 lectures by Graeme Segal about quantum field theory and geometry. Different topics covered by other lecturers included (stable) homotopy theory, bordism, positively curved manifolds, foliations, low dimensional manifolds and knot theory, group representations, algebraic K- and L-theory.

Between and after the talks there was plenty of time for discussions and research. This opportunity has been widely used and appreciated by the participants, and together with the pleasant atmosphere of the institute contributed to the success of the meeting.

Abstracts

Norbert A'Campo

Ideal triangulations on knot complements, spines and monodromy

A tree Γ defines on the metric space, which is spanned by its vertices, a quadratic form. To each vertex corresponds a reflection. The product of these reflections is the Coxeter element C_Γ of Γ. The eigenvalues lie on $S^1 \cup \mathbb{R}_+$ and the Coxeter group is finite if $\lambda + \frac{1}{\lambda} < 2$ for each eigenvalue λ. The Coxeter diffeomorphism D_Γ is defined for a planar tree Γ as a diffeomorphism of the surface Σ_Γ which one obtains by plumbing annuli, such that we have one annulus for each vertex and one annulus for each edge. D_Γ is the product of the right Dehn twist along the core curves of the plumbing. We only consider trees Γ such that Σ_Γ has one boundary component. In this case, the diffeomorphism D_Γ is the monodromy of the slalom knot associated to the tree Γ.

Theorem. *D_Γ is pseudo-Asonov if Γ has 4 or more terminal vertices, or if Γ' contains strictly the tree E_8, where the tree Γ' is obtained by subdividing with a node each edge of Γ.*

We discuss how to read off the fibration and the monodromy of a fibred hyperbolic knot from an ideal triangulation of its complement. The main ingredient uses a 1-cochain on the Matveev spine of the complement.

We give combinatorial evidence for a positive answer to the following question:

Let $\theta\colon S^3 - K \to S^1$ be the harmonic representative for the essential map from the complement of the knot K to the circle, harmonic for the complete hyperbolic metric of finite volume on $S^3 - K$. Is the map θ a fibration if the knot K is a fibred knot?

Arthur Bartels

On the isomorphism conjecture in algebraic K-theory

(joint work with Tom Farrell, Lowell Jones, and Holger Reich)

For a torsionfree group Γ and a regular ring R the Isomorphism Conjecture states that the assembly map $A\colon H_i(B\Gamma; \mathbb{K}R) \to K_i(R\Gamma)$ is an isomorphism. Here $\mathbb{K}R$ denotes the K-theory spectrum of R. It implies for example that $K_0(R\Gamma) = K_0(R)$. For general rings a different assembly map has to be considered.

Theorem. *Let M be a compact Riemannian manifold with strictly negative sectional curvature and $\Gamma := \pi_1(M)$. Let R be a ring. Then for $i \leq 1$ the assembly map*

$$A_{\mathcal{C}}\colon H_i^\Gamma(E\Gamma(\mathcal{C}); \mathbb{K}R) \to K_i(R\Gamma)$$

is an isomorphism.

This can be used to show that in the same situation for regular rings A is also an isomorphism. The proof of the theorem uses controlled topology. The map $A_{\mathcal{C}}$ can be expressed as a "forget control" map. Then a transfer argument, the geodesic flow and a foliated control theorem are used to regain control.

Jim Bryan

Introduction to BPS-state counts and Gromov-Witten invariants

Gromov-Witten theory can be studied from the point of view of topology, analysis, algebra, geometry, or physics. The interplay between these subjects has given the subject a remarkable robustness. Recent advances in M-theory in physics has suggested that the Gromov-Witten invariants should be equivalent to a simpler set of invariants which in physics are obtained from BPS state counts but are mathematically unknown. In this talk, we give an introduction to Gromov-Witten invariants with an eye toward describing how the "BPS invariants" should be defined mathematically and how they should be related to the Gromov-Witten invariants.

Anand Dessai

Elliptic genera and manifolds with positive curvature

Let M be a simply connected closed Riemannian *Spin*-manifold of positive sectional curvature. By the Lichnerowicz formula the index of its Dirac operator vanishes. We show that the index of certain twisted Dirac operators (e.g. the Rarita-Schwinger operator) vanishes provided the dimension of M is > 8 and its symmetry rank is ≥ 2. If one restricts to metrics with a prescribed lower bound on the symmetry rank these results may be used to exhibit simply connected manifolds (of arbitrary large dimension) with small Betti numbers which admit a metric of positive Ricci curvature but no metric of positive sectional curvature. The proof uses the rigidity theorem for elliptic genera, intersection properties of totally geodesic submanifolds of positively curved manifolds and elementary properties of codes.

Ian Hambleton

Topological equivalence of linear representations

(joint work with Erik Pedersen)

Let G be a finite group and V, V' finite dimensional real orthogonal representations of G. Then V is said to be *topologically equivalent* to V' (denoted $V \sim_t V'$) if there exists a homeomorphism $h\colon V \to V'$ which is G-equivariant. If V, V' are topologically equivalent, but not linearly isomorphic, then such a homeomorphism is called a non-linear similarity. These notions were introduced and studied by de Rham, and developed extensively by Cappell and Shaneson, Steinberger and West, Hsiang and Pardon, and Madsen and Rothenberg in the 1980's.

We say that V_1 and V_2 are *stably topologically similar* ($V_1 \approx_t V_2$) if there exists a G-representation W such that $V_1 \oplus W \sim_t V_2 \oplus W$. Let $R_{Top}(G) = R(G)/R_t(G)$ denote the quotient group of the real representation ring of G by the subgroup $R_t(G) = \{[V_1] - [V_2] \mid V_1 \approx_t V_2\}$. The following result gives the first complete calculation of $R_{Top}(G)$ for any group which admits non-linear similarities.

Theorem. *Let $G = C(2^r)$ be a cyclic group of order 2^r, with $r \geq 4$. Then $R_{Top}(G) = \bigoplus_{K\subset G} R^{free}_{Top}(G/K)$ and $R^{free}_{Top}(G) = \mathbb{Z} \oplus \widetilde{R}^{free}_{Top}(G)$ where*

$$\widetilde{R}^{free}_{Top}(G) = \langle \alpha_1, \alpha_2, \ldots, \alpha_{r-2}, \beta_1, \beta_2, \ldots, \beta_{r-3}\rangle$$

subject to the relations $2^s\alpha_s = 0$ for $1 \leq s \leq r-2$, and $2^{s-1}(\alpha_s + \beta_s) = 0$ for $2 \leq s \leq r-3$, together with $2(\alpha_1 + \beta_1) = 0$.

The generators for $r \geq 4$ are given by the elements

$$\alpha_s = t - t^{5^{2^{r-s-2}}} \quad \text{and} \quad \beta_s = t^5 - t^{5^{2^{r-s-2}}+1}.$$

We remark that $\widetilde{R}^{free}_{Top}(C(8)) = C(4)$ is generated by $t - t^5$.

Kathryn Hess

Algebraic models in homotopy theory

In the mid 1990's N. Dupont and I developed a theory of NONCOMMUTATIVE ALGEBRAIC MODELS OF FIBER SQUARES, which we have since applied to modelling free loop spaces and homotopy pullbacks.

1. The theoretical framework

We work in the category whose objects, which we call cochain algebras, are cochain complexes over a field $\mathbb{K}$ endowed with a (cochain) homotopy-associative product and whose morphisms are *quasi-algebra maps*, i.e., (essentially) cochain maps inducing algebra homomorphisms on cohomology. There is a special class of maps in this category, known as *twisted algebra extensions*, analogous to the KS-extensions of rational homotopy theory, that are used for modelling topological fibrations. If (A,d) and (B,d) are cochain algebras, then a twisted algebra extension of (A,d) by (B,d), denoted $(A,d)\odot(B,d)$, is a cochain algebra (C,D) such that $C\cong A\otimes B$ as graded R-modules and such that the inclusion map $(A,d)\to(C,D)$ and the projection map $(C,D)\to(B,d)$ are both strict algebra morphisms.

Theorem (A). *Suppose there is a commuting diagram over a field* $\mathbb{K}$,

$$\begin{array}{ccccc} (A,d)\odot(C,e) & \xleftarrow{\iota} & (A,d) & \xrightarrow{\varphi} & (\bar{A},\bar{d}) \\ {\scriptstyle\beta}\downarrow{\scriptstyle\simeq} & & {\scriptstyle\alpha}\downarrow{\scriptstyle\simeq} & & {\scriptstyle\gamma}\downarrow{\scriptstyle\simeq} \\ C^*E & \xleftarrow{C^*q} & C^*B & \xrightarrow{C^*f} & C^*X \end{array}$$

in which $\bar{A}$ is a free algebra, φ admits a cochain algebra section σ, α and γ are algebra maps, and β may be only a quasi-algebra map. Then there exist a twisted algebra extension $(\bar{A},\bar{d})\to(\bar{A},\bar{d})\odot(C,e)$ (almost entirely explicitly defined) and a quasi-algebra quasi-isomorphism $(\bar{A},\bar{d})\odot(C,e)\xrightarrow{\simeq} C^*(E\underset{B}{\times}X)$.

2. Applications

2.1. Free loop space. We have applied Theorem A to the construction of a model of the free loop space E^{S^1} over any field $\mathbb{K}$, where E is simply connected and of finite type. The most delicate stage of the construction is the definition of a twisted algebra extension that models the fibration $q\colon E^I\to E\times E$.

Our input data for the construction consist of an Adams-Hilton model for E over $\mathbb{Z}$, together with a homotopy-coassociative and homotopy-cocommutative coproduct on this model. We proved that there exists a twisted algebra extension $\Omega(C,\widehat{\partial})\to\Omega(C,\widehat{\partial})\odot(C,\widehat{\partial})$ (almost entirely explicitly defined) and a quasi-algebra quasi-isomorphism $\Omega(C,\widehat{\partial})\odot(C,\widehat{\partial})\xrightarrow{\simeq} C^*(E^{S^1})$, where Ω denotes the cobar construction and $(C,\widehat{\partial})$ the dual of such an "enriched" Adams-Hilton model.

We have provided explicit formulas for the entire structure of $\Omega(C,\widehat{\partial})\odot(C,\widehat{\partial})$ when the product on $(C,\widehat{\partial})$ is strictly commutative and associative, e.g., when E is a r-connected finite CW-complex of dimension at most pr and $\operatorname{char}\mathbb{K}=p$. For $p>2$ we could then construct a commutative cochain algebra over $\mathbb{F}_p$ that models the free loop space. For spaces that are also p-formal, such as spheres and projective spaces, we defined an even simpler commutative model that applies for all primes p. We then used the simplified model to compute the cohomology algebras of a number of free loop spaces explicitly.

2.2. Homotopy fiber products and homotopy fibers. Applying Theorem A and the model of q above, we have constructed a model for the homotopy pullback of continuous maps $f\colon X \to E$ and $g\colon Y \to E$. If $(C,\widehat{\partial})$, $(C',\widehat{\partial}')$, and $(C'',\widehat{\partial}'')$ are dual to enriched Adams-Hilton models of E, X, and Y, respectively, then the homotopy pullback of f and g has a model of the form $\big(\Omega(C',\widehat{\partial}') \otimes \Omega(C',\widehat{\partial}')\big) \odot (C,\widehat{\partial})$.

We plan to employ this new model in the study of homotopy fibers of interesting topological cofibrations, such as cell attachments. It may also be useful for determining when maps of p-compact groups are monomorphisms.

Gerd Laures

The E_∞-structure of the $K(1)$-local spin-bordism

A classical result says that spin manifolds are determined up to bordism by their KO characteristic and Stifel-Whitney numbers. This leads to an additive splitting of the spin bordism $MSpin$ which allows the computation of the bordism groups. The cartesian product of manifolds gives spin bordism in addition a highly commutative and associative (E_∞) ring structure. So far there is not much known about it since it is not compatible with the mentioned splitting.

In the talk we concentrate on the part of spin bordism which can be investigated with KO-theoretical tools. We develop an E_∞ splitting of this localized spin bordism:

$$L_{K(1)}MSpin \cong T_\zeta \wedge \bigwedge_{i=1}^{\infty} TS^0.$$

Here, T_ζ is the E_∞-cone over a generator ζ of $\pi_{-1}L_{K(1)}S^0$ and TS^0 is the free E_∞-spectrum generated by the sphere. Together with a result of M. Hopkins this formula gives the desired $O\langle 8\rangle$-orientation of the $K(1)$-local elliptic cohomology TMF.

Eric Leichtnam

On the cut and paste property of higher signatures of closed manifolds

(joint work with Wolfgang Lück)

We extend the notion of the symmetric signature $\sigma(\overline{M},r) \in L^n(R)$ for a compact n-dimensional manifold M without boundary, a reference map $r\colon M \to BG$ and a homomorphism of rings with involutions $\beta\colon \mathbb{Z}G \to R$ to the case with boundary ∂M, where $(\overline{M},\overline{\partial M}) \to (M,\partial M)$ is the G-covering associated to r. We need the assumption that $C_*(\overline{\partial M}) \otimes_{\mathbb{Z}G} R$ is R-chain homotopy equivalent to a R-chain complex D_* with trivial m-th differential for $n = 2m$ resp. $n = 2m+1$. We prove a gluing formula, homotopy invariance and additivity for this new notion.

Let Z be a closed oriented manifold with reference map $Z \to BG$. Let $F \subset Z$ be a cutting codimension one submanifold $F \subset Z$ and let $\overline{F} \to F$ be the associated G-

covering. Denote by $\alpha_m(\overline{F})$ the m-th Novikov-Shubin invariant and by $b_m^{(2)}(\overline{F})$ the m-th L^2-Betti number. If for the discrete group G the Baum-Connes assembly map is rationally injective, then we use $\sigma(\overline{M}, r)$ to prove the additivity (or cut and paste property) of the higher signatures of Z, if we have $\alpha_m(\overline{F}) = \infty^+$ in the case $n = 2m$ and, in the case $n = 2m+1$, if we have $\alpha_m(\overline{F}) = \infty^+$ and $b_m^{(2)}(\overline{F}) = 0$.

This additivity result had been proved (by a different method) by Leichtnam-Lott-Piazza when G is Gromov hyperbolic or virtually nilpotent. We give new examples, where these conditions are not satisfied and additivity fails. Our work is greatly motivated by and partially extends some of the work of Leichtnam-Lott-Piazza, Lott, and Weinberger.

Ran Levi

Spaces of equivalences between classifying spaces of finite groups completed at a prime p

(joint work with Carles Broto and Bob Oliver)

We study homotopy equivalences of p-completions of classifying spaces of finite groups. To each finite group G and each prime p, we associate a finite category $\mathcal{L}_p^c(G)$ with the following properties. Two p-completed classifying spaces $BG_p^\wedge$ and $BG'^\wedge_p$ have the same homotopy type if and only if the associated categories $\mathcal{L}_p^c(G)$ and $\mathcal{L}_p^c(G')$ are equivalent. The topological monoid $Aut(BG_p^\wedge)$ of self equivalences of $BG_p^\wedge$ is determined by the self equivalences of the associated category $\mathcal{L}_p^c(G)$. Both the question of whether $BG_p^\wedge$ and $BG'^\wedge_p$ are homotopy equivalent and the homotopy type of $Aut(BG_p^\wedge)$ can be approximated by means of fusion preserving isomorphisms of the Sylow p subgroup of G. We explain these approximations and use a recent theorem of Oliver to show that in fact at odd primes, they are precise. This theorem also settles a conjecture of Martino and Priddy at odd primes.

Ib Madsen

A topologist's view of a conjecture of Mumford

Let $\Gamma_{g,1+1}$ be the mapping class group of genus g surfaces with two boundary components, $\Gamma_{g,1+1} = \pi_0 \operatorname{Diff}(F_{g,1+1}, \partial)$. Adding a genus one surface with two boundary components to F induces a map from $\Gamma_{g,1+1}$ to $\Gamma_{g+1,1+1}$. On the classifying space level one obtains maps from $B\Gamma_{g,1+1}$ to $B\Gamma_{g+1,1+1}$, and a limit space which we denote by $B\Gamma_\infty$. A theorem of Ulrike Tillmann asserts that after applying Quillen's plus construction, $B\Gamma_\infty$ becomes an infinite loop space $B\Gamma_\infty^+$. The space $B\Gamma_{g,1+1}$ classifies surface bundles $E \to X$. The Thom-Pontrjagin construction gives a map $X \to \Omega^\infty \operatorname{Th}(-T^vE)$, and universally a map

$$\alpha_\infty : B\Gamma_\infty^+ \to \Omega^\infty \mathbb{C}P_{-1}^\infty .$$

The talk discussed evidence for the following

Conjecture. *The map α_∞ is a homotopy equivalence.*

In geometric terms the conjecture asserts that a map $f : E^{n+2} \to M^n$ between manifolds, with the extra property that TE^{n+2} is stable equivalent to $f^*TM^n \oplus \xi$ for some oriented 2-plane bundle ξ over E, up to cobordism can be deformed into a submersion.

Rationally the conjecture is equivalent to the so-called Mumford conjecture that

$$H^*(B\Gamma_\infty; \mathbb{Q}) \cong \mathbb{Q}[\kappa_1, \kappa_2, \cdots].$$

After localization at an odd prime p,

$$\Omega^\infty \mathbb{C}P^\infty_{-1} \simeq B_0 \times \cdots \times B_{p-3} \times W_{p-2}$$

where the B_i are factors of $\Omega^\infty S^\infty(\mathbb{C}P^\infty_+)$. Theorems of Tillmann and the speaker assert that α is an infinite loop map and that $B_0 \times \cdots \times B_{p-3}$ is a factor of $B\Gamma^+_\infty$ after p-completion. This gives a host of new cohomology, all p^n-torsion, in $H^*(B\Gamma_\infty; \mathbb{Z})$. Moreover there is a map η from $\Omega^\infty \mathbb{C}P^\infty_{-1}$ into $Sp/\mho$ such that $\eta \circ \alpha_\infty$ cohomologically behaves like the standard map

$$B\Gamma_\infty \to BSp(\mathbb{Z}) \to Sp/\mho.$$

The talk represents joint work with Ulrike Tillmann.

Birgit Richter

Taylor-approximations and cubical constructions of Gamma-modules

Topological André-Quillen homology is a generalization of usual André-Quillen homology (AQ for short) to commutative ring spectra. One version of such a theory is Gamma homology – H^Γ – defined by A. Robinson and S. Whitehouse. T. Pirashvili and the speaker identified Gamma homology of Eilenberg-MacLane spectra with stable homotopy of a functor. This functor $\mathcal{L}(A, M)$ depending on a commutative k-algebra A (k a field) and an A-module M sends a finite pointed set $\{0, 1, \ldots, n\}$ to $M \otimes A^{\otimes n}$.

Theorem. $H^\Gamma_*(A, M) \cong \pi^{st}_*(\mathcal{L}(A, M))$.

In the context of Taylor-approximations of functors from the category of finite pointed sets Γ to k-modules I proved these groups to be isomorphic to the homology of the linearization of $\mathcal{L}(A, M)$. This approach leads to an Atiyah-Hirzebruch spectral sequence $E^2_{p,q} = H^\Gamma_q(k[x], k) \otimes AQ_p(A, k) \Longrightarrow H^\Gamma_{p+q}(A, k)$ with A augmented over k. Moreover H^Γ of the "basepoint" $k[x]$ is shown to be isomorphic to $Hk_*H\mathbb{Z}$. The methods in this subject were sketched by calculating $H^\Gamma(\mathbb{F}_2[x], \mathbb{F}_2)$ as a graded module—which is well-known, in order to indicate how these methods may be helpful in other cases.

John Rognes

Algebraic K-theory of finitely presented ring spectra

The talk introduced the notion of a G-Galois extension $A \to B$ of commutative S-algebras, or E_∞ ring spectra, and presented the Galois descent problem for algebraic K-theory in this context. With E_n the Lubin–Tate spectrum with

$$E_{n*} = \mathbb{WF}_{p^n}[[u_1, \dots, u_{n-1}]][u, u^{-1}]$$

and $\overline{E}_n$ a maximal connected pro-Galois extension of E_n, we optimistically conjecture that

$$L_{K(n+1)}K(\overline{E}_n) = E_{n+1}\,.$$

For L_p the Adams summand of p-complete topological K-theory and ℓ_p its connective cover, C. Ausoni and the speaker have computed the mod p and v_1 homotopy of $K(\ell_p)$ using topological cyclic homology, the answer being a free $P(v_2)$-module on $4p+4$ generators. This leads to the conjectural formula

$$4p + 4 = \sum_{i=1}^{p^2-1} \sum_{n=0}^{\infty} \dim_{\mathbb{F}_p} H^n(\mathrm{Gal}(\overline{L}_p/L_p); \mathbb{F}_{p^2}(i))$$

with $\overline{L}_p = \overline{E}_1$. The talk indicated how algebraic K-theory of topological K-theory is a form of elliptic cohomology. More generally we presented the available evidence for the following:

Chromatic red-shift problem. *Let E be an S-algebra of pure fp-type n. Does $TC(E;p)$ have pure fp-type $n+1$?*

Daniel Ruberman

Moduli spaces of metrics of positive scalar curvature on 4-manifolds

Suppose that a manifold X admits a metric of positive scalar curvature. Then one can ask for topological properties of $\mathcal{M}^+(X)$, the space of all such metrics. Alternatively, one can consider topological properties of the moduli space of positive scalar metrics, $\mathcal{M}^+(X)/\mathrm{Diff}(X)$.

The simplest question about these spaces is whether they are connected. Results of Hitchin, Gromov–Lawson, and more recently Gilkey–Botvinnik, Stolz, and others show that these spaces can indeed be disconnected, when the dimension of X is greater than or equal to 5. We extend these results to dimension 4 in the following theorems.

Theorem (A). *There are simply connected smooth 4–manifolds X for which $\mathcal{M}^+(X)$ is disconnected.*

Theorem (B). *There are (non-orientable) smooth 4–manifolds X for which $\mathcal{M}^+(X)/\mathrm{Diff}(X)$ is disconnected.*

The metrics in Theorem A are of the form g_0 and f^*g_0, where f is a certain diffeomorphism, which is homotopic to the identity. It follows from the method of proof (which uses the Seiberg–Witten equations) that f is not isotopic to the identity map of X. The components of $\mathcal{M}^+/\mathrm{Diff}$ in Theorem B are detected by η–invariants associated to Pin^c structures, as in the work of Gilkey–Botvinnik.

Thomas Schick

Atiyah's conjecture about the integrality of L^2-Betti numbers

The Atiyah conjecture for a torsion-free discrete group π states that the L^2-Betti numbers of a finite CW-complex with fundamental group π are integers.

The L^2-Betti numbers are defined in terms of harmonic L^2-cycles (in the cellular chain-complex tensored over $\mathbb{Z}\pi$ with the Hilbert space $l^2\pi$). There exists a real valued normalized dimension function for such Hilbert spaces with unitary π-action (going back to Murray and von Neumann), and the L^2-Betti numbers are these π-dimensions of the spaces of harmonic L^2-cycles.

The Atiyah conjecture for a torsion-free group π implies the Kaplanski conjecture that there are no non-trivial zero divisor in the ring $\mathbb{Z}\pi$.

In the talk, certain classes of groups were presented for which the Atiyah conjecture is true. Among them are torsion-free elementary amenable groups, residually torsion-free elementary amenable groups, and poly-free groups (proved by Linnell, Schick, Dicks-Schick, respectively).

The last part of the talk dealt with the question how to handle finite extensions of groups for which the Atiyah conjecture is true. As an example, the following result of Linnell-Schick was presented:

Theorem. *Assume Γ has a finite classifying space, $H^*(\hat{\Gamma}^p,\mathbb{Z}/p)\to H^*(\Gamma,\mathbb{Z}/p)$ is an isomorphism for all primes p, and $\Gamma/\gamma_n(\Gamma)$ is torsion-free for infinitely many $n\in\mathbb{N}$.*

Assume $1\to\Gamma\to\pi\to F\to 1$ is an exact sequence of groups, π is torsion-free and F is finite. If the Atiyah conjecture is true for Γ, then it is also true for π.

Here $\hat{\Gamma}^p$ is the pro-p-completion of Γ (i.e. the inverse limit of all the quotients of Γ which are finite p-groups), $H^*(\hat{\Gamma}^p,\mathbb{Z}/p)$ its continuous cohomology (i.e. the direct limit of the cohomology of the quotients defining $\hat{\Gamma}^p$), and $\gamma_n(\Gamma)$ is the n-th lower central series subgroup.

As an example, it is proved that the pure braid groups satisfy all the conditions of the theorem. It follows that the full braid groups fulfill the Atiyah conjecture.

Very similar methods apply to the Baum-Connes conjecture, to give e.g. a proof of the Baum-Connes conjecture (with coefficients) for the full braid group.

Moreover, the methods of the proof of the theorem imply that $\pi/\gamma_n(\Gamma)$ is torsion-free for infinitely many $n \in \mathbb{N}$. This implies in particular that all pure braid groups have many non-trivial torsion-free quotients; and disproves a conjecture of Lin which stated that no such quotients exist.

Stefan Schwede

Uniqueness results for stable homotopy theory

We study the question of whether or not the stable homotopy category admits "exotic" models. The precise formulation uses the framework of Quillen's "closed model categories"; so we ask whether there exists a model category whose homotopy category is equivalent—as a triangulated category—to the stable homotopy category, but which is not (Quillen-)equivalent to the standard model for spectra.

A Quillen equivalence of model categories implies an equivalence of homotopy categories, but is in general a much stronger condition. Loosely speaking, a Quillen equivalence preserves all "higher order" homotopy information.

An important motivation for our study comes from work of J. Franke on chromatic localizations of the stable homotopy category. For a prime p and a chromatic level n such that $n^2+n < 2p-2$, he constructs an algebraic model for the homotopy category of $E(n)$-local spectra. The objects in this model are certain complexes of $E(n)_*E(n)$-comodules, and the category is *not* Quillen-equivalent to $E(n)$-local spectra.

On the other hand, we expect that without localizing there is only one model for the stable homotopy category up to Quillen-equivalence. The problem can be studied one prime at a time, and we show that the model is indeed unique at the primes 2 and 3. We expect that the uniqueness holds for all primes, but a proof requires refined techniques.

Graeme Segal

Quantum field theory and geometry

I Deligne or Cheeger-Simons cohomology

An electromagnetic field on a space-time manifold X is a complex hermitian line bundle on X equipped with a unitary connection. The field is nearly determined by its "field strength", the curvature 2-form of the connection, but it carries additional holonomy information. Two other structures of the same type have been prominent in string theory recently.

(i) String theory is a theory of gravity, but reduces in its classical limit to a theory of manifolds equipped not only with a Riemannian metric but also with a "B-field".

The latter is roughly described by a closed 3-form with integral periods, but carries additional holonomy information.

(ii) Supergravity is a theory of 11-dimensional manifolds with Riemannian metric and an additional closed 4-form with integral periods.

In fact, on a smooth manifold X one can define (for $p \geq 0$) a category $\mathcal{C}_p(X)$ of "p-objects", and a category $\mathcal{C}_p^{\mathrm{conn}}(X)$ of "p-objects with connection". The set of isomorphism-classes of objects of $\mathcal{C}_p(X)$ is $H^p(X;\mathbb{Z})$, while that of $\mathcal{C}_p^{\mathrm{conn}}(X)$ is a group $H_D^p(X)$ which fits into two exact sequences

$$0 \longrightarrow \Omega^{p-1}(X)/Z_{\mathbb{Z}}^{p-1}(X) \longrightarrow H_D^p(X) \longrightarrow H^p(X;\mathbb{Z}) \longrightarrow 0$$

$$0 \longrightarrow H^{p-1}(X;\mathbb{T}) \longrightarrow H_D^p(X) \longrightarrow Z_{\mathbb{Z}}^p(X) \longrightarrow 0$$

where $Z_{\mathbb{Z}}^p(X)$ denotes the closed p-forms with integral periods.

I described the main features of these categories, especially the multiplicative structure and Poincaré duality, the existence of a direct-image functor, and the lifting of the p-th Chern class of a vector bundle with unitary connection to an object of $\mathcal{C}_{2p}^{\mathrm{conn}}(X)$.

II Twisted K-theory and the Verlinde algebra

For any space X an element of $H^3(X;\mathbb{Z})$ can be represented by a bundle P on X whose fibres P_x are projective spaces $\mathbb{P}_{\mathbb{C}}^{\infty}$. For such P there is a bundle $\mathrm{Fred}(P)$ whose fibre at $x \in X$ is the space of Fredholm operators in the fibre P_x. The twisted K-group $K_P(X)$ is defined as the group of vertical homotopy classes of sections of $\mathrm{Fred}(P)$. I described some properties of these groups, including the Atiyah-Hirzebruch spectral sequence relating them to $H^*(X;\mathbb{Z})$. In fact $K_P(X) \otimes \mathbb{Q} \cong \ker(c)/\operatorname{im}(c)$, where $c\colon H^i(X;\mathbb{Q}) \to H^{i+3}(X;\mathbb{Q})$ is multiplication by the class c of P.

An equivariant version of this theory can be defined when a compact Lie group G acts on X: the twisting is then by elements of $H_G^3(X;\mathbb{Z})$.

The central extensions by $\mathbb{T}$ of the loop-group $\mathcal{L}G$ correspond to elements $c \in H_G^3(G;\mathbb{Z})$, where G acts on itself by conjugation. Recently D. Freed, M. Hopkins, and C. Teleman have proved a remarkable theorem identifying the Verlinde algebra of positive energy projective representations of $\mathcal{L}G$ with class c with the twisted K-group $K_{G,P}^*(G)$, where the twisting P is simply related to c. The multiplication in the Verlinde algebra—essentially defined by concatenation of loops—corresponds to the direct-image map in twisted K-theory induced by the product $G \times G \to G$.

III Two-dimensional topological field theories

In string theory Riemannian manifolds X (with B-fields) correspond to conformal theories with certain properties. As a toy model of these structures we can consider two-dimensional topological field theories, which are known to correspond precisely to commutative Frobenius algebras. Recently it has emerged that string theory should allow more general space-times X whose metrics have singularities along submanifolds Y of X known as D-branes: the string description is to supplement the closed strings of the usual theory by open strings with end-points on the D-brane Y. It is believed that D-branes in X have "charges" lying in the twisted K-theory $K_c(X)$, where c is the B-field of X.

Jointly with G. Moore I have analysed the toy topological version of this situation, showing that the possible D-brane theories compatible with a commutative Frobenius algebra A of "closed strings" correspond to non-commutative Frobenius algebra B related to A in a specific way. If A is semisimple these are indeed classified by elements of the K-theory of the spectrum of A. A less trivial model, which illuminates the role of the B-field, is obtained by considering gauged topological field theories, making use of results of Turaev.

Vladimir Turaev

Homotopy field theory in dimension 3 and group-categories

We apply the idea of a topological quantum field theory (TQFT) to maps from manifolds into topological spaces. This leads to a notion of a $(d+1)$-dimensional homotopy quantum field theory (HQFT) which may be described as a TQFT for closed d-dimensional manifolds and $(d+1)$-dimensional cobordisms endowed with homotopy classes of maps into a given space. For a group π, we introduce cohomological HQFT's with target $K(\pi, 1)$ derived from cohomology classes of π and its subgroups of finite index. The main part of the talk is concerned with $(1+1)$-dimensional HQFT's. We classify them in terms of so-called crossed group-algebras. In particular, the cohomological $(1+1)$-dimensional HQFT's over a field of characteristic 0 are classified by simple crossed group-algebras. We also discuss $(2+1)$-dimensional HQFT's and derive such HQFT's from so-called crossed braided categories.

Elmar Vogt

Tangential Lyusternik-Shnirelman category of foliations

(joint work with Wilhelm Singhof)

Call an open set U of a foliated manifold M categorical if U can be homotoped inside M via a homotopy which moves points only inside their leaves to a map which contracts every leaf of the foliation induced on U to a point. The tangential category of the foliation is the smallest cardinal of an open covering of M by categorical sets. This concept was introduced by Hellen Colman in her 1998 Santiago thesis and is an invariant of foliated homotopy type. As with the usual category there is a cup-length lower bound using foliated cohomology (which is usually hard to compute).

We have the following results.

A. The tangential category is at most equal to dim(foliation)+1.

B. Tangential category is upper semi-continuous on the space of foliations of a manifold. (We use the C^1-topology on plane fields of C^2-foliations.)

C. A cup–length criterion using ordinary cohomology instead of foliated cohomology:

   if there are $x_1, x_2, \ldots, x_r \in H^{>\operatorname{codim}\mathcal{F}}(M; A)$ with $x_1 x_2 \cdots x_r \neq 0$, then the tangential category of the foliation $\mathcal{F}$ is greater than r.

D. A characterization of C^2-foliations of codimension 1 with tangential category ≤ 2 on closed n-manifolds, $n \geq 3$:

   these are exactly the fibre bundles over S^1 with homotopy spheres as fibres.

In particular this determines the tangential category of all foliations on closed 2- and 3-manifolds.

Edited by Marco Varisco

Schedule

Monday, September 25, 2000

9:30–10:30 Graeme Segal
"Quantum Field Theory and Geometry I"

11:00–12:00 Vladimir Turaev
"Homotopy field theory in dimension 3 and group-categories"

16:00–17:00 Eric Leichtnam
"On the cut and paste property of higher signatures of closed manifolds"

17:15–18:15 Arthur Bartels
"On the isomorphism conjecture in algebraic K-theory"

Tuesday, September 26, 2000

9:30–10:30 Graeme Segal
"Quantum Field Theory and Geometry II"

11:00–12:00 Ran Levi
"Spaces of equivalences between classifying spaces of finite groups completed at a prime p"

16:00–17:00 Ib Madsen
"A topologist's view of a conjecture of Mumford"

17:15–18:15 Daniel Ruberman
"Moduli spaces of positive scalar curvature on 4-manifolds"

Wednesday, September 27, 2000

9:30–10:30 Graeme Segal
"Quantum Field Theory and Geometry III"

10:45–11:25 Anand Dessai
"Elliptic genera and manifolds with positive curvature"

11:40–12:20 Birgit Richter
"Taylor-approximations and cubical constructions of Gamma-modules"

12:25 photo

13:30 excursion

Thursday, September 28, 2000

9:30–10:10 Kathryn Hess
"Algebraic models in homotopy theory"

10:25–11:05 Gerd Laures
"The E_∞-structure of the $K(1)$-local spin-bordism"

11:20–12:00 Stefan Schwede
"Uniqueness results for stable homotopy theory"

16:00–17:00 Jim Bryan
"Introduction to BPS-state counts and Gromov-Witten invariants"

17.15–18:15 Norbert A'Campo
"Ideal triangulations on knot complements, spines and monodromy"

Friday, September 29, 2000

9:30–10:30 Thomas Schick
"Atiyah's conjecture about the integrality of L^2-Betti numbers"

11:00–12:00 Ian Hambleton
"Topological equivalence of linear representations"

16:00–17:00 John Rognes
"Algebraic K-theory of finitely presented ring spectra"

17.15–18:15 Elmar Vogt
"Tangential Lyusternik-Shnirelman category for foliations"

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